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Activity design for improving mathematical understanding

Matematiksel anlamayı geliştirmeye yönelik etkinlik tasarımı

Rahime Çelik GÖRGÜT [1]

[1] Kastamonu University, Kastamonu, Türkiye

Abstract: This study aims to present a conceptual framework on the design of activities to improve comprehension and evaluate the impact of activities developed on students' mathematical understanding. The design of the study is a case study. While the situation examined in the study is mathematical activities, the unit of analysis is the structure of the activities within the scope of achieving understanding and the functionality of these activities. In the study, the easily accessible sampling method, one of the purposeful sampling methods, was used to determine the participants, and the study was conducted with 21 8th-grade students. The obtained data were analyzed descriptively according to the students' correct use of algorithms, production of multiple solutions, logical inference and discovery, connection with daily life, use of multiple representations, establishment of relations between concepts, and their ability to recognize the need felt in history for a concept and the production of solutions for it. The study findings show that the designed activities provide students with the opportunity to engage in a series of meaningful mathematical thoughts. As a result, it is thought that the activities developed within the framework presented in the current study will enable mathematics to be learned with understanding.

Keywords: *Activity design, mathematical understanding, mathematical understanding activities, understanding.*

Özet: Bu çalışmada matematiksel anlamayı geliştirmeye yönelik etkinliklerin tasarımı üzerine kavramsal bir yapı sunulması ve bu bağlamda geliştirilen etkinliklerin öğrencilerdeki anlama üzerine etkisinin değerlendirmesi amaçlanmıştır. Çalışmanın deseni durum çalışması olarak nitelendirilmektedir ve çalışmada incelenen durum matematiksel etkinlikler iken analiz birimi anlamayı gerçekleştirme ve geliştirme kapsamındaki etkinliklerin yapısı ile bu etkinliklerin işlerlikleridir. Çalışmada katılımcıların belirlenmesi için amaçlı örnekleme yöntemlerinden kolay ulaşılabilir örnekleme yöntemi kullanılmıştır ve çalışma 21 tane 8. sınıf öğrencisi ile yürütülmüştür. Elde edilen veriler öğrencilerin algoritmaları doğru bir biçimde kullanmaları, çoklu çözüm yolları üretmeleri, mantıksal çıkarımlarda ve keşifte bulunmaları, günlük yaşamla ilişki kurmaları, çoklu temsillerden yararlanmaları, kavramlar arasında ilişki kurulmaları ve bir kavrama ilişkin tarihte duyulan ihtiyacı fark edebilmeleri ve ona dair çözüm üretebilmeleri durumlarına göre betimsel olarak analiz edilmiştir. Çalışma bulguları tasarlanan etkinliklerin öğrenciler için bir dizi anlamlı matematiksel düşünceyle meşgul olmaları için fırsat verdiğini göstermektedir. Sonuç olarak şimdiki çalışmada sunulan çerçevede geliştirilen etkinliklerin matematiğin anlaşılarak öğrenilmesine olanak sağlamada önemli bir adım olacağı düşünülmektedir.

Anahtar Kelimeler: *Anlama, etkinlik tasarlama, matematiksel anlama, matematiksel anlama etkinlikleri*

1. Introduction

Encouraging students to learn mathematics by understanding is one of the core aims of mathematics teaching, as this equips them with knowledge that can be flexibly implemented in the real world (National Council of Teachers of Mathematics [NCTM], 2000; Yao et al., 2021). Understanding involves knowing concepts and their interconnections with other concepts (Krathwohl, 2002). Therefore, understanding is an important factor in learning mathematics (NCTM, 2000), which is based on building new knowledge by utilizing prior knowledge and experiences and transferring them to new and different situations (Ministry of National Education [MoNE], 2018). To support the development of

Corresponding author: rahime.celik.gorgut@gmail.com
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understanding among students, the learning process should be enriched with active learning methods (Yatim et al., 2022). This can be achieved through activities that enable students to construct their own mathematical understanding (Lai et al., 2019). (Lai et al., 2019).

Activities are considered as a multidimensional structure that enables students to build their personal knowledge in maths lessons, is related to daily life, requires creativity, involves different ways of reaching solutions, and involves processes such as the use of mathematical symbols and expressions, modelling, logical inference and the ability to abstract (Suzuki & Harnisch, 1995). Therefore, activities increase students' understanding of mathematics by requiring a high level of mathematical reasoning and thinking. However, as well as encouraging students to make higher-order cognitive demands (Chapman, 2013), activities may also lead students to use formulas, procedures or algorithms in forms that do not have a meaningful connection and to apply previously memorized facts (Stein et al., 2000). Therefore, some basic principles should be considered in both the design and implementation of understanding-based mathematical activities (Bingölbali & Özmantar, 2014). For this reason, the present study aims to present a conceptual framework on the design of activities to improve comprehension and evaluate the impact of activities developed on students' mathematical understanding. Thus, it is aimed to accelerate the researches in which mathematical understanding will be realized through activities and to contribute to the related literature to improve students' mathematical understanding.

1.1. Conceptual Framework

Mathematical Understanding

Mathematical understanding is the ability to make connections between mathematical concepts and facts, link newly acquired information with prior knowledge, and integrate disparate parts of knowledge into a cohesive whole (Sierpinska, 2013). Within the domain of mathematics education study, mathematical understanding is predominantly considered in two distinct manners: procedural and conceptual understanding. Procedural understanding entails the acquisition of a plan to solve a given problem (Kadijevich, 2018). Conceptual understanding is explanationed as the knowledge of concepts and their interrelationships (Cai & Ding, 2017). These two forms of mathematical comprehension are mutually reinforcing and equally indispensable for attaining mathematical expertise (NCTM, 2000). Besides, mathematical understanding is also considered with different components related to conceptual and procedural understanding. For instance, Usiskin (2012) scrutinized mathematical comprehension across five distinct dimensions: algorithm, property-proof, use-application, representation-metaphor, and history-culture. The algorithm dimension encompasses the utilization and discovery of algorithms, while the property-proof dimension entails an understanding of the properties of concepts and the discerning the proofs of new findings. The use-application dimension encompasses the application of a concept to mathematical models and the discovery of new models, while the representation-metaphor dimension encompasses the representation of an idea, the analysis of said representations, and the discovery of new representations. Lastly, the history-culture dimension explores the emergence and development of concepts within the historical process, as well as their functionality within diverse cultures. In this context, in the study, a person who fully understands a mathematical concept is expected to be able to demonstrate skills such as the ability to represent mathematics in different ways, communicate mathematically, solve problems and reason (Garner, 2007), develop connections and relationships between different aspects of mathematical knowledge, formulate and discuss hypotheses and ideas about mathematics, and develop the same phenomenon from multiple mathematical perspectives (NCTM, 2000). This can be achieved by providing students with activities that require high levels of mathematical thinking (Stylianides & Stylianides, 2008).

Principles of mathematical activity and design

A mathematical activity is described as a concept that enables students to generate knowledge and transfer this to new situations, is associated with daily life, requires creativeness, involves different ways of reaching solutions, includes the processes of using maths symbols, modelling and making logical conclusions (Powell et al., 2009). Therefore, mathematical activities are expected to encourage students to do mathematics, construct meaning, understand, make connections, think, and make decisions (Goos et al., 2013). Besides, they should have some properties such as modelling real-world events, having multiple strategies, having different ways to reach a solution, showing that mathematics is a whole, linking several concepts for a solution, and noticing the incompatibility between an instinctive solution and a mathematical solution (Kieran et al., 2015). In this regard, Swan (2014) identified a framework of principles that should be targeted when designing activities for mathematics lessons. In this framework, goal 1 is to develop procedural fluency, goal 2 is to develop conceptual understanding, goal 3 is to improve the skill of solving non-routine problems and extend this to solving real-world problems, and goal 4 is to involve students in working on the mathematical products created. The Turkish mathematics curriculum states that when structuring activities, it is necessary to consider the properties that will develop mathematical thinking processes such as “appropriate to the level of the students, based on problem solving and modelling, associating mathematical knowledge with different disciplines, focusing on the historical development of mathematical concepts, providing students with formal mathematical knowledge through real- life problems, making assumptions and generalizations, using multiple representations, allowing students to transfer their knowledge to new situations and making synthesis” (MoNE, 2013). In this study, a mathematical activity will be used as activities that require situations such as using algorithms, making applications, constructing concepts, making exemplifications, using representations, making generalizations, establishing relationships between concepts and different disciplines, making logical inferences, establishing a problem situation, producing multiple solutions, and associating concepts with daily life, in line with our definition of mathematical understanding.

Mathematical activity and mathematical understanding

The activities to be implemented in mathematics lessons to develop mathematical knowledge, mathematical reasoning processes, modelling and problem solving are undoubtedly among the activities to be conducted for the realisation of mathematical understanding (Kieran et al., 2015). Based on the idea that in-depth understanding in mathematics is possible through the construction and implementation of mathematical activities in teaching, Swan (2014) states that an activity for mathematics lessons should include the components of classifying mathematical objects, using multiple representations, making connections between them and interpreting them, evaluating mathematical expressions, that is, deciding whether the given expressions are always, sometimes or never true, creating problems and analysing solutions. Considering these components and the definitions of mathematical activity, an activity designed for mathematical understanding requires the ability to use algorithms effectively and fluently, enables students to create their own conceptual structures and then apply, adapt and transfer these structures to new and different situations, requires establishing relationships with both the concepts within mathematics and different disciplines, is related to daily life, is interesting and intriguing, encourages creativity by producing different questions and solutions, It is considered as a structure that includes the ability to discover and generalise, involves the use of mathematical expressions and symbols, requires multiple representations and establishing relationships between them, involves processes such as abstraction, making logical inferences, and aims to know how mathematical concepts appeared in history and how they were applied in different cultures.

Framework of Activities to Develop Mathematical Understanding (ADMU)

An activity designed for mathematical understanding is supposed to include an approach to reveal and improve students' mathematical procedural abilities (NCTM, 2000; MoNE, 2013) such as communication, modelling, reasoning-proof, and association. In addition, when the mathematical comprehension dimensions (Usiskin, 2012) and mathematical activity definitions and the characteristics (Swan, 2014) they should have are related to each other, an activity framework that will be used in mathematics teaching in general and aimed at realising and developing a deep mathematical understanding is considered as a multidimensional structure based on student-centred active participation aiming to; *Algorithm*: classifying mathematical objects, using mathematical expressions and symbols, using algorithms flexibly, effectively and fluently; *Property:* knowing the properties of concepts, creating a conceptual structure, generating different questions/problems and solutions, creativity, discovery and generalisation, establishing cause-effect relationships, abstraction, and making logical inferences; *Using-applying*: applying, adapting and transferring mathematical concepts to new and different situations, making connections with daily life; *Relating-Representation*: relating both to concepts within mathematics and to other disciplines, using multiple representations and establishing relationships between them, using situations that show different representations of the same mathematical idea together; *History-Culture*: to know how mathematical concepts emerged in history and how they are used in different cultures.

This framework effectively bridges theory and practice by linking each dimension of mathematical understanding to its practical counterparts in activity design. Its theoretical foundation is grounded in Usiskin's (2012) five-dimensional approach to mathematical understanding. Building upon this, the principles of mathematical activity design proposed by Swan (2014) and Kieran et al. (2015) serve as fundamental components for creating student-centered activities that foster versatile thinking. Additionally, processes such as problem-solving, modeling, reasoning, communication, and inter-conceptual linking-highlighted by the NCTM (2000) and the Ministry of National Education (MEB, 2013)-are integrated with these dimensions and principles, thereby establishing a practical and comprehensive framework for structuring activities. Within this context, the ADMU framework not only illustrates how mathematical activities can be developed from a solid theoretical base but also provides a holistic roadmap to support students' deep mathematical understanding.

Aim and significance of the research

This study aims to provide a conceptual framework for the design of activities for the multidimensional development of mathematical understanding. Designed specifically for this purpose, the ADMU Framework offers a comprehensive roadmap-both theoretical and practical-on how to structure activities that support students' mathematical thinking and understanding in a multi-dimensional way. This constitutes the original contribution of the study by proposing a holistic framework for meaning-based activity design in mathematics teaching. The focus of mathematics teaching is on the deep understanding of mathematical concepts and relationships, and for the purpose of to achieve this, it is expected to create learning activities that will ensure understanding among students. In addition, studies in the related field show that teachers have conceptual deficiencies regarding mathematical understanding (Cai & Ding, 2017), that designing diverse activities for understanding is quite difficult for mathematics teachers (Fujita & Yamamoto, 2011) and that there are some deficiencies in the activities designed by teachers to improve understanding (Dempsey & O'shea, 2017).

Therefore, it is thought that presenting a conceptual framework on the design of activities to improve mathematical understanding will firstly provide a better understanding of the concept of activities for understanding by mathematics educators, set an example for the elimination of deficiencies in this context, be a guiding material for teachers, and it is also thought that it will make an important contribution in terms of enabling teachers to design and implement activities that will realise mathematical understanding in all its dimensions in their lessons and to raise students with mathematical literacy targeted in the curriculum. It is thought that it will contribute to future research and professional development activities in which mathematical understanding will be realised through activities and will add momentum to the studies to be carried out in this direction.

2. Methodology

2.1. Research Design

The present study is characterised as a case study. In general, case study is a methodological approach that involves in-depth investigation of a limited system, uses systematic data collection tools on how the system works or functions, and is used to answer how and why questions in situations where the current and researcher is no control over the variables (Creswell & Poth, 2018). While the situation examined in the present study is mathematical activities, the unit of analysis is the structure, characteristics and functioning of activities aimed at realising and developing comprehension.

2.2. Participants

In the current study, convenience sampling method, which is one of the purposive sampling methods, was used to select the participants during the implementation of the activities designed within the scope of the proposed ADMU framework. The study was conducted with a total of 21 students studying in the 8th grade of a secondary school in Türkiye. The activities were applied to the students by mathematics teachers. This process was recorded with a camera.

2.3. Data Collection

In the current study, mathematical understanding development activities were used as data collection tools. Under this heading, the process of designing mathematical understanding development activities is mentioned and the designed activities are included.

The process of designing mathematical understanding development activities

The activities designed to be implemented in the current study were first developed within the scope of the ADMU framework. In addition, the purpose of the activity, the tools and materials to be used within the scope of the activity, the activity having more than one starting point, teacher and student roles, students' prior knowledge, expectations from students, and the time allocated for the activity were also emphasised (Bingölbali & Özmantar, 2014). In this stage, the steps below were followed.

1. *Literature Review:* A literature review was conducted on topics such as understanding, mathematical understanding, activity design and principles, mathematical activity.
2. *Determining the Structure of the Activities:* The framework of the activities for developing mathematical understanding (ADMU) was determined.
3. *Determination of the Subject:* By considering the secondary school mathematics curriculum, it was determined in which subjects the activities for developing mathematical understanding would be designed.

4. *Determination of Objectives and Student Outcomes:* The objectives that would be the source of the activities were determined.

5. *Designing the Activities:* Activities were designed by the researcher in accordance with the ADMU framework in line with the gains targeted in the activities.

6. *Obtaining expert opinions:* The suitability of the designed activities to the dimensions of mathematical understanding and activity design principles was first evaluated by an expert with a doctoral degree in mathematics education. With the help of the feedback from the expert, it was clarified which understanding dimension the designed activities were intended to reveal. For example, the expert evaluated activity-1 in the using- applying dimension since it required making connections with daily life. However, during the discussions with the expert, it was concluded that activity-1 also requires generating different questions and solutions and making logical inferences. In addition, a 21- year mathematics teacher with a master's degree in mathematics education also evaluated which understanding dimensions the designed activities contained and whether they could develop mathematical understanding sufficiently. The expert teacher stated that the activities would contribute to the development of mathematical understanding. Besides, to determine the age and level appropriateness of the designed activities and their clarity and comprehensibility, the activities were applied to a total of 3 students selected voluntarily from 9th grade students.

Activities developed within the framework of ADMU

Examples of the developed activities are given below.

Figure 1

Activity-1

Task	Area of the circle section
Course	Mathematics
Grade	8
Learning area	Geometry and Measurement
Learning sub-area	Circle
Recommended Duration	20′
Teaching-Learning Methods and Techniques	Learning by doing, question-answer, discovery learning
Core Skills	Communication, association, transferring, exploration
Key Concepts	Circle, Centre Angle, Arc, Circle Piece, Circle Section
Tools	Smart board, model house, toy horse, rope in different lengths
Outcomes	1. Calculates the area of a circle and a circle section.
Expectations from Students for the Activity	Students are expected to make some calculations using the formula for the area of a circle section.

Activity: Ahmet wants to graze his horse. Ahmet has tied the halter of his horse to a pole in the corner of the stable.

Question: Find out in what area the horse tied to the corner of the stable can graze.

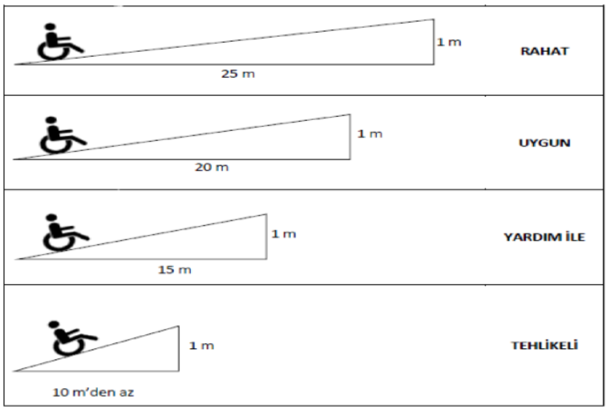
In the activity, how a horse tied with a rope to a pole in the corner of a barn can graze in a certain area is asked. The fact that the lengths of the rope and the barn to which the horse is tied are not given in the activity is expected to push students to different thoughts. Students are expected to predict that the rope and the barn may have different widths/lengths and accordingly the regions where the horse can graze may change. Thus, students are expected to produce different problems and solutions. For example, what kind of a region can the horse graze in if the length of the rope is longer than the length of the barn but shorter than its width or if the length of the rope is equal to the length and width of the barn? In addition, through this activity, students are expected to discover that the set of points equidistant to a point in the plane is a circle and generalise this situation. In these aspects, Activity 1 was handled both in the using- applying and property dimensions within the framework of ADMU.

Figure 2

Activity-II

Task	Handicapped Ramp.
Course	Mathematics
Grade	8
Learning area	Algebra
Learning sub-area	Linear Equations.
Recommended Duration	20’
Teaching-Learning Methods and Techniques	Learning by doing, question-answer, discovery learning
Core Skills	Communication, associating, transferring
Key Concepts	Slope
Tools	Smart board
Outcomes	Explains the slope of the line with models, associates linear equations and graphs with slope
Expectations from Students for the Activity	Students are expected to make calculations by reaching the concept of slope.

Activity: The "General Directorate of Disabled and Elderly Services", which wants to develop measurement standards for disabled ramps and issue a regulation, has reached the following results in its research.



- According to the given information, find the ramp lengths required to reach the heights given in the table below.

HEIGHT	RAMP LENGTH	EASE OF CLIMBING
120 cm		Easy
180 cm		Suitable
240 cm		With help

- Can you help them to determine how the directive should be for the regulation to be issued by the General Directorate of Disabled and Elderly Services?

Activity-II is a daily-life problem and the first item of the activity requires basic algorithms that can be solved with the help of ratio, proportion and similarity in triangles. With this property, Activity-II is evaluated in the using-applying and algorithm dimension of the ADMU framework. In the second part of the activity, students were not given any information about the concept of slope. However, students are expected to reach generalisations by proportioning the length of the opposite right side to the length of the neighbouring right side in a right triangle, that is, students are expected to reach the concept of slope. In this case, Activity-II is evaluated in the property dimension within the framework of ADMU.

Figure 3

Activity-III

Task	Aircraft.
Course	Mathematics
Grade	8
Learning area	Algebra
Learning sub-area	Linear Equations.
Recommended Duration	20’
Teaching-Learning Methods and Techniques	Learning by doing, question-answer, discovery learning
Core Skills	Communication, associating, transferring
Key Concepts	Slope
Tools	Smart board
Outcomes	Explains the slope of the line with models, associates linear equations and graphs with slope
Expectations from Students for the Activity	Students are expected to make calculations using the concept of slope. Students are expected to transfer their knowledge about speed.

Activity: An aeroplane belonging to brand A airline company can take off with a slope of 40% after reaching the maximum speed to take off.

- When this aeroplane reaches an altitude of 1800 m, how many km will it have travelled from a bird's eye view?
- How do you think this aeroplane should change its take-off slope in order to reach an altitude of 1800 m in a shorter time?

Note: The aircraft follows a linear route.

Activity-III is a daily-life problem and the first item of the activity requires the use of algorithms related to the concept of slope. Thus, the activity is handled in the algorithm dimension within the framework of ADMU. The second item of the activity requires students to transfer their knowledge about speed and to establish a relationship with different disciplines. In this respect, the activity includes the relationship building representation dimension of the ADMU framework. In addition, the activity requires students to realise that the length of the hypotenuse will shorten as the slope increases and to make generalisations in this context. Therefore, it is considered in the property dimension of the ADMU framework.

Figure 4

Activity-IV

Task	Linear Equations.
Course	Mathematics
Grade	8
Learning area	Algebra
Learning sub-area	Linear Equations.
Recommended Duration	20’
Teaching-Learning Methods and Techniques	Learning by doing, question-answer, discovery learning
Core Skills	Communication, associating, transferring
Key Concepts	Linear Equation
Tools	Smart board
Outcomes	Creates and interprets equations, tables and graphs of real-life situations involving linear relationships.
Expectations from Students for the Activity	Students are expected to make calculations with the help of equations, tables or graphs.

Activity: The price lists of gasoline and diesel options for brand A car are given in the table below.

Model	Gasoline	Diesel
A	240000	30000

On a day when vehicle prices are as above, fuel prices are given in the table below.

Gasoline (lt)	Diesel (lt)
8 TL	6 TL

Assuming that fuel prices are constant, after how long will the sum of car and fuel costs of vehicle owners who consume 80 litres of fuel per month be equal to each other?

Activity-IV can be solved with the help of algebraic expressions, tables and graphs. The activity contains multiple representations. Therefore, Activity-IV is evaluated in both using-applying and relating-representation dimensions within the framework of ADMU.

Figure 5

Activity-V

Activity: Agricultural activities of human beings date back to approximately ten thousand years ago. While people used to benefit from the plants that grew spontaneously in nature, later they started to grow their own plants. In time, they learnt where, when and how to plant and grow which seeds.

In the table below, an inscription from an ancient civilization shows people's observations about how many days it rained in March in some years.

Years	Week 1	Week 2	Week 3	Week 4
Year of the Bull	5 days	1 day	3 days	5 days
Year of the Fish	4 days	2 days	4 days	5 days
Year of the Ram	3 days	2 days	2 days	6 days
Year of the Sheep	5 days	1 day	2 days	4 days
Year of the Chicken	4 days	3 days	4 days	5 days

- This civilization knows that seeds sown in March will rot if they are too wet. Accordingly, which week could this civilization have chosen as the sowing week?

Activity-V is a daily-life problem. In the activity, students are made to feel a need similar to the needs of past civilisations and they are expected to make some probability calculations to meet this need. In this respect, Activity-V is an activity that arises from a need in the past and leads students to discovery. Therefore, it is evaluated in terms of both using-applying, property and history-culture dimensions.

Figure 6

Activity-VI

Task	Pine tree
Course	Mathematics
Grade	8
Learning area	Numbers and operations
Learning sub-area	Linear Equations.
Recommended Duration	20’
Teaching-Learning Methods and Techniques	Learning by doing, question-answer, discovery learning
Core Skills	Communication, associating, transferring
Key Concepts	Very large and very small numbers, scientific notation
Tools	Smart board
Outcomes	Understands the basic rules of exponential expressions and forms equivalent expressions. Finds the positive integer factors of given positive integers, writes the positive integer factors of positive integers as the product of exponential expressions.
Expectations from Students for the Activity	Students are expected to calculate the integer powers of whole numbers.

Activity: Pine trees double the number of leaves every 3 years. Since a pine seedling has about 256 leaves when it is planted, calculate how many leaves can be found on average in a 120-year-old pine tree.

- Pine trees absorb a large part of the dust generated in cities. Thanks to these features, they have a great importance in terms of preventing environmental pollution and human health. It is known that approximately 1 hectare of pine area absorbs 30 tonnes of dust in a year. Accordingly, find approximately how much dust a pine tree absorbs in 1 year. (A pine tree covers an area of approximately 30 m2)

Activity-VI is a daily- life problem and requires students to make basic calculations related to exponents. In this respect, the activity develops algorithm skills and is evaluated in both using-applying and algorithm dimensions within the framework of ADMU.

2.4. Data Analysis

The data collected through the implementation of the activities were subjected to descriptive analysis within the scope of the ADMU framework. In this process, students' responses were analysed according to dimensions of algorithm, property, using-applying, relating- representation and history-culture. An instance of data analysis is presented in Table 1. (See Appendix-1)

2.5. Validity and reliability

In order to ensure the validity and reliability of the activity design process of the present study, the operational steps followed in the development of the activities are given in detail and expert opinions were taken for each stage. For the implementation process of the activities, the criteria of credibility, transferability and reproducibility were considered (Lincoln & Guba, 1985). In order to ensure the credibility of the study, expert opinions were received both in the process of preparing the activities and analysing the data obtained during the implementation of the activities. In addition, participant control was applied (Creswell & Poth, 2018). The data obtained during the implementation of the activities were first analyzed by the researcher according to the ADMU framework. Then, an expert with a PhD in mathematics education was asked to analyze the activities according to the ADMU framework. Later, the expert and the researcher came together and decided on the ADMU dimensions that emerged in the activities by consensus. In order to ensure the transferability of the study, appropriate and sufficient participation in the data collection process was searched. The data obtained in the study were handled completely in the analysis process and direct quotations were included to describe the findings in detail, and all operation steps were tried to be defined for other scientists who might want to repeat the research. Finally, for confirmability, the results and interpretations in the study were presented without researcher bias.

3. Findings

In the present study, the activities were applied to 21 8th grade students at a secondary school for 5 lesson hours during 1 week. Details about the implementation process of the activities are presented below.

Implementation process of Activity-1

The findings of the study show that the majority of the students (16 students) made appropriate solutions for Activity I. Figure 7 (See Appendix-2) shows that the students made different interpretations for the area where the horse can graze. For example, S1and S3 stated that the length of the rope to which the horse was tied would be the radius of the area where the horse could graze and the area of the circle should be calculated accordingly. He stated that the area of the region where the horse can graze can be found by subtracting the area of the barn from the area of the circle to be calculated. In the solution of S4, it is seen that S4 specified different regions for the area where the horse can graze according to different rope lengths. S11 stated that the horse could run a few laps. This situation shows that the students discovered that the region where the horse can graze will be a circular region. During the activity, it is also seen that the students predicted that the regions where the horse can graze can change according to the different widths/lengths of the rope and the barn, and they were able to produce different solutions accordingly. In this context, it can be said that Activity-I helps students discover that the set of points equidistant to a point in the plane will indicate a circle.

Implementation process of Activity-II

The findings of the study show that the majority of the students (18 students) made appropriate solutions for Activity II. Students were confronted with a daily-life problem and it was observed that students used algorithms to solve the first item of the activity. This shows that Activity-II provides the algorithm and use-application dimensions of the ADMU framework. In the second item of Activity-II, students were not given any information about the concept of slope. The students made comments about increasing the length, decreasing the height and narrowing the angle. However, the fact that the students stated that for a suitable ramp, it would not only be sufficient to reduce the height, but also the length should be increased and the angle should be reduced shows that they reached the concept of slope. Figure 8 presents sample student solutions for Activity II.(See Appendix-3)

Implementation process of Activity-III

The findings of the study show that the majority of the students (15 students) made right solutions for Activity III. Figure-9 (See Appendix-4) shows that, students made solutions by using algorithms for the first item of the activity. This shows that Activity-III was effective in providing the algorithm dimension of the ADMU framework. In the second item of the activity, it is seen that the students discovered that in order for the plane to reach the height of 1800 m in a shorter time, the angle of the plane with the ground should be increased, that is, the slope should be increased in order to shorten the length of the hypotenuse in a right triangle. Therefore, it can be said that Activity-III is a useful activity in providing the property dimension within the framework of ADMU. In addition, Activity-III is a daily-life problem and it is seen that the students transferred their knowledge about speed to it.

Implementation process of Activity-IV

During the implementation of Activity-IV, the students were confronted with a daily-life problem and it was observed that the students mostly (18 students) used algebraic expressions to form equations and reached the solution in this way. Besides, it is seen that there were students who tried to reach the solution by creating simple tables (S8 and S12) and used different representations for the activity. Figure 10 presents sample student solutions for Activity IV.(See Appendix-5)

Implementation process of Activity-V

The findings of the study show that the majority of the students (19 students) made right solutions for Activity V. Activity-V is a daily life problem and students were not given any information about the concept of probability during the implementation of the activity. In this process, students were made to feel a need similar to the need of past civilisations for the concept of probability and it was observed that students made calculations to meet this need. It is seen that the students calculated the probability of a simple event by establishing a ratio in their calculations. In this respect, the activity is an activity that arises from a past need and leads students to discovery. Figure 11 presents sample student solutions for Activity V.(See Appendix-6)

Implementation process of Activity-VI

The findings of the study show that the majority of the students (17 students) made right solutions for Activity VI. It was observed that the students made basic calculations by using exponents in a daily life problem. In order to solve the first question of the activity, it was observed that the students were able to divide a number into its prime factors and write it as exponential expressions, and then they were able to multiply exponential numbers. For the second question of the activity, it was seen that they could reach the solution by establishing simple proportion. Figure 12 presents sample student solutions for Activity V.(See Appendix-7)

4. Discussion and Conclusion

The present study, an activity design framework for the multidimensional development of mathematical understanding (ADMU) was presented. It is thought that the ADMU framework will be useful in designing effective activities to provide opportunities for students to develop their mathematical thinking skills and understanding and will be an important step in enabling the development of targeted students.

An activity considered within the framework of ADMU is expected to develop students' flexible, effective and fluent use of mathematical expressions, symbols and algorithms. In the related literature, among the principles that should be targeted when designing activities for mathematics courses, the development of procedural fluency is targeted (Swan, 2014). This is because the activities in which students algorithmise require thinking (Rasmussen et al., 2005) and exposure to routine activities influence students' ability to reason, answer unusual questions, and transfer their understanding (Boesen et al., 2010). It is also hard to imagine that students who are not good at mathematical procedures will enthusiastically pursue mathematical problem solving (Goldin, 2018). Therefore, it would be wrong to move away from situations where routine procedures and algorithms are used in the activities given to students. In addition, there are studies in the related literature reporting that the most commonly used mathematical activities are those that focus on procedural skills, and that the most common way for students to represent mathematics is symbolic representations (Hatisaru, 2020). However, the dominance of such maths activities is of concern because the development of students' mathematical understanding is as crucial as the development of their operational proficiency and students need to participate in a sufficient richness and variety of maths activities.

Secondly, within the framework of ADMU, the property dimension that requires knowing the properties of concepts, creating a conceptual structure, generating different problems and solutions, creativity, exploring and generalising, establishing cause-effect relationships and making logical inferences is proposed. In the relevant literature, it is stated that mathematical activities should include dimensions such as creating examples, making analysis, evaluating mathematical expressions, making assumptions and generalizations, visualizing and using definitions (Breen & O'Shea, 2019). In particular, since concepts and principles are the basic building blocks of mathematics and understanding a concept is seen as a prerequisite for making complex inferences or achieving any scientific study related to it, they should be included in the activities (Mi et al., 2020). In addition, it is thought that activities should encourage understanding the essence of mathematical concepts and mathematical discovery (Chen & Weng, 2003). However, studies in the related field generally show that the design and implementation of such activities are quite difficult for educators (Paolucci & Wessels, 2017).

The third dimension in the ADMU framework is the using-applying dimension, which requires applying and transferring mathematical concepts to new and different situations and making connections with daily life. When designing activities for mathematics lessons, it is aimed to develop students' ability to solve non-routine problems and extend this to solving real-world problems (Swan, 2014). There are studies in the relevant literature that conclude that associating the activities carried out in mathematics lessons with daily life increases students’ interest in the lesson and increases the retention of what is learned (Boaler, 2011). In addition, it is also seen that teachers take into account the association of the activities they develop and implement with daily life (Koç, 2019). However, there are also studies showing that activities based on problems that require students' cognitive activation, direct them to think critically, and include multiple solutions based on real life are rarely implemented in classrooms (Hatisaru, 2020). However, since mathematics is closely related to real life, it is expected to carry out course activities in a way that is connected to students’ real lives (Yang, 2016).

The fourth dimension within the framework of ADMU is the relating-representation dimension, which requires establishing relationships both with the concepts within mathematics and with different disciplines, using multiple representations and establishing relationships between them. Since the essence of mathematical understanding is to connect mathematical facts and concepts and to relate newly acquired knowledge to what is already known (Sierpinska, 2013), students are expected to establish connections and relationships between different aspects of mathematical knowledge, to develop the same phenomenon from multiple mathematical perspectives, and to develop a deeper understanding of mathematical concepts and relationships by creating, using and comparing various representations (NCTM, 2000). The literature supports that students can achieve good learning by making connections between the procedures and relationships underlying concepts (Berry & Nyman, 2003). In addition, working with rich mathematical activities and representations contributes to students' flexible thinking and problem-solving development (Huinker, 2015). It is suggested that teaching and learning activities in effective mathematics classrooms should be structured in a way that both uses different forms of representation and enables students to translate between these forms of representation (Lesh et al., 2003).

Finally, within the framework of ADMU, the history-culture dimension, which requires knowledge of how mathematical concepts emerged in history and how they were used in different cultures, is proposed. Activities from the history of mathematics contribute to students' learning mathematical concepts in a more meaningful way by showing how mathematical concepts are come out (Fauvel, 1991; Özdemir & Yıldız, 2015). Because the activities carried out with a historical perspective serve as a bridge for students to transition from theoretical structure to practical thinking (Gulikers & Blom, 2001). It is thought that the history of mathematics should be included in the lessons, especially because seeing the problems that emerged in the development of mathematics helps students to solve the problems they currently encounter and this contributes to the development of students' mathematical thinking (Liu, 2003). In the related literature, there are studies concluding that courses with activities designed based on the history of mathematics have an effect on students' mathematics achievement (İdikut, 2007).

In the process of implementing the activities designed within the framework of ADMU, it was seen that each activity provided an opportunity for students to engage in a series of meaningful mathematical thoughts. The students' responses to the activities provide evidence that the activities had the desired effect in general. It was observed that the students who participated in the study were interested in being engaged in such activities and that these activities usually had multiple solution methods or required students to explain their thoughts, thus encouraging students to generate different ideas. For example, students stated that the non-routine nature of the designed activities made them think more and they tried to construct meaning by making connections between concepts, providing justifications, logical inferences and generalisations. Therefore, unlike traditional verbal problems, activities within the ADMU framework, in which students develop their own mathematical ideas, justify their approaches throughout the solution process, establish relationships between mathematical concepts and develop solutions to a real- life situation, are considered to be appropriate materials that can be used to develop multidimensional understanding in mathematics lessons. In the related literature, there are studies showing the effectiveness of the results of the applications of such activities (O'Shea & Breen, 2021). Therefore, in order to achieve mathematical understanding in students in all its dimensions, activities within the framework of ADMU that will enable students to create and use formulas, make generalisations, make predictions, make discoveries, produce multiple solutions, use multiple representations, establish relationships between mathematical concepts and different disciplines, and translate real-life situations into mathematical language by making use of the emergence of mathematics in history can be included from primary school and students can be encouraged to participate in all kinds of mathematical activities.

Conflict of Interest

The study was conducted by the author and I declare that there is no conflict of interest.

Declaration of Generative AI Use

No generative artificial intelligence tools (ChatGPT, Bard, Claude, Copilot, etc.) were used in the preparation of this study. All text, analysis, and content were produced by the author(s) with human input.

Ethical Statement

The study involved observational classroom activities conducted with middle school students. Written permission was obtained from parents, and informed consent forms were obtained from the students. The study was approved by the

Kastamonu University Faculty of Science and Engineering Scientific Research and Publication Ethics Committee (Ethics Committee Approval Number: 3/2 / Date: 27.02.2023).

Author Contributions

The study was conducted by the author and no other author contributed.

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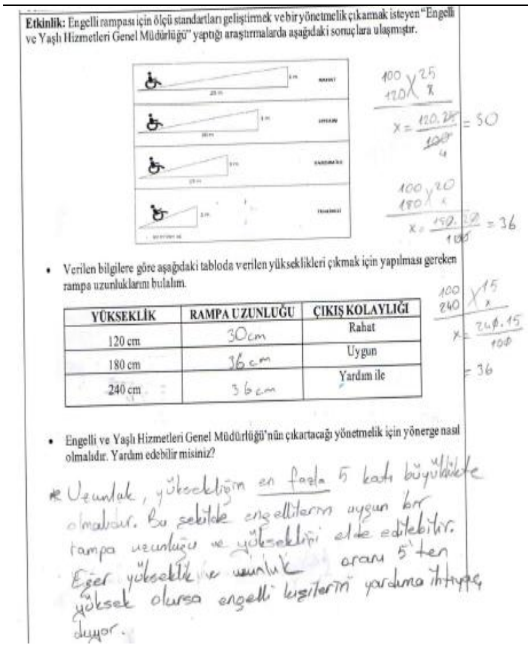
Appendix 1

Table 1

Sample Data Analysis

Sample Activity

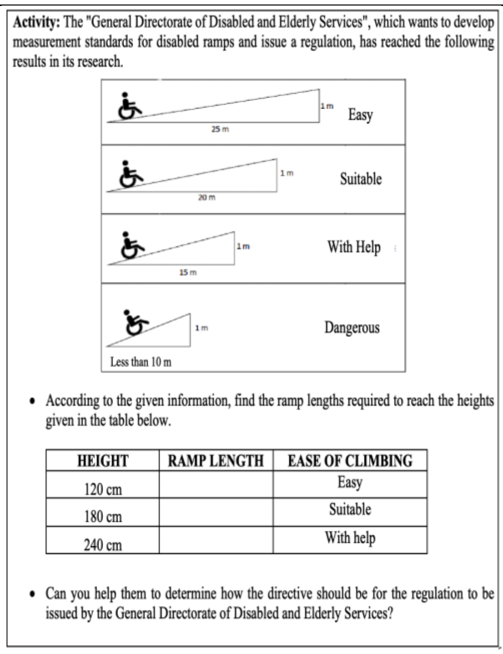


Etkinlik: Engelli rampası için ölçü standartları geliştirmek ve bir yönetmelik çıkarmak isteyen "Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü" yaptığı araştırmalarda aşağıdaki sonuçlara ulaşmıştır.

- Verilen bilgilere göre aşağıdaki tabloda verilen yükseklikleri çıkmak için yapılması gereken rampa uzunluklarını bulalım.

YÜKSEKLİK	RAMPA UZUNLUĞU	ÇIKIŞ KOLAYLIĞI
120 cm	30cm	Rahat
180 cm	36 cm	Uygun
240 cm	36 cm	Yardım ile

- Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü'nün çıkartacağı yönetmelik için yönerge nasıl olmalıdır. Yardım edebilir misiniz?



Activity: The "General Directorate of Disabled and Elderly Services", which wants to develop measurement standards for disabled ramps and issue a regulation, has reached the following results in its research.

- According to the given information, find the ramp lengths required to reach the heights given in the table below.

HEIGHT	RAMP LENGTH	EASE OF CLIMBING
120 cm		Easy
180 cm		Suitable
240 cm		With help

- Can you help them to determine how the directive should be for the regulation to be issued by the General Directorate of Disabled and Elderly Services?

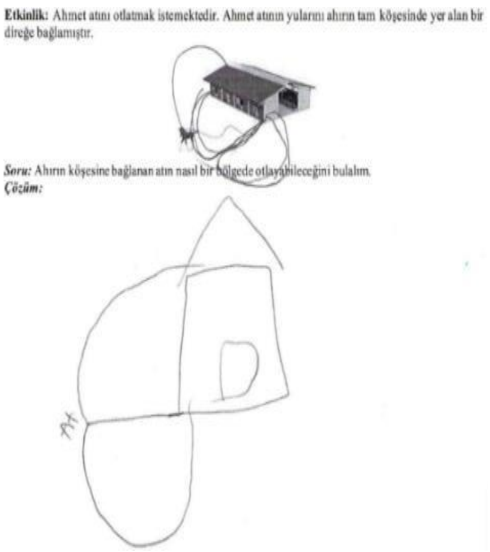
Length should be no more than 5 times the heigh. In these conditions, a ramp length and height suitable for the disabled can be obtained. If the ratio of height to length is more than 5 disabled people need help.

S7's solution for Activity 2	
Data Analysis	
Algorithm	The student used mathematical symbols and algorithms. He/she transferred his/her knowledge about ratio-proportion.
Property	The student reached the concept of slope by making generalizations on the ratio between length and height.
Using-applying	The student established a relationship with daily life.
Relating-Representation	The student did not establish a relationship with different disciplines or did not use multiple representations
History-Culture	The student did not establish a relationship with the history of mathematics

Appendix 2

Figure 7

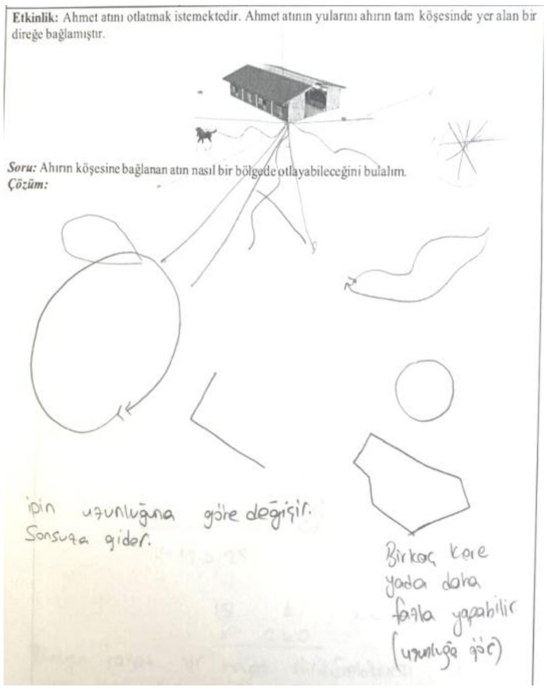
Students' solutions for Activity-I



S4's solution for Activity 1

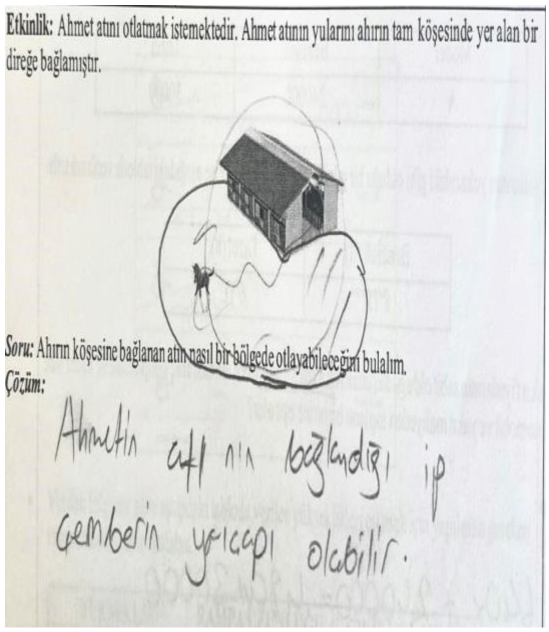
Let's find the length of the rope. Use the length of the rope as the radius of the circle and find the area of the circle. Let's subtract the area of the barn from the area of the circle

S1's solution for Activity 1



Depends on the length of the rope,
Can take a few turns

S11's solution for Activity 1



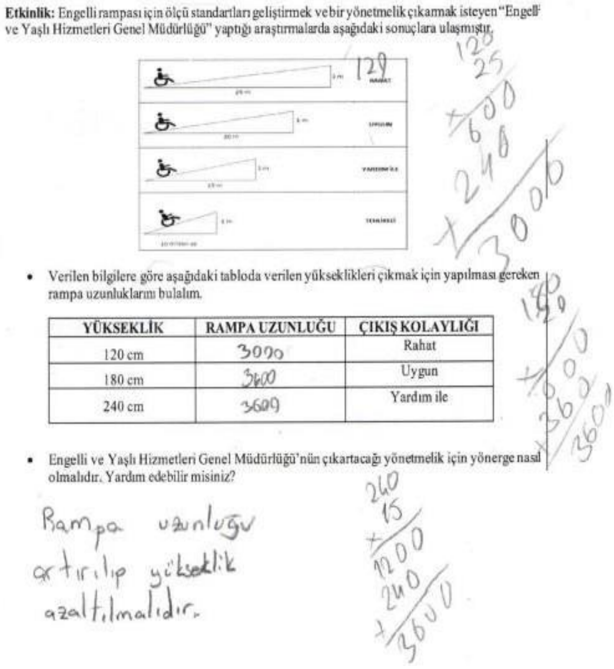
The rope that Ahmet tied the horse to could be the radius of the circle

S3's solution for Activity 1

Appendix 3

Figure 8

Students' solutions for Activity-II



Etkinlik: Engelli rampası için ölçü standartları geliştirmek ve bir yönetmelik çıkarmak isteyen "Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü" yaptığı araştırmalarda aşağıdaki sonuçlara ulaşmıştır.

- Verilen bilgilere göre aşağıdaki tabloda verilen yükseklikleri çıkmak için yapılması gereken rampa uzunluklarını bulalım.

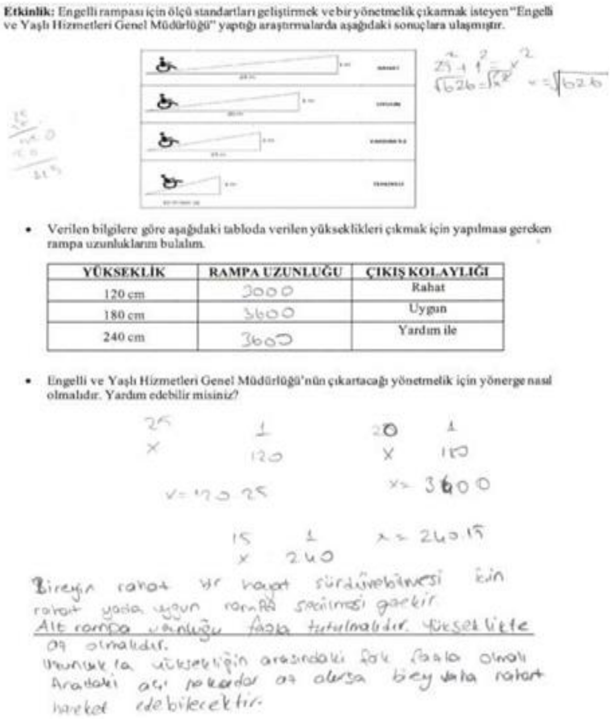
YÜKSEKLİK	RAMPA UZUNLUĞU	ÇIKIŞ KOLAYLIĞI
120 cm	3000	Rahat
180 cm	3600	Uygun
240 cm	3600	Yardım ile

- Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü'nün çıkartacağı yönetmelik için yönerge nasıl olmalıdır. Yardım edebilir misiniz?

Rampa uzunluğu artırılıp yükseklik azaltılmalıdır.

Ramp length should be increased and height should be decreased.

S5's solution for Activity 2



Etkinlik: Engelli rampası için ölçü standartları geliştirmek ve bir yönetmelik çıkarmak isteyen "Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü" yaptığı araştırmalarda aşağıdaki sonuçlara ulaşmıştır.

- Verilen bilgilere göre aşağıdaki tabloda verilen yükseklikleri çıkmak için yapılması gereken rampa uzunluklarını bulalım.

YÜKSEKLİK	RAMPA UZUNLUĞU	ÇIKIŞ KOLAYLIĞI
120 cm	3000	Rahat
180 cm	3600	Uygun
240 cm	3600	Yardım ile

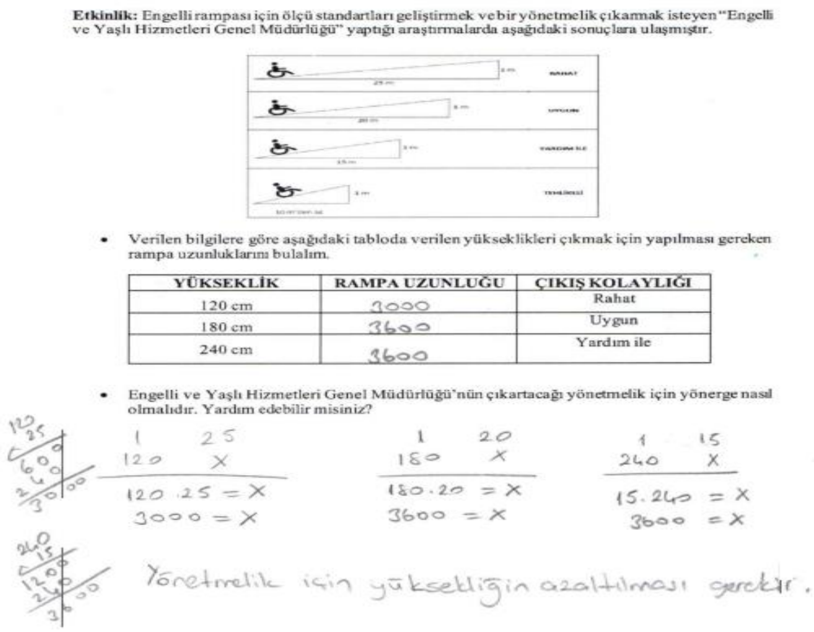
- Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü'nün çıkartacağı yönetmelik için yönerge nasıl olmalıdır. Yardım edebilir misiniz?

25 — 1, x — 120, x = 120.25
20 — 1, x — 180, x = 3600
15 — 1, x — 240, x = 240.15

Bireyin rahat bir hayat sürdürebilmesi için rahat yada uygun rampa seçilmesi gerekir.
Alt rampa uzunluğu fazla tutulmalıdır. Yükseklikte az olmalıdır.
Uzunluk ile yüksekliğin arasındaki fark fazla olmalı. Aradaki açı ne kadar az olursa birey daha rahat hareket edebilecektir.

An individual should choose a comfortable or suitable ramp in order to lead a comfortable life. In order for the individual to lead a comfortable life, he or she must choose a comfortable or suitable ramp.
The rapping length should not be excessive. The height should also be less. The difference between length and height should be large. The smaller the angle, the more comfortable the individual will move.

S11's solution for Activity 2



Etkinlik: Engelli rampası için ölçü standartları geliştirmek ve bir yönetmelik çıkarmak isteyen "Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü" yaptığı araştırmalarda aşağıdaki sonuçlara ulaşmıştır.

- Verilen bilgilere göre aşağıdaki tabloda verilen yükseklikleri çıkmak için yapılması gereken rampa uzunluklarını bulalım.

YÜKSEKLİK	RAMPA UZUNLUĞU	ÇIKIŞ KOLAYLIĞI
120 cm	3000	Rahat
180 cm	3600	Uygun
240 cm	3600	Yardım ile

- Engelli ve Yaşlı Hizmetleri Genel Müdürlüğü'nün çıkartacağı yönetmelik için yönerge nasıl olmalıdır. Yardım edebilir misiniz?

1 — 25, 120 — x, 120.25 = x, 3000 = x
1 — 20, 180 — x, 180.20 = x, 3600 = x
1 — 15, 240 — x, 15.240 = x, 3600 = x

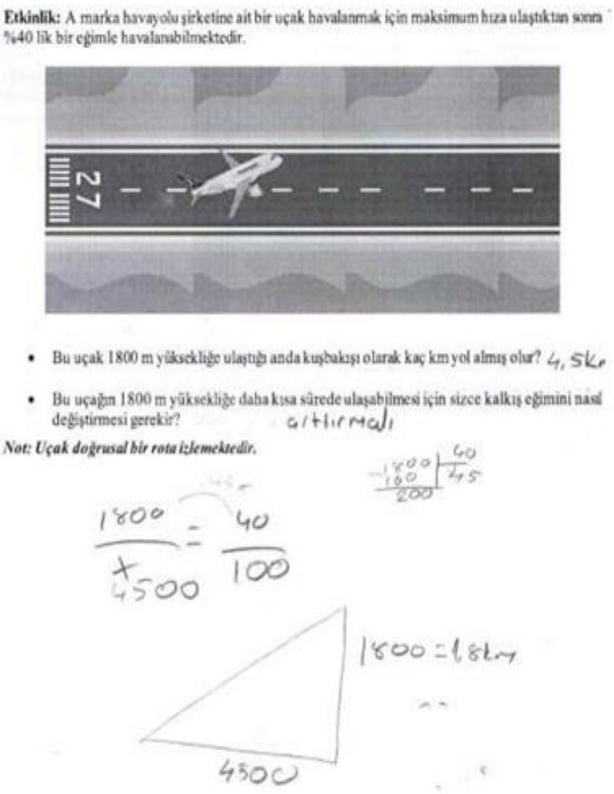
Yönetmelik için yüksekliğin azaltılması gerekir.

For regulation, the height must be reduced

S19's solution for Activity 2

Appendix 4

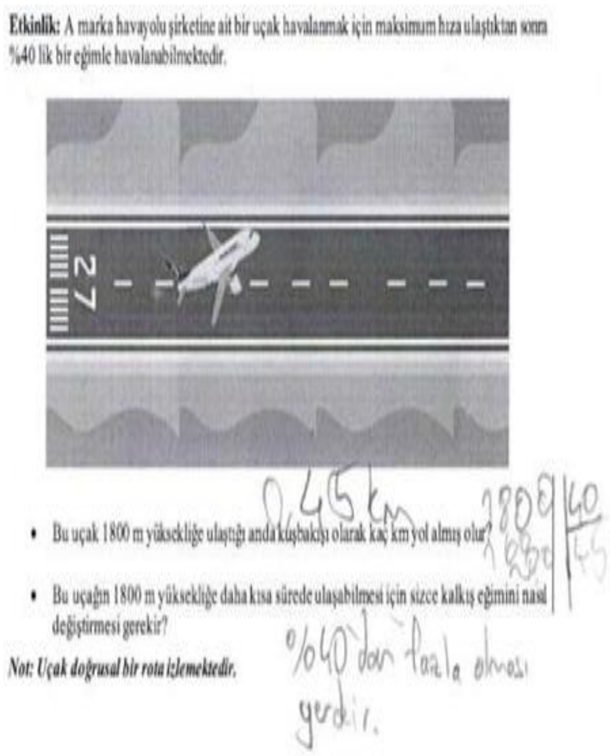
Figure 9



Etkinlik: A marka havayolu şirketine ait bir uçak havalanmak için maksimum hıza ulaştıktan sonra %40 lık bir eğimle havalanabilmektedir.

- Bu uçak 1800 m yüksekliğe ulaştığı anda kuşbakışı olarak kaç km yol almış olur? 4,5km
- Bu uçağın 1800 m yüksekliğe daha kısa sürede ulaşabilmesi için sizce kalkış eğimini nasıl değiştirmesi gerekir? artırmalı

Not: Uçak doğrusal bir rota izlemektedir.



Etkinlik: A marka havayolu şirketine ait bir uçak havalanmak için maksimum hıza ulaştıktan sonra %40 lık bir eğimle havalanabilmektedir.

- Bu uçak 1800 m yüksekliğe ulaştığı anda kuşbakışı olarak kaç km yol almış olur? 0,45 km
- Bu uçağın 1800 m yüksekliğe daha kısa sürede ulaşabilmesi için sizce kalkış eğimini nasıl değiştirmesi gerekir?

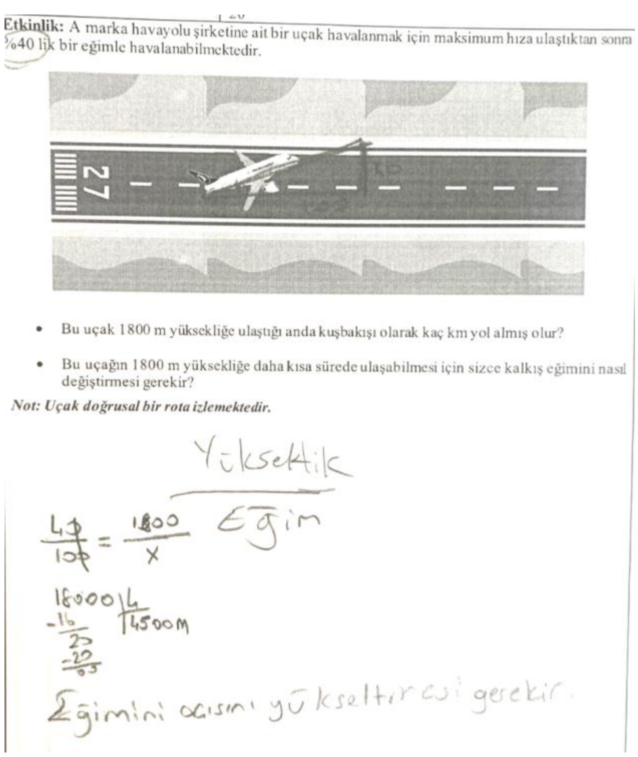
Not: Uçak doğrusal bir rota izlemektedir.

%40'dan fazla olması gerekir.

It should be more than 40%

S10's solution for Activity III

S20's solution for Activity III



Etkinlik: A marka havayolu şirketine ait bir uçak havalanmak için maksimum hıza ulaştıktan sonra %40 lık bir eğimle havalanabilmektedir.

- Bu uçak 1800 m yüksekliğe ulaştığı anda kuşbakışı olarak kaç km yol almış olur?
- Bu uçağın 1800 m yüksekliğe daha kısa sürede ulaşabilmesi için sizce kalkış eğimini nasıl değiştirmesi gerekir?

Not: Uçak doğrusal bir rota izlemektedir.

Yükseklik / Eğim

Eğimini açısını yükseltirsi gerekir.

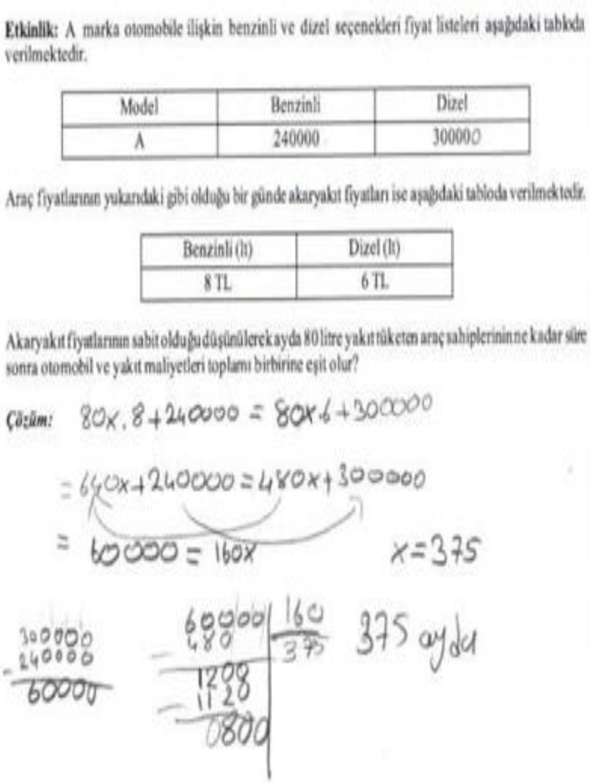
It needs to increase its slope and angle

S17's solution for Activity III

Appendix 5

Figure 10

Students' Solutions for Activity-IV



Etkinlik: A marka otomobile ilişkin benzinli ve dizel seçenekleri fiyat listeleri aşağıdaki tabloda verilmektedir.

Model	Benzinli	Dizel
A	240000	300000

Araç fiyatlarının yukarıdaki gibi olduğu bir günde akaryakıt fiyatları ise aşağıdaki tabloda verilmektedir.

Benzinli (lt)	Dizel (lt)
8 TL	6 TL

Akaryakıt fiyatlarının sabit olduğu düşünülerek ayda 80 litre yakıt tüketen araç sahiplerinin ne kadar süre sonra otomobil ve yakıt maliyetleri toplamı birbirine eşit olur?

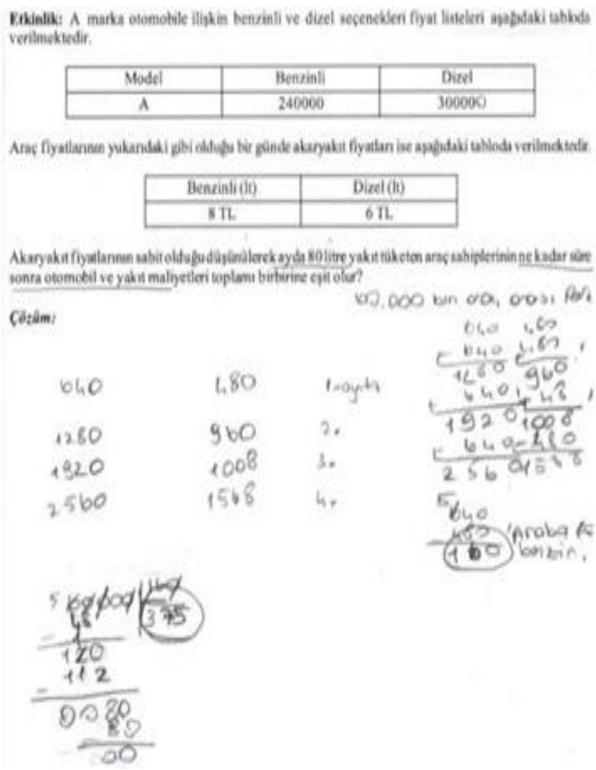
Çözüm: 80x.8+240000 = 80x6+300000

= 640x+240000 = 480x+300000

= 60000 = 160x   x=375

375 ayda

S10's solution for Activity IV



Etkinlik: A marka otomobile ilişkin benzinli ve dizel seçenekleri fiyat listeleri aşağıdaki tabloda verilmektedir.

Model	Benzinli	Dizel
A	240000	300000

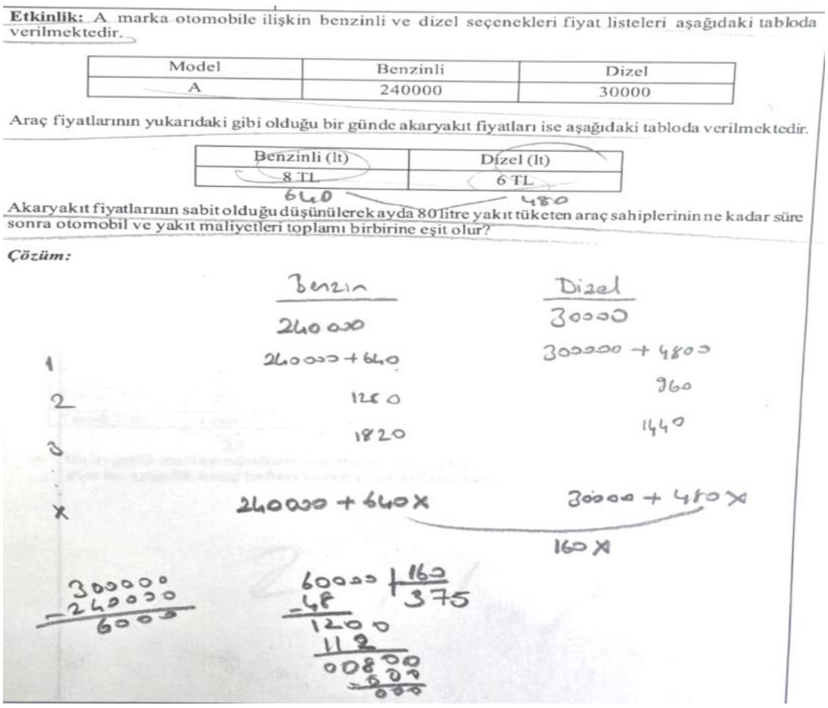
Araç fiyatlarının yukarıdaki gibi olduğu bir günde akaryakıt fiyatları ise aşağıdaki tabloda verilmektedir.

Benzinli (lt)	Dizel (lt)
8 TL	6 TL

Akaryakıt fiyatlarının sabit olduğu düşünülerek ayda 80 litre yakıt tüketen araç sahiplerinin ne kadar süre sonra otomobil ve yakıt maliyetleri toplamı birbirine eşit olur?

Çözüm:

S12's solution for Activity IV



Etkinlik: A marka otomobile ilişkin benzinli ve dizel seçenekleri fiyat listeleri aşağıdaki tabloda verilmektedir.

Model	Benzinli	Dizel
A	240000	30000

Araç fiyatlarının yukarıdaki gibi olduğu bir günde akaryakıt fiyatları ise aşağıdaki tabloda verilmektedir.

Benzinli (lt)	Dizel (lt)
8 TL	6 TL

Akaryakıt fiyatlarının sabit olduğu düşünülerek ayda 80 litre yakıt tüketen araç sahiplerinin ne kadar süre sonra otomobil ve yakıt maliyetleri toplamı birbirine eşit olur?

Çözüm:

Benzin | Dizel

240000 | 30000

240000+640 | 300000+480

240000+640x | 30000+480x

160x

S8's solution for Activity IV

Appendix 6

Figure 11

Students' solutions for Activity-V

Etkinlik: İnsanoğlunun tarımsal faaliyetleri yaklaşık olarak on bin yıl öncesine dayanmaktadır. İnsanlar önceleri doğada kendiliğinden yetişen bitkilerden faydalanırken sonraları kendi bitkilerini yetiştirmeye başlamışlardır. Zamanla hangi tohumun nerede, ne zaman ve nasıl ekilip yetiştirilmesi gerektiğini öğrenmişledir.

Aşağıdaki tabloda eski bir uygarlığa ait yazıtta bazı yıllarda mart ayında kaç gün yağmur yağdığına ilişkin insanların gözlemlerini göstermektedir.

Yıllar	1.Hafta	2.Hafta	3.Hafta	4. Hafta
Boğa Yılı	5 gün	1 gün	3 gün	5 gün
Balık Yılı	4 gün	2 gün	4 gün	5 gün
Koç Yılı	3 gün	2 gün	2 gün	6 gün
Koyun Yılı	5 gün	1 gün	2 gün	4 gün
Tavuk Yılı	4 gün	3 gün	4 gün	5 gün

- Bu uygarlık mart ayında ekilen tohumların çok ıslak kaldığında çürüyeceğini bilmektedir. Buna göre bu uygarlık hangi haftayı tohum ekim haftası olarak seçilmiş olabilir?

1.hafta=21 gün
2.hafta=9 gün
3.hafta=15 gün
4.hafta=25 gün

2.hafta

1st week = 21 days
2st week = 9 days
3st week = 15 days
4st week = 25 days

2st week

S10's solution for Activity V

Etkinlik: İnsanoğlunun tarımsal faaliyetleri yaklaşık olarak on bin yıl öncesine dayanmaktadır. İnsanlar önceleri doğada kendiliğinden yetişen bitkilerden faydalanırken sonraları kendi bitkilerini yetiştirmeye başlamışlardır. Zamanla hangi tohumun nerede, ne zaman ve nasıl ekilip yetiştirilmesi gerektiğini öğrenmişledir.

Aşağıdaki tabloda eski bir uygarlığa ait yazıtta bazı yıllarda mart ayında kaç gün yağmur yağdığına ilişkin insanların gözlemlerini göstermektedir.

Yıllar	1.Hafta	2.Hafta	3.Hafta	4. Hafta	
Boğa Yılı	5 gün	1 gün	3 gün	5 gün	=14
Balık Yılı	4 gün	2 gün	4 gün	5 gün	15
Koç Yılı	3 gün	2 gün	2 gün	6 gün	13
Koyun Yılı	5 gün	1 gün	2 gün	4 gün	12
Tavuk Yılı	4 gün	3 gün	4 gün	5 gün	16
5 hafta = 21	9	15	25		

- Bu uygarlık mart ayında ekilen tohumların çok ıslak kaldığında çürüyeceğini bilmektedir. Buna göre bu uygarlık hangi haftayı tohum ekim haftası olarak seçilmiş olabilir?

Haftanın 5 yada 6 gün yağmur yağması çok fazla
2. haftadır Çünkü yağışlı gün sayısı az.
Yağmur yağma oranı : 1.hafta 21/5 = 4,2
2.hafta 9/5 = 1,8
3.hafta 15/5 = 3
4.hafta 25/5 = 5

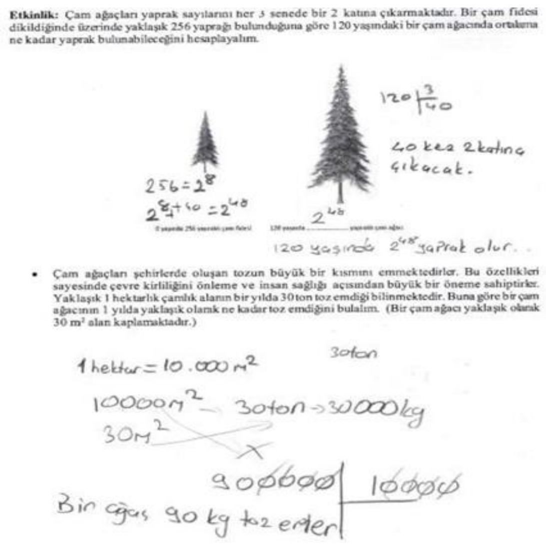
It's too much to rain 5 or 6 days a week. 2st week. Because the number of rainy days in the 2nd week is the least. Rainfall rate: 1st week 21/5=4,2
2st week=9/15=1,8
3st week 15/5=3
4st week 25/5=5

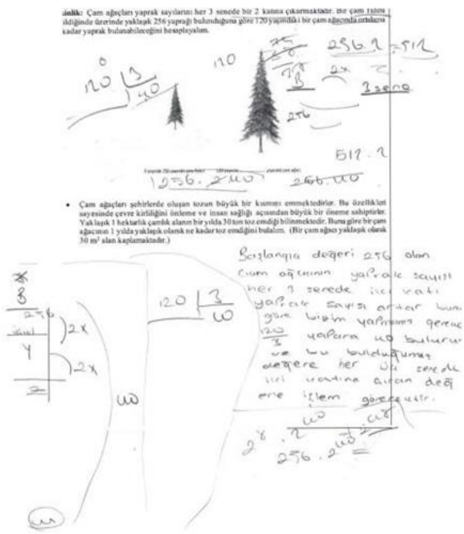
S11's solution for Activity V

Appendix 7

Figure 12

Students' Solutions For Activity-VI

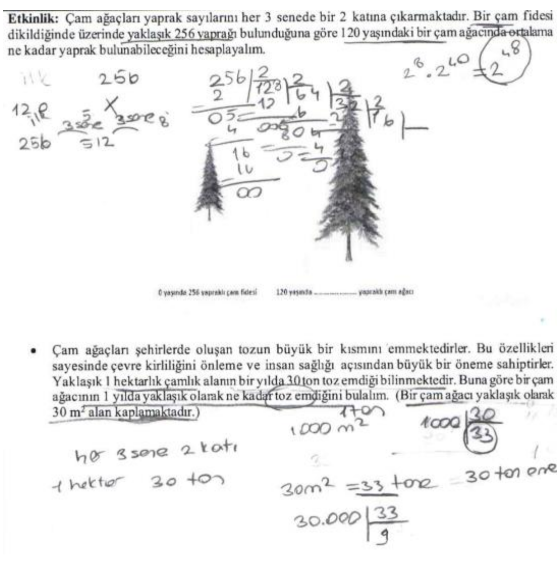




A pine tree with an initial value of 256 increases the number of leaves by 2 times every 3 years. In this case, what we need to do is 120/3=40. We need to trade between this value we found and the value that doubles every three years.

S10's solution for Activity VI

S13's solution for Activity VI



S12's solution for Activity VI