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Comparison of Fourier and Trigonometric Transform based Multicarrier Modulations for Visible Light Communication

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Abstract: It is predicted that the radio frequency spectrum will be insufficient in the near future due to the increase in wireless data. Visible Light Communication (VLC) is an alternative solution, which promises high speeds. Similar to other wireless communication systems, VLC systems prefer Multicarrier Modulation (MCM), but the signals are converted to be real and unipolar before transmission for optical communication. In this paper, two optical MCM groups that utilize Discrete Fourier Transform (DFT) and Discrete Trigonometric Transform (DTT) are questioned with respect to Bit Error Rate (BER), spectral efficiency and complexity. DFT based techniques use complex mapped signals together with their Hermitian symmetries to obtain real output signals, while DTT based techniques already output real signals when the input signal is real mapped. It is seen that DFT based techniques have lower BERs because of used mapping. DTT based techniques improve spectral efficiency, but they are limited to real mappings with higher error rates. For both transformations, the real signals are made unipolar by adding a bias (DCO-MCM), by asymmetrically clipping (ACO-MCM) or by sending positive and negative values separately (UnO-MCM). It is shown that, adding a dc bias (DCO-MCM) increases BERs, where ACO-MCM and UnO-MCM have close performances with lower BERs.

Key words: Bit error rate, discrete Fourier transform, discrete trigonometric transform, multicarrier modulation, visible light communication.

Görünür Işık Haberleşmesi için Fourier ve Trigonometrik Dönüşüm Tabanlı Çok Taşıyıcılı Modülasyonların Karşılaştırılması

Öz: Kablosuz verideki artış nedeniyle yakın gelecekte radyo frekans spektrumunun yetersiz kalması ön görülmektedir. Görünür Işık Haberleşmesi (VLC) yüksek hız vaat eden alternatif bir çözümdür. VLC sistemleri, diğer kablosuz haberleşme sistemlerine benzer şekilde, Çoklu Taşıyıcılı Modülasyon (MCM) tercih ederler, ancak optik iletişim için iletimden önce sinyaller gerçek ve tek kutuplu olacak şekilde dönüştürülürler. Bu makalede, Ayrık Fourier Dönüşümü (DFT) ve Ayrık Trigonometrik Dönüşümü (DTT) kullanan iki optik MCM grubu Bit Hata Oranı (BER), spektral verimlilik ve karmaşıklık açısından sorgulanmaktadır. DFT tabanlı teknikler, gerçek çıkış sinyali elde etmek için karmaşık eşlemeli sinyaller ve onların Hermit simetrilerini kullanırlar, DTT tabanlı tekniklerde ise giriş sinyali gerçek eşlenmiş ise çıkış zaten gerçek olacaktır. DFT tabanlı tekniklerin kullanılan eşleme nedeniyle daha düşük BER'lere sahip oldukları görülmektedir. DTT tabanlı teknikler spektral verimliliği arttırır ancak bu teknikler daha yüksek hata oranlarına sahip gerçek eşlemelerle sınırlıdırlar. Her iki dönüşüm için de gerçek sinyaller, bir öngerilim eklenerek (DCO-MCM), asimetrik olarak kırpılarak (ACO-MCM) veya pozitif ve negatif değerleri ayrı ayrı göndererek (UnO-MCM) tek kutuplu hale getirilir. Dc bileşen eklemek (DCO-MCM) BER'leri arttırırken, ACO-MCM ve UnO-MCM in daha düşük BER'lerle yakın performanslara sahip oldukları gösterilmiştir.

Anahtar kelimeler: Bit hata oranı, ayrık fourier dönüşüm, ayrık trigonometrik dönüşüm, çok taşıyıcılı modülasyon, görünür ışık haberleşmesi.

1. Introduction

There is a tremendous amount of increase in wireless data traffic due to the rise in the number of users with demands of high speed communication for services such as social media, video streaming, cloud storage and game streaming. Thus, wireless systems using radio frequency bands become more and more saturated every day. Visible Light Communication (VLC), a kind of optical communication, seems to be a strong candidate for future wireless communication. VLC systems use already existing Light Emitting Diode (LED) based illumination infrastructure for communication, and as LEDs become widespread, VLC systems seem to increase rapidly. The health friendly, low cost VLC has many advantages such as wide band, high data rate, high security and no interference to the radio frequency band. Therefore, many interesting indoor and outdoor applications have come into view. i.e., high speed data transmission in houses and offices, communication in hospitals and airplane cabins, traffic management and underwater communication.

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VLC systems, as well as other optical communication systems, use the Intensity Modulation / Direct Detection (IM/DD) technique [1]. Thus, the intensity of light is modulated with the LED's input current which is controlled by modulated signal. After transmission of the signal, the demodulation process is carried out in the receiver by an optical detector, generally a photodiode, which outputs an electrical signal related to the detected light intensity.

Multicarrier Modulation (MCM) is attractive in wireless communication for high data rates and its resilience to Intersymbol Interference. MCM schemes are divided into two groups: Discrete Fourier Transform (DFT) based techniques (they are also named Orthogonal Frequency Division Multiplexing, OFDM) and Discrete Trigonometric Transform (DTT) based techniques. MCM is also used in optical communication but with modifications. The output signals of MCM are complex and/or bipolar, but the input of IM/DD block should be real and unipolar [2] because light intensity cannot be complex and bipolar. Various approaches are proposed in the literature to solve this problem. For real outputs, techniques implement DFT input symbols together with their Hermitian symmetries, whereas techniques that use DTT already output real signals in case of real input. Following, real signals are made unipolar, by adding a bias, by clipping, or positive and negative values are sent separately.

In the literature, these optical MCM techniques are investigated. For example, the authors of [3] survey the evolution of Optical OFDM (O-OFDM) designed for both VLC and optical fiber. In [4], Direct Current O-OFDM and Asymmetrically Clipped O-OFDM are analyzed in terms of signal clipping under the constraints of maximum and non-negative optical power. In [5], an innovative unipolar transceiver system is proposed and the authors enhance the data rate, system complexity, power and spectrum efficiency. The authors in [6] investigate optical MCM systems based on discrete Hartley transform for VLC. Spectral and energy efficiency of Asymmetrically Clipped O-OFDM is investigated in [7]. In [8], an enhanced OFDM scheme is proposed for VLC. A hybrid O-OFDM scheme for VLC with adaptive bias is proposed in [9] to improve spectral efficiency and power efficiency.

In this paper, Bit Error Rate (BER) performances of DFT and DTT based techniques are investigated under additive white Gaussian Noise (AWGN) channel assumption, together with their complexity and spectral efficiency. The remainder of this paper is organized as follows. First, DFT and DTT based optical MCM techniques are introduced in Section 2. Then BER performances of these techniques are demonstrated by simulations in Section 3. Finally, the conclusions of the study are given.

2. Optical Multicarrier Modulation

A block diagram of an optical MCM system is given in Figure 1. Note that, the receiver side is the same as the conventional ones, but two steps are added to the transmitter. For obtaining real signals, if Inverse Discrete Fourier Transform (IDFT) is used, mapped complex signals together with their Hermitian symmetries are fed to the block. If Inverse Discrete Trigonometric Transform (IDTT) is used, real mapping is preferred and Hermitian symmetries are not needed. Following, parallel to serial conversion is done and cyclic prefix (CP) is appended to the signal to mitigate the effect of Intersymbol Interference. Next, to obtain unipolar signals, one of the three methods is applied: An appropriate dc bias is added and negative peaks are clipped at zero, or after mapping zero padding is done and asymmetrically clipping is performed, or positive and negative values are sent separately.

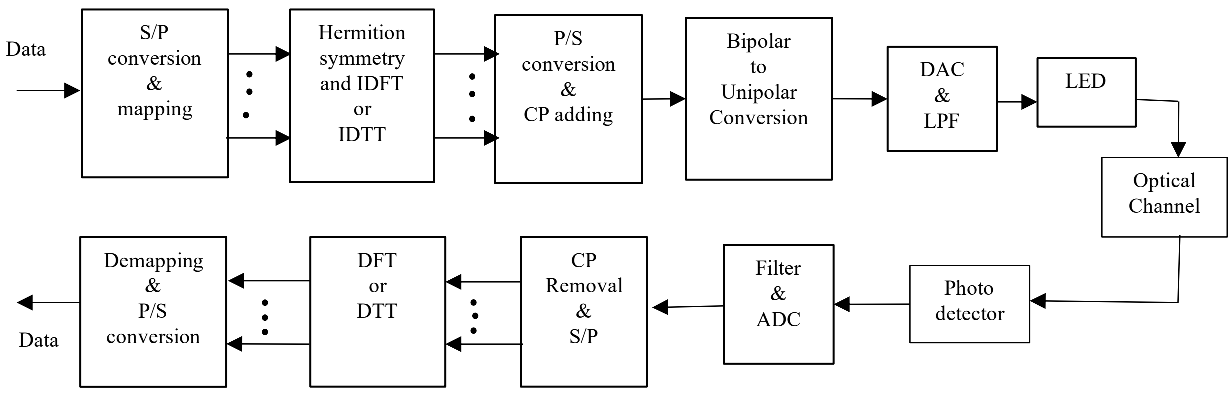

Figure 1. Optical MCM system block diagram.

2.1. Real MCM signals

The used mapping in DFT based MCM generally outputs complex values. To obtain real signals, the IDFT block is fed with complex mapped symbols together with their Hermitian symmetries [10], as given in Equation 1:

$$X_{DFT,(N-k)} = X^*_{DFT,k} \qquad k = 1, 2, \dots, \frac{N}{2} - 1 \tag{1}$$

where $X_{DFT,k}$ is the input of IDFT block and N denotes the subcarrier number. Then the input vector of IDFT is given in Equation 2:

$$\boldsymbol{X_{DFT}} = [0,\ X_{DFT,1},\ X_{DFT,2},\ \dots\ ,\ X_{DFT,\frac{N}{2}-1},\ 0,\ X^*_{DFT,\frac{N}{2}-1},\ \dots\ ,\ X^*_{DFT,2},\ X^*_{DFT,1}] \tag{2}$$

The output of IDFT is given by Equation 3 [10]:

$$x_{DFT,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{DFT,k} \exp\left(\frac{j2\pi kn}{N}\right), \quad n = 0, 1, 2, \dots, N-1 \tag{3}$$

Where n is subcarrier indices. Then, DFT equation is known to be as given in Equation 4:

$$X_{DFT,k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{DFT,n} \exp\left(\frac{-j2\pi kn}{N}\right), \quad k = 0, 1, 2, \dots, N-1 \tag{4}$$

Note that because of the Hermitian symmetry given in Equation 1, it can easily be shown that Equation 3 simplifies to Equation 5 [11]:

$$x_{DFT,n} = \frac{1}{\sqrt{N}} \left[\sum_{k=1}^{\frac{N}{2}-1} 2X_{DFT,real,k} \cos\left(\frac{2\pi nk}{N}\right)\right], \ n = 0, 1, 2, \dots, N-1 \tag{5}$$

Thus, the output of IDFT block is real so that it can be transferred via IM/DD after bipolar to unipolar conversion.

The second group of optical MCM techniques uses one of the well-known DTTs, namely Discrete Hartley Transform (DHT), Discrete Sine Transform (DST) and Discrete Cosine Transform (DCT) [11-15]. DTT already outputs real valued signals when input signals are real. Thus, DTT based MCM techniques are limited to real mappings.

N point Inverse DST (IDST) can be expressed as Equation 6 [11],

$$x_{DST,n} = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} X_{DST,k} \, sin\left(\frac{\pi(2n+1)(2k+1)}{4N}\right) \quad n = 0, 1, 2, \dots, N-1 \tag{6}$$

where $X_{DST,k}$ is output of mapping and it is clear that the output of IDST block $x_{DST,n}$ is going to be real in case of real mapping. DST equation is given in Equation 7:

$$X_{DST,k} = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x_{DST,n} \, sin\left(\frac{\pi(2n+1)(2k+1)}{4N}\right) \quad k = 0, 1, 2, \dots, N-1 \tag{7}$$

DCT also outputs real valued signals when input is real. Inverse DCT (IDCT) is expressed as Equation 8 [14],

$$x_{DCT,n} = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} \varepsilon_k \, X_{DCT,k} \, cos\left(\frac{\pi k(2n+1)}{2N}\right) \quad n = 0, 1, 2, \dots, N-1 \tag{8}$$

The coefficient ε_k is defined in Equation 9:

$$\varepsilon_k = \begin{cases} \sqrt{0.5}, & k = 0 \\ 1, & k = 1, 2, \dots, N-1 \end{cases} \tag{9}$$

DCT equation (Equation 10) can be expressed as:

$$X_{DCT,k} = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} \varepsilon_n \, x_{DCT,n} \, cos\left(\frac{\pi k(2n+1)}{2N}\right) \quad k = 0, 1, 2, \ldots, N-1 \tag{10}$$

The output of Inverse DHT (IDHT) is given by Equation 11 [15],

$$x_{DHT,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{DHT,k} \left[\sin\left(\frac{2\pi nk}{N}\right) + \cos\left(\frac{2\pi nk}{N}\right)\right], n = 0,1,2, \ldots, N-1 \tag{11}$$

where $X_{DHT,k}$ is the output of mapping. Then, DHT equation is (Equation 12)

$$X_{DHT,k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{DHT,n} \left[\sin\left(\frac{2\pi nk}{N}\right) + \cos\left(\frac{2\pi nk}{N}\right)\right], k = 0,1,2, \ldots, N-1 \tag{12}$$

Note that the equations of DHT and IDHT are identical, as in DST and DCT. Thus, same digital signal processing unit may be used both in the transmitter and in receiver which reduces system complexity [15-17].

2.2. Unipolar MCM signals

Real but bipolar MCM signals should be made unipolar before IM. Three basic techniques are found in the literature: The first one is adding a bias, (Direct Current Biased Optical MCM, DCO-MCM), the second one is clipping (Asymmetrically Clipped Optical MCM, ACO-MCM) and the third is sending positive and negative values separately (UnO-MCM).

DCO-MCM is one of the popular approaches that is used in VLC. In DCO-MCM, an appropriate dc bias is added to the bipolar MCM signal with a cyclic prefix and then all remaining negative peaks are clipped at zero [18, 19]. Thus, dc bias addition is given in Equation 13,

$$x_{DC}(t) = x_{cp}(t) + B_{DC} \tag{13}$$

where $x_{cp}(t)$ is the cyclic prefix added MCM signal and B_{DC} is the dc bias. Adding a constant dc bias is a choice but selecting a dc bias related to the power of the signal will be more appropriate, i.e. [20]:

$$B_{DC} = k\sqrt{E\{(x_{cp}(t)^2\}} \tag{14}$$

Where in Equation 14 *k* is the proportional constant of clipping factor and E{} denotes statistical expectation. The approximate increase in energy dissipation of DCO-MCM in decibels is given in Equation 15 [20]:

$$B_{DC,dB} = 10\log_{10}(k^2 + 1) \tag{15}$$

Note that, all negative parts may not be eliminated even after adding the dc bias, therefore clipping at zero should be applied as in Equation 16:

$$x_{DCO}(t) = \begin{cases} x_{DC}(t), & x_{DC}(t) \geq 0 \\ 0, & x_{DC}(t) < 0 \end{cases} \tag{16}$$

DCO-MCM is easy to implement, but deciding to the dc bias is critical because high dc bias may result with the clipping of the signal peaks which are out of the led dynamic range. Further, high positive peaks may be observed which requires high power. Thus, Peak to Average Power Rate increases which lowers the power efficiency [21]. But low dc bias selection also results with high clipping noise.

ACO-MCM is introduced as a power efficient technique, where only odd subcarriers are used for data transmission and even subcarriers are set to zero [21]. Then, for example in an ACO-DFT system, the input vector of IDFT is expressed as in Equation 17:

$$\boldsymbol{X} = [0, X_1, 0, X_2, \ldots, X_{\frac{N}{2}-1}, 0, X^*_{\frac{N}{2}-1}, 0, X^*_{\frac{N}{2}-3}, \ldots, 0, X^*_1] \tag{17}$$

Finally, negative parts are clipped to ensure nonnegativity (Equation 18):

$$x_{ACO}(t) = \begin{cases} x_{cp}(t), & x_{cp}(t) \geq 0 \\ 0, & x_{cp}(t) < 0 \end{cases} \tag{18}$$

Note that all the intermodulation caused by clipping is on the even subcarriers because only the odd subcarriers are modulated. Thus, the data-carrying odd subcarriers are not affected.

In UnO-MCM, for unipolarization, the time-domain real signal (one bipolar frame) is transformed into two separate frames and then transmitted one after the other [19, 22]. The first frame includes the positive samples where the places of negative ones are set to zero. Similarly, the second frame holds the absolute values of the negative samples with zeros in the places of positive ones. At the receiver, the negative frame is subtracted from the positive frame to obtain the original bipolar frame. Indeed, this may combine the noise in both frames.

The illustrations of MCM symbol structures of related schemes for the same amount of sending data are given in Figure 2.

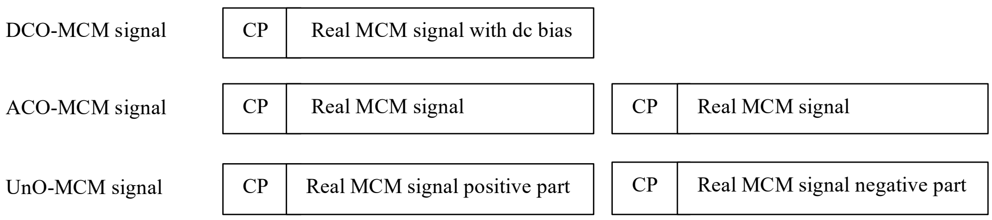

Figure 2. Illustration of the transmitted signals for DCO-MCM, ACO-MCM and UnO-MCM.

The spectral efficiency of DCO-MCM with *M* level of mapping is calculated by Equation 19 [23],

$$\eta_{DCO-MCM} = l\frac{N-2}{(N+N_{cp})}log_2(M) \tag{19}$$

Where N_{cp} is the length of the cyclic prefix. The constant l is equal to 0.5 for DFT based MCM (Half of the subcarriers is allocated to Hermitian symmetry which halves the data carrying subcarriers.) and 1 for the DTT based MCM. Thus, DTT based MCM spectral efficiency is better than DFT based MCM.

In ACO-MCM, the use of only odd subcarriers results in $N/2$ independent complex input values, then spectral efficiency of ACO-OFDM is given in Equation 20 [23],

$$\eta_{ACO-MCM} = l\frac{N-2}{2.(N+N_{cp})}log_2(M) \tag{20}$$

In UnO-MCM, transformation of positive and negative parts halves the achievable data rate, i.e., the spectral efficiency becomes as Equation 21 [22]:

$$\eta_{UnO-MCM} = l\frac{N-2}{2.(N+N_{cp})}log_2(M) \tag{21}$$

As seen, spectral efficiency is halved in ACO-MCM and UnO-MCM compared to DCO-MCM. However, they have a significant energy advantage over DCO-MCM [24].

3. Results and Discussion

For the BER performances, numerical results are obtained using computer based Monte Carlo simulations with MATLAB software. In the simulations, AWGN channel is assumed, i.e., a robust line-of-sight path exists, which is commonly the case for indoor and outdoor optical wireless systems. The number of subcarriers is 1024 and guard interval subcarriers are 96. The most popular mapping are selected for both transformations, i.e., Quadrature Amplitude Modulation (QAM) mapping for DFT based MCMs and Pulse Amplitude Modulation (PAM) mapping for DTT based MCMs.

Comparing the speeds of Fast Fourier Transform (FFT) algorithm proposed in [25] and Fast Hartley Transform (FHT) proposed in [26], FFT needs a few less operations [26]. However since FFT based MCM needs to calculate Hermitian symmetry, FHT based MCM has less computational complexity. Further, DTT based MCM systems use same digital signal processing unit for both receiver and transmitter due to self-inverse property, therefore complexity and cost are reduced compared to DFT based MCM systems.

In the simulations, the effect of selected dc bias is investigated first. The BER graphs of DCO-DHT for 2, 4 and 8 PAM mapping levels as a function of Signal to Noise Ratio (SNR) are given in Figure 3. The added dc biases are selected to be the minimum of the signal (solid lines without markers), the proportional constant of clipping factor k=1.5 (lines with circle) and k=5 (lines with stars). As seen from the figure, adding larger dc biases increases the BERs, especially for low SNRs. But note that adding small bias values results with small BERs, if clipping noise is tolerable, as in 2-PAM. But low dc bias results with high BERs for SNR values greater than 25 dB for 4-PAM and 8-PAM. This is because of insufficient dc bias which results in high clipping noise. It is interesting that especially for lower modulation levels, adding the minimum of the signal as dc bias may result in higher BER, especially for low SNR values.

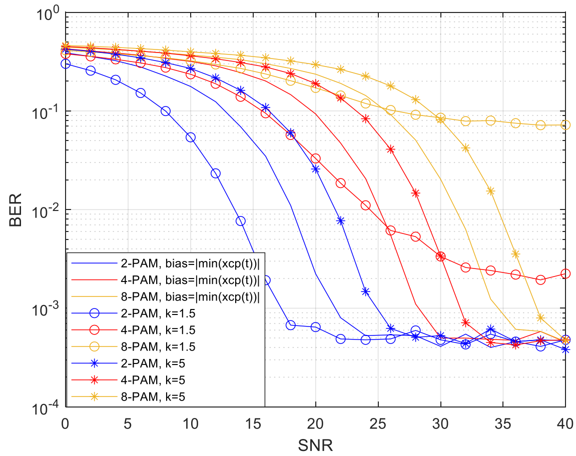

Figure 3. BER of DCO-DHT, effect of dc bias selection.

The BER graphs of DCO-DFT are given in Figure 4, for 2, 4 and 8 QAM levels. The added dc biases are the same as in the case of DCO-DHT. As seen from the figure, adding larger dc biases increases the BERs, especially for low SNRs. Clipping noise is observed for 8-QAM when SNR is larger than 25 dB only. It is concluded that the added bias is critical for the performance of DCO-MCMs. Maximum bias should be the minimum value of the signal, indeed lower values can be found for decreasing the power but care is needed for clipping noise, especially for high SNRs. The optimum k value depends on modulation level and SNR, obtaining the value is out of the scope of this study. The dc bias is selected to be the minimum of the signal power in the following simulations.

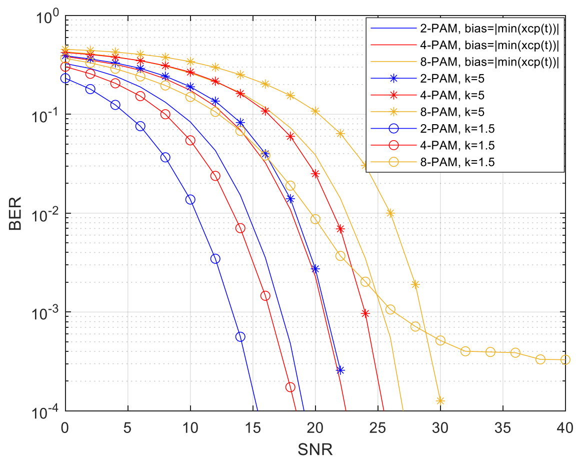

Figure 4. BER of DCO-DFT, the effect of dc bias selection.

BER performances of DCT, DHT and DFT based MCMs for three unipolarization schemes are given in Figure 5, Figure 6 and Figure 7, respectively. Mapping levels are selected to be 2, 4 and 8, for both PAM and QAM. In all figures, solid lines are used for DCO-MCM (dc bias is selected to be the minimum of the signal power), dashed line is for ACO-MCM and starred line is used for UnO-MCM schemes. For all MCM techniques, ACO-MCMs and UnO-MCMs give very close results (two curves overlap on the graphs) and DCO-MCMs result with higher BER values compared to ACO-MCMs and UnO-MCMs. For all O-MCM techniques, as the modulation level increases, the BER also increases, as expected. Comparing the BER graphs, DCT-MCM and DHT-MCM have very close performances, but DFT-MCM performance is better because of the used mapping.

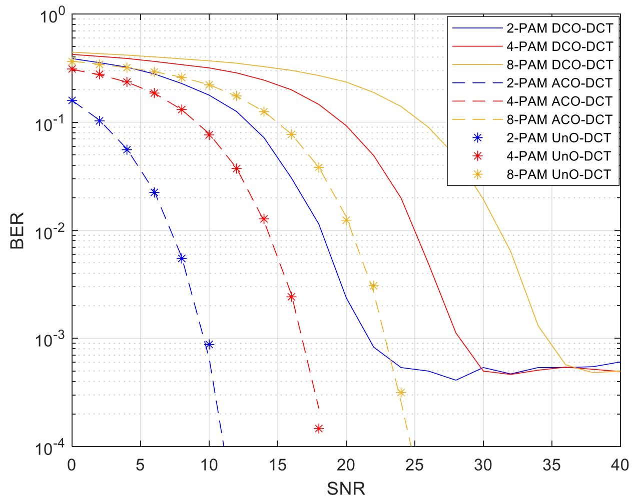

Figure 5. BERs of DCO-DCT, ACO-DCT and UnO-DCT.

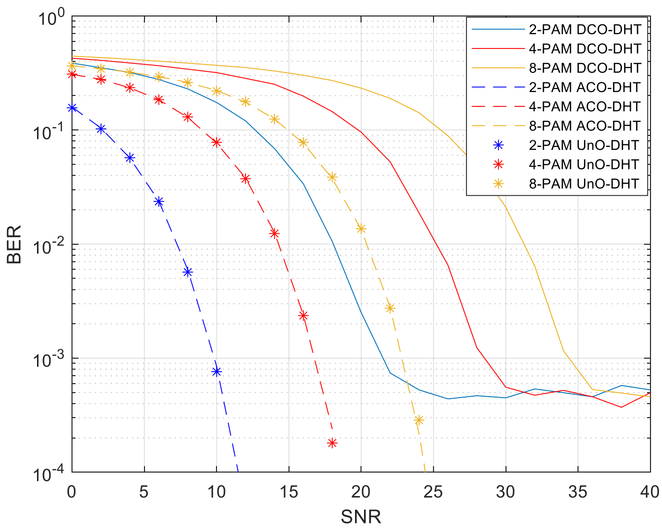

Figure 6. BERs of DCO-DHT, ACO-DHT and UnO-DHT

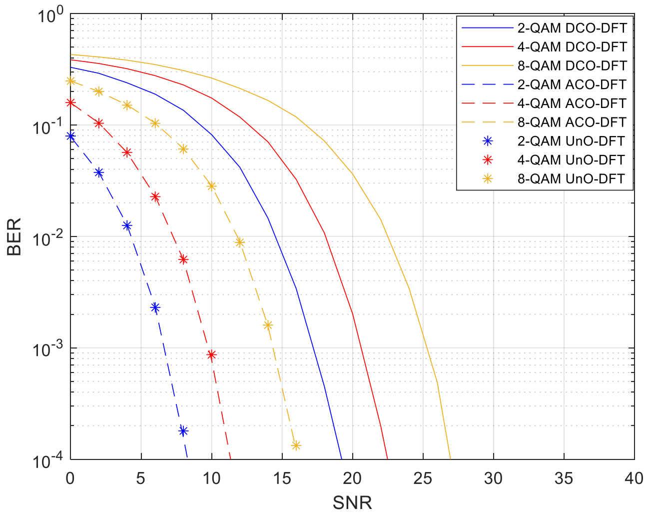

Figure 7. BERs of DCO-DFT, ACO-DFT and UnO-DFT

4. Conclusion

VLC systems use optical MCM techniques, where the complex and bipolar MCM signals are made real and unipolar so that they can be transmitted over optical channels by intensity modulation. In this paper, optical MCM schemes that use DFT and DTT with various unipolarization schemes are compared in terms of their spectral efficiency, complexity and BERs. DTT-MCM has reduced complexity, but these systems are limited to using real mappings only which results in poor BER performance. BER performance of DFT-MCM is shown to be better, because of the mapping used. Comparing unipolarization techniques, namely DCO_MCM, ACO-MCM and UnO-MCM, DCO-MCM has the best spectral efficiency with a simple structure but requires higher power. Furthermore, the BER performance of DCO-MCM is poor compared to the others. Arranging the bias is also a critical task, i.e., high dc bias increases the power but reduces the clipping noise or vice versa. ACO-MCM and UnO-MCM have similar performances in means of spectral efficiency and BER. Finally, as an optimum decision, ACO-DTT systems come forward with their relatively simple structures and acceptable BER performances.

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