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SOLUTION OF A MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM HAVING ROUGH INTERVAL COEFFICIENTS USING ZERO-SUM GAME

KABA ARALIKLI KATSAYILARA SAHIP ÇOK AMAÇLI DOĞRUSAL PROGRAMLAMA PROBLEMİNİN SIFIR TOPLAMLI OYUN İLE ÇÖZÜMÜ

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Abstract

In this paper, a set of compromise solutions is found for the multi-objective linear programming with rough interval coefficients (MOLPRIC) problem by proposing a two-phased algorithm. In the first phase, the MOLPRIC problem is separated into single-objective LPRIC problems considering the number of objective functions, and the rough optimal solution of each LPRIC problem is found. In the second phase, a zero-sum game is applied to find the rough optimal solution. Generally, the weighted sum method is used for determining the trade-off weights between the objective functions. However, it is quite inapplicable when the number of objective functions increases. Thus, the proposed algorithm has an advantage such that it provides an easy implementation for the MOLPRIC problems having more than two objective functions. With this motivation, applying a zero-sum game among the distinct objective values yields different compromise solutions.

Keywords: Compromise solution, game theory, multi-objective linear programming problem, rough interval coefficient.

Öz

Kaba sayılardan oluşan aralıklara sahip katsayılar içeren, çok amaçlı doğrusal programlama (MOLPRIC) problemi için bir çözüm önerisinde bulunulmuştur. Bu çalışmada ele alınan probleme uzlaşmacı çözümler kümesi önerilmiş olup çözüm algoritması iki aşamalı olarak düzenlenmiştir. İlk aşamada, MOLPRIC probleminin barındırdığı amaç fonksiyonlarının sayısı dikkate alınarak her bir tek amaçlı LPRIC kaba optimal çözümü bulunmuştur. İkinci aşamada ise MOLPRIC probleminin kaba optimal çözümünü bulmak üzere sıfır toplamlı oyundan yararlanılmıştır. Çok amaçlı problemlerin çözüm sürecinde amaç fonksiyonları arasındaki ödünleşim ağırlıklarının belirlenmesinde genellikle ağırlıklı toplam yöntemi kullanılmaktadır. Ancak amaç fonksiyonlarının sayısı arttığında bu geleneksel yöntem uygulamada zorluk çıkarabilmektedir. Dolayısıyla önerilen algoritmanın özgünlüğü, ikiden fazla amaç fonksiyonuna sahip MOLPRIC problemlerine kolay uygulanabilir olmasıdır. Bu motivasyonla, farklı amaç değerleri arasında sıfır toplamlı oyunun uygulanması, farklı uzlaşık çözümlerin bulunmasını sağlamaktadır.

Anahtar Kelimeler: Çok amaçlı doğrusal programlama problemi, kaba sayılı aralık, oyun teorisi, uzlaşık çözüm.

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1. INTRODUCTION

Linear Programming (LP) is a type of mathematical programming problem generally used in decision-making and optimization. It deals with finding unique or alternative optimal solutions under linear constraints and aims to maximize or minimize a linear objective function. LP has a wide application area such as economy, industry, marketing, military, transportation, and technology. When the parameters of an LP problem are crisp numbers, classical methods are used to find the solution. However, the data collected from real life may contain some errors or have uncertainty arising from lack of information, these parameters and the variables cannot be expressed precisely. Therefore, it is more appropriate to represent these variables and parameters by randomness, fuzziness, or roughness.

Pawlak (1982) presented an approach as an alternative to the fuzzy theory and tolerance theory and coined a term *rough set*. Rough set theory expresses the vagueness by defining a boundary region of a set, and accordingly, a vague concept is represented by a pair of precise concepts known as lower and upper approximations. The lower approximation includes all elements that are surely in the vague concept whereas the upper approximation contains all elements that possibly belong to the concept. The rough concept has a great importance in areas such as medicine (Fibak & Pawlak, 1986; Pawlak et al., 1986), data mining (Munakata, 1997), uncertainty reasoning (Düntsch & Gediga, 1998), civil engineering (Arciszewski & Ziarko, 1999), pattern recognition (Mitatha et al., 2003), rule extraction (Apolloni et al., 2006), data envelopment analysis (Xu et al., 2009), knowledge reduction (Li et al., 2013), transportation (Akilbasha et al., 2017; Das et al., 2016; Roy et al., 2018), decision analysis (El-Feky & Abou-El-Enien, 2019; Singh & Huang, 2020), feature selection (Zhao et al., 2020), and granular computing (Velázquez-Rodríguez et al., 2020). Rough concept is applied to the linear fractional programming (Omran et al., 2016; Khalifa, 2018b) and quadraic programming (Saad et al., 2014) problems, game theory (Ammar & Brikaa, 2019; Brikaa et al., 2021), and multi-criteria decision-making (Greco et al., 2001; Tanackov et al., 2022).

In some real-life problems, the complexity of the social and economic environment and trade-offs between these environments require the consideration of multiple objective functions, which is known as a multi-objective programming problem. Since there is more than one objective function in a multi-objective problem, an optimal solution is found for each of these objectives. However, the aim is to find a single optimal solution for a multi-objective problem. In other words, the found solution may not satisfy all the objectives to the same satisfaction degree. Thus, the concept of optimal solution is replaced by the concept of non-dominant solution or non-inferior solution in multi-objective programming problems, and it would be useful to generate a set of compromise solutions to offer them to the decision maker (DM).

The rough concept is also applied to multi-objective programming, and it is called rough multi-objective programming problem (Youness, 2006). Tao and Xu (2012) presented a general model for a rough multiple-objective programming problem and established an application for a solid transportation problem. In the study, compromise solutions were found by using the interactive fuzzy satisfying method and proposed the rough simulation-based genetic algorithm for solving the rough multiple objective solid transportation problem. Atteya (2016) focused on characterizing and solving the rough multiple objective programming problems and contributed to the data mining process confined only to the "post-processing stage". They investigated a multiple-objective programming problem that had a rough decision set, and all the objectives were crisp functions. Hamzehee et al. (2016) presented a set of multi-objective programming problems in a rough environment. They studied that all the quadratic objective functions were crisp, and the feasible region was a rough set. To find solutions to rough multi-objective programming problems, they used a scalarization method. Khalifa (2018a) considered a multi-objective nonlinear programming problem having rough intervals in the constraints. The problem was converted into two classical multi-objective nonlinear programming problems which are the lower and the upper approximation problems, and these were solved by using the weighting method. Garg and Rizk-Allah (2021) studied the solution of the multi-objective transportation problems taking the transportation cost and demand of the product as rough interval coefficients. Their proposed approach exploited the merits of the weighted sum method to find the non-inferior solutions.

This paper aims to find compromise solutions for the multi-objective linear programming with rough interval coefficients (MOLPRIC) problem. In the study, to deal with partially unknown or ill-defined parameters and variables, rough intervals proposed by Robolledo (2006), which are a particular case of rough sets since the rough sets could not represent continuous values, are utilized. To find the compromise solutions for the MOLPRIC problem, a two-phased algorithm is proposed. In the first phase, the MOLPRIC problem is divided into single-objective LPRIC problems according to the number of objective functions. The rough optimal solution of each LPRIC problem, if it exists, is found by applying the method proposed in (Hamzehee et al., 2014). In the second phase, the approach proposed in (Temelcan, 2023) is used to find the rough optimal solution by applying a zero-sum game. The proposed algorithm can contribute to the literature by providing ease of implementation when there are more than two objective functions. Accordingly, while it is quite applicable to use the weighted-sum or scalarization method in two-objective MOLP problems, finding the appropriate combination of the weights between more than two objective functions will be difficult. With this motivation, applying a zero-sum game among the distinct objective values yields finding compromise solutions. Therefore, it can be presented as the originality of the paper that a set of compromise solutions for a multi-objective linear programming problem including rough interval coefficients can be found using a zero-sum game to determine the weights.

The framework of this paper is presented as follows. The preliminaries including the rough set concept, linear programming problem, and their shared usages are given in Section 2. The solution algorithm for the MOLPRIC problem is explained in detail in two separate phases in Section 3. Illustrative numerical examples are given in Section 4. Finally, the conclusion and future studies are declared in Section 5.

2. PRELIMINARIES

In this section, some definitions taken from (Hamzehee et al., 2014) and (Temelcan, 2023), are given of the rough set concept, linear programming problem, and their shared use.

Definition 1. Consider an LP with interval coefficients (LPIC) problem as follows:

$$max \sum_{j=1}^{n} [\underline{c}_j, \overline{c_j}] x_j \tag{1.a}$$

s.t.

$$\sum_{j=1}^{n} [\underline{a}_{ij}, \overline{a_{ij}}] x_j \leq [\underline{b}_i, \overline{b_i}], \quad i = 1, \dots, m \tag{1.b}$$

$$x_j \geq 0, \quad j = 1, \dots, n \tag{1.c}$$

where $[\underline{c}_j, \overline{c_j}]$ is the coefficients (profit for *max*; cost for *min*) of objective function, $[\underline{a}_{ij}, \overline{a_{ij}}]$ and $[\underline{b}_i, \overline{b_i}]$ are parameters of constraints and all these coefficients are closed intervals on real numbers. $x = (x_1, x_2, \dots, x_n)^T$ is the vector of decision variables.

The best optimal solution to the LPIC problem (1) is obtained by solving the following LP problem:

LP_{best}:
$$max \sum_{j=1}^{n} \overline{c_j}\, x_j \tag{2.a}$$

s.t.

$$\sum_{j=1}^{n} \underline{a}_{ij} x_j \leq \overline{b_i}, \quad \forall i \tag{2.b}$$

$$x_j \geq 0, \quad \forall j \tag{2.c}$$

where the problem has the maximum value range inequalities (constraints) and the most favorable value (objective function) (Hamzehee et al., 2014).

The worst optimal solution to the LPIC problem (1) is found by solving the following LP problem:

LP_{worst}:
$$max \sum_{j=1}^{n} \underline{c}_j x_j \tag{3.a}$$

s.t.

$$\sum_{j=1}^{n} \overline{a_{ij}}\, x_j \leq \underline{b}_i, \quad \forall i \tag{3.b}$$

$$x_j \geq 0, \quad \forall j \tag{3.c}$$

where the problem has the minimum value range inequalities (constraints) and the least favorable value (objective function) (Hamzehee et al., 2014, p.1182).

According to LP_{best} and LP_{worst} problems, the solution of LPIC problem (1) can be found considering the following cases:

- If each LP_{best} and LP_{worst} problem has optimal solution, then LPIC problem (1) has a finite bounded optimal range,
- If the solution of LP_{worst} problem is unbounded, then LP_{best} problem has an unbounded solution,
- If the solution of LP_{best} problem is infeasible, then LP_{worst} problem has an infeasible solution.

Definition 2. Let A be a qualitative value, A_* and A^* are lower and upper approximation (closed) intervals of A, respectively. $A = (A_*, A^*)$ is called a rough interval if the following properties are satisfied:

- If $x \in A_*$, then $x \in A$ (i.e. x is surely in A),
- If $x /\in A^*$, then $x /\in A$ (i.e. x is not surely in A),
- If $x \in A^*$, then x is probably in A.

It is important to note that the lower approximation interval must be defined inside of the upper approximation interval, i.e. $A_* \subseteq A^*$, however, they are not complements of each other.

Definition 3. Consider an LP with rough interval coefficients (LPRIC) problem as

$$max \sum_{j=1}^{n}\left(\left[\underline{c}_j^l, \underline{c}_j^u\right], \left[\overline{c_j^l}, \overline{c_j^u}\right]\right)x_j \quad (4.a)$$

s.t.

$$\sum_{j=1}^{n}\left(\left[\underline{a}_{ij}^l, \underline{a}_{ij}^u\right], \left[\overline{a_{ij}^l}, \overline{a_{ij}^u}\right]\right)x_j \leq \left(\left[\underline{b}_i^l, \underline{b}_i^u\right], \left[\overline{b_i^l}, \overline{b_i^u}\right]\right), \quad i = 1, \dots, m \quad (4.b)$$

$$x_j \geq 0, \quad j = 1, \dots, n \quad (4.c)$$

where $\left(\left[\underline{c}_j^l, \underline{c}_j^u\right], \left[\overline{c_j^l}, \overline{c_j^u}\right]\right)$, $\left(\left[\underline{a}_{ij}^l, \underline{a}_{ij}^u\right], \left[\overline{a_{ij}^l}, \overline{a_{ij}^u}\right]\right)$, and $\left(\left[\underline{b}_i^l, \underline{b}_i^u\right], \left[\overline{b_i^l}, \overline{b_i^u}\right]\right)$ are rough interval coefficients of the objective function and the constraints, respectively, and $x = (x_1, x_2, \dots, x_n)^T$ is the vector of decision variables.

Consider the following LPIC problem:

$$max \sum_{j=1}^{n}\left[\underline{c}_j^l, \underline{c}_j^u\right]x_j \quad (5.a)$$

s.t.

$$\sum_{j=1}^{n}\left[\underline{a}_{ij}^l, \underline{a}_{ij}^u\right]x_j \leq \left[\underline{b}_i^l, \underline{b}_i^u\right], \quad i = 1, \dots, m \quad (5.b)$$

$$x_j \geq 0, \quad \forall j. \quad (5.c)$$

If the optimal range of the LPIC problem (5) exists, then it is also the surely optimal range $\left[\underline{z}^{l*}, \underline{z}^{u*}\right]$ of the LPRIC problem (4).

Similarly, consider the LPIC problem as

$$max \sum_{j=1}^{n}\left[\overline{c_j^l}, \overline{c_j^u}\right]x_j \quad (6.a)$$

s.t.

$$\sum_{j=1}^{n}\left[\overline{a_{ij}^l}, \overline{a_{ij}^u}\right]x_j \leq \left[\overline{b_i^l}, \overline{b_i^u}\right], \quad i = 1, \dots, m \quad (6.b)$$

$$x_j \geq 0, \quad \forall j. \quad (6.c)$$

If the optimal range of the LPIC problem (6) exists, then it is also the possibly optimal range $\left[\overline{z^{l*}}, \overline{z^{u*}}\right]$ of the LPRIC problem (4) (Osman et al., 2011).

As a result, the rough interval $\left(\left[\underline{z}^{l*}, \underline{z}^{u*}\right], \left[\overline{z^{l*}}, \overline{z^{u*}}\right]\right)$ is called the rough optimal range of the LPRIC problem (4). Moreover, the optimal solution of each corresponding LP problem of the LPRIC problem which its optimal value belongs to $\left[\underline{z}^{l*}, \underline{z}^{u*}\right]$, $\left(\left[\overline{z^{l*}}, \overline{z^{u*}}\right]\right)$ is called a completely (rather) satisfactory solution of the LPRIC problem.

Definition 4. Considering Definition 1 and Definition 3, we can construct the following LP problems:

$Surely\ Best\ LP: LP_{SBest}$

$$max \sum_{j=1}^{n} \underline{c}_j^u x_j \quad (7.a)$$

s.t.

$$\sum_{j=1}^{n} \underline{a}_{ij}^{l} x_j \leq \underline{b}_i^{u}, \quad \forall i \tag{7.b}$$

$Surely\ Worst\ LP: LP_{SWorst}$

$$max \sum_{j=1}^{n} \underline{c}_j^{l} x_j \tag{8.a}$$

s.t.

$$\sum_{j=1}^{n} \underline{a}_{ij}^{u} x_j \leq \underline{b}_i^{l}, \quad \forall i \tag{8.b}$$

$Possibly\ Best\ LP: LP_{PBest}$

$$max \sum_{j=1}^{n} \overline{c_j^{u}}\, x_j \tag{9.a}$$

s.t.

$$\sum_{j=1}^{n} \overline{a_{ij}^{l}}\, x_j \leq \overline{b_i^{u}}, \quad \forall i \tag{9.b}$$

$Possibly\ Worst\ LP: LP_{PWorst}$

$$max \sum_{j=1}^{n} \overline{c_j^{l}}\, x_j \tag{10.a}$$

s.t.

$$\sum_{j=1}^{n} \overline{a_{ij}^{u}}\, x_j \leq \overline{b_i^{l}}, \quad \forall i \tag{10.b}$$

where $x_j \geq 0\ \forall j$. The solution set of these LP problems is written as

$$LP_{PWorst} \subset LP_{SWorst} \subset LP_{PBest} \subset LP_{SBest}. \tag{11}$$

For proof, please check the study (Hamzehee et al., 2014, p.1184).

Definition 5. Consider a Multi-Objective LP with rough interval coefficients (MOLPRIC) problem as

$$max\ z_k = \sum_{j=1}^{n}\left(\left[\underline{c}_{kj}^{l}, \underline{c}_{kj}^{u}\right], \left[\overline{c_{kj}^{l}}, \overline{c_{kj}^{u}}\right]\right) x_j, \quad k = 1, \dots, q \tag{12.a}$$

s.t.

$$\sum_{j=1}^{n}\left(\left[\underline{a}_{ij}^{l}, \underline{a}_{ij}^{u}\right], \left[\overline{a_{ij}^{l}}, \overline{a_{ij}^{u}}\right]\right) x_j \leq \left(\left[\underline{b}_i^{l}, \underline{b}_i^{u}\right], \left[\overline{b_i^{l}}, \overline{b_i^{u}}\right]\right), \quad i = 1, \dots, m \tag{12.b}$$

$$x_j \geq 0, \quad j = 1, \dots, n \tag{12.c}$$

where $\left(\left[\underline{c}_{kj}^{l}, \underline{c}_{kj}^{u}\right], \left[\overline{c_{kj}^{l}}, \overline{c_{kj}^{u}}\right]\right)$, $\left(\left[\underline{a}_{ij}^{l}, \underline{a}_{ij}^{u}\right], \left[\overline{a_{ij}^{l}}, \overline{a_{ij}^{u}}\right]\right)$, and $\left(\left[\underline{b}_i^{l}, \underline{b}_i^{u}\right], \left[\overline{b_i^{l}}, \overline{b_i^{u}}\right]\right)$ are rough interval parameters of the objective functions and the constraints, respectively, and $x = (x_1, x_2, \dots, x_n)^T$ is the vector of decision variables.

It is seen that the MOLPRIC problem (12) has q −LPRIC problems, so there are $2q$-LPIC problems, and $4q$-LP problems accordingly. Since each parameter in each LPRIC problem belongs to the upper approximation interval or lower approximation interval of its rough intervals, different LPIC problems emerge.

Definition 6. According to the properties of a rough interval, the components of each rough interval coefficient are sorted as

$$\overline{c_{kj}^{l}} \leq \underline{c}_{kj}^{l} \leq \underline{c}_{kj}^{u} \leq \overline{c_{kj}^{u}}$$

$$\overline{a_{ij}^{l}} \leq \underline{a}_{ij}^{l} \leq \underline{a}_{ij}^{u} \leq \overline{a_{ij}^{u}}$$

$$\overline{b_{i}^{l}} \leq \underline{b}_{i}^{l} \leq \underline{b}_{i}^{u} \leq \overline{b_{i}^{u}}$$

for all $i, j, and\ k$.

3. SOLUTION ALGORITHM FOR MOLPRIC PROBLEM

The solution process is separated into two main phases and each phase is explained using the following steps.

3.1. First Phase

In this phase, the MOLPRIC problem (12) is separated into q −LPRIC problem for each objective function, and the optimal rough solution of each LPRIC problem, if there exists, is found by applying the method proposed in (Hamzehee et al., 2014). The following steps are iterated for q −LPRIC problems, independently, as follows:

Step 0. Consider a MOLPRIC problem as given in (12).

Step 1. Separate the MOLPRIC problem into q −LPRIC problem for each objective function.

Step 2. Construct each LPRIC problem of the MOLPRIC problem as given in (4). Here, the decision variables belong to X^{S+} which is the set of variables of which at least one of the coefficients is a rough interval. Moreover, the variables in this set are sign restricted as $x_j \geq 0$ $(j = 1, \ldots, n)$, for details check (Hamzehee et al., 2014).

Step 3. Find possibly optimal range $\left[\overline{z_k^{l*}}, \overline{z_k^{u*}}\right]$ by solving the LPIC problem as in (6). If the LPIC problem (6) is infeasible, go to Step 5.

Step 4. Find surely optimal range $\left[\underline{z}_k^{l*}, \underline{z}_k^{u*}\right]$ by solving the LPIC problem given in (5).

Step 5. There are three possible cases for an LPRIC problem:

- If (5) and (6) have optimal ranges, then the LPRIC problem has a rough optimal range as $\left(\left[\underline{z}^{l*}, \underline{z}^{u*}\right], \left[\overline{z^{l*}}, \overline{z^{u*}}\right]\right)$.
- If the LPIC problem (5) has an unbounded range, then the LPRIC problem has an unbounded range.
- If the LPIC problem (6) is infeasible, then the LPRIC problem is infeasible.

In the first phase, the algorithm is iterated from Step 2 to Step 5 for each LPRIC problem, as explained at the beginning.

3.2. Second Phase

In this phase, a rough optimal range of the MOLPRIC problem (12) is found by applying the game theory approach proposed in (Temelcan, 2023) using the study (Temelcan et al., 2020). Distinct zero-sum games are constructed taking the satisfactory solutions as players and their objective function values as strategies. After solving each game, weights are found, and they are used to form a single-objective LP problem to evaluate the bounds of the rough optimal range of the MOLPRIC problem.

Step 6. Construct a payoff matrix such that rows are filled with elements taking each solution of LP_{SBest} problems, and columns are their corresponding objective function values.

Step 7. Find the weights by solving the zero-sum game. Then, construct a single-objective LP problem multiplying the weights by the corresponding objective functions of LP_{SBest} problems, and solve.

Step 8. Find the compromise rough range of the MOLPRIC problem as $\left(\left[\underline{z_1}^{l*}, \underline{z_1}^{u*}\right], \left[\overline{z_1}^{l*}, \overline{z_1}^{u*}\right]\right)$.

In this phase, these steps are applied for each LP problem, that is LP_{SWorst}, LP_{PWorst} , and LP_{PBest} problems. Payoff matrices are constructed via the solutions of each LP problem, and the weights are found. These weights are utilized to form a single-objective LP problem. Then distinct optimal solutions are found by solving each weighted single-objective LP problem. Each optimal solution, that is exactly four points, is recorded to determine the compromise rough optimal range of the MOLPRIC problem (12).

To represent the process of the algorithm, a flowchart is given in Figure 1.

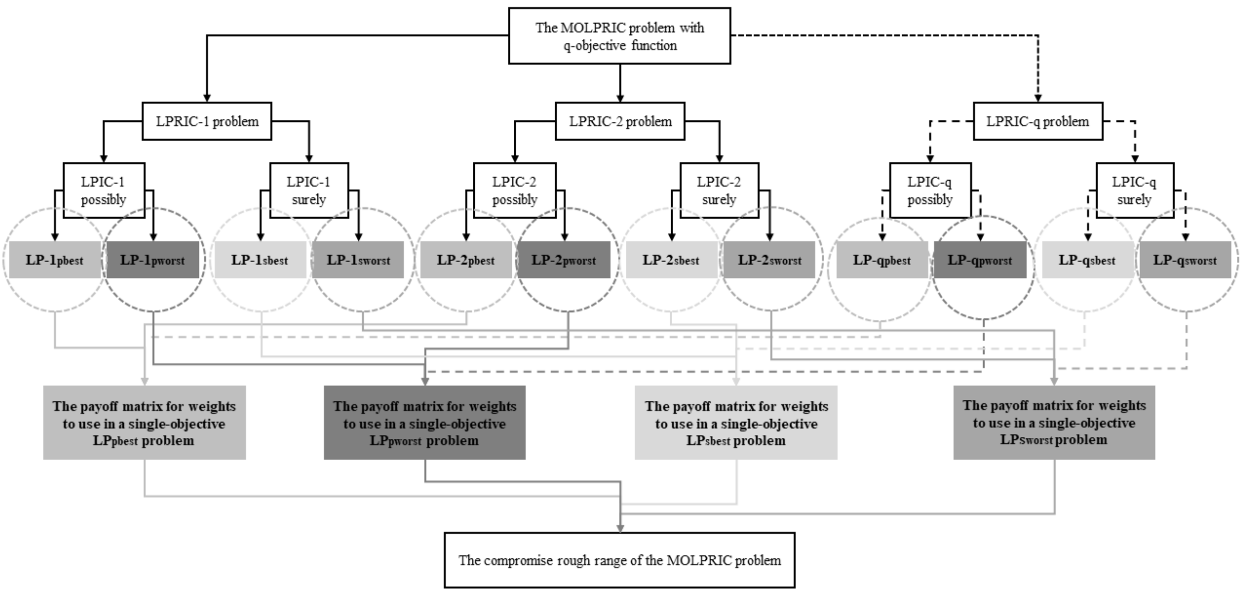

Figure 1. Flowchart of The Solution Algorithm for A MOLPRIC Problem

To obtain several compromise solutions, we can apply different techniques in the second phase. One of these techniques is making all the entries of the payoff matrix positive. This means that if there is any negative term in any cell of the payoff matrix, find a value making that term positive and add it to all entries, and thus we can construct a positive payoff matrix. Then we can solve the zero-sum game. Another technique is taking the ratio of rows. Another technique is proposed in the study Sivri et al. (2019) taking the ratio of rows. After obtaining a positive payoff matrix, we can take the ratio of each term in the first row to its corresponding term in the second row. This technique decreases the number of rows by one.

It is seen that the proposed algorithm has an ease of implementation for multi-objective problems. While it is quite applicable to use the weighted-sum method in two-objective MOLP problems, finding the appropriate combination of the weights between more than two objective functions will be challenging. Accordingly, applying a zero-sum game among the distinct objective values would yield finding compromise rough solutions.

4. NUMERICAL EXAMPLES

Example 4.1. Consider a MOLPPRIC example modified from the study Sivri et al. (2019). Since the example in (Sivri et al., 2019) is a crisp LP problem, the coefficients of the objective functions and left-hand side parameters are prepared to provide symmetry. Accordingly, the lower approximation intervals are symmetric by one unit while the upper approximation intervals are symmetric by two units. Similarly, the crisp numbers on the right-hand side of the constraints are arranged to remain within the lower approximation interval. The upper approximation interval is designed by taking widths of five units from the left and right sides of the lower approximation interval.

$$Max \quad z_1 = ([-2,0],[-3,1])x_1 + ([1,3],[0,4])x_2$$
$$Max \quad z_2 = ([1,3],[0,4])x_1 + ([0,2],[-1,3])x_2$$

s.t.

$$([-2,0],[-3,1])x_1 + ([2,4],[1,5])x_2 \leq ([20,25],[15,30])$$
$$([0,2],[-1,3])x_1 + ([2,4],[1,5])x_2 \leq ([25,30],[20,35])$$
$$([3,5],[2,6])x_1 + ([2,4],[1,5])x_2 \leq ([40,50],[35,55])$$
$$([2,4],[1,5])x_1 + ([0,2],[-1,3])x_2 \leq ([25,35],[20,40])$$
$$x_1, x_2 \geq 0.$$

The following phases are applied to find the rough optimal range of the MOLPRIC problem.

First phase: The MOLPRIC problem is separated into two LPRIC problems according to the number of objective functions. Therefore, the first LPRIC problem is obtained by taking the objective function z_1 and the constraints, which is labeled as $LPRIC_1$. From the $LPRIC_1$ problem, an $LPIC_{1P}$ problem is formed as

$$Max \quad [-3,1]x_1 + [0,4]x_2$$

s.t.

$$[-3,1]x_1 + [1,5]x_2 \leq [15,30]$$
$$[-1,3]x_1 + [1,5]x_2 \leq [20,35]$$
$$[2,6]x_1 + [1,5]x_2 \leq [35,55]$$
$$[1,5]x_1 + [-1,3]x_2 \leq [20,40]$$
$$x_1, x_2 \geq 0$$

and it has a possibly optimal range whereas another $LPIC_{1S}$ problem is

$$Max[-2,0]x_1 + [1,3]x_2$$

s.t.

$$[-2,0]x_1 + [2,4]x_2 \leq [20,25]$$
$$[0,2]x_1 + [2,4]x_2 \leq [25,30]$$
$$[3,5]x_1 + [2,4]x_2 \leq [40,50]$$

$[2,4]x_1 + [0,2]x_2 \leq [25,35]$
$x_1, x_2 \geq 0$

and it has a surely optimal range.
To find the possibly optimal range, the LPIC1P problem is divided into the following LP problems:

LP_{PWorst}
$Max\overline{z_1}^l = -3x_1$

s.t.

$x_1 + 5x_2 \leq 15$
$3x_1 + 5x_2 \leq 20$
$6x_1 + 5x_2 \leq 35$
$5x_1 + 3x_2 \leq 20$
$x_1, x_2 \geq 0$

and

LP_{PBest}
$Max\overline{z_1}^u = x_1 + 4x_2$

s.t.

$-3x_1 + x_2 \leq 30$
$-x_1 + x_2 \leq 35$
$2x_1 + x_2 \leq 55$
$x_1 - x_2 \leq 40$
$x_1, x_2 \geq 0$

where $\overline{z_1}^l$ and $\overline{z_1}^u$ are the lower and upper bounds of the possibly optimal range of LPRIC1. By solving the LP_{PWorst} problem, the optimal solution is found as $x_1 = 0, x_2 = 0$ and optimal value is $\overline{z_1}^l = 0$. The optimal solution of LP_{PBest} problem is $x_1 = 6{,}667, x_2 = 41{,}667$ and optimal value is $\overline{z_1}^u = 173{,}333$.

Similarly, for finding the surely optimal range, the LPIC1S problem is separated into two LP problems as follows:

LP_{SWorst}
$Max\underline{z_1}^l = -2x_1 + x_2$

s.t.

$4x_2 \leq 20$
$2x_1 + 4x_2 \leq 25$
$5x_1 + 4x_2 \leq 40$
$4x_1 + 2x_2 \leq 25$
$x_1, x_2 \geq 0$

and

LP_{SBest}
$Max\underline{z_1}^u = 3x_2$
s.t.

$-2x_1 + 2x_2 \leq 25$
$2x_2 \leq 30$
$3x_1 + 2x_2 \leq 50$
$2x_1 \leq 35$
$x_1, x_2 \geq 0$

where $\underline{z_1}^l$ and $\underline{z_1}^u$ are the lower and upper bounds of the surely optimal range of LPRIC$_1$. By solving the LP_{SWorst} problem, the optimal solution is found as $x_1 = 0, x_2 = 5$ and optimal value is $\underline{z_1}^l = 5$. The optimal solution of LP_{SBest} problem is $x_1 = 2{,}5\ , x_2 = 15$ and optimal value is $\underline{z_1}^u = 45$.

The same process is iterated for the second LPRIC problem, labeled as LPRIC$_2$, which is obtained taking the objective function z_2 and the same constraints. Therefore, the optimal solution of

- LP_{PWorst} problem is $(0{,}0)$ and the optimal value is 0,
- LP_{PBest} problem is $(6{,}67\ , 41{,}67)$ and the optimal value is $151{,}67$,
- LP_{SWorst} problem is $(6{,}25\ , 0)$ and the optimal value is $6{,}25$,
- LP_{SBest} problem is $(16{,}67\ , 0)$ and the optimal value is 50.

For both LPRIC problems, the completely and rather satisfactory solutions, and the rough optimal range are presented in Table 1.

Second phase: Payoff matrices are constructed to determine weights for finding single-objective LP problems. One of these payoff matrices is shown in detail below.

The first payoff matrix is constructed taking the optimal solutions of LP_{SWorst} problems and their objective function values. The weights of $\underline{z}_1^l$ and $\underline{z}_2^l$ are found by solving the zero-sum game. The payoff matrix and the weights are shown in Table 2.

Table 1. Results for LPRIC Problems of the MOLPRIC Problem

Results for LPRIC$_1$		
Possibly optimal range	$\left[\overline{z_1}^l, \overline{z_1}^u\right] = [0\ , 173{,}33]$	
Surely optimal range	$\left[\underline{z_1}^l, \underline{z_1}^u\right] = [5{,}45]$	
Rough optimal range	$\left(\left[\underline{z_1}^l, \underline{z_1}^u\right], \left[\overline{z_1}^l, \overline{z_1}^u\right]\right) = ([5{,}45], [0{,}173.33])$	
Rather satisfactory solutions	$(0{,}0)$	$(6{,}67\ , 41{,}67)$
Completely satisfactory solutions	$(0{,}5)$	$(2{,}5\ , 15)$
Results for LPRIC$_2$		
Possibly optimal range	$\left[\overline{z_2}^l, \overline{z_2}^u\right] = [0\ , 151{,}67]$	
Surely optimal range	$\left[\underline{z_2}^l, \underline{z_2}^u\right] = [6{,}25\ , 50]$	
Rough optimal range	$\left(\left[\underline{z_2}^l, \underline{z_2}^u\right], \left[\overline{z_2}^l, \overline{z_2}^u\right]\right) = ([6{,}25\ , 50], [0\ , 151{,}67])$	
Rather satisfactory solutions	$(0{,}0)$	$(6{,}67\ , 41{,}67)$
Completely satisfactory solutions	$(6{,}25\ , 0)$	$(16{,}67\ , 0)$

Table 2. The Payoff Matrix of Lower Bounds of Surely Optimal Range

	$\underline{z}_1^l$	$\underline{z}_2^l$
(0,5)	5	0
(6,25 , 0)	−12,5	6,25
weights	0,26	0,74

The weights are used to find the lower bound of the surely optimal solution of the MOLPRIC problem. Therefore, the objective function of the following LP problem is constructed by multiplying the weights by the corresponding objective functions, and the constraints are taken from the LP_{SWorst} problem:

$max\underline{z}^l = 0{,}26(-2x_1 + x_2) + 0{,}74x_1$

s.t.

$4x_2 \leq 20$
$2x_1 + 4x_2 \leq 25$
$5x_1 + 4x_2 \leq 40$
$4x_1 + 2x_2 \leq 25$
$x_1, x_2 \geq 0$

is solved and the completely satisfactory solution is found as $(x_1, x_2) = (4{,}167\ , 4{,}167)$ and lower bound of surely optimal range is $\underline{z}^{l*} = 2$.

Similarly, the other payoff matrix is constructed taking the optimal solutions of LP_{SBest} problems and their objective function values. The weights of $\underline{z}_1^u$ and $\underline{z}_2^u$ are found via zero-sum game. This payoff matrix and the weights are shown in Table 3.

A single-objective LP problem is constructed using the weights in the objective function and taking the same constraints of LP_{SBest} problems. As a result, the completely satisfactory solution is $(x_1, x_2) = (6{,}67\ , 15)$and the upper bound of the surely optimal range is $48{,}9$.

Table 3. The Payoff Matrix of The Upper Bound of The Surely Optimal Range

	$\underline{z}_1^u$	$\underline{z}_2^u$
(2,5 , 15)	45	37,5
(16,67 , 0)	0	50
weights	0,22	0,78

Since the rather satisfactory solution $(0{,}0)$ is identical, the weights of $\overline{z}^u$ and $\overline{z}^u$ are found 0, and thus the lower bound of the possibly optimal range is 0. Moreover, the other rather satisfactory solution is $(6{,}67\ , 41{,}67)$, and it is identical, their values in the objective functions $\overline{z}_1^u$ and $\overline{z}_2^u$ are the same. Thus, whichever has the higher objective function value, that function takes the weight 1, that is, the weight of $\overline{z}_2^u$ is 1. As a result, the rather satisfactory solution is $(x_1, x_2) = (6{,}67\ , 41{,}67)$ and the upper bound of the possibly optimal range is 151.67.

Consequently, the compromise rough range of the MOLPRIC problem is found as $\left(\left[\underline{z_1}^{l*}, \underline{z_1}^{u*}\right], \left[\overline{z_1}^{l*}, \overline{z_1}^{u*}\right]\right) = ([2\ , 48{,}9], [0\ , 151{,}67])$. The completely satisfactory solutions are $(4{,}167\ , 4{,}167)$ and $(6{,}67\ , 15)$ whereas the rather satisfactory solutions are $(0{,}0)$ and

$(6{,}67\ ,41{,}67)$. It is seen from the comparison with the study (Sivri et al., 2019) that all the crisp compromise solutions they found are contained in the lower approximation of the compromise rough range found above. It can also be expressed that the width of the ranges constructed at the modification determines the width of the possibly optimal range of the rough range.

Example 4.2. Consider the MOLPPRIC example solved in the study (Garg & Rizk-Allah, 2021):

$Max\ z_1 = ([1,3],[0,5])x_1 + ([1,2],[1,3])x_2$
$Max\ z_2 = ([3,5],[2,6])x_1 + ([2,6],[1,7])x_2$

s.t.

$([2,3],[1,3])x_1 + ([2,4],[1,5])x_2 \leq ([7,9],[5,10])$
$([2,3],[1,5])x_1 + ([1,2],[0,6])x_2 \leq ([5,8],[3,9])$
$x_1, x_2 \geq 0$

In the first phase, the MOLPRIC problem is divided into two LPRIC problems, and these LPRIC problems are solved to find the possibly and surely optimal solutions. Table 4 presents the possibly and surely optimal solutions for each LPRIC problem.

According to Table 4, the completely satisfactory solutions of LPRIC problems 1 and 2 are $(1,1), (3{,}5\ ,1)$ and $(1{,}667\ ,0), (0\ ,4{,}5)$, respectively. On the other hand, the rather satisfactory solutions of LPRIC problems 1 and 2 are found $(0\ ,0{,}5), (9,1)$ and $(0{,}6\ ,0), (0,10)$ and the first phase is finalized.

Table 4. The Possibly and Surely Optimal Solutions of Distinct LPRIC Problems

LPRIC problem 1	
Possibly optimal solution	
$LP_{PWorst}: z = x_2$	$LP_{PBest}: z = 5x_1 + 3x_2$
$x_1 = 0\ \ x_2 = 0{,}5$	$x_1 = 9\ \ x_2 = 1$
Surely optimal solution	
$LP_{SWorst}: z = x_1 + x_2$	$LP_{SBest}: z = 3x_1 + 2x_2$
$x_1 = 1\ \ x_2 = 1$	$x_1 = 3{,}5\ \ x_2 = 1$
LPRIC problem 2	
Possibly optimal solution	
$LP_{PWorst}: z = 2x_1 + x_2$	$LP_{PBest}: z = 6x_1 + 7x_2$
$x_1 = 0{,}6\ \ x_2 = 0$	$x_1 = 0\ \ x_2 = 10$
Surely optimal solution	
$LP_{SWorst}: z = 3x_1 + 2x_2$	$LP_{SBest}: z = 5x_1 + 6x_2$
$x_1 = 1{,}667\ x_2 = 0$	$x_1 = 0\ \ x_2 = 4{,}5$

For the second phase, the optimal solutions of each LP problem and their objective function values are taken in the cells of payoff matrices. These payoff matrices and their weights are given in Table 5.

Table 5. Payoff Matrices and The Weights

LP_{SWorst}	$x_1 + x_2$	$3x_1 + 2x_2$
$(1,1)$	2	5
$(1,667\ ,0)$	$1,667$	4
Weights	1	0

LP_{SBest}	$3x_1 + 2x_2$	$5x_1 + 6x_2$
$(3,5\ ,1)$	$12,5$	$23,5$
$(0\ ,4,5)$	9	27
Weights	1	0

LP_{PWorst}	x_2	$2x_1 + x_2$
$(0\ ,0,5)$	$0,5$	$0,5$
$(0,6\ ,0)$	0	$1,2$
Weights	1	0

LP_{PBest}	$5x_1 + 3x_2$	$6x_1 + 7x_2$
$(9,1)$	48	61
$(0,10)$	30	70
Weights	1	0

As a result, a compromise rough optimal range of the MOLPRIC problem can be found as $\left(\left[\underline{z_1}^{l*}, \underline{z_1}^{u*}\right], \left[\overline{z_1}^{l*}, \overline{z_1}^{u*}\right]\right) = ([2,12.5], [0.5,48])$ where the completely satisfactory solutions are $(3,5\ ,1)$ and $(1,1)$ and the rather satisfactory solutions are $(9,1)$ and $(0\ ,0,5)$. Since the study (Garg and Rizk-Allah, 2021) used the weighted sum method, they suggested a set of compromise solutions. However, the proposed algorithm produced a solution that matches one of their solutions. For the comparison, it can be put forward that constructing different payoff matrix structures such as taking ratios of the rows or normalization of the values generates different compromise solutions.

5. CONCLUSION

The algorithm proposed in this paper helps to find a set of compromise solutions for the multi-objective linear programming with rough interval coefficients (MOLPRIC) problem. To find a compromise solution, a two-phased algorithm is constructed. In the first phase, the MOLPRIC problem is separated into single-objective LPRIC problems under the number of objective functions. The rough optimal solution of each LPRIC problem, if exists, is found by applying the method proposed in (Hamzehee et al., 2014). The second phase works using the game theory approach. Here, the approach proposed in (Temelcan, 2023) is used to find the rough optimal solution by applying a zero-sum game. Since there is a MOLPRIC problem, it would be possible to determine the players (objective functions) and their strategies (objective function values). The solution of the game gives the weight of each objective function, and thus the weighted sum method is used for determining the trade-offs between the objective functions.

The advantage of the proposed algorithm can be presented when the number of objective functions increases. It is quite applicable to use the weighted-sum method in two-objective MOLP problems, finding the appropriate combination of the weights between more than two objective functions will be difficult. In this case, the proposed algorithm provides an easy implementation for finding a compromise solution to the MOLPRIC problems having more than two objective functions.

Limitations of the study can be given as the difficulty of finding the solution in the case of negative or mixed rough intervals. Thus, searching different algorithms to solve any MOLPRIC problems and comparing their results can be a subject for further research. Application of the proposed algorithm to real-world problems can also lead to further considerations.

Statement of Research and Publication Ethics
Research and publication ethics were complied with in the study.

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