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Effect of source anisotropy on the intensity correlations of stochastic electromagnetic beams in free space communications

Kaynak anizotropisinin serbest uzay iletişimde stokastik elektromanyetik ışınların yoğunluk korelasyonları üzerindeki etkisi

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Abstract

Correlations between stochastic electromagnetic beams' intensity variations which are produced by sources with anisotropic crossspectral density matrices are explored. Recently introduced spectral degree of cross-polarization, spectral degree of coherence, and the intensity fluctuations of electromagnetic sources are analyzed upon propagation by considering the source as an anisotropic beam. An analytical formula is given for the cross-spectral density matrix of an anisotropic electromagnetic Gaussian Schell beam moving across free space and extended to longitudinal and transverse directions. The analysis shows that the cross-polarization of two beams having same source parameters might increase or decrease on propagation depending on the anisotropy ratio a. Numerical results are presented to show that the beam anisotropy might be one of the convenient methods for modeling partially coherent sources.

Keywords: Optical communications, Partial coherence, Physical optics, Electromagnetic waves.

1 Introduction

After the initial examination of radiation from sources using the anisotropic Gaussian Schell model [1], it quickly becomes a field of research which is because of the fact that it produces identical radiant intensity with a completely coherent as well as rotationally symmetric laser source when the source parameters are chosen properly. In addition to this fact, anisotropic Gaussian beams have received appreciable interest because they produce the resonant modes of laser cavities with astigmatic mirrors [2]. On the other hand, the concept of Correlations among intensity fluctuations is of special interest in relation with the Hanbury Brown-Twiss effect [3]-[5]. Thus, it is our aim here to study the correlations between intensity fluctuations in stochastic electromagnetic beams by considering the source beam as an anisotropic beam.

Partially coherent Gaussian correlated sources of anisotropic kind have been crucial in current coherence theory research. The former research results show that different types of anisotropic Gaussian Schell-model beams [6],[7] have wide applications in modern topics such as in turbulent atmosphere [8],[9], in turbulent ocean [10], in optical systems [11], and in apertured astigmatic optical systems [12].

Öz

Anizotropik çapraz spektral yoğunluk matrislerine sahip kaynaklar tarafından üretilen stokastik elektromanyetik ışınların yoğunluk değişimleri arasındaki korelasyonlar araştırıldı. Geçtiğimiz yıllarda tanıtılan spektral çapraz polarizasyon derecesi, spektral tutarlılık derecesi ve elektromanyetik kaynakların yayılma üzerine yoğunluk dalgalanmaları, kaynağı bir anizotropik ışın olarak ele alarak analiz edildi. Boş uzayda ilerleyen ve boyuna ve enine yönlere uzanan bir anizotropik elektromanyetik Gauss Schell ışınının çapraz spektral yoğunluk matrisi için analitik bir formül verilmiştir. Analiz, aynı kaynak parametrelerine sahip iki ışının çapraz polarizasyonunu, anizotropi oranı a'ya bağlı olarak yayılma sırasında artabileceğini veya azalabileceğini göstermektedir. Işın anizotropisinin kısmen uyumlu kaynakları modellemek için uygun yöntemlerden biri olabileceğini göstermek için sayısal sonuçlar sunulmuştur.

Anahtar kelimeler: Optik komünikasyon, Kısmi tutarlılık, Fiziksel optik, Elektromanyetik dalgalar.

The spatial correlations between random fluctuations of light fields will affect their propagation characteristics. The modeling of spatial correlation functions of random optical beams, generating a variety of radiating fields with prescribed intensity patterns, has different methods. Source anisotropy is one of them. Varied anisotropic beams with the same spectral density and polarization state in the source plane were demonstrated to exhibit different beam shapes and polarization states on propagation [13]. Furthermore, it was demonstrated that stochastic electromagnetic beams might propagate with varying degrees of polarization despite sharing the same coherence characteristics in the plane of origin [14]. The impact of beam source anisotropy on the crosspolarization, coherence, and fluctuations of the light beams on free space propagation is examined in this article in addition to these two results.

The main contribution of this article is the research of correlations between intensity fluctuations in anisotropic light fields. Correlations between intensity fluctuations in stochastic beams is a recent finding and has not been studied for the anisotropic stochastic beams to the best of our knowledge.

The rest of the paper is organized as follows. In section 2, the anisotropic electromagnetic Gaussian Schell-model source is propagated via extended Huygens-Fresnel integral method.

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Once the cross-spectral density matrix is obtained, the longitudinal and the transverse expressions of the beam are found. In section 3, the spectral degree of cross-polarization, the spectral degree of coherence and the correlations between intensity fluctuations are analyzed based on the propagated correlation matrix.

2 Materials and method

Degree of coherence (DOC) between two vibrations of light is one of the concepts to study correlations between the fluctuations. For example, the effect of coherence on anisotropic electromagnetic Gaussian Schell model beams is worked in [13] which pertains to a single plain (in tensor notation). Also, in [14], the impact of source anisotropy on the spectral degrees of polarization is investigated. Another main concept is the degree of polarization (DOP). A generality of degree of polarization to two spatial points in time domain introduced by [15] and named complex degree of mutual polarization. The same problem is considered in frequency domain by [3] and the spectral degree of cross-polarization (SDCP) is introduced.

The theory acquires explicitly the so called electric correlation matrix. The definition of the electric correlation matrix whose elements are pieces of the electric field that are mutually orthogonal is given by

$$W_{ij}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \langle E_i^*(\boldsymbol{r}_1, \omega) E_j(\boldsymbol{r}_2, \omega) \rangle, \quad i, j = x, y$$
(1)

 $E_x(\mathbf{r}, \omega)$ and $E_y(\mathbf{r}, \omega)$ are realizations of suitably constructed wide sense stationary statistical ensembles which are formed in the plane ($\mathbf{r} = \boldsymbol{\rho}$; z = 0) and propagates ($\mathbf{r}, z > 0$, see Figure 1). In Eqn. 1, the complex conjugate is indicated by an asterisk, while the ensemble average is shown by angular brackets $\langle ... \rangle$.



Figure 1. Illustration of the concept. Two detectors D1 and D2 are located on perpendicular planes to the beam axis.

The diagonals of the electric correlation matrix ($\vec{W} = W_{ij}$) give the average intensity of the beams as shown ("tr" is the trace operation)

$$\langle I(\boldsymbol{r},\omega)\rangle = tr\left[\,\overrightarrow{W}(\boldsymbol{r},\boldsymbol{r},\omega)\,\right] \tag{2}$$

Based on this definition together with the supplementary tools, the correlations between the intensity fluctuations on position of two field points is found to be [3]

$$\mu(\mathbf{r}_{1}, \mathbf{r}_{2}, \omega) = \langle I(\mathbf{r}_{1}, \omega) I(\mathbf{r}_{2}, \omega) \rangle$$

= $tr[\vec{w}^{\dagger}(\mathbf{r}_{1}, \mathbf{r}_{2}, \omega) \cdot \vec{w}(\mathbf{r}_{1}, \mathbf{r}_{2}, \omega)]$
= $\frac{1}{2} \langle I(\mathbf{r}_{1}, \omega) \rangle \langle I(\mathbf{r}_{2}, \omega) \rangle [1$
+ $P^{2}(\mathbf{r}_{1}, \mathbf{r}_{2}, \omega)] |\eta(\mathbf{r}_{1}, \mathbf{r}_{2}, \omega)|^{2}$ (3)

In Eqn. 3 the dagger denotes the Hermitian adjoint, $P(r_1, r_2, \omega)$ is the spectral degree of cross-polarization, and $\eta(r_1r_2, \omega)$ is the

spectral degree of coherence [16] whose equations are as follows.

$$P(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sqrt{1 - \frac{4det[\vec{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)]}{[tr[\vec{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)]]^2}}$$
(4)

$$\eta(\mathbf{r}_{1},\mathbf{r}_{2},\omega) = \frac{tr[\vec{W}(\mathbf{r}_{1},\mathbf{r}_{2},\omega)]}{\sqrt{tr[\vec{W}(\mathbf{r}_{1},\mathbf{r}_{1},\omega)]}\sqrt{tr[\vec{W}(\mathbf{r}_{2},\mathbf{r}_{2},\omega)]}}$$
(5)

Spectral degree of cross polarization is first derived by studying the correlations of intensity fluctuations considering the Brown Twiss experiment. When the points coincide (i.e. $r_1, r_2 = r$) Eqn.4 leads us to the usual degree of polarization (see [17] for definitions of degree of polarization, and see [18],[19] for its difference between space-time and space-frequency). SDCP for isotropic Gaussian Schell-Model beams in free space is analyzed by [20] and in turbulent atmosphere is investigated by [21]. Although it is a recent concept, a method is proposed to experimentally investigate the spectral degree of cross polarization [22]. In addition, an experimental method is proposed to measure the spectral degree of coherence for stochastic electromagnetic fields in [23].

Note that in Eqs. 4 and 5 " ρ " can be taken instead of "r" to see the polarization in the source plane. The degree of crosspolarization given by Eqn. 4 takes the determinant (det) and trace (tr) of the electric correlation matrix (Eqn. 1). To apply Eqs.(3)-(5) to arbitrary points outside the source, we first have the free space paraxial propagation of the incident field $W_{ij}(\rho_1, \rho_2, \omega)$ which is given by the extended Huygens-Fresnel integral [20]:

$$W_{ij}(\mathbf{r}_{1}, z_{1}, \mathbf{r}_{2}, z_{2}, \omega) = \left(\frac{k}{2\pi}\right)^{2} \frac{1}{z_{1}z_{2}} \int \int \int \int W_{ij}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}, \omega) exp\left[-ik\left(\frac{(r_{1} - \rho_{1})^{2}}{2z_{1}}\right) - \frac{(r_{2} - \rho_{2})^{2}}{2z_{2}}\right)\right] d^{2}\boldsymbol{\rho}_{1} d^{2}\boldsymbol{\rho}_{2}$$
(6)

Where "*k*" is the wave number $(2\pi/\lambda)$ and the "*i*" in the exponential is the complex unit number different from the subindices of correlation matrices \vec{W} . This expression is the free-space propagation, while propagation in homogeneous and isotropic atmospheric turbulence can be found in [15].

The anisotropic electromagnetic Gaussian Schell-model source, which consists of Gaussian beams with uneven widths along two mutually perpendicular directions to the beam axis, will be used as the source's model, which is given by:

$$W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \sqrt{I_i I_j} B_{ij} exp \left[-\frac{\rho_1^2}{4\sigma_i^2} \right] exp \left[-\frac{\rho_2^2}{4\sigma_j^2} \right] exp \left[-\frac{|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|^2}{2\delta_{ij}^2} \right]$$
(7)

Eqn. 7 represents planar, anisotropic source of any state of coherence. And, this type of anisotropic beam produces the same intensity distribution in the far-zone as an entirely coherent laser source under suitable source beam variables [1]. Here, I_i , I_j are the spectral intensity distributions of x and y components of the electric field in the source plane, B_{ij} are correlation coefficients between the two components E_x and E_y of the electric field-vector at the points ρ_1 and ρ_2 in the source plane. σ_i^2 , σ_j^2 are the variances of the intensity distributions, δ_{ij} are the variances of the correlations. On substituting from

Eqn.7 into Eqn.6 we find the beam in the output planes z_1 , z_2 (see [20] for the integration method):

$$\begin{split} W_{ij}(\mathbf{r}_{1}, z_{1}, \mathbf{r}_{2}, z_{2}, \omega) \\ &= \frac{\sqrt{I_{i}I_{j}}B_{ij}}{\Delta_{ij}^{2}(z_{1}, z_{2})} exp\left[-\left(\frac{1}{16\sigma_{i}^{2}} + \frac{1}{16\sigma_{j}^{2}} + \frac{1}{16\sigma_{j}^{2}} + \frac{1}{2\sigma_{i}^{2}\sigma_{jj}^{2}} + \frac{1}{2\sigma_{i}^{2}\sigma_{ij}^{2}} + \frac{1}{2\sigma_{ij}^{2}(z_{1}, z_{2})} \right] exp\left[-\frac{(\mathbf{r}_{1} - \mathbf{r}_{2})^{2}}{2\Delta_{ij}^{2}(z_{1}, z_{2})} \left(\frac{1}{8\sigma_{i}^{2}} + \frac{1}{8\sigma_{j}^{2}} + \frac{1}{\delta_{ij}^{2}} \right) \right] exp\left[-i\frac{k(\mathbf{r}_{1}^{2} - \mathbf{r}_{2}^{2})}{2R_{ij}(z_{1}, z_{2})} \right] \end{split}$$
(8)

Where the parameters entering this formula are

$$\Delta_{ij}^{2}(z_{1}, z_{2}) = 1 + \frac{z_{1}z_{2}}{k^{2}} \left(\frac{1}{4\sigma_{i}^{2}\sigma_{j}^{2}} + \frac{1}{2\sigma_{i}^{2}\delta_{ij}^{2}} + \frac{1}{2\sigma_{j}^{2}\delta_{ij}^{2}} \right) \\ + \frac{i}{k} \left(z_{2} \left[\frac{1}{2\sigma_{j}^{2}} + \frac{1}{\delta_{ij}^{2}} \right] - z_{1} \left[\frac{1}{2\sigma_{i}^{2}} + \frac{1}{\delta_{ij}^{2}} \right] \right)$$
(9)

and

$$R_{ij}(z_1, z_2) = \sqrt{z_1 z_2} \left(1 + \frac{k^2}{z_1 z_2} \left[\frac{1}{4\sigma_i^2 \sigma_j^2} + \frac{1}{2\sigma_i^2 \delta_{ij}^2} + \frac{1}{2\sigma_j^2 \delta_{ij}^2} \right]^{-1} \right)$$
(10)

Eqn. 8 is the main expression of this article for the evolution of anisotropic beam throughout the planes (r_1, z_1) and (r_2, z_2) . By applying Eqn. 8 into Eqs. 3-5 we can now find DOC, SDCP, and the correlations between the intensity fluctuations on any point throughout the planes. However, special cases appear when we consider the longitudinal and transverse evolutions of the beam. Thus, we now split the analysis into two different cases $W_{ij}(\mathbf{r}, z_1, \mathbf{r}, z_2, \omega)$ and $W_{ij}(\mathbf{r}_1, z, \mathbf{r}_2, z, \omega)$ which represent the longitudinal and transverse forms along the propagation, respectively. The longitudinal beam during its evolution, i.e. the longitudinal electric correlation matrix becomes

$$W_{ij}(\mathbf{r}, z_1, \mathbf{r}, z_2, \omega) = \frac{\sqrt{l_i l_j B_{ij}}}{\Delta_{ij}^2(z_1, z_2)} exp \left[-\left(\frac{1}{16\sigma_i^2} + \frac{1}{16\sigma_j^2} + i\frac{z_2 - z_1}{2k} \left\{ \frac{1}{4\sigma_i^2 \sigma_j^2} + \frac{1}{2\sigma_i^2 \delta_{ij}^2} + \frac{1}{2\sigma_i^2 \delta_{ij}^2} + \frac{1}{2\sigma_i^2 \delta_{ij}^2} \right\} \right]$$

$$\left. + \frac{1}{2\sigma_i^2 \delta_{ij}^2} \right] \frac{\mathbf{r}^2}{\Delta_{ij}^2(z_1, z_2)} \right]$$
(11)

With the same parameter $\Delta_{ij}^2(z_1, z_2)$ given by Eqn.9, and we mention that the phase factor R_{ij} does not involve in the formula in this specific case.

On applying the condition $z_1 = z_2 = z$ we get the transverse electric correlation matrix as follows

$$\begin{split} W_{ij}(\mathbf{r}_{1}, z, \mathbf{r}_{2}, z, \omega) \\ &= \frac{\sqrt{I_{i}I_{j}B_{ij}}}{\Delta_{ij}^{2}(z)} exp \left[-\left(\frac{1}{16\sigma_{i}^{2}}\right. \\ &+ \frac{1}{16\sigma_{j}^{2}}\right) \frac{(\mathbf{r}_{1} + \mathbf{r}_{2})^{2}}{\Delta_{ij}^{2}(z)} \right] exp \left[-\frac{(\mathbf{r}_{1} - \mathbf{r}_{2})^{2}}{2\Delta_{ij}^{2}(z)} \left(\frac{1}{8\sigma_{i}^{2}} + \frac{1}{8\sigma_{j}^{2}}\right. \\ &+ \frac{1}{\delta_{ij}^{2}} \right) \right] exp \left[-i \frac{k(\mathbf{r}_{1}^{2} - \mathbf{r}_{2}^{2})}{2R_{ij}(z)} \right] \end{split}$$
(12)

With the parameters supplied by

$$\Delta_{ij}^{2}(z) = 1 + \frac{z^{2}}{k^{2}} \left(\frac{1}{4\sigma_{i}^{2} \sigma_{j}^{2}} + \frac{1}{2\sigma_{i}^{2} \delta_{ij}^{2}} + \frac{1}{2\sigma_{j}^{2} \delta_{ij}^{2}} \right) + \frac{i}{k} \left(\left[\frac{z}{2\sigma_{j}^{2}} - \frac{z}{2\sigma_{i}^{2}} \right] \right)$$
(13)

and

$$R_{ij}(z) = z \left(1 + \frac{k^2}{z^2} \left[\frac{1}{4\sigma_i^2 \sigma_j^2} + \frac{1}{2\sigma_i^2 \delta_{ij}^2} + \frac{1}{2\sigma_j^2 \delta_{ij}^2} \right]^{-1} \right)$$
(14)

3 The Research findings and results

With the help of Eqs. 3, 4, 5, and 8, we will now carry out numerical calculations that yield quantitative information about the effect of source anisotropy on DOC, on SDCP, and on intensity fluctuations. We note here that, the anisotropy characteristics of the field often rely on the source parameters σ_i , σ_i and δ_{ii} , as well as on the cross-section and on the distance from the source. The complete statistics and due to the dependence on so many variables, there are many different scenarios that are difficult to categorize in a straightforward manner. In Figure 2 we show the dependence of SDCP (Eqn. 8 in Eqn. 4) on the source anisotropy $\alpha = \sigma_i / \sigma_i$. Note that, in Figure 2(a), the degree of cross-polarization rises in harmony with the increase in α on the propagating direction. For a proper choice of parameters, Figure 2(b), it increases and decreases with α increment, however, this change stays stable only for $\alpha = 1,2$.



Figure 2. Changes in the spectral degree of cross-polarization as

determined by Eqn. 4. The parameters are as given: $\lambda = 630 \text{ nm}, I_x = I_y = 1, B_{xx} = B_{yy} = 1, B_{xy} = B_{yx} = 0.2, \delta_{xx} = \delta_{yy} = 1.225 \text{ nm}, \delta_{xy} = \delta_{xy} = 1.25 \text{ nm}, \sigma_x = 1 \text{ cm}, \sigma_x/\sigma_y = \alpha.$ (a): $r_1 = r_2 = 1 \text{ cm},$ (b): $r_1 = 1 \text{ cm}, r_2 = 5 \text{ cm}.$

In the graphical display Figure 3, we present the transverse and longitudinal propagation of the cross-polarization. Figure 3(a) reveals that the cross-polarization is unbounded different than polarization which is also shown in [20]. In Figure 3(b) we see a non-monotonic change in the cross-polarization which results in an asymptotical decay.



Figure 3. Changes in the spectral degree of cross-polarization as determined by Eqn. 4. The parameters are as given: $\lambda=630$ nm, $I_x=I_y=1, B_{xx}=B_{yy}=1, B_{xy}=B_{yx}=0.2, \delta_{xx}=\delta_{yy}=1.225$ mm, $\delta_{xy}=\delta_{xy}=1.25$ mm, $\sigma_x=1$ cm, $\sigma_x/\sigma_y=\alpha$. (a): Transverse propagation; $z_1=z_2=1$ m, $r_1=0$ m, $r_2=r$, (b): Longitudinal propagation; $r_1=r_2=0$ cm, $z_1=100$ m, $z_2=z$.

Figure 4 illustrates the evolution of DOC by taking into account the anisotropic ratio α (see also [24] for α). On the cross-section, Figure 4(a), a single oscillation takes part in the decay of DOC for $\alpha > 2$. In Figure 4(b), some large α values keep the beam from being coherent on long distance propagation in general depending on the parameters.



Figure 4. Variations in the spectral degree of coherence as determined by Eqn. 5. The parameters are taken as: $\lambda = 630 \text{ nm}$, $I_x = I_y = 1$, $B_{xx} = B_{yy} = 1$, $B_{xy} = B_{yx} = 0.2$, $\delta_{xx} = \delta_{yy} = 1.225 \text{ nm}$, $\delta_{xy} = \delta_{xy} = 1.25 \text{ nm}$, $\sigma_x = 1 \text{ cm}$, $\sigma_y = \alpha$. (a) $z_1 = 1 \text{ m}$, $z_2 = 100 \text{ m}$, $r_1 = 0 \text{ m}$, $r_2 = r$, (b) $r_1 = 1 \text{ cm}$, $r_2 = 5 \text{ cm}$, $z_1 = 1 \text{ m}$, $z_2 = z$.

We want to see the impact of anisotropic source on correlations between intensity fluctuations in Figure 5 (average intensities are normalized). When Figure 5(a), and Figure 5(b) are compared, it is seen that the correlations increase both on the cross-sectional and propagational directions. Figure 5(a) shows us that large α causes a quick increase in the correlations on radial propagation, but it is the opposite on longitudinal propagation as shown in Figure 5(b).



Figure 5. Variations in the correlations between the intensity fluctuations as computed from Eqn. 3. The parameters are taken as: $\lambda = 630 \text{ nm}, \text{ I}_x = \text{ I}_y = 1, \text{ B}_{xx} = \text{ B}_{yy} = 1, \text{ B}_{xy} = \text{ B}_{yx} = 0.2, \delta_{xx} = \delta_{yy} = 1.225 \text{ nm}, \delta_{xy} = \delta_{xy} = 1.25 \text{ nm}, \sigma_x = 1 \text{ cm}, \sigma_x / \sigma_y = \alpha. (a): z_1 = 1 \text{ m}, z_2 = 100 \text{ m}, r_1 = 1 \text{ cm}, r_2 = r,$

(b): $r_1 = 1 \text{ cm}, r_2 = 5 \text{ cm}, z_1 = 1 \text{ m}, z_2 = z$.

4 Discussion

In summary, analytical formula for the anisotropic electromagnetic Gaussian-Schell beams moving in free space which generalizes to two spatial points in free space is derived and analyzed. General propagation formula for anisotropic light beams is narrowed down to transverse and longitudinal directions. We make use of derived formula in statistical propagation analysis. We mainly concentrate on the newly obtained theory of cross-polarization of stochastics light beams. It is found that the two beams having same source parameters might have totally different cross-polarization evolution on free space propagation because of the anisotropy. Lastly, it is applied to the intensity correlations of stochastic electromagnetic beams at the detectors, where such applications with anisotropic beam sources may supply additional insight in light beam modeling and overcoming turbulence effects in different environments.

5 Author contribution statements

Within the scope of this study, Serkan ŞAHİN contributed to the formation of the idea, design and literature review, evaluation of the results obtained, provision of the materials used and examination of the results; writing and supervision of the article in terms of content.

6 Ethics committee approval and conflict of interest statement

There is no need for any permission from ethics committee for the article prepared.

The authors declare that they have no conflict of interest for the article prepared.

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