Determining the Production Amounts in Textile Industry with Fuzzy Linear Programming

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ABSTRACT

In the early part of the last century, making the mathematical model of the real problems encountered became one of the most important issue of engineering. At this point, operations research has begun to be applied in decision making problems. However, as a result of daily life in the developing process, very complex problems have been encountered. Fuzzy set theory introduced by Zadeh tried to cope with complexity caused by uncertainty and lack of information. In this study, a fuzzy linear programming model was constructed using data from a factory which produces cloth types in textile industry. Solution of the model gives the amount of production for each cloth type in order to get optimal solution for profit maximizing. In this way, problems that may occur such as cost, waste of time, overstock and customer loss will be prevented.

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Introduction

Decision making (DM) helps executives for determining the best alternative in any decision problem which comprise of procedures and criteria. DM generally depend on Decision Support Systems (DSS) tools (Nucci, Cavallo, and Grieco 2007). Selecting and applying of decision making method which is the most appropriate with structure of the problem provide a useful insight to executives for giving rational decision. Structure of the problem must be understood in order to select the appropriate decision support method. If there is any imprecision, uncertainty or incompleteness situation in the problem, decision support tools integrated fuzzy logic should be used for getting better solution.

Zadeh and Bellman propounded the notion of maximizing the decision for decision making problems. A fuzzy approach concerning multi-objective Linear Programming (LP) problems was introduced by Zimmermann. Studies in recent years suggest new techniques with the purpose of ranking fuzzy numbers and coming to an optimal solution (Gani, Duraisamy, and Veeramani 2009).

Many authors proposed several approaches and solved their own problems with FLP model. For example; Abdullah and Abidin are used FLP with single objective function for getting optimal solutions and profits of red meat production problem. They successfully obtained to the profit of red meat production with the variability of fuzzy memberships in FLP (Abdullah and Abidin 2016). Kalaf et al., 2015 have developed a fuzzy multi-objective model for solving aggregate production planning problems that contain multiple both periods and products in fuzzy environments. They adopted a new method that utilizes a Zimmermans approach. This proposed model attempts to minimize total production costs and labor costs synchronically (Kalaf et al. 2015). Elamvazuthi et al. solved a FLP problem in which the parameters involved are fuzzy quantities with logistic membership functions. They determined monthly profit and production planning quotas via numerical example of home-textile group to explore the applicability of the method (Elamvazuthi et al. 2010). Demiral used FLP model for production planning problem of a dairy industry because of the uncertain supply of milk and demand of dairy products. Results of his study was shown that FLP is more realistic than LP (Demiral 2013). Herath and Samarathunga are presented a fuzzy multi-criteria mathematical programming model. This study was undertaken to find out the optimal allocation for profit maximizing and cost minimizing subjected to the utilizing of ‘water and demand’ constraint. They achieved to get optimal production plan with this model (Herath and Samarathunga 2015). Tan et al. developed a fuzzy linear programming enterprise input–output model to determine optimal adjustments in production levels of multi-product systems when a crisis is induced by a loss of resource inputs (Tan et al. 2015). Afzali, Rafsanjani, and Saed 2016 used fuzzy multi-objective linear programming model for supplier selection problem. They utilised group decision making. Ebrahimnejad and Tavana developed a new method for solving FLP problems. In this method, the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers. However, the elements of the coefficient matrix are represented by real numbers (Ebrahimnejad and Tavana 2014).

In this study, FLP method is used for determining amounts of 5 different cloth types under uncertainty environment. The objective of this paper is to determine amounts of these products for obtaining maximum profit and minimum deviation of demand by taking capacities of labour and production into consideration.

This paper is organized as follows. Section 2 and 3 respectively describes FLP and types of FLP. Zimmermann Method is described in Section 4. A case study and its results presented in Section 5. Finally, conclusions are given in Section 6.

Fuzzy Linear Programming

Linear Programming

A LP problem is a special case of Mathematical Programming problem. From an analytical perspective, a mathematical program attempts to identify a minimum or maximum point of a function, which furthermore satisfies a set of constraints. Objective function and problem constraints are linear in LP (Dervişoğlu Toprak 2005).

A classical model of LP, also called a crisp LP model, may have the following formulation:

\[
\begin{align*}
\text{Max } \quad & \quad Cx \\
\text{s.t. } \quad & \quad Ax \leq b, \quad i = 1, \ldots, m
\end{align*}
\]  

in which x is an n × 1 alternative set, C is a 1 × n coefficients of an objective function, Ai is an m × n matrix of coefficients of constraints and bi is an m × 1 right-hand sides.
The traditional problems of LP are solved with LINDO optimization software and obtain the optimal solution in a precise way. If coefficients of constraints, objective function or the right-hand sides are imprecise, in other words, being fuzzy numbers, traditional algorithms of LP are unsuitable to solve the fuzzy problem and to obtain the optimization.

In the real world, the coefficients are typically imprecise numbers because of insufficient information, for instance, technological coefficients. Many researchers formed FLP of various types, invented approaches to convert them into crisp LP, and finally solved the problems with available software (Lee and Wen 1996).

**Fuzzy Linear Programming**

FLP follows from the fact that classical LP is often insufficient in practical situations. In reality, certain coefficients that appear in classical LP problems may not be well-defined, either because their values depend on other parameters or because they cannot be precisely assessed and only qualitative estimates of these coefficients are available. FLP is an extension of classical LP and deals with imprecise coefficients by using fuzzy variables (Ren and Sheridan 1994).

We consider the FLP Problem

\[ \text{Max } \widetilde{Z} = \widetilde{C}^T x \]

\[ \text{s.t. } \]

\[ \widetilde{A} x \leq \widetilde{b} \]

\[ x \geq 0 \]

The solution of this problem is to find the possibility distribution of the optional objective function \( Z \). Many researchers had handled this problem by converting the fuzzy objective function and the fuzzy constraints into crisp ones (Yenilmez, Rafail, and Gasimor 2002).

**Types of Fuzzy Linear Programming**

FLP model divides into parts in terms of fuzzy coefficients. For instance, while objective function is fuzzy, constraints cannot be fuzzy. Combinations of possible situations are briefly introduced as below:

**Objective Function is Fuzzy**

In a real life, there are many situations that parameters of objective function (profit and cost) are imprecise. FLP model of this was propounded by Verdegay.

**Right-Hand Sides are Fuzzy**

There are two approach for this type of problem. While first approach concerning asymmetric models belongs to Verdegay, second approach concerning symmetric model belongs to Werners.

**Right-Hand Sides and Coefficients of Constrains are Fuzzy**

Negoita and Sularia developed an approach for this type of FLP model.

**Objective Function and Constrains are Fuzzy**

As it is understood the title, in this model, both objective function and constrains involve fuzziness. Zimmermann and Chanas have different approaches about it.

**All Coefficients are Fuzzy**

Sometimes, all coefficients can be fuzzy in the problem. Carlsson and Korhonen developed the approach for this.

**Zimmermann Method**

A LP with a fuzzy objective function and fuzzy inequalities shown by Zimmermann is indicated as follows: [13]

\[ \text{Max } \widetilde{Z} = \widetilde{C}^T x \]

\[ \text{s.t. } \]

\[ \widetilde{A} x \leq \widetilde{b} \]

\[ x \geq 0 \]

Inequality is a symmetrical model of which the objective function becomes one constraint. To write a general formulation, inequality is converted to a matrix form as [13]:

\[ -\widetilde{C}^T x \leq \widetilde{b} \]

In which

\[ B = \begin{bmatrix} \widetilde{C} \\ -A \end{bmatrix} \]

\[ b = \begin{bmatrix} -\widetilde{b} \\ \widetilde{b} \end{bmatrix} \]
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The degree of violation is represented by membership function as [13]:
\[
\mu_b(x) = \begin{cases} 
0 & \text{if } Cx \leq b_0 - d_0 \\
1 - \frac{b_0 - Cx}{d_0} & \text{if } b_0 - d_0 \leq Cx \leq b_0 \\
1 & \text{if } Cx \leq b_0 
\end{cases} 
\]
(6)

\[
\mu_i(x) = \begin{cases} 
0 & \text{if } (Ax_i) \geq b_i + d_i \\
1 - \frac{(Ax_i) - b_i}{d_i} & \text{if } b_i \leq (Ax_i) \leq b_i + d_i \\
1 & \text{if } (Ax_i) \leq b_i 
\end{cases} 
\]
(7)

In which \( d \) is a matrix of admissible violation.

This problem can be transformed by introducing the auxiliary variable \( \lambda \) as follows:
\[
\mu_b(x) \geq \lambda \\
\mu_i(x) \geq \lambda \\
\lambda \in [0,1] 
\]
(8)

This problem can be stated as LP as follows:
\[
\begin{align*}
\text{Max } & \lambda \\
\text{s.t. } & \mu_b(x) \geq \lambda \\
& \mu_i(x) \geq \lambda \\
& \lambda \in [0,1] 
\end{align*} 
\]
(9)

This problem was shown with membership functions of fuzzy objective function and fuzzy constrains as follows:
\[
\begin{align*}
\text{Max } & \lambda \\
\text{s.t. } & 1 - \frac{b_0 - Cx}{d_0} \geq \lambda \\
& 1 - \frac{(Ax_i) - b_i}{d_i} \geq \lambda, \forall i \\
& \lambda \in [0,1] \\
& x \geq 0 
\end{align*} 
\]
(10)

After some simplification, FLP model obtains as follows:
\[
\begin{align*}
\text{Max } & \lambda \\
\text{s.t. } & C^T x - \lambda d_b \geq b_0 - d_0 \\
& (Ax) + \lambda d \leq b_1 + d_1, \forall i \\
& \lambda \in [0,1] \\
& x \geq 0 
\end{align*} 
\]
(11)

Table 1. Data for FLP Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit profits (TRY per meter)</td>
<td>1.44</td>
<td>0.5</td>
<td>0.16</td>
<td>0.11</td>
<td>0.39</td>
</tr>
<tr>
<td>Expected demands (meter per month)</td>
<td>19860 2460 2280 1370 5390</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tolerances for demands (meter per month)</td>
<td>950 410 110 220 390</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labour usage (hour per meter)</td>
<td>0.28 0.03 0.03 0.02 0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected profit (TRY)</td>
<td>41667</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tolerance for profit (TRY)</td>
<td>9500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly production capacity (meter)</td>
<td>60120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monthly labour capacity (hour)</td>
<td>54,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Application

Problem Definition

Data used for the application was obtained a factory in a textile industry. It produces 5 different cloth types. Since the expected profit and the demand of the product types are uncertain the problem is built as FLP model in order to determine production amounts per month for each product type for maximizing the profit. Data about the production and its constraints are given in Table I.

FLP Model

Problem was modelled as monthly basis. The FLP model of the problem is given below:
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\[ C^T x = 1.44x_1 + 0.5x_2 + 0.16x_3 + 0.11x_4 + 0.39x_5 \]
\[ b_0 = 41667 \quad d_0 = 9500 \]
\[ b_1 = 19860 \quad d_1 = 950 \]
\[ b_2 = 2460 \quad d_2 = 410 \]
\[ b_3 = 2280 \quad d_3 = 110 \]
\[ b_4 = 1370 \quad d_4 = 220 \]
\[ b_5 = 5390 \quad d_5 = 390 \]

(12)

\[ \text{Max } \lambda \]
\[ \text{s.t.} \]
\[ 1.44x_1 + 0.5x_2 + 0.16x_3 + 0.11x_4 + 0.39x_5 - 9500\lambda \geq 32,167 \]
\[ x_1 + 950\lambda \leq 20810 \]
\[ x_2 + 410\lambda \leq 2870 \]
\[ x_3 + 110\lambda \leq 2390 \]
\[ x_4 + 220\lambda \leq 1590 \]
\[ x_5 + 390\lambda \leq 5780 \]
\[ x_1 + x_2 + x_3 + x_4 + x_5 \leq 60120 \]
\[ 0.28x_1 + 0.03x_2 + 0.03x_3 + 0.02x_4 + 0.08x_5 \leq 54,000 \]
\[ \lambda \in [0,1] \]
\[ x_i \geq 0 \]
\[ i = 1, 2, ..., 5 \]
\[ \forall x_i \in Z^+ \]

(13)

**Problem Solution**

FLP model of the problem has been solved using Lindo optimization software. Results of the solution are given in Table II.

As can be seen from the solution, the factory should produce 20721 meter \( x_1 \), 2831 meter \( x_2 \), 2379 meter \( x_3 \), 1569 meter \( x_4 \), 5743 meter \( x_5 \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.093</td>
<td>0</td>
</tr>
<tr>
<td>( X_1 )</td>
<td>20721</td>
<td>-0.000152</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>2831</td>
<td>-0.000016</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>2379</td>
<td>-0.000017</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>1569</td>
<td>-0.000012</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>5743</td>
<td>-0.000041</td>
</tr>
</tbody>
</table>

(14)

Total profit of the factory can be calculated as follows:

\[ (1.44 \times 20721 + 0.5 \times 2831 + 0.16 \times 2379 + 0.11 \times 1569 + 0.39 \times 5743) = 34046.74 \text{TRY} \]

**Conclusion**

This paper has discussed the use of FLP for solving a production planning problem in a textile industry. It can be concluded that this method introduced is a promising method for solving such problems. Because the FLP model has fuzziness in both objective function and constraints, it was solved by using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of production for each cloth type in order to gain maximum profit. The study illustrates how particular problems of real production systems can be treated by the theory on fuzzy sets.

**References**


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