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Optimizing logistics warehouse space utilization with minimum total transportation

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Abstract

Logistics warehouses are integral to supply chain management, enabling the efficient storage and movement of goods. However, the dynamic operational nature of these facilities, characterized by high product turnover, often results in suboptimal space utilization. This study addresses the inefficiency caused by partially filled pallets and the honeycombing effect, which leads to substantial storage capacity loss. Focusing on a third-party logistics warehouse managing apparel boxes, the uniqueness of each box introduces specific challenges in space optimization. To mitigate these issues, two integer linear programming models were developed. The first model is utilized for emptying and reallocating products from the predetermined low-capacity shelf cells to new locations. The second model simultaneously identifies both the shelf cells to be vacated and the optimal relocation destinations. Both models aim to minimize the total transportation costs. The first model is suited for rapid reallocation and efficient short-term solutions, whereas the second model offers a more holistic approach to long-term space optimization. These models provide a systematic, data-driven solution for enhancing warehouse space management. The problem is also considered bi-objective, with the objectives of maximizing the number of empty shelf cells and minimizing the total transportation costs. The bi-objective mathematical model was scalarized using the epsilon-constraint method and solved for different epsilon values. This process yielded 39 Pareto-optimal solutions. The results indicate that as more cells are emptied, both the total cost and cost per cell increase. Considering the problem in a bi-objective form has also been advantageous for offering the decision-maker not just one solution but a solution set with different numbers of cells to be emptied and different transportation costs.
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Keywords: Logistics warehouse; Integer programming; Warehouse space utilization.

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1. Introduction

Logistics is the physical flow of goods from the place of supply to the receiving place [1]. Logistics warehouses are structures used to store and process products before they reach customers. Warehouse operations broadly include the receiving of products, storage, order-picking, accumulation, sorting, and shipping operations [2]. Logistics warehouses are essential to the supply chain and help businesses deliver fast and efficient customer service. Today, corporations employ warehouse efficiency as a strategic weapon or a center of competency. A well-built warehouse can meet customer requirements quickly and improve company performance [3].

Warehouse management includes methods like product placement, stock tracking, storage, material flow, and personnel management. A key part of this is location optimization, which helps allocate inventory in the best way. This means placing high-turnover items for easy access, reducing travel times, and cutting transportation costs. Proper location optimization makes sure materials are in the right place, easy to find, and aligned with operational needs. This helps improve overall warehouse efficiency.

Warehouse storage capacity is defined as the amount of storage space needed to accommodate the materials to be stored in order to meet a desired service level that specifies the degree of storage space availability [4]. Storage and addressing methods organize the storage and location of materials. Storage methods include shelves, drawers, and pallets while addressing methods include labeling and coding systems that identify the location of products. These methods help maximize storage space efficiency and allow for quick product location.

Efficient space management is a crucial aspect of logistics warehouses due to the constant movement of goods. Unlike standard factory warehouses, logistics warehouses handle frequent product turnover, resulting in a dynamic flow of incoming and outgoing shipments. The product stored in the warehouse in our case study consists of boxes. The boxes on pallets are picked and shipped according to the customer's order. In a case like the one in our article, where the boxes are unique, half-loaded pallets with missing boxes will appear in the warehouse. However, even if a pallet is half-loaded, it still occupies a full shelf space. This significantly reduces the efficiency of storage space utilization and the amount of free shelf slots. A common issue contributing to this inefficiency is honeycombing, where partially filled storage areas leave unused gaps between items. These empty spaces result in wasted storage capacity, leading to a reduction in overall warehouse efficiency and higher operational costs.

The uniqueness of each box in the warehouse, which is the focus of this paper, presents a distinct challenge that requires a specific solution. A continuous and systematic process of reallocation and merging is necessary. It is crucial that the implementation effectively addresses the lack of free space and minimizes transportation costs. Although warehouse space efficiency is a crucial and widely studied topic, this uniqueness complicates the search for directly related studies.

Our search query focused on papers related to optimizing warehouse efficiency and maximizing space utilization. Using Google Scholar, we searched with keywords such as 'optimizing warehouse efficiency', 'maximizing space utilization', 'storage space management', and 'maximizing free shelf slots'. Studies addressing these topics, particularly those involving 'reallocation,' 'defragmentation,' and 'merging' implementations, have been especially relevant to our work.

The article by Chen, G. et al. [5] explores the challenges faced in radio-shuttle compact storage systems, particularly in managing the honeycombing effect, where partially filled channels limit storage space utilization. To address this, the authors propose a shared storage policy and develop a Mixed-Integer Linear Programming (MILP) model to minimize the relocation of items during retrieval. The study compares different system layouts, revealing that while the radio-shuttle system increases the number of SKUs stored, it remains a compromise between accessibility and efficiency. By optimizing channel configurations and applying the MILP model, the authors demonstrate significant improvements in storage space utilization and retrieval efficiency.

Perera et al. [6] present a simple and effective linear programming model to use warehouse storage space by efficient palletizing. Their study proposed a general and flexible model and stated that linear programming maximizes logistics warehouse storage space. In their paper, Kang et al. [7] build an optimization model using integer programming to solve space allocation problems in a breakbulk terminal. To simultaneously decide on cargo allocation and reallocation to have both minimum transportation cost and maximum available space, creating a mathematical optimization model that fully uses rows or otherwise leaves them empty and assigns the units with the same departure date to the same rows, they managed to create space for newly arriving cargo.

Georgise et al. [8] provided a solution to the problem of cargo volume exceeding the available warehouse volume. Alternative layouts were compared using the analytical hierarchical procedure, and the Double-deep shelf layout design, which creates the maximum free space, was applied to increase warehouse space utilization. Derhami et al. [9] addressed inefficiencies in space utilization within block-stacking warehouses. The existing model in the warehouse was found to underestimate accessibility waste, leading to suboptimal layouts. To overcome this, the researchers introduced a new analytical model incorporating a waste function that more accurately estimates storage volume waste. The model allowed flexibility in bay depths and stock-keeping units (SKUs) allocations, utilizing a mixed integer programming approach to optimize these parameters. The study demonstrated the effectiveness of the proposed model in solving large-scale problems, showing improved space utilization.

Kim et al. [10] propose a two-phase redesign model reducing warehouse space waste in the long term and a reshuffling method in the short term. Their study aims to optimize storage efficiency under inventory uncertainty - item sizes and quantities - by balancing block stacking and racking. The research evaluates proposed methods through theoretical directions and a real-world warehouse case. The findings indicate that reshuffling is an effective short-term solution for space issues, while redesign offers long-term space savings by minimizing waste.

Derhami et al. [11] introduce a simulation-based optimization algorithm to find the best layout considering both space utilization and material handling costs. Additionally, the paper presents a closed-form solution for determining the optimal number of aisles. A case study in the beverage industry illustrates that the optimized layout can lead to up to ten percent savings in operational costs for a warehouse. The paper highlights the algorithm's computational efficiency and provides insights through extensive experimental analysis covering warehouses of various sizes.

Zhou et al. [12] explore the impact of routing strategies on warehouse picking efficiency, introducing three heuristic methods (S-shape, return, and composite) to reduce travel distance in leaf-layout warehouses. It demonstrates that the composite strategy significantly outperforms the others, especially as the number of storage locations increases. Wang et al. [13] present a study on the performance of a robotic mobile fulfillment system (RMFS), developing a model to predict robot travel time for various layouts and comparing it with actual data. It finds that optimizing the number of horizontal aisles improves picking efficiency and space utilization, and storing high-demand (A-class) goods centrally in the storage area minimizes expected travel time, enhancing overall system performance.

The study by Ahmed et al. [14] addresses inefficiencies in the kitting process within Industry 4.0 smart factories, utilizing Process Failure Modes Effects Analysis (PFMEA) to identify and optimize the root causes of excess travel, retrieval time, and inventory issues. By reorganizing the kitting process based on part commonality, standardization, and co-occurrence frequency, the study has achieved significant reductions in kitting times. Yerlikaya and Arıkan [15] addressed the storage location assignment problem in their study and proposed a mixed integer nonlinear programming model to solve the problem. The proposed model focuses on reducing transportation distances and maximizing storage efficiency by strategically allocating storage locations to products. Zhang et al. [16] proposed a multi-objective mathematical model based on the weight, usage frequency and category of products in their study, which aimed to achieve better storage stability and improve storage and retrieval efficiency. Within the scope of the study, the operational efficiency of the stacker crane and the turnover rate of products were improved. In addition, the overall stability of the shelves was ensured and the management efficiency of the warehouse was improved.

In the literature on optimizing warehouse space, it is generally assumed that pallets are not divided, and in these studies, it is determined where they should usually be placed. In this study, for the first time, the problem of deciding which half-loaded pallets should be combined to create free space in a logistics warehouse is addressed, and two new mathematical models are proposed for the problem.

The considered problem and proposed mathematical models are detailed in the subsequent section of the paper. Section 3 outlines a sample problem, while Section 4 delves into the experimental outcomes. The final section offers conclusions and suggestions.

2. Considered problem and proposed mathematical models

This section consists of two parts. The first part is the definition of the problem. Initially, the operating system of the logistics warehouse, where the study is carried out, is explained, and then the problem is described. The second part presents the mathematical models created to solve the problem. The reason for making more than one model for the solution is explained in the related section.

2.1. The considered problem

The site of the study is a third-party logistics (3PL) company. The company provides warehouse services for the products of an apparel brand. The logistics company stores boxes containing apparel. Almost all the boxes are formed by the brand's stores and sent to the warehouse. Each box is assigned a unique barcode, and the apparel contained within the box is associated with this barcode during placement. The apparel in the boxes is not organized by type but is mixed. The boxes stored in the warehouse are identical in size, with dimensions of 40 cm in width, 60 cm in length, and 40 cm in height (40×60×40 cm). Similarly, the pallets used for transportation and storage are uniform, with dimensions of 80 cm in width and 120 cm in length (80×120 cm). Pallets of sixteen identically sized boxes are delivered to the warehouse by the stores. The pallets are given a unique barcode at the warehouse, and the boxes on the pallet are registered to this barcode. This ensures that the current address of the boxes is known. The pallet is ready to be placed on the shelf from this stage. The sections of the shelves that one pallet fits in are called cells.

The pallets are placed on the first available cell in the warehouse by forklift truck, with addressing. Since the apparel in the boxes may differ, and each box is unique, a layout plan in which similar boxes are classified and placed together cannot be applied in the warehouse layout. During the shipment stage, the barcodes of the boxes are transmitted to the warehouse and requested to be shipped. Since the boxes are unique, the addresses requested to be shipped differ. The boxes to be shipped are collected from their addresses in the warehouse and transferred to the shipment area. As a result, the number of boxes in the shelf cells decreases. Since the new boxes arriving at the warehouse are already palletized and ready to be placed on the shelf, the shelf cell with fewer boxes is in a status where a new placement process cannot be applied. Therefore, empty spaces in the shelf cells cannot be utilized. A completely empty cell is required to place the new pallet in it. The logistics company tries to free up space in the warehouse by moving the boxes in the shelf cells with a few boxes left to the nearest available cells. However, the current solution has deficiencies. The purely verbal nature of the process means that the data is not recorded and analyzed.

Furthermore, the solution has no specific boundaries or rules. Hence, the process is unsystematic. The results are questionable in terms of consistency and effectiveness.

2.2. Mathematical models

Two different mathematical models were developed to solve the problem. The reason for this is to develop solution alternatives that can adapt to the variable input-output flow of the warehouse. Both models have a straightforward and clear objective: To provide the maximum number of empty shelf cells with the minimum transportation cost. However, the models are distinct from each other by the differences in their parameters and constraints.

The first model is an integer linear programming model in which the cells with a low capacity utilization ratio are predetermined and notified to the model as a parameter, i.e., the model knows which cells will be emptied and determines the assignment locations by considering the transportation cost. The second model is more complex. When the number of cells to be emptied (α) is given, it determines both the assignment locations and which cells are to be emptied.

Model 1:

index :
i,j: cells $i, j \in \{1,2 \dots, n\}$
note: *i* indicates the cell to be emptied, and *j* indicates the target cell.

parameters:
q_i : number of boxes in cell *i*
k_j : available space in cell *j*
b_i : 1 if the cell is to be emptied, 0 otherwise
c_{ij} : transportation cost from cell *i* to cell *j*

decision variables:
x_{ij} : number of boxes transported from cell *i* to cell *j*

constraints:

$$\sum_{i \mid i \neq j,\ b_i=1}^{m} x_{ij} \leq k_j \qquad \forall j \mid b_j = 0 \tag{1}$$

$$\sum_{j \mid i \neq j,\ b_j=o}^{m} x_{ij} = q_i \qquad \forall i \mid b_i = 1 \tag{2}$$

$$x_{ij} \geq 0 \ , \qquad integer \qquad \forall i, j \tag{3}$$

objective function:

$$min\, z_{M1} = \sum_{i \mid b_i=1}^{m} \ \sum_{j \mid i \neq j,\ b_j=0}^{m} c_{ij} x_{ij} \tag{4}$$

Constraint (1) guarantees that the available space of the target cell is not exceeded. Constraint (2) enforces that all boxes in the cell to be emptied must be transported and relocated without leaving any boxes behind. Constraint (3) is an integer constraint. Objective (4) is to minimize the total transportation cost.

Model 2:

index :
i, j: cells $i, j \in \{1,2 \dots, n\}$
note: *i* indicates the cell to be emptied, and *j* indicates the target cell.

parameters:
q_i : number of boxes in cell *i*
k_j : available space in cell *j*
α: number of cells to be emptied
c_{ij} : transportation cost from cell *i* to cell *j*

decision variables:
x_{ij} : number of boxes transported from cell *i* to cell *j*
y_i : 1 if the cell is to be emptied, 0 otherwise

constraints:

$$\sum_{i \mid i \neq j}^{m} x_{ij} \leq k_j \qquad \forall j \tag{5}$$

$$\sum_{j \mid i \neq j}^{m} x_{ij} \geq q_i - M(1 - y_i) \qquad \forall i \tag{6}$$

$$\sum_{j \mid i \neq j}^{m} x_{ij} \leq q_i + M(1 - y_i) \qquad \forall i \tag{7}$$

$$\sum_{i}^{m} y_i = \alpha \tag{8}$$

$$x_{ij} \leq M\left(1 - y_j\right) \qquad \forall i, j \tag{9}$$

$$x_{ij} \geq 0 \, , integer \qquad \forall i, j \tag{10}$$

$$y_i \in \{0,1\} \qquad \forall i \tag{11}$$

objective function:

$$min \, z_{M2} = \sum_{i}^{m} \sum_{j \mid i \neq j}^{m} c_{ij} x_{ij} \tag{12}$$

Constraint (5) guarantees that the available space of the target cell is not exceeded. Constraints (6) and (7) enforce that all boxes in the cell to be emptied must be transported and relocated without leaving any boxes behind. Constraint (8) sets the number of cells to be emptied equally to 'α'. Constraint (9) prevents transportation to the cell to be emptied. Constraint (10) is an integer constraint. Constraint (11) is a binary variable constraint. Objective (12) is to minimize the total transportation cost.

3. Experimental results

In this section, firstly, a sample problem is generated to simplify the understanding of the considered problem and proposed mathematical models. Then, the real-life problem is solved with the proposed mathematical models and the obtained results are discussed. The bi-objective version of the problem is also presented and the contributions of considering the problem as bi-objective are discussed. Finally, it has been investigated the proposed mathematical models can provide feasible solutions to which size of the problems.

The proposed mathematical models were coded in the GAMS 45.7.0 and solved by using Cplex solver of GAMS. All tests were performed on a computer with an Intel Core i9 CPU 3.60GHz processor and 32GB of memory.

3.1. Sample problem

Creating a sample problem, i.e., a small-size problem, makes the results easier to understand by simplifying complex concepts, describing the parameters, and explaining the outcomes. The parameter values are generated randomly in Excel.

As previously described, the task of Model 1 is relatively simple compared to Model 2. Cells with low capacity utilization, i.e., cells that will take a short time to empty, are predetermined, and Model 1 is informed that these cells will be emptied. Thus, all that Model 1 has to do is choose the assignment locations of the boxes by considering the cost of transportation between cells.

Table 1 displays the cells' row and floor information and the number of boxes on the pallet in this cell. The number of boxes in the cells was randomly generated between 0 and 16. A cell can hold a maximum of 16 boxes, with the pallet carrying these boxes. The available spaces in the cells are calculated based on the number of boxes in that cell. In this sample, cells with five or fewer boxes were determined as 'cells to be emptied'.

Table 1. q_i, k_i, b_i parameters of the sample problem

Rows (R)	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4
Floor (F)	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Cell number (i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Box count (q_i)	6	3	1	9	0	14	4	4	14	2	16	5	15	8	10	1
Available space (k_i)	10	13	15	7	16	2	12	12	2	14	0	11	1	8	6	15
Will be emptied (b_i)	0	1	1	0	1	0	1	1	0	1	0	0	0	0	0	1

Transportation costs (c_{ij}) of the sample problem have been given in Table 2.

Table 2. c_{ij} parameter of the sample problem

i/j	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
1	1	2	3	4	2	3	4	5	3	4	5	6	4	5	6	7
2	2	4	6	8	3	5	7	9	4	6	8	10	5	7	9	11
3	3	6	9	12	4	7	10	13	5	8	11	14	6	9	12	15
4	4	8	12	16	5	9	13	17	6	10	14	18	7	11	15	19
5	2	3	4	5	1	2	3	4	2	3	4	5	3	4	5	6
6	3	5	7	9	2	4	6	8	3	5	7	9	4	6	8	10
7	4	7	10	13	3	6	9	12	4	7	10	13	5	8	11	14
8	5	9	13	17	4	8	12	16	5	9	13	17	6	10	14	18
9	3	4	5	6	2	3	4	5	1	2	3	4	2	3	4	5

10	4	6	8	10	3	5	7	9	2	4	6	8	3	5	7	9
11	5	8	11	14	4	7	10	13	3	6	9	12	4	7	10	13
12	6	10	14	18	5	9	13	17	4	8	12	16	5	9	13	17
13	4	5	6	7	3	4	5	6	2	3	4	5	1	2	3	4
14	5	7	9	11	4	6	8	10	3	5	7	9	2	4	6	8
15	6	9	12	15	5	8	11	14	4	7	10	13	3	6	9	12
16	7	11	15	19	6	10	14	18	5	9	13	17	4	8	12	16

The sample problem was solved by Model 1, and the optimum solution was obtained within one second. The results are given in Fig. 1. In Fig. 1, the before and after status of the shelves are visualized. In the pre-transportation state, it is seen that six cells, which are light green, had less than five boxes. These cells are pre-defined as to be emptied. The boxes in these cells were moved to the new cells to which they were assigned with minimum transportation costs. In the post-transportation state, the number of empty cells, which became available for new pallet placement, increased from 1 to 7. Occupied cells decreased from 15 to 9 while capacity utilization rates increased.

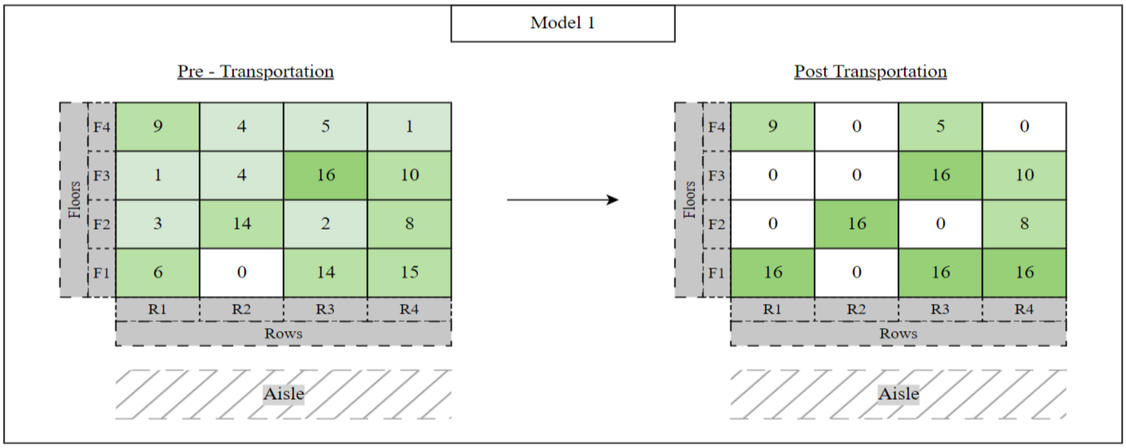

Fig. 1. Status of the shelves

Table 3 shows the cells the boxes were taken from, the cells they were moved to, and the quantity of boxes transported. In the lower right corner of the 'cell' text, row and floor information is provided. This table not only highlights the details of box movements but also provides a basis for understanding the operational stages involved in implementing the proposed models, which are described below.

For the implementation of the proposed models, the operational stages are as follows: First, the pallet in the cell designated to be emptied is lowered to the corridor floor using a forklift. Next, the pallet in the target cell, where the boxes will be relocated, is similarly lowered to the floor. The worker then manually transfers the boxes to their new pallet. Finally, the forklift places the pallet, now containing the relocated boxes, back into its cell. These steps ensure the efficient execution of the relocation process and contribute to the systematic management of warehouse space.

Table 3. Transportation of the boxes between cells of Model 1

from	*quantity*	*to*	*from*	*quantity*	*to*
$Cell_{12}$	3	$Cell_{11}$	$Cell_{23}$	2	$Cell_{22}$
$Cell_{13}$	1	$Cell_{11}$	$Cell_{32}$	2	$Cell_{31}$
$Cell_{23}$	2	$Cell_{11}$	$Cell_{44}$	1	$Cell_{41}$

Cell_{24}	4	Cell_{11}

In the analysis of Model 2, the same sample problem has been used. The only difference is that there are no parameters for emptying the cells. The reason is that Model 2, unlike Model 1, is a model that determines which cells to be emptied. As can be comprehended when the mathematical models are examined, α parameter is specific to Model 2. This is α parameter that guides the model to decide on the cells to be emptied. Table 4 includes the number of shelf cells, the total number of boxes within those cells, and the number of cells, denoted as 'α', that the model selects for emptying in the sample problem. As shown, the model sets 'α' to 9. Consequently, after the process, 9 cells will be emptied, leaving 7 cells occupied.

Table 4. Data set of the sample problem

Parameter	*Value*
Number of cells	16
Number of boxes	112
α	9

The sample problem was solved by Model 2, and the optimum solution was found within one second. The obtained results are given in Fig.2. In Fig.2, the before and after status of the shelves are visualized. As can be seen, the model meets its objectives. All boxes fit into the minimum number of cells. The number of occupied cells decreased from 15 to 7. The number of empty cells increased from 1 to 9.

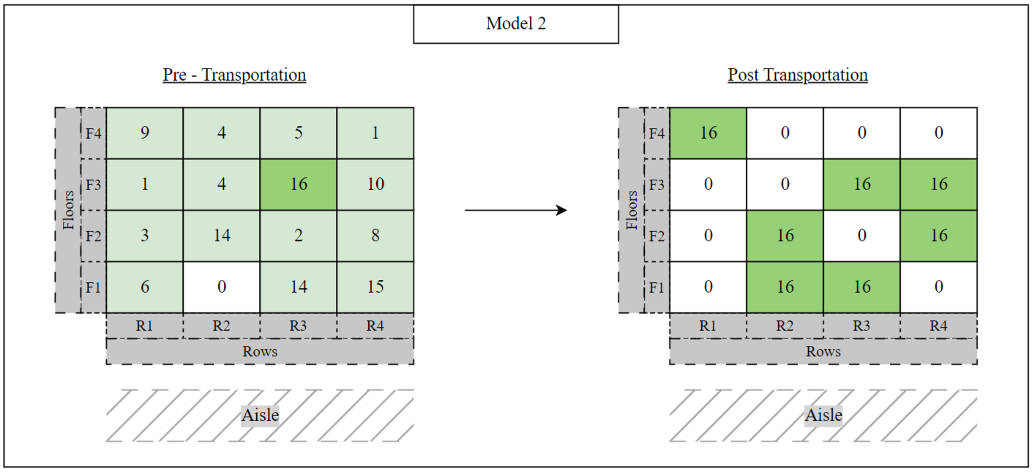

Fig.2. Status of the shelves

Table 5 shows the cells the boxes were taken from, the cells they were moved to, and the quantity of boxes transported. In the lower right corner of the 'cell' text, row and floor information is provided.

Table 5. Transportation of the boxes between cells of Model 2

from	*quantity*	*to*	*from*	*quantity*	*to*
Cell_{11}	6	Cell_{14}	Cell_{32}	1	Cell_{22}

Cell_{12}	1	Cell_{14}	Cell_{34}	3	Cell_{21}
Cell_{12}	2	Cell_{21}	Cell_{34}	2	Cell_{31}
Cell_{13}	1	Cell_{21}	Cell_{41}	1	Cell_{22}
Cell_{23}	4	Cell_{21}	Cell_{41}	8	Cell_{42}
Cell_{24}	4	Cell_{21}	Cell_{41}	6	Cell_{43}
Cell_{32}	1	Cell_{21}	Cell_{44}	1	Cell_{21}

3.2. Real-life problem

The real-life problem is solved with the proposed mathematical models M1 and M2. As in the sample problem, cells with five or fewer boxes are determined as 'cells to be emptied' for Model 1. The optimum solutions were found within one second for both of them. The obtained results are analyzed and compared. Their effectiveness and advantages are identified and discussed. The problem was generated from a block shelf situated within the warehouse. This block shelf spans approximately 70 meters long and stands 7 meters high, constituting a 2-dimensional space. Both models utilized the same problem, enabling a fair and accurate comparison of their results. Table 6 contains overall information about the real-life problem and the results. The left part of the table includes parameters indicating the current status of the shelves. The right part contains the solution results of the models.

Table 6. Results of the real-life problem

			Model 1	*Model 2*
Cells	96	*Emptied Cells*	20	42
Boxes	810	*Transported Boxes*	60	240
Occupied Cells	89	*Occupied Cells*	69	51
Empty Cells	7	*Empty Cells*	27	45
		Transportation Cost	196	902

The obtained results of the real-life problem are visualized in Fig. 3 and Fig. 4. Fig. 3 compares the number of occupied and empty cells before and after implementing Model 1 and Model 2. Initially, the shelf harbors 96 cells. Of these, 89 are occupied, leaving seven empty. Following the implementation of Model 1, the number of empty cells increased to 27. After applying Model 2, the number of empty cells increased to 45.

In Fig. 4, the transportation costs obtained with Model 1 and Model 2 have been compared. Model 1 has a low transportation cost of 196 since it only empties the cells with a few boxes. Model 2 has a considerably higher cost of 902 compared to Model 1. This significant increase in transportation cost is due to the problem the models focus on solving, as described in the related section. Briefly, Model 2 is designed to optimize box placement and aims to merge boxes into as few cells as feasible. Hence, the number of boxes it transports is significantly higher than Model 1.

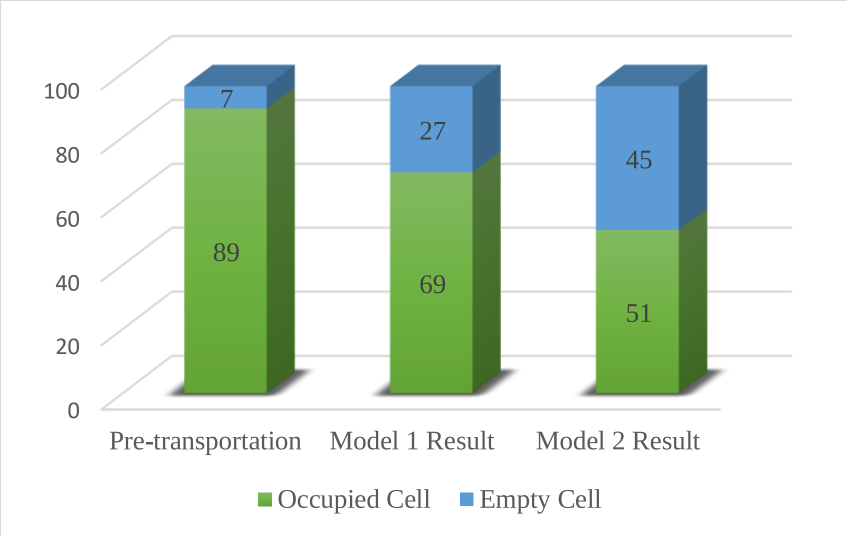

Fig. 3. Status of cells before and after transportation

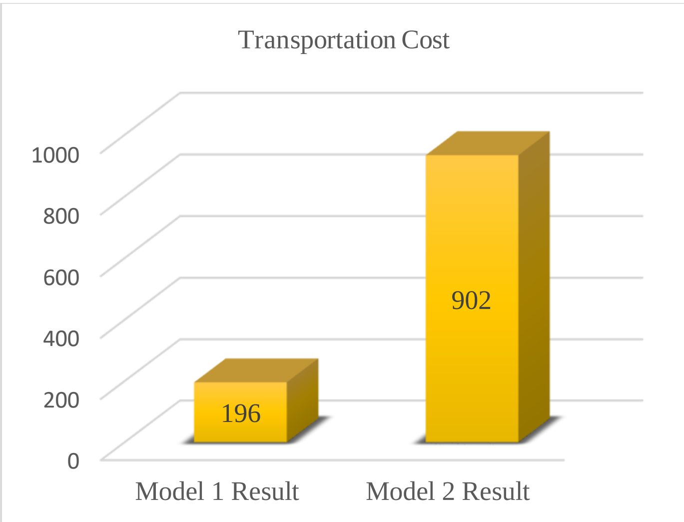

Fig. 4. Transportation costs obtained with Model 1 and Model 2

Considering the sufficient increase in the number of empty cells in the short term and the considerably lower transportation cost compared to Model 2, it can be said that the Model 1 solution method provides an effective solution in a short period. Model 2 offers the most effective solution to the problem of the lack of empty cells in the warehouse. However, considering the required amount of transportation, the solution of this model is unused if there is a limited time for transportation. Since the solution of Model 1 includes a lower number of empty cells, it is better suited for periods of high flow in the warehouse when time is limited (e.g., seasonal transitions) for transportation and empty cells are required. Model 2 is a method to be preferred when time is not limited for transportation, and the whole warehouse is to be organized. It is very suitable for preparation for periods when the input-output flow will increase in the warehouse.

3.3. Multi-objective version of the real-life problem

Both models proposed in the previous section are single-objective and minimize the total transportation cost. The first model is for the emptying of the determined cells, and the second model is for the emptying of α cells determined by the model. The problem can also be considered in a multi-objective structure where the transportation costs are minimized and the number of cells to be evacuated is maximized, simultaneously. The objective functions of the multi-objective problem can be defined as $f_1 = \sum_i^m \sum_{j,\ i\neq j}^m c_{ij}x_{ij}$ while $f_2 = \sum_i^m y_i$. The constraints are the (5)-(7), (9)-(11) numbered constraints. This problem is solved with the epsilon constraint method, which is one of the most widely used solution approaches in the literature for solving multi-objective problems. In the epsilon constraint method, one of the objective functions is selected as the objective function of the scalarized model, and the others are converted to epsilon (ε) constraints. For this problem, f_1 is determined as the objective function. f_2 is converted to epsilon constraint as given in equation (13).

$$\sum_i^m y_i \geq \varepsilon \tag{13}$$

ε can take values in the range $[7\ , \alpha]$. α is calculated with the formula given in the equation (14).

$$\alpha = m - \sum_i^m \left\lceil \frac{q_i}{16} \right\rceil \tag{14}$$

The real-life problem is solved for all ε values in this range, and the obtained solutions are given in Table 7.

Table 7. Multi-Objective Results

ε	f_1	f_2	ε	f_1	f_2	ε	f_1	f_2
7	0	7	20	92	20	33	307	33
8	2	8	21	102	21	34	336	34
9	6	9	22	112	22	35	366	35
10	10	10	23	124	23	36	401	36
11	16	11	24	136	24	37	437	37
12	22	12	25	149	25	38	474	38
13	30	13	26	162	26	39	518	39
14	38	14	27	178	27	40	565	40
15	46	15	28	195	28	41	620	41
16	55	16	29	214	29	42	675	42
17	64	17	30	234	30	43	743	43
18	73	18	31	256	31	44	812	44
19	82	19	32	280	32	45	902	45

As can be seen from Table 7, the real-life problem was solved for 39 epsilon values by changing the epsilon between 7 and 45. When the obtained results are examined, it is seen that seven cells are already empty in the current situation, transportation cost is zero for the first epsilon (=7). As expected, it is observed that transportation costs increase as the number of cells to be emptied increases. When the problem is solved as bi-objective, options with different numbers of cells to be emptied and transportation costs can be presented to the decision maker. Thus, the decision maker has the chance to choose the most suitable one from these options.

The obtained Pareto optimum points are also presented in the graph given in Figure 5. The total transportation cost (f_1) is on the horizontal axis of the graph, and the number of cells to be emptied (f_2) is on the vertical axis.

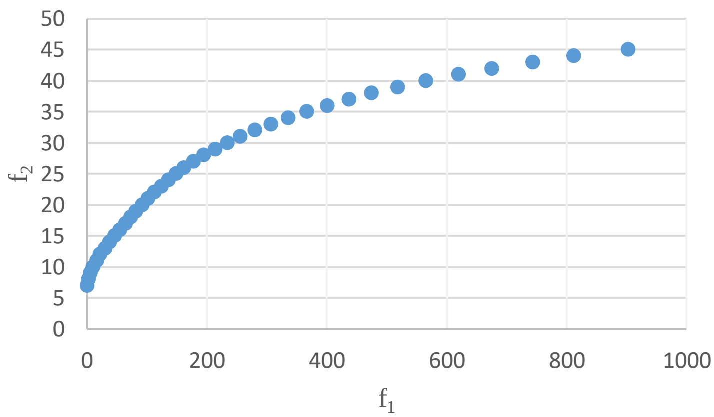

Fig. 5. Pareto Optimum Points

As can be seen from Figure 5, the cost of emptying a cell is low when the number of cells to be emptied is small. As the number of cells to be emptied increases, not only the total cost but also the cost of emptying a cell has increased.

3.4. Large-sized problems

Additional tests were conducted to demonstrate the performance of the proposed mathematical models for problems larger than real-life problem. For Model 1, 9 very large-sized test problems were generated in the range of 500 cells-3730 boxes and 4500 cells-33730 boxes. All problems were solved with Model 1 and the obtained solutions are given in Table 8.

Table 8. Results of Large-Sized Problems obtained with Model 1

n	$\sum_i p_i$	z_{M1}	t_{M1}
500	3730	7820	0.11
1000	7476	15664	0.45
1500	11238	23596	0.94
2000	15000	31600	2.22
2500	18730	39420	3.38
3000	22476	47264	5.53
3500	26238	55196	7.14
4000	30000	63200	8.08
4500	33730	-	-

Table 6 consists of four columns. The first column shows the number of cells (n), the second column shows the number of boxes $(\sum_i p_i)$, the third column shows the objective function value (z_{M1}), and the last column shows the solution times (t_{M1}) in seconds. As can be seen from the table, only the 4500 cells-33730 boxes problem could not be solved due to GAMS giving an out of memory error. The optimum solutions of all other problems were reached in less than 9 seconds. This problem size is large enough to represent large-sized problems that can be encountered in real life. Similarly, 6 large-sized test problems were generated for Model 2 in the range of 100 cells-730 boxes and 600 cells-4475 boxes. All problems were solved with Model 2 and the obtained results are given in Table 9.

Table 9. Results of Large-Sized Problems obtained with Model 2

n	$\sum_i p_i$	z_{M2}	t_{M2}
100	730	3548	2.62
200	1476	7216	31.8
300	2238	11344	156.8
400	3000	15048	604.3
500	3730	18596	1391.34
600	4476	-	-

Table 9 consists of four columns. The first column shows the number of cells (n), the second column shows the number of boxes $(\sum_i p_i)$, the third column shows the objective function value (z_{M2}), and the last column shows the solution times (t_{M2}) in seconds. As can be seen from the table, only the 600 cells-4475 boxes problem could not be solved due to GAMS giving an out of memory error. The optimum solutions of all other problems were reached in less than 1392 seconds.

5. Conclusion

Efficient warehouse management is crucial for increasing productivity and reducing costs, encouraging businesses to adopt effective strategies and solutions to optimize warehouse operations. Unlike factory warehouses, logistics warehouses handle a continuous flow of products regularly. Given that products are only placed in existing warehouse spaces, there is a need to consolidate used pallets to free up space periodically. The working system in the logistics warehouse, the case of our study, creates the problem of lack of free space to store new incoming cargo. This study introduces two different mathematical models to overcome the problem. In the first one, we decide beforehand which locations to free up, while the second model figures out these positions on its own. The first model stands out for its rapid transportation time, while the second model delivers high-quality solutions. If the priority is a quick and effective solution with a notable increase in empty cells and lower transportation costs in the short term, Model 1 is the advisable option. Otherwise, Model 2 is the most effective choice for addressing the shortage of empty cells in the warehouse with maximum free space. Model 1 is best suited for warehouses experiencing high activity levels and stringent schedules, such as those undergoing seasonal transitions, where rapid solutions are essential. Conversely, Model 2 is more appropriate for periods without strict time constraints, permitting a thorough organization of the entire warehouse. This model is particularly advantageous when preparing for anticipated increases in input-output flow. Additional tests have been conducted to see the success of the proposed mathematical models in solving large-sized problems. The results of the large-sized problems tests have shown that Model 1 can find optimum solutions for very large problems up to 4000 cells and Model 2 problems up to 500 cells in reasonable times. The real-life problem has also been considered in a bi-objective form where the number of cells to be evacuated is maximized and the transportation costs are minimized, simultaneously. The bi-objective problem was scalarized using the epsilon constraint method, solved with 39 different epsilon values ranging from 7 to 45, and 39 Pareto optimum points were obtained. Obtained results show that when the number of cells to be emptied increases, not only the total cost but also the cost of emptying a cell has increased. Addressing the problem in a bi-objective form has also been beneficial in terms of presenting different options to the decision-maker.

The models in this study apply to warehouses with multi-level shelving and uniformly sized boxes, regardless of their contents. Whether storing clothing, electronics, or other boxed goods, the models optimize space by addressing inefficiencies like honeycombing and partial loads. This ensures broad applicability across various industries seeking to enhance warehouse efficiency. Future research could focus on developing mathematical models tailored to warehouses with different storage characteristics. Additionally, matheuristic or metaheuristic solution approaches can be developed for very large-scale problems that cannot be solved by mathematical models.

Author Contributions

B.Ö and T.S. actively contributed to all stages of the research, including conducting the experimental studies and writing the manuscript.

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