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RESEARCH PAPER

Mathematical modelling of using renewable energy in the power sectors for the sustainable environment

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Abstract

Currently, human-caused greenhouse gas emissions are one of the main causes of global warming. Burning fossil fuels (such as coal, oil, and gas) have become a climate change due to the uptake of heat-trapping gases. A lot of CO_2 is produced from this, which helps in the creation of greenhouse gases. On the other hand, global electricity demand has been rising for decades, such to rising populations, increasing industrialization, and higher incomes. The power sector is the biggest source of carbon dioxide emissions because of fossil fuel, the main source of energy used for power generation all over the world that's why climate change as well as increased global warming. Therefore, most countries have set targets for the use of renewable energy (RE) to reduce their electricity and need for energy and carbon emissions. In this study, RE is used to keep the environment sustainable, where the system of ODEs has been formed using different types of parameters to analyze the mathematical structure of four variables associated with RE. Positivity test, stability analysis, and bifurcation analysis are examined to prove the truth for the sustainability of the environment. The model plays a special role in increasing electricity production and reducing greenhouse gases in the environment. This study emphasizes the significance of employing RE in the power sector for environmental sustainability.

Keywords: Renewable energy; environmental sustainability; bifurcation analysis; logistic model

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1 Introduction

The environment is an important component of human civilization. Since the development of civilization, people have gradually developed their environment. Just as humans are exploiting

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nature for their own needs or using natural resources, nature is also ready to oppose humans and the entire life force in its fragmented and wounded form. At the turn of the 21st century, when mankind is at the pinnacle of civilization, the environment is pushing us toward catastrophe. A 'greenhouse reaction' has occurred in the environment, which is changing the climate by increasing global temperatures day by day. Human activities have reached such a level that the environment can't meet the needs of billions of people. Among man-made activities, industries play a major role in the emission of various types of air pollutants into the atmosphere. With the increase in population unplanned urbanization, new industrial areas, thermal power generation vehicles, etc. have developed, where most of the electricity is used. Electric power plays an important role in people's lives because nowadays all-important activities require electricity directly or indirectly and it provides an increasing share of energy production and consumption in all countries and its growth continues for transportation and thermal electrification energy applications as well as digital connectivity devices and air conditioning. A large portion of the world's energy is used to build transport and power buildings. The use of fossil fuels is a major cause of environmental pollution and climate change. Most of the world's electricity comes from fossil fuel-based power plants. Carbon dioxide and other hothouse feasts are responsible for global warming because such fuel can provide a large amount of energy. It is widely used in everything from power generation to vehicles. Because of the high concentration of carbon in these fuels, the use of these fuels releases a large amount of carbon into the atmosphere, contributing to the production of various greenhouse gases, including carbon dioxide, carbon monoxide, and methane. This command to global warming and climate change.

The Paris Climate Agreement, adopted in 2015, is a landmark international treaty aiming to limit global warming to below 2 degrees Celsius, preferably to 1.5 degrees Celsius, compared to pre-industrial levels. The agreement requires all signatories to submit Nationally Determined Contributions (NDCs) that outline their plans to reduce greenhouse gas emissions and adapt to climate impacts [1]. Among the latest developments from the COP meeting COP26 (2021) - pledging to phase out unabated coal power and end international financing for coal in Glasgow, more than 100 countries committing to reduce methane emissions by 30% by 2030, agreement to increase financial support to help developing countries adapt to climate impacts and transition to renewable energy, and comprehensive review of collective progress towards the goals of the Paris Agreement at COP28 (2023) - Dubai, highlighting the urgent need for accelerated action, many countries submitting more ambitious NDCs, pledging to cut deep emissions and increase investment in renewable energy, significant discussions were held on the role of green hydrogen as a key component of decarbonization efforts. The latest regulation on renewable energy and sustainability is a package of policies in the European Union that aims to reduce net greenhouse gas emissions by at least 55% by 2030 compared to 1990 levels and amends the binding target for the share of renewable energy in the EU's energy mix to 40% by 2030 [2]. In the United States, reducing emissions by 40% by 2030 includes significant investments in renewable energy, electric vehicles, and other green technologies. China is focused on raising the share of non-fossil fuels in primary energy consumption to about 20% by 2025 and to peak carbon emissions before 2030.

Renewable energy for Bangladesh's position in climate action aims to generate 10% of its energy from renewable sources by 2025, with a long-term target of 40% by 2041. Significant investments in solar power, including the development of large-scale solar parks and off-grid solar home systems, have made Bangladesh a leader in off-grid solar solutions. Emerging focus on wind and biomass energy to diversify the renewable energy portfolio. Bangladesh is highly vulnerable to the effects of climate change, such as sea level rise and extreme weather events. The country has implemented robust adaptation measures, including cyclone shelters, improved drainage systems, and climate-tolerant infrastructure. Actively seek international climate finance to support adaptation and mitigation efforts. Bangladesh has been vocal in favour of establishing a loss and damage fund at COP27. The Bangladesh Climate Change Strategy and Action Plan (BCCSAP) is a comprehensive plan outlining the country's approach to climate adaptation, mitigation, and capacity building. The Renewable Energy Policy (2008) provides incentives and a framework for promoting renewable energy development, including tariff benefits, subsidies, and feed-in tariffs. Despite the high risk of climate change impacts, Bangladesh is making significant progress in renewable energy and climate adaptation. The country's proactive policies, international cooperation, and ambitious goals align with the global objectives set out in the Paris Agreement and the COP meeting [2]. As global regulations and commitments intensify, Bangladesh's efforts in renewable energy and sustainability are crucial to its resilience and contribution to global climate action.

Although fossil fuels have been around for millions of years, human civilization began using them only 200 years ago. But it's hard to prognosticate how long it will last because fossil fuel reserves are being discovered almost every year. Some research reports have predicted the depletion of fossil energy reserves. Therefore, people today are using renewable energy to combat climate change, fossil fuel conservation, and environmental pollution, which are produced through the use of a variety of natural resources including sunlight, wind, water wave and tidal energy, bioenergy, geothermal, ocean waves, ocean heat, tidal, hydrogen fuel cells, and biogas. The electricity and thermal energy sectors are more affected by climate change than any other sector of the global economy. According to a report by think tank Ember, the electricity sector is the largest source of Carbon dioxide emissions [3]. But the International Energy Agency 2022 says the decarbonization of the power sector is underway, and to achieve this milestone, wind and solar will need to make up 41 percent of global electricity generation by 2030, with solar up 24 percent and wind up 17 percent over the previous year [4]. World leaders are working together on this problem. According to experts, there is no alternative to renewable energy to combat climate change. With the development of civilization, the use of electricity and heat is increasing every day. It has been described in various papers below that a large amount of greenhouse gases is being emitted in the power sector today, which is a threat to the environment in the coming days. Therefore, researchers are using different types of renewable energy in the power sector and have developed models showing that using renewable energy compared to fossil fuels reduces carbon emissions reduces greenhouse gas emissions, and also helps to keep the environment sustainable.

To strengthen research on sustainable and renewable energy models, models often estimate the continuous availability of renewable resources, such as sunlight, wind, and water flow, by making a more thorough examination of the model's assumptions, constraints, and possible extensions. Assumptions about the efficiency and performance of renewable energy technologies can significantly influence the model's results. Incorporates realistic efficiency rates and degradation over time into models. The model often assumes the constant availability of wind resources. Changes in the model can be observed by including temporal and spatial variability of wind speed. Includes historical data and climate projections to account for seasonal and geographic variations. One of the limitations is the integration of renewable energy into the existing power grid. The models have to deal with the technical challenges of grid stability, energy storage, and transmission losses. The availability of land and water resources for renewable energy projects such as solar farms and hydropower creates practical limitations. Availability of suitable land for wind farms and their environmental impacts. Assesses land use conflicts, impacts on wildlife, and community acceptance. Offshore wind can be considered as an alternative. Expanding the models to include hybrid renewable energy systems can improve reliability and efficiency. For example, combining solar, wind, and biomass can provide more consistent energy and reduce dependence on a single source. Combine wind energy with solar, biomass, and other renewable

sources. Increases reliability and streamlines supply. Dynamic simulation models can optimize such hybrid systems.
By critically examining and incorporating these assumptions, limitations, and extensions, researchers can develop more accurate and comprehensive models for sustainable and renewable energy systems. This holistic approach will better inform policy-making, investment decisions, and strategic planning of renewable energy infrastructure.
Proposed a mathematical model in which the Bangladeshi analyzed the greenhouse gas emission rate as a result of the use of fuel in various power stations. It collected parameters related to greenhouse gas emissions from various power stations in Bangladesh and in the simulation using HOMER software found that the majority of greenhouse gas emissions in the power sector are dependent on coal [5]. Increasing the use of renewable energy in these sectors will meet the demand for electricity or heat on the one hand, while reducing the adverse impact on the environment. For this, humanity has to increase the use of renewable energy by changing our dependence on fossil fuel energy for electricity and heat generation. A power generation and transmission company, especially suitable for CELECEP, has started to work together with the agreements reached by countries in the 2030 Agenda, taking into account all the Sustainable Development Goals [6]. Among electric companies, SDG priorities are matched by efficient energy, innovation, and adequate infrastructure that is environmentally friendly, and responsibly combating climate change. If this continues, it will be possible to reduce global greenhouse gas emissions by 2050. The production of such coal-based electricity will reduce fossil fuels on the one hand and pollute the environment on the other. Some research reports have predicted the depletion of fossil energy reserves and briefly described that due to limited fossil fuel and environmental pollution, fossil fuel-based power plants are declining worldwide today. Saving the world from this will be realized only when renewable energy is replaced by electricity and renewable energy is compared with non-renewable energy, if the price of fossil fuel can be increased by making the price of renewable energy friendly, then it can be easily replaced There are different types of renewable energy such as wind, water, solar, wave, tidal, etc. which play a huge role in generating electricity today. These energies emit some carbon, but very little in quantity, which helps to keep the environment sustainable. The review showed that offshore wind technologies emit the lowest GHG emissions (average life cycle GHG emissions can be 5.3 to 13 g Carbon dioxide-eq/kWh). Results comparing GHG projections by fossil fuel heat and electricity indicate that life cycle GHG emissions are relatively high compared to renewable sources, excluding nuclear-based power generation [7]. This study has shown that the analysis of GHG emissions in the life cycle is an effective tool for assessing the environmental impacts of renewable energy technologies.
Recent advances in environmental sustainability and renewable energy modeling have focused on improving the efficiency, integration, and resilience of renewable energy systems. Combining different renewable energy sources, such as solar, wind, and biomass, in a hybrid system can improve reliability and efficiency. Studies on hybrid systems show that they can provide more consistent energy, reduce dependence on fossil fuels, and reduce greenhouse gas emissions. Dynamic simulation models help optimize the operation of these hybrid systems by accounting for different resource availability and demand patterns, which leads to more effective energy management [8]. Power-to-X (PtX) technology converts surplus renewable electricity into other energy, such as hydrogen or synthetic fuel, which can then be stored and used. These technologies address the mismatch between renewable energy production and demand, increasing system flexibility and reliability. Recent advances in PtX include more efficient electrolyzers for hydrogen production and innovative methods of carbon capture and utilization in the production of synthetic fuels. Integrating renewable energy systems into the urban environment is a growing field of research. This includes the use of building-integrated photovoltaics (BIPV) urban wind turbines

and decentralized energy systems to create smart, sustainable cities. The studies emphasize the importance of urban planning and policy support in facilitating the adoption of renewable energy technologies in cities, addressing challenges such as space constraints and aesthetic considerations [9].

The developed world is now turning to electric vehicles to prevent environmental pollution and increased costs of fossil fuels. Electric vehicles are making the future of transportation easier. Electric vehicles have now appeared on the streets of Bangladesh such as autos, rickshaws, vans, etc. Greenhouse gas emissions, especially Carbon dioxide are increasing due to the rapid growth of EVs in Bangladesh. So, proposing an EVCS model based on solar and biogas found that a conventional grid-based charging station reduces Carbon dioxide emissions by 34.68% compared to conventional charging stations [10–12]. As the use of electricity increases, the residents of Bangladesh are suffering from load shedding, and the use of renewable energy can contribute a lot to meet this demand. In addition to energy production, Bangladesh needs to increase consumer knowledge and reduce power wastage to conserve energy, which will reduce the pressure on the power supply [13]. Briefly describe the PV power plant produced electricity from PV technology depending on the solar radiation and other meteorological variables in the desert climate conditions per hour and is the combination of the ambient air temperature and relative humidity of the PV power output of all technologies is related to the sky conditions [14].

To provide a comprehensive analysis of the importance of renewable energy in sustainable power generation, the model can include various renewable energy sources such as solar and hydroelectric, but only wind energy has been used in this paper. Solar energy is one of the most abundant and widely used forms of renewable energy. Advances in photovoltaic (PV) technology have significantly improved the efficiency and cost-effectiveness of solar power. Research indicates that combining solar energy with other renewable sources can increase the stability and reliability of power grids. For example, a study on hybrid systems consisting of PV and thermal solar collectors Carbon dioxide emissions and high profitability. Hydroelectricity remains the basis of renewable energy due to its ability to generate consistent and large-scale electricity [8]. Recent advances focus on small-scale hydropower systems that can be integrated into existing water infrastructure, reducing environmental impact while maximizing energy output. The versatility of hydropower, from large dams to small micro-hydro systems, makes it an important component of a sustainable energy mix. Biomass energy, derived from organic matter, provides a renewable source that can be used for both electricity generation and heating. The development of advanced biomass conversion technologies such as pyrolysis and gasification increases the efficiency and environmental benefits of biomass energy. Combining biomass with other renewable sources can help solve breakage problems and provide a stable energy supply. Advanced modeling and simulation techniques are crucial for optimizing the integration of multiple renewable energy sources. Dynamic simulation models, such as those used for hybrid renewable energy plants, help design and operate systems that can adapt to different energy demand and supply conditions. These models are essential for effectively planning and managing renewable energy resources. By incorporating a diverse range of renewable energy sources into their models, researchers can develop more robust and sustainable power generation systems. This holistic approach ensures that the benefits of each type of renewable energy are maximized, contributing to a more reliable and environmentally friendly energy landscape.

Carbon emissions have become a major cause of extreme environmental pollution, with negative implications for human life whether a country's economy is developed or underdeveloped. Therefore, reducing such emissions in developing countries is critical to sustaining economic growth [15]. South Asia's transnational capital flows are reducing Carbon dioxide, while energy consumption and economic growth are hurting the environment. The study, which takes into account FDI

and trade, inverted U-shaped concluded that there is an inverted U-shaped relationship between economic growth and Carbon dioxide emissions in the short and long term for Malaysia. To achieve long-term environmental and economic goals, governments must take transformational initiatives towards green energy and less polluting economic growth sectors. He has proposed and analyzed a mathematical model to increase the production capacity by absorbing GHG by sowing seeds in a dynamic system of green buildings [16].
Greenhouse gases capture heat and warm the Earth. Almost all of the rise in greenhouse gases in the atmosphere over the past 150 years has been attributed to human activity. In the United States of America, the majority of greenhouse gases released from human activity are the use of fossil fuels for energy production, heating, and transportation. Figure 1 represents the percentage of

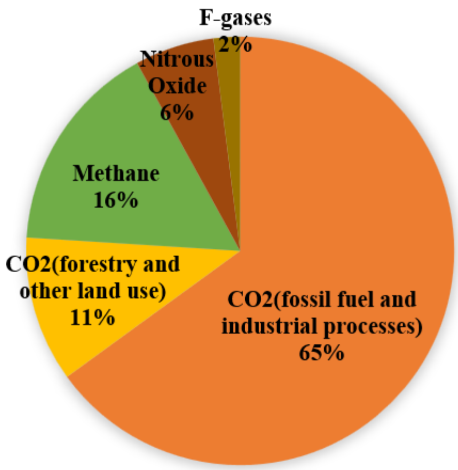

Figure 1. Global GHGs emissions by gas [17]

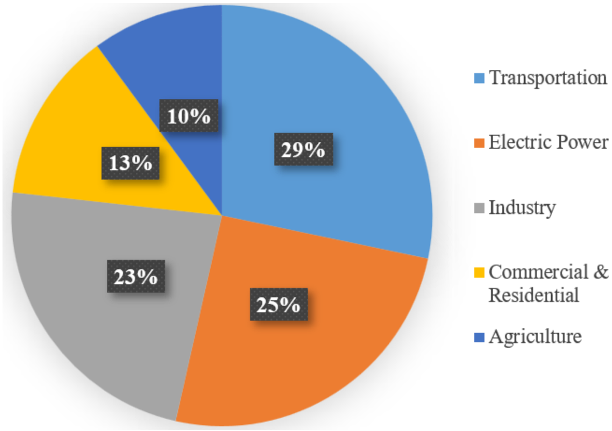

Figure 2. U.S. Greenhouse gases emissions by different sectors in 2021 [18]

common GHGs in the environment in 2022 which implies that Carbon dioxide is the main gas of GHGs. In Figure 2, it is illustrated that the yearly study estimates total national greenhouse gas emissions from human activities. In Figure 3, for the fiscal year 2021–22, the total electricity output

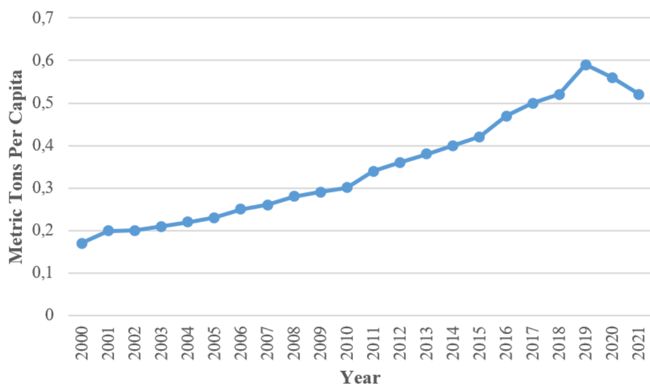

Figure 3. Bangladesh carbon CO_2 emissions [18]

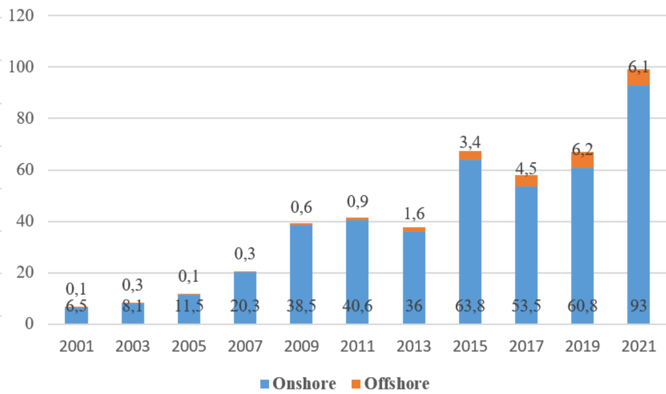

Figure 4. New installations of global wind power [18]

was 85,607 GWh, which was the previous year's net production of 80,423 GWh [13]. Figure 4 shows the historic expansion of new global wind power projects (2001-2021). In 2021, worldwide wind energy produced a total of 837 GW (more than 780 GW from onshore and 57 GW from offshore), a progress of 12% compared to 2020, with roughly 93.6 GW (72.5 GW from onshore and the rest from offshore) of new capacity added to the global grids. To improve the presentation

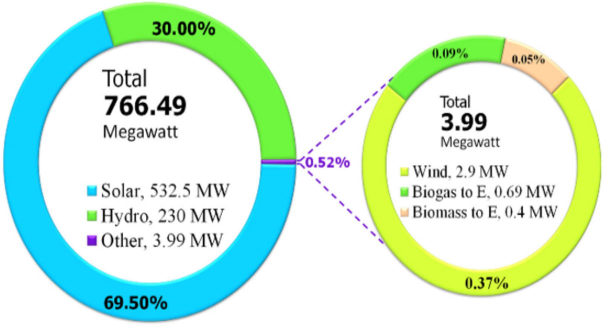

Figure 5. Current status of RE production capacity of the country [13]

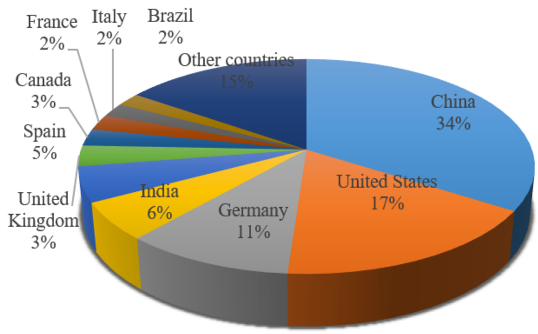

Figure 6. Top 10 countries of worldwide total wind power installation in 2021 [18]

of numerical findings and better illustrate the behavior of models and the impact of renewable energies, various types of visual aids can be effectively used such as a time series graph, this graph shows the daily change in the production of solar and wind energy compared to the demand in a month. Highlights how renewable energy production and demand change over time, daily or seasonal variations in solar and wind energy production compared to energy demand, and periods of surplus or deficit, helping to understand the need for energy storage and grid management [8]. Pie chart and bar graph, this pie chart illustrates the percentage contribution of solar, wind, hydro, and biomass to the total energy mix. Representing the contribution of different energy sources to the total energy mix, showing the percentage of energy generated from solar, wind, hydropower, and biomass, provides a clear snapshot of the energy mix and can be used to compare different scenarios or regions. A Geographic Information System (GIS) map, showing spatial data related to renewable energy resources and infrastructure, showing the geographical distribution of solar radiation, wind speed, or location of renewable energy installations, shows the regional potential for renewable energy projects and increases the planning burden.

Many theoretical models were described and many models were proposed considering one or two species related to the work. In this research, we have developed a model taking into account the above discussions to analyze the impact of rapidly increasing GHGs from the electricity sector and after using renewable energy in the electricity sector. Then based on some basic assumptions and based on the formulation of newly separated optimal control problems for reducing emissions, renewable energy has been used to the maximum. Renewable energy sources come from naturally occurring sources that regenerate themselves through natural factors. Renewable energy sources have been identified as the key answer to mitigation of greenhouse gas emissions climate change, and environmental pollution. As the world's population and economy keep expanding, so does the energy need, a scenario that naturally boosts the demand and consumption of conventional sources of energy, particularly fossil fuels. Fossil fuels constitute the principal type of energy sources that generate serious environmental pollution. Therefore, replacing fossil fuels with renewable energy sources in electricity generation is a significant step to minimize carbon emissions. The wind has been used by man for a very long period to operate windmills, pumps, sailing ships, and mechanical energy for industrial activities. Wind turbine generators are used to create electric power and provided around 6% of global electricity in 2019. Through sustainable energy, the dependence on fossil fuel sources is reduced while increasing the usage of renewable sources of energy thus reducing greenhouse gases. Researchers are using the Lotka Volterra model to perform various tasks such as the biographies of people near coastal areas as a result of climate change [19]. A combined plankton-oxygen kinetics model showed that the rate of oxygen production gradually changes over time due to ocean warming [20]. A non-linear mathematical model has been formulated with the assistance of such different papers and it consists of four species to describe the positive effects of increasing the use of renewable energy sources. To show the impacts of the other dynamical model by increasing renewable energy, analytical analysis has been conducted for the expanded dynamical model and also numerical simulations have been performed.

2 Mathematical formulation of the model

To describe the effect of environmental sustainability because of increasing the use of renewable energy in the power sectors. The consequences and preventative measures that the government should take in response to environmental problems can be readily ascertained by examining these interconnections. A progressive system has been considered, involving four dynamical variables the density of greenhouse gases *G(t)*, electricity produced i.e. coal-based *P(t)*, renewable energy *i.e.* wind *R(t)* environmental sustainability *E(t)*. Renewable energy sources have been identified as the main solution for mitigation of greenhouse gas emissions climate change, and environmental pollution. The interrelationship of the previously described dynamical system is shown in Figure 7.

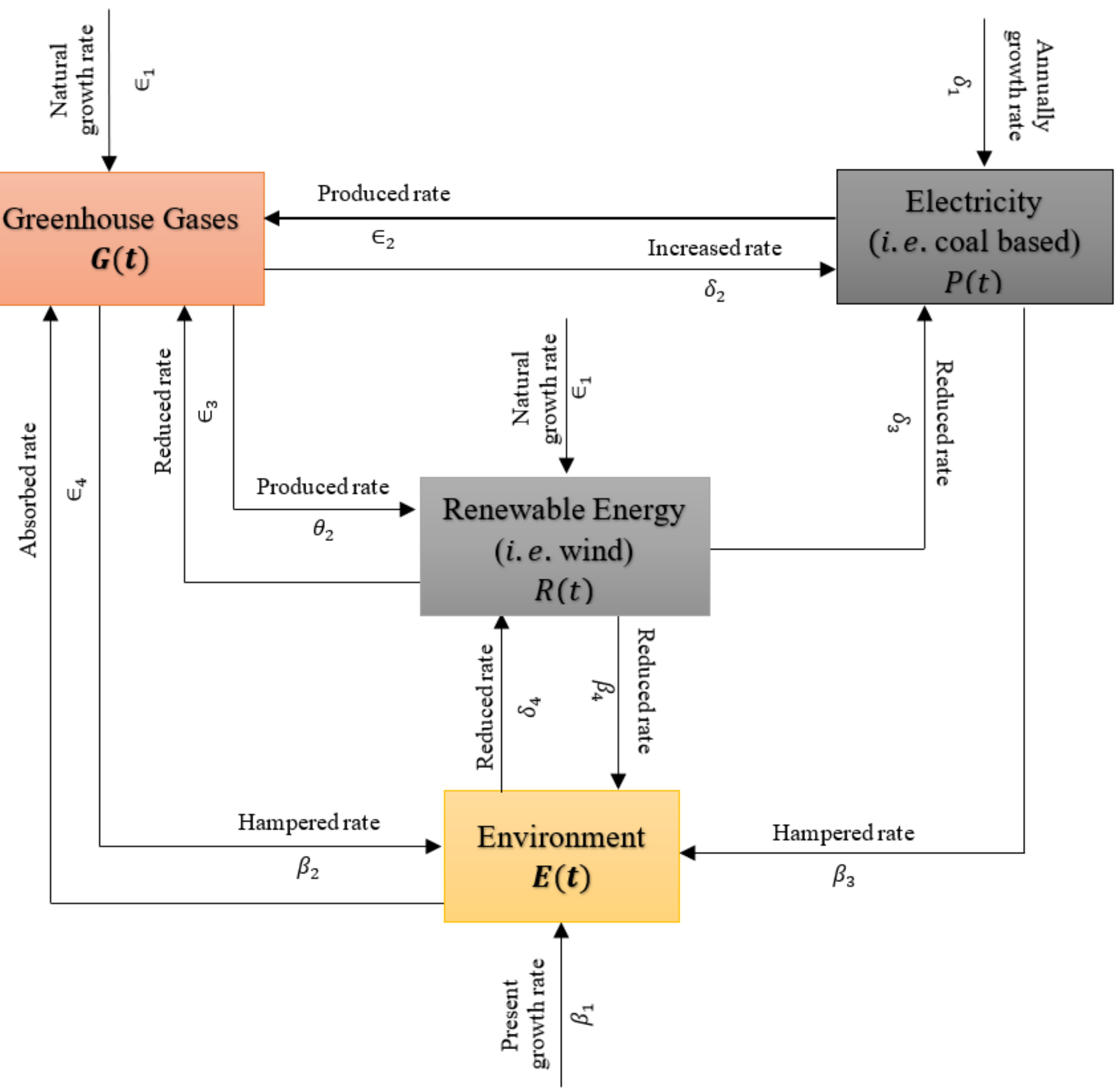

Figure 7. Impacts of the use of renewable energy on the GHGs, coal-based electricity and environment in the power sector

$$
\begin{aligned}
\frac{dG}{dt} &= \in_1 G + \in_2 GP - \in_3 GR - \in_4 GE, \\
\frac{dP}{dt} &= \delta_1 P\left(1 - \frac{P}{k}\right) + \delta_2 PG - \delta_3 PR, \\
\frac{dR}{dt} &= \theta_1 R + \frac{\theta_2 R}{a+G} - \theta_3 RE + \theta_4 R, \\
\frac{dE}{dt} &= \beta_1 E - \beta_2 EG - \beta_3 EP + \beta_4 ER + \beta_5 E,
\end{aligned} \tag{1}
$$

with initial conditions $G_0 = G(0) > 0,\ P_0 = P(0) \geq 0,\ R_0 = R(0) > 0,\ E_0 = E(0) > 0$. In model (1), $G(t), P(t), R(t)$ and $E(t)$ are denoted by the greenhouse gas, the electricity produces i.e. coal-based, renewable energy i.e wind, and environmental sustainability, respectively. The first equation $\frac{dG}{dt}$ represents the rate of change greenhouse gases where $\in_1$ is the natural growth rate of the greenhouse gas; $\in_2$ is the producing rate of the greenhouse gas by the electricity produced i.e. burning coal; $\in_3$ is the reducing rate of greenhouse gas by using renewable energy i.e. wind in the electricity; $\in_4$ is the net absorbing rate of greenhouse gas by the environment where $\in_4 = \in_5 - \in_6$ (here $\in_5$ is the natural absorption rate of the greenhouse gas and is the producing rate of the greenhouse gas after a natural disaster). The second governing equation $\frac{dP}{dt}$ represents the rate of change of coal-based electricity production where δ_1 is the annually produced rate of electricity; δ_2 is the increasing rate of electricity demand for the effect of temperature due to increasing GHGs; δ_3 is the net reducing rate of coal-based electricity due to using renewable energy i.e. wind, where $\delta_3 = \delta_5 - \delta_6$ (here δ_5 is the reducing rate of coal-based electricity due to using renewable energy i.e. wind and is the increasing rate of electricity by using renewable energy i.e. wind).
The third governing equation $\frac{dR}{dt}$ represents the rate of change of renewable energy i.e. wind whereθ_1 is the annually produced rate of renewable energy; θ_2 is the increasing changing rate of renewable energy (i.e. wind energy) by the effect of climate change due to increasing GHGs; θ_3 is the reducing potential rate of renewable energy sources i.e. wind due to (adverse) environmental factors; θ_4 is the highest producing rate of renewable energy i.e. wind in April where $\theta_4 = \theta_5 - \theta_6$ (here θ_5 is the highest producing rate of renewable energy i.e. wind in April and is the lowest producing rate of renewable energy i.e. wind in November). The fourth governing equation $\frac{dE}{dt}$ represents the rate of change of environmental sustainability where β_1 is the present environmental quality; β_2 is the hampering rate of environment quality by the greenhouse gas; β_3 is the hampering rate of environment quality by producing electricity (i.e. burning coal); β_4 is the produced rate of environmental sustainability (or, reducing the rate of environment pollution) by the use of renewable energy in the electricity sector; β_5 is the increasing rate of environmental pollution due to without electricity i.e. coal-based.

3 Analytical analysis

In this section, the positivity test of all variables, boundedness of the system, stability analysis at equilibrium points and sensitivity analysis are demonstrated.

Positivity test

Lemma 1 *Let $G(0) > 0,\ P(0) \geq 0,\ R(0) > 0,\ E(0) > 0$, and $(G(t),\ P(t),\ R(t),\ E(t)) \in R_4^+$, then the solutions $G(t),\ P(t),\ R(t), E(t)$ of the model are non-negative.*

Proof To verify the lemma, system (1) has been employed. For this, the first equation of the system is considered as given

$$\frac{dG}{dt} = \in_1 G + \in_2 GP - \in_3 GR - \in_4 GE. \quad (2)$$

To find the positivity, Eq. (2) can be written as $\frac{dG}{dt} \geq (\in_1 + \in_2 P)\, G \Rightarrow \frac{dG}{dt} \geq A_1 G$, where $A_1 = \in_1 + \in_2 P$. Then $\Rightarrow \frac{dG}{G} \geq A_1 dt$ that yields $\Rightarrow \ln G \geq A_1 t + \ln d_1$, where d_1 is an integrating constant,

$$G(t) \geq d_1 e^{A_1 t}. \quad (3)$$

Now applying the initial condition at $t = 0$, $G(0) = G_0 > 0$,then from Eq. (3), we have $G(0) = G_0 \geq d_1$ Putting the value of $d_1$in Eq. (3), we obtain $G(t) \geq G_0 e^{A_1 t}$ When $t \to \infty, G(t) > 0$, means that $G(t)$ is positive for all $t \geq 0$. By following the same way, it can be obtained for other compartments. Therefore, $G(t) \geq 0,\ P(t) \geq 0,\ R(t) \geq 0\, and\, E(t) \geq 0, \forall t \geq 0$. Hence, the lemma is proved.

Boundedness of the equation

Now, it is established that the proposed system is bounded. By the following Lemma 2, it is started to be proved.

Lemma 2 *The set*

$$\Psi = \left\{ (G, P, R, E) \in \mathfrak{R}_4^+ : h(t) = G(t) + P(t) + R(t) + E(t),\ 0 \leq h(t) \leq \frac{\omega}{\rho} \right\},$$

is a region of attraction for each solution and initially all the variables are positive, where ρ is a constant.

Proof Let

$$h(t) = G(t) + P(t) + R(t) + E(t),$$

and $\rho > 0$ be a constant. Then it can be written

$$\frac{dh}{dt} = \frac{dG}{dt} + \frac{dP}{dt} + \frac{dR}{dt} + \frac{dE}{dt}.$$

$$\begin{aligned} \frac{dh}{dt} &= \in_1 G + \in_2 GP - \in_3 GR - \in_4 GE + \delta_1 P \left(1 - \frac{P}{k}\right) + \delta_2 PG - \delta_3 PR \\ &\quad + \theta_1 R + \frac{\theta_2 R}{a + G} - \theta_3 RE + \theta_4 R + \beta_1 E - \beta_2 EG - \beta_3 EP + \beta_4 ER + \beta_5 E, \end{aligned}$$

$$\begin{aligned} \frac{dh}{dt} + \rho h &= (\in_1 + \rho)\, G + (\delta_1 + \rho)\, P + (\theta_1 + \theta_4 + \rho)\, R + (\beta_1 + \beta_5 + \rho)\, E \\ &\quad + (\in_2 P - \in_3 R - \in_4 E + \delta_2 P - \beta_2 E)\, G - \frac{\delta_1 P^2}{k} \\ &\quad - (\delta_3 R + \beta_3 E)\, P + \left(\frac{\theta_2}{a + G} - \theta_3 E + \beta_4 E\right) R, \end{aligned}$$

$$\begin{aligned}\frac{dh}{dt}+\rho h &\leq (\in_1+\rho)\,G+(\delta_1+\rho)\,P+(\theta_1+\theta_4+\rho)\,R+(\beta_1+\beta_5+\rho)\,E\\&\quad+(\in_2 P-\in_3 R+\delta_2 P-\beta_2 E)\,G-\frac{\delta_1 P^2}{k}-(\delta_3 R+\beta_3 E)\,P+\left(\frac{\theta_2}{a+G}+\beta_4 E\right)R,\\&\leq \omega. \end{aligned} \tag{4}$$

Applying the theory of inequality, we have $h \leq \frac{\omega}{\rho}+C_0 e^{-\rho t}$. For $t \to \infty$ it has been $0 \leq h \leq \frac{\omega}{\rho}$. Hence the solution of the system is bounded in Ψ.

Equilibrium points

The equilibrium points of the system can be obtained by setting, $\frac{dG}{dt}=0, \frac{dP}{dt}=0, \frac{dR}{dt}=0, \frac{dE}{dt}=0$. The system produces two dynamic equilibrium points $R_i(\overline{G}, \overline{P}, \overline{R}, \overline{E})$, where $i=$ 1, 2 and these are shown as

(i) $R_1(G,P,R,E)=R_P\left(\frac{I_1R^*+I_2}{I_3},\frac{I_7R^*+I_8}{I_3\beta_3},\frac{\beta_1\delta_2+\beta_5\delta_2}{\beta_2\delta_3}-\frac{\beta_1+\beta_5}{2\beta_4},\frac{I_4R^*+I_5}{I_6+\theta_3 I_1 R^*}\right)$.

(ii) $R_2(\bar{G},0,\bar{R},\bar{E})=R_P\left(\frac{\beta_1+\beta_4\bar{R}+\beta_5}{\beta_2},0,\frac{\theta_3\in_1-\theta_1\in_4-\theta_4\in_4}{2\theta_3\in_3}-\frac{a\beta_2+\beta_1+\beta_5}{2\beta_4},\frac{\in_1-\in_3\bar{R}}{\in_4}\right)$.

Stability analysis

Now, the system of nonlinear differential equations given by model (1) can be evaluated into the Jacobian matrix given as,

$$J_{(x_1,x_2,x_3,x_4)}=\begin{bmatrix}\in_1+\in_2P-\in_3R-\in_4E & \in_2G & -\in_3G & -\in_4G\\ \delta_2P & a_{22} & -\delta_3P & 0\\ -\frac{\theta_2R}{(a+G)^2} & 0 & a_{33} & -\theta_3R\\ -\beta_2E & -\beta_3E & \beta_4E & a_{44}\end{bmatrix}, \tag{5}$$

where $a_{22}=\delta_1\left(1-\frac{2P}{k}\right)+\delta_2G-\delta_3R$, $a_{33}=\theta_1+\frac{\theta_2}{a+G}-\theta_3E+\theta_4$, and $a_{44}=\beta_1-\beta_2G-\beta_3P+\beta_4R+\beta_5$.

Theorem 1 *The system's coexisting equilibrium point is stable or locally stable under certain conditions but unstable otherwise.*

Proof The Jacobian matrix given by (5) at the co-existing equilibrium point $R_1(G,P,R,E)=R_P(G^*,P^*,R^*,E^*)$ takes the following form

$$\begin{aligned}J_{(x_1,x_2,x_3,x_4)} &= \begin{bmatrix}\in_1+\in_2P^*-\in_3R^*-\in_4E^* & \in_2G^* & -\in_3G^* & -\in_4G^*\\ \delta_2P^* & a_{22} & -\delta_3P^* & 0\\ -\frac{\theta_2R^*}{(a+G^*)^2} & 0 & a_{33} & -\theta_3R^*\\ -\beta_2E^* & -\beta_3E^* & \beta_4E^* & a_{44}\end{bmatrix}\\ &= \begin{bmatrix}\in_1+\in_2P^*-\in_3R^*-\in_4E^* & \in_2G^* & -\in_3G^* & -\in_4G^*\\ 0 & b_{22} & b_{23} & b_{24}\\ 0 & 0 & c_{33} & c_{34}\\ 0 & 0 & 0 & d_{44}\end{bmatrix}.\end{aligned}$$

The characteristic equation of the system takes the form as
$(\in_1+\in_2P^*-\in_3R^*-\in_4E^*-\lambda_1)\,(b_{22}-\lambda_2)\,(c_{33}-\lambda_3)\,(d_{44}-\lambda_4)=0.$

Hence the eigenvalues are

$$\lambda_1 = \in_1 + \in_2 P^* - \in_3 R^* - \in_4 E^*,$$
$$\lambda_2 = b_{22} = \delta_1 \left(1 - \frac{2P^*}{k}\right) + \delta_2 G^* - \delta_3 R^* + \frac{\delta_2 P^* \in_2 G^*}{\in_1 + \in_2 P^* - \in_3 R^* - \in_4 E^*},$$
$$\lambda_3 = c_{33} = b_{33} - \eta_4 b_{23},$$
$$\lambda_4 = d_{44} = c_{44} - \eta_6 c_{34}.$$

Therefore, the co-existing equilibrium may be a locally stable point when

$$\in_1 + \in_2 P^* > \in_3 R^* + \in_4 E^*,$$
$$\delta_1 + \delta_2 G^* + \frac{\delta_2 P^* . \in_2 G^*}{\in_1 + \in_2 P^* - \in_3 R^* - \in_4 E^*} > \delta_1 \frac{2P^*}{k} + \delta_3 R^*, b_{33} > \eta_4 b_{23}, c_{44} > \eta_4 c_{34},$$

or, it may be stable when

$$\in_1 + \in_2 P^* < \in_3 R^* + \in_4 E^*, \ \delta_1 + \delta_2 G^* + \frac{\delta_2 P^* \in_2 G^*}{\in_1 + \in_2 P^* - \in_3 R^* - \in_4 E^*} < \delta_1 \frac{2P^*}{k} + \delta_3 R^*,$$

$b_{33} < \eta_4 b_{23}$, $c_{44} < \eta_4 c_{34}$, otherwise, it is unstable.

Theorem 2 *The system's coal-free power plant equilibrium point is stable or locally stable under certain conditions but unstable otherwise.*

Proof The Jacobian matrix given in Eq. (5) at the coal-free electricity equilibrium point $R_2(G, 0, R, E) = R_P(\bar{G}, 0, \bar{R}, \bar{E})$ takes the following form:

$$J_{(x_1,x_2,x_3,x_4)} = \begin{bmatrix} \in_1 - \in_3 \bar{R} - \in_4 \bar{E} & \in_2 \bar{G} & -\in_3 \bar{G} & -\in_4 \bar{G} \\ 0 & \delta_1 + \delta_2 \bar{G} - \delta_3 \bar{R} & 0 & 0 \\ -\frac{\theta_2 \bar{R}}{(a+\bar{G})^2} & 0 & a_{33} & -\theta_3 \bar{R} \\ -\beta_2 \bar{E} & -\beta_3 \bar{E} & \beta_4 \bar{E} & a_{44} \end{bmatrix}$$
$$= \begin{bmatrix} \in_1 - \in_3 \bar{R} - \in_4 \bar{E} & \in_2 \bar{G} & -\in_3 \bar{G} & -\in_4 \bar{G} \\ 0 & \delta_1 + \delta_2 \bar{G} - \delta_3 \bar{R} & 0 & 0 \\ 0 & 0 & b_{33} & b_{34} \\ 0 & 0 & 0 & d_{44} \end{bmatrix}.$$

The characteristic equation of the above matrix takes the following form:

$$|J_{R_P} - \bar{\lambda} I| = 0,$$

$$\Rightarrow \begin{bmatrix} \in_1 - \in_3 \bar{R} - \in_4 \bar{E} - \lambda_1 & \in_2 \bar{G} & -\in_3 \bar{G} & -\in_4 \bar{G} \\ 0 & \delta_1 + \delta_2 \bar{G} - \delta_3 \bar{R} - \lambda_2 & 0 & 0 \\ 0 & 0 & b_{33} - \lambda_3 & b_{34} \\ 0 & 0 & 0 & d_{44} - \lambda_4 \end{bmatrix} = 0,$$

$$\Rightarrow (\in_1 - \in_3 \bar{R} - \in_4 \bar{E} - \lambda_1)(\delta_1 + \delta_2 \bar{G} - \delta_3 \bar{R} - \lambda_2)(b_{33} - \lambda_3)(d_{44} - \lambda_4) = 0.$$

Hence the eigenvalues are

$$\lambda_1 = \in_1 - \in_3\bar{R} - \in_4\bar{E}, \quad \lambda_2 = \delta_1 + \delta_2\bar{G} - \delta_3\bar{R},$$

$$\lambda_3 = b_{33} = \theta_1 + \frac{\theta_2}{a+\bar{G}} - \theta_3\bar{E} + \theta_4 + \frac{-\theta_2\bar{R}\in_3\bar{G}}{(\in_1 - \in_3\bar{R} - \in_4\bar{E})\,(a+\bar{G})^2}, \text{ and}$$

$$\lambda_4 = d_{44} = b_{44} - \eta_{11}b_{34}.$$

Therefore, the coal-free electricity equilibrium may be a locally stable point when

$$\in_1 > \in_3\bar{R} + \in_4\bar{E}\,,\ \delta_1 + \delta_2\bar{G} > \delta_3\bar{R}\,,$$

$$\theta_1 + \frac{\theta_2}{a+\bar{G}} + \theta_4 > \theta_3\bar{E} + \frac{\theta_2\bar{R}\in_3\bar{G}}{(\in_1 - \in_3\bar{R} - \in_4\bar{E})\,(a+\bar{G})^2}\,,\ b_{44} > \eta_{11}b_{34},$$

or, it may be stable when the following conditions hold, otherwise, it is unstable:

$$\in_1 < \in_3\bar{R} + \in_4\bar{E}\,,\ \delta_1 + \delta_2\bar{G} < \delta_3\bar{R}\,,\ b_{44} < \eta_{11}b_{34}\,,$$

$$\theta_1 + \frac{\theta_2}{a+\bar{G}} + \theta_4 > \theta_3\bar{E} + \frac{\theta_2\bar{R}\in_3\bar{G}}{(\in_1 - \in_3\bar{R} - \in_4\bar{E})\,(a+\bar{G})^2}\,.$$

4 Numerical simulations

Changes in environmental sustainability, driven by the impact of renewable energy on the power sector, as well as alterations in both greenhouse gas emissions and coal-based power plants are depicted in the dynamics of the model shown in Table 1. The trend of electricity consumption is gradually increasing in the country where most of the fossil fuels such as natural gas, coal, oil, etc. are produced by burning and most of the carbon dioxide is emitted from coal, which is spoiling the environment and increasing greenhouse gases. Coal-based electricity generation typically results in an increase in greenhouse gases and a decrease in environmental sustainability. The depicted figure observed that, if electricity is produced by reducing coal-based electricity production and increasing the use of renewable energy such as wind, solar, hydro, etc., it reduces greenhouse gases and increases environmental sustainability. Table 1 shows the changing rate of all dynamical variables of the model.

Table 1. Description and respective values of parameters for the proposed model

Sym	Descriptions of the parameters	Values (year^{-1})	Refs.
$\in_1$	The natural growth rate of the greenhouse gas	1.741e(-8) Gt/GWh	[17, 19]
$\in_2$	Producing rate of the greenhouse gas by the electricity produced i.e. burning coal	1.60e(-5) Gt/GWh	[7, 13, 21]
$\in_3$	Reducing the rate of greenhouse gas by using renewable energy i.e. wind in the electricity	0.0001021 Gt/GWh	[7, 13]
$\in_4$	Net absorbing rate of greenhouse gas by the environment	6.56e(-6) Gt/GWh	[19, 22]

Table 1. Description and respective values of parameters for the proposed model - continued

Sym	Descriptions of the parameters	Values ($year^{-1}$)	Refs.
$\in_5$	Naturally absorption rate of the greenhouse gas	0.00676 kg/km^2	[17, 22]
$\in_6$	Producing rate of greenhouse gas after a natural disaster	0.0005 kg/km^2	[17, 22]
δ_1	Annually produced rate of electricity	0.0645 GWh	[13, 23]
k	carrying capacity of the electricity	10000	[est.]
δ_2	Increasing rate of electricity demand due to increasing GHGs	9.969e(-7) GWh	[23, 24]
δ_3	Reducing the rate of the coal-based electricity (power plant) due to using renewable energy i.e. wind	0.063495 GWh	[19, 21, 25]
δ_4	Reducing rate of the coal-based electricity (power plant) due to environmental pollution	0.0636 GWh	[19, 25]
δ_5	Increasing the rate of electricity by using renewable energy i.e. wind	0.0001048 GWh	[15, 26]
θ_1	Annually growth rate of renewable energy	0.1078 GWh	[13, 26]
a	Saturated constant	0.005	[est.]
θ_2	Increasing changing rate of wind energy by the effect of climate change due to increasing GHGs	0.0495 GWh	[7, 13, 17]
θ_3	Reducing the potential rate of renewable energy sources i.e. wind due to (adverse) environmental factors	0.0290 GWh	[5, 13]
θ_4	April has the highest rate of renewable energy	2.474e(-6) GWh	[5, 13]
β_1	Present environmental quality	0.15	[est.]
β_2	Hampered rate of environmental quality by the greenhouse gas	0.105	[24, 27]
β_3	Hampered rate of environment quality by producing electricity (i.e. burning coal)	0.0122	[19, 28]
β_4	Produced rate of environmental sustainability by the use of renewable energy in the electricity sector	7.27e(-8)	[17, 19]
β_5	The increasing rate of environmental pollution due to electricity i.e. coal-based (normally pollution)	0.023467	[17, 19]

In this section, the increasing changing rate of renewable energy (i.e. wind energy) is described by the effect of climate change, attributed to the rising GHGs levels in the power sector (θ_2), in relation to the other dynamics of the model shown in Figure 9, Figure 10, Figure 11 and Figure 12. If the wind energy changes continue to increase due to greenhouse gases, the change of greenhouse gases rapidly with different values of wind energy shown in Figure 9.

With the increasing wind energy in the power sector, then coal-based electricity is decreasing rapidly with different values of θ_2 shown in Figure 10. Climate change is having a clear impact on the weather. Experts say due to this change three seasons are now visible in the country of six seasons. Among them, summer is the most prominent. It is understood that the wind speed is higher in summer than at other times, but in extreme heat, this wind feels less on our bodies. Climate change is changing the wind speed sometimes more or sometimes less thereby affecting wind power that is shown in Figure 11. While the increase in renewable energy in the power sector is reducing coal-based electricity, it is reducing the amount of air pollution in the environment and protecting animals from various problems, reducing the amount of greenhouse gases, thereby helping to keep the environment sustainable that is shown in Figure 12.

The reducing potential rate of renewable energy sources i.e. wind due to (adverse) environmental factors. Due to human activities, the climate is changing, resulting in various problems in the environment such as low pressure and natural disasters. The main source of solar energy production is the sun, if there is a low pressure or the sky is cloudy then our solar energy is interrupted, on the other hand, due to frequent natural disasters the wind speed changes. So, it can be said that an adverse environment destroys the source of renewable energy. Here, Figure 13 represents the decreasing GHGs with different values of θ_3 when increasing renewable energy due to adverse environmental factors being decreased.

Figure 14 represents the increasing renewable energy after fewer environmental factors in the coal-based power sector are decreased. Figure 15 represents the increasing potential rate of renewable energy sources i.e.

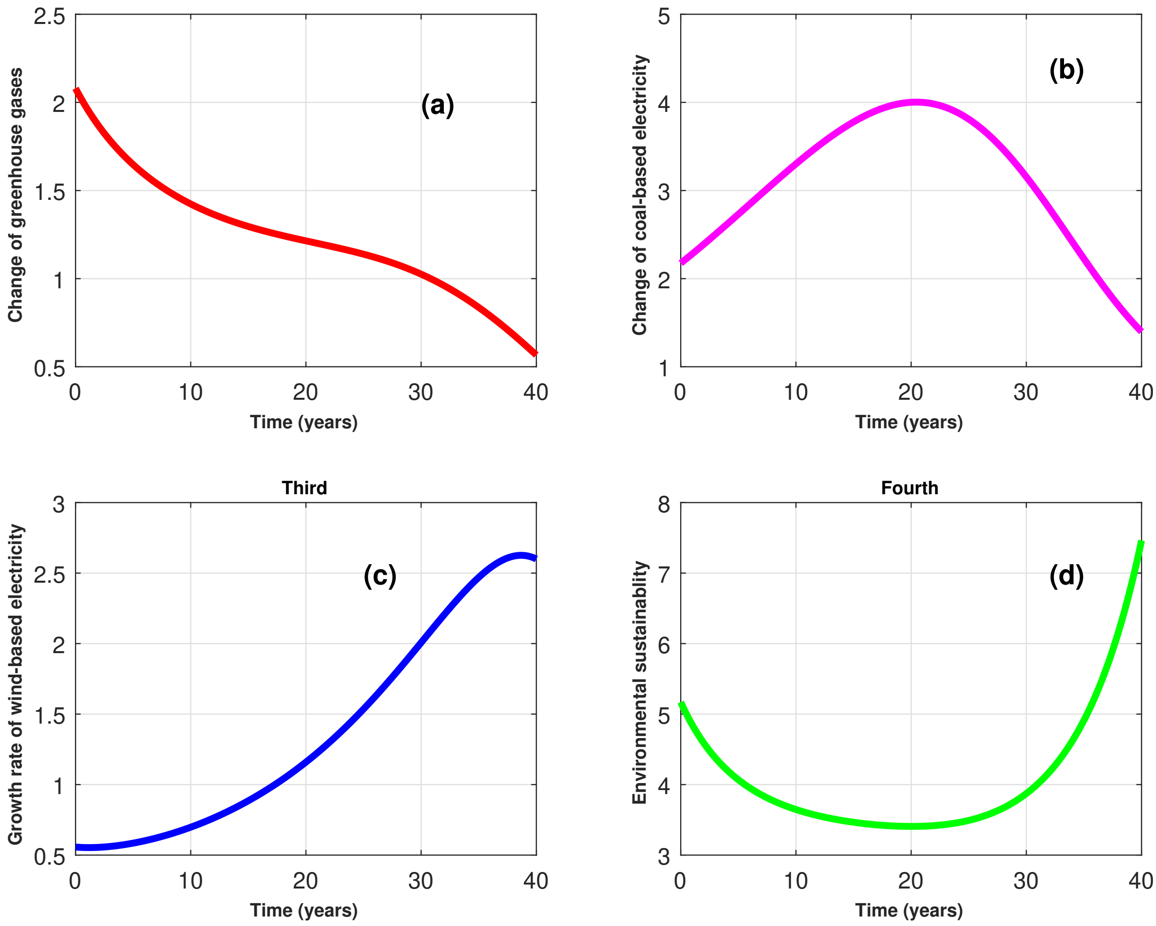

Figure 8. Impact of using renewable energy in power sector on the greenhouse gases, coal-based electricity and environmental sustainability

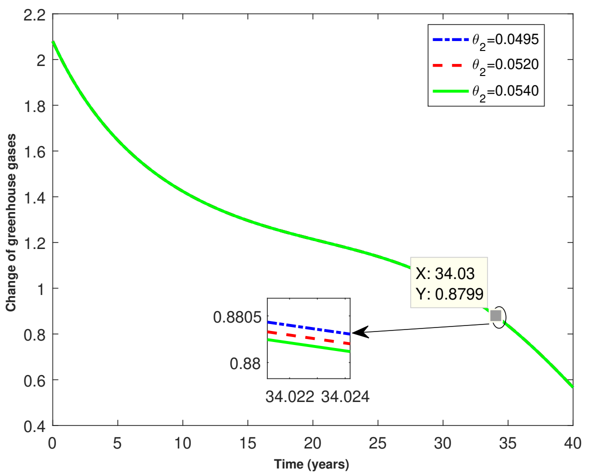

Figure 9. Change of greenhouse gases for different values of θ_2

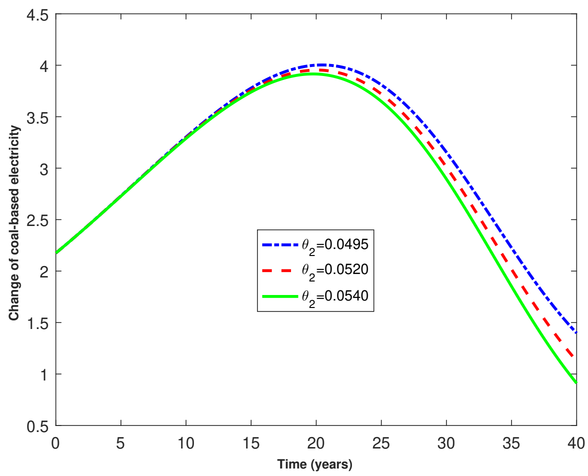

Figure 10. Change of coal-based electricity for different values of θ_2

wind when adverse environmental factors are being decreased. Figure 16 represents the increasing potential rate of renewable energy sources i.e. wind when adverse environmental factors are being decreased. The

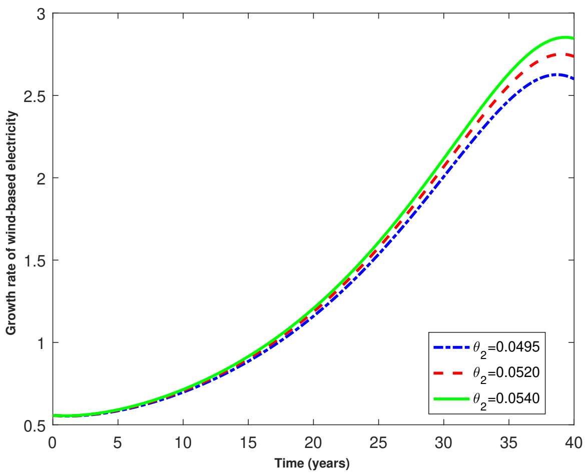

Figure 11. Change of wind-based electricity for different values of θ_2

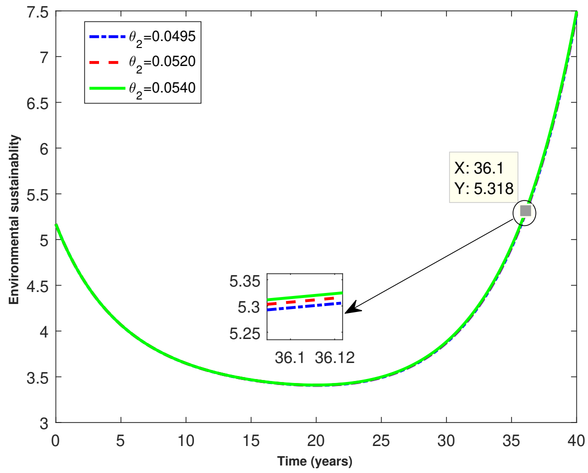

Figure 12. Change of ES for different values of θ_2

amount of load shedding is increasing due to the reduction in power generation due to grid disasters and energy crises. In this situation, it is being planned to reduce load shedding by keeping the amount of renewable energy in full production.

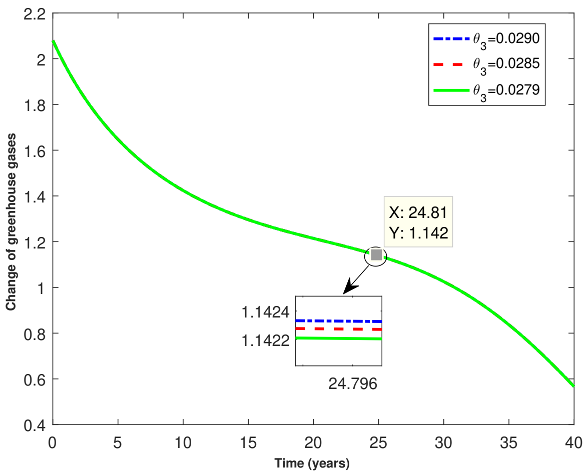

Figure 13. Change of greenhouse gases for different values of θ_3

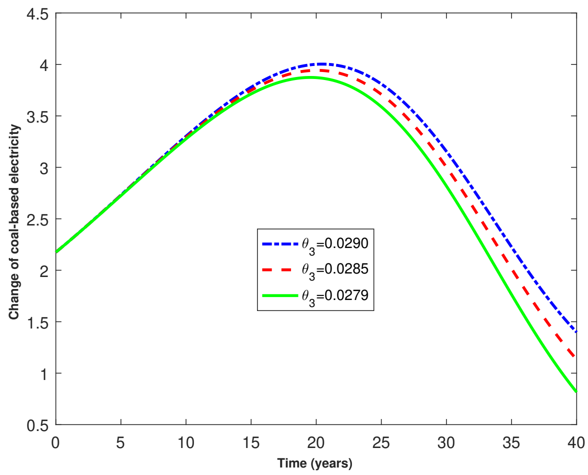

Figure 14. Change of coal-based electricity for different values of θ_3

Using wind power energy, in Figure 17, it is seen that the greenhouse gases are decreasing with different values of $\in_3$. The use of renewable energy in the power sector not only reduces greenhouse gas emissions but also helps to sustain the environment as shown in Figure 18, Figure 19 and Figure 20.

5 Bifurcation diagram for different variables

Bifurcation analysis is a qualitative shift in the behaviour of dynamics for the verification of one or more parameters which investigates how an ODE depends on parameters. However, a bifurcation diagram depicts the change of equilibrium number concerning parametric variables. For bifurcation analysis, we need the Jacobian matrix of model (1). According to Eq. (5), the characteristic equation of the Jacobian matrix $J = DF(x)$ is $Jv = \lambda v$, where λ and v are eigenvalues and eigenvectors, respectively. Then at any

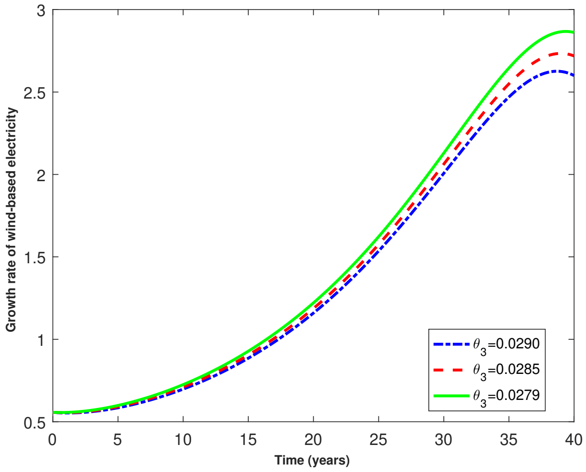

Figure 15. Change of wind-based electricity for different values of θ_3

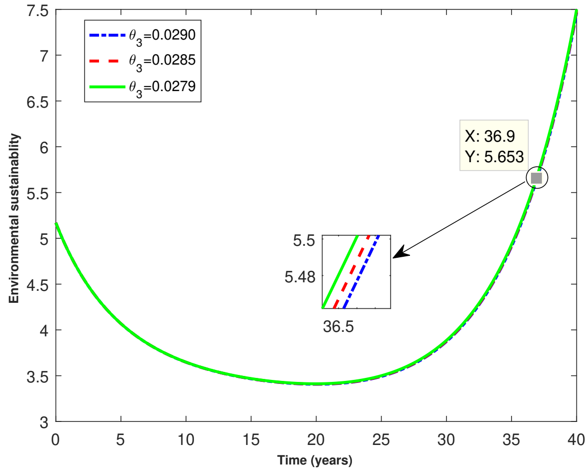

Figure 16. Growth rate of ES for different values of θ_3

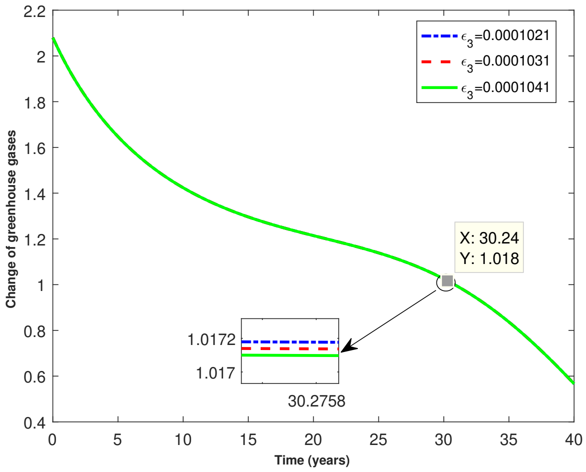

Figure 17. Change of greenhouse gases for different values of $\in_3$

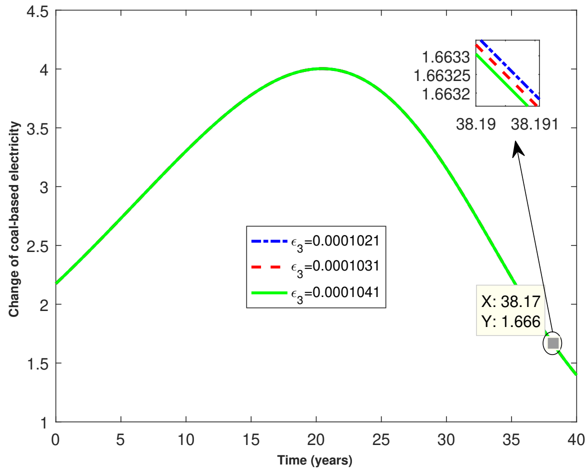

Figure 18. Change of coal-based electricity for different values of $\in_3$

equilibrium point $(\tilde{G}, \tilde{P}, \tilde{R}, \tilde{E})$, the following equation is provided:

$$\begin{bmatrix} \in_1 + \in_2\tilde{P} - \in_3\tilde{R} - \in_4\tilde{E} & \in_2\tilde{G} & -\in_3\tilde{G} & -\in_4\tilde{G} \\ \delta_2\tilde{P} & a_{22} & -\delta_3\tilde{P} & 0 \\ -\frac{\theta_2\tilde{R}}{(a+\tilde{G})^2} & 0 & a_{33} & -\theta_3\tilde{R} \\ -\beta_2\tilde{E} & -\beta_3\tilde{E} & \beta_4\tilde{E} & a_{44} \end{bmatrix} \begin{bmatrix} v_G \\ v_P \\ v_R \\ v_E \end{bmatrix} = \lambda \begin{bmatrix} v_G \\ v_P \\ v_R \\ v_E \end{bmatrix}. \quad (6)$$

The eigenvalues give information on the direction and strength of the attraction and repulsion of the orbit. If the eigenvalues are complex numbers, we obtain a spiral equilibrium. But if the eigenvalues are real numbers, we get a node. Further, by assessing the $tr(J)$ and $\det(J)$ at the equilibrium points, we can easily establish whether a bifurcation happens or not and also the type of the equilibrium points i.e. stable, node, saddle, or unstable.

We have a saddle when $\det(J) < 0$. But for $\det(J) > 0$, we need to determine the $tr(J)$ value. If $tr(J) < 0$, we have a stable point but if $tr(J) > 0$, we have an unstable point. Here four diagrams have been tested for bifurcation using 3D numerical simulations. Three alternative dynamics are investigated for various bifurcation analyses. Figure 21 depicts the bifurcation analysis of the proposed model.

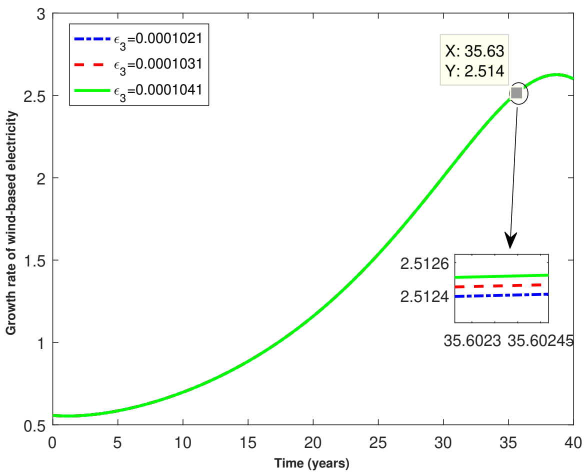

Figure 19. Change of wind-based electricity for different values of $\in_3$

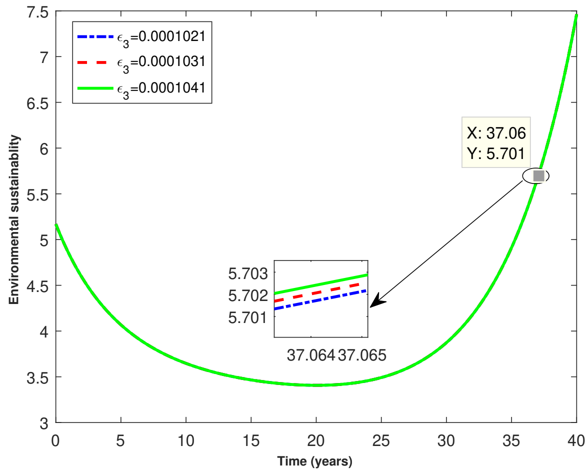

Figure 20. Change of ES for different values of $\in_3$

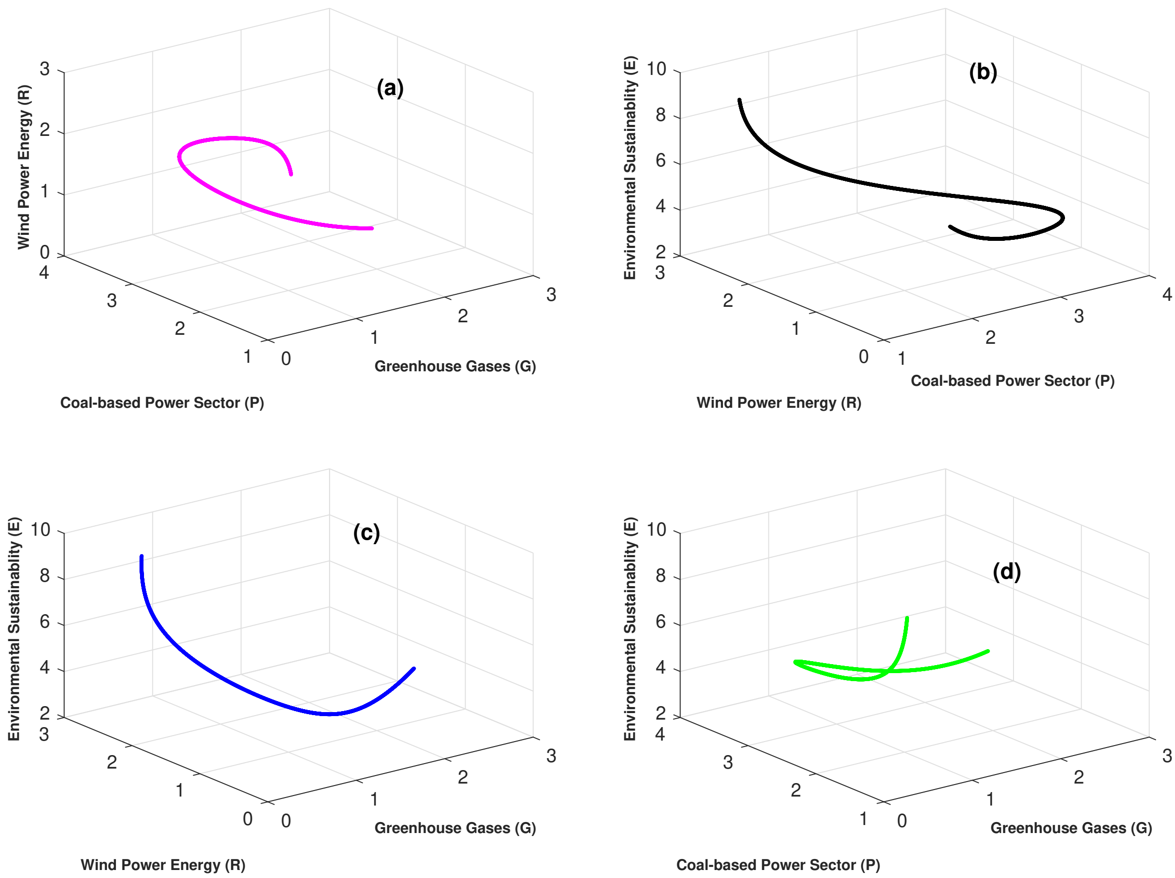

Figure 21. Bifurcation diagram of model (1) for Figure 21(a) greenhouse gases $G(t)$, coal-based power sector $P(t)$ and wind power energy $R(t)$. Figure 21(b) coal-based power sector $P(t)$, wind power energy $R(t)$ and environmentally sustainable $E(t)$. Figure 21(c) greenhouse gases $G(t)$, wind power energy $R(t)$ and environmentally sustainable $E(t)$. Figure 21(d) greenhouse gases $G(t)$, coal-based power sector $P(t)$ and environmentally sustainable $E(t)$

6 Sensitivity analysis

In the context of mathematical modelling, sensitivity analysis is a technique used to understand how changes in the input parameters of a model affect the output or outcomes of the model. It helps quantify the influence of individual parameters on the overall behaviour of the system being modelled. Sensitivity analysis is particularly valuable for assessing the robustness, reliability, and credibility of a model's results, especially when dealing with complex systems where uncertainties exist. The basic reproduction number, often denoted as $\mathbf{R}_0$, is a critical concept in epidemiology and mathematical modeling. In our model, the basic reproduction number can be calculated using the method of next-generation matrix. First, let's find the coal-free equilibrium point by setting all compartments except R to zero: $G_0 = 0,\ P_0 = 0,\ R_0 = R,\ E_0 = 0$.

$$FV = \begin{bmatrix} \in_1 - \in_3 R_0 & 0 & 0 & 0 \\ 0 & \delta_1 - \delta_3 R_0 & 0 & 0 \\ -\in_3 R_0 & -\delta_3 R_0 & \theta_1 + \frac{\theta_2}{a} + \theta_4 & \beta_4 R_0 \\ 0 & 0 & 0 & \beta_1 + \beta_4 R_0 + \beta_5 \end{bmatrix}.$$

In this case, the basic reproduction number $\mathbf{R}_0$ would be

$$\mathbf{R}_0 = \max(\in_1 - \in_3 R_0,\ \delta_1 - \delta_3 R_0\,,\ \theta_1 + \frac{\theta_2}{a} + \theta_4,\ \beta_1 + \beta_4 R_0 + \beta_5). \tag{7}$$

It is essential to conduct a sensitivity study on the model parameters to understand the reliability. It system-

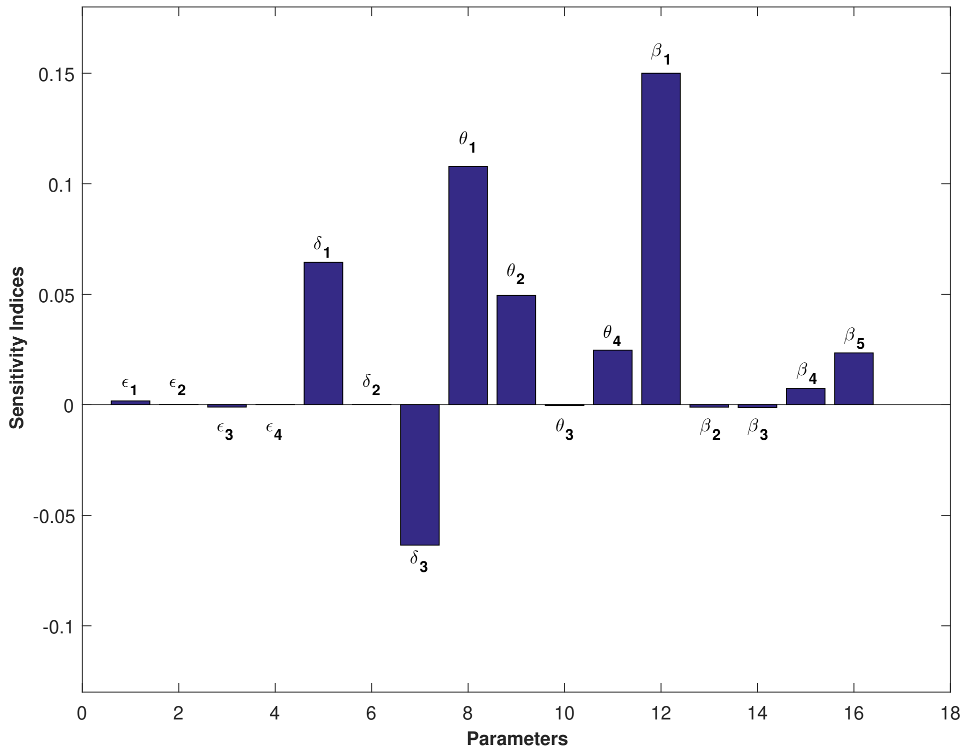

Figure 22. Sensitivity diagram for different parameter values of model (1)

atically involves various key parameters to monitor their impact on the model outputs. Here a sensitivity analysis indicates how sustainable energy can increase the reliability of models. The performance rate of wind turbines and energy storage systems can be adjusted to see how changes in technical performance affect the overall system outputs. This is very important for deterioration and technological progress over time. Running the model in best-case and worst-case scenarios for key parameters provides insight into the

range of possible outcomes.
It can help to identify the conditions under which the model works optimally or fails to meet the energy demand. Using simulations to change multiple parameters at random allows a broader understanding of the robustness of the model. This statistical method can provide probability distributions for different outcomes, highlighting the most likely scenarios.
Extending the model to include hybrid renewable energy systems (e.g., wind) can be tested through sensitivity analysis to determine the best mix for reliability and efficiency. Assessing the sensitivity of models to climate variables (e.g., temperature, precipitation) can help understand how future climate conditions may affect renewable energy production and system resilience [8]. In conclusion, a sensitivity study provides important insights into the reliability and robustness of renewable energy models. By systematically changing key parameters, researchers can identify the most influential factors, measure uncertainties, and optimize system design for better performance and resilience.

7 Conclusion

Model (1) has been built to represent the influence of coal-based electricity and greenhouse gases by using renewable energy to reduce the significant quantity of carbon emissions from the power industry or to keep the environment sustainable. Then, the constraints and positives for the solution of the model have been studied and two equilibrium positions have been found. After that, the stability at each equilibrium point has been assessed by the next-generation matrix approach. Numerical simulations have been also performed for the model to highlight the dynamic behaviour of the species.

Declarations

Use of AI tools

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Data availability statement

All data generated or analyzed during this study are included in this article.

Ethical approval (optional)

The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

Consent for publication

Not applicable

Conflicts of interest

The authors declare that they have no conflict of interest.

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Author's contributions

M.S.I.: Conceptualization, Methodology, Data Curation, Writing - Original Draft, Writing - Review & Editing. M.S.K.: Methodology, Software, Validation, Writing - Review & Editing, Visualization, Supervision. M.H.A.B.: Validation, Formal Analysis, Writing - Original Draft, Writing - Review & Editing, Visualization. All authors discussed the results and contributed to the final manuscript.

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Not applicable

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