Dönüşüm Geometrisi Üzerine Bir Araştırma: Cevaplanmış ve Cevaplanmamış Sorular

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Öz

Bu çalışma, geometrik dönüşüm araştırmalarındaki mevcut bilgi durumunu sentezlemekte ve gelecekteki çalışmalar için yeni araştırma alanları ve araştırma soruları önermektedir. Çalışmada spesifik olarak, aşağıdaki beş soru ele alınmaktadır: (1) Geometrik dönüşüm nedir?, (2) Geometrik dönüşüm okul matematiğinde neden önemlidir?, (3) Geometrik dönüşümü anlamak hakkında ne biliyoruz?, (4) Geometrik dönüşüm etkinliklerinde teknoloji nasıl kullanılabilir?, (5) Geometrik dönüşüm etkinlikleri matematik ders kitaplarına nasıl dahil edilir? Bu soruların her biri geometrik dönüşümler alanında verimli bir araştırma alanı sunmaktadır. Bu alandaki mevcut bilgilere genel bir bakışın ardından, her bir soru için, araştırma topluluğunun ek değerlendirmesini gerektiren ilgili cevaplanmamış araştırma soruları incelenmeye devam edilecektir.

Keywords: Geometrik Dönüşüm, Öteleme, Yansıma, Dönme, Teknoloji

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Bu makale 27-30 Eylül 2023 tarihlerinde Kafkas Üniversitesi'nde gerçekleştirilen 15. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi'nde özet sözel bildiri olarak sunulmuştur.

GENİŞLETİLMİŞ ÖZET

Giriş

Bu çalışma, geometrik dönüşüm konusunda yapılan araştırmalarda yer alan mevcut bilgi durumunu sentezleyerek, gelecekte yapılacak olan geometrik dönüşüm çalışmalarına yeni araştırma soruları önermektedir. Özellikle, geometrik dönüşümler alanında kapsamlı bir araştırma alanı sunan beş soruyu ele almıştır. Bu sorular şunlardır:

(1) Geometrik dönüşüm nedir?

(2) Geometrik dönüşüm okul matematiğinde neden önemlidir?

(3) Geometrik dönüşüm anlayışı hakkında neler biliyoruz?

(4) Teknoloji nasıl geometrik dönüşüm etkinliklerinde kullanılabilir?

(5) Geometrik dönüşüm etkinlikleri matematik ders kitaplarına nasıl dâhil edilir?

Yöntem

Bu çalışma bir derleme çalışması olduğundan, matematik eğitimi alanındaki önemli bilimsel veri tabanlarında yapılan geniş kapsamlı bir literatür taramasının başlangıcını ve ardından elde edilen bulguların analizini içermektedir. Literatür araştırması son 15 yılda gerçekleştirilmiş geometrik dönüşüm çalışmaları ele alınmış, türkçe ve yabancı alan dergilerinin veritabanları konu ile ilgili İngilizce ve Türkçe anahtar kelimeler üzerinden taranarak gerçekleştirilmiştir. Bulgular, araştırma sorularını ele almak için sistemli bir şekilde analiz edilmiş ve sonrasında literatür sentezlenmiştir. Ayrıca, gelecekteki çalışmalara rehberlik etmek amacıyla cevaplanmamış sorular belirlenmiştir.

Bulgular

Geometrik Dönüşüm Nedir?

Martin (1982), geometrik dönüşümü, bir düzlemdeki noktalar kümesinin kendisiyle birebir eşleniği olduğunu belirterek kavramsal bir tanım yapmıştır. Bu tanım, geometrik dönüşümün tanımını ve tanım kümesini anlama ve geometrik dönüşümde birebir ve örten kavramlarının anlamını keşfetme konusunda iki önemli perspektif olduğunu vurgulamaktadır: hareket ve eşleştirme perspektifi. Geometrik dönüşümde, 'bire-bir' ifadesi düzlemdeki tüm noktaların kendisi ile birebir karşılıklı eşlenmiş noktalara sahip olduğunu belirtir. Dolayısıyla, geometrik dönüşüm sadece nesneye uygulanmaz, aynı zamanda düzlemi oluşturan tüm noktalara da uygulanmış olur. Geometrik dönüşümün tanımı ve kavramsal olarak anlaşılması, geometrik dönüşümleri öğrenme için önemli bir ölçüt haline gelir.

Geometrik Dönüşüm Neden Önemlidir?

Geometrik dönüşümü öğrenmek, öğrencilerin görselleştirme becerilerini, analitik stratejilerini ve eleştirel düşünme yeteneklerini geliştirmelerine katkı sağlar. Diğer yandan matematikle diğer bilim dalları arasındaki ilişkilerin keşfedilmesine olanak tanır, böylece öğrencilerin disiplinler arası çalışma becerilerini geliştirmelerine yardımcı olurken öğrencilerin çeşitli becerilerini geliştirmek ve bu becerileri etkin bir şekilde kullanmalarını sağlamak için müfredatlara entegre edilmesi gereken önemli bir konudur.

Geometrik Dönüşümleri Anlama Konusunda Neler Biliyoruz?

Araştırmalar, öğrencilerin geometrik dönüşümleri anlama konusunda iki perspektife sahip olduklarını göstermiştir; hareket ve eşleme perspektifi (Akarsu, 2018; Flanagan, 2001; Yanik, 2006). Çalışmalar, hareket ve eşleştirme perspektifine göre geometrik dönüşümlerin anlaşılmasında üç kritik parametre olduğunu göstermektedir, bunlar simetri ekseni, simetrinin tanım kümesi ve düzlemdir. Öğrenciler hareket perspektifine sahip olduklarında parametrelerin rolünü (yansıma çizgisi, vektör ve dönme merkezi vb.) dönüşüm uygulamalarında kullanamazlar. Örneğin, öğrenciler yansıma dönüşümü uygularken yansıma çizgisini, diklik ve eşit uzaklık özelliklerini dikkate almazlar. Öte yandan, eşleştirme perspektifinde, öğrenciler yansıma dönüşümü uygularken yansıma çizgisinin diklik ve eşit uzaklık özelliklerini kullanarak şekli doğru yerde konumlandırırlar.

Geometrik Dönüşüm Etkinliklerinde Teknoloji Nasıl Kullanılabilir?

Ulusal Matematik Öğretmenleri Konseyi (NCTM) (2000), teknolojinin matematik öğretiminde ve öğreniminde temel olduğunu ve öğrencilerin öğrenme sürecini olumlu etkilediğini belirtmiştir. NCTM (2000) dinamik geometri yazılımlarının rolünü, öğrencilerin geometrik dönüşümleri öğrenirken şekilleri manipüle edebilmeleri ve her manipülasyonun şeklin görüntüsünü nasıl etkilediğini gözlemleyebilmelerine imkân vermesi şeklinde açıklamıştır. Örneğin, interaktif şekiller aracılığıyla öğrenciler, bir yansımanın dönüştürülen şeklin, orijinal şekle göre yansıma çizgisine eşit uzaklıkta olduğunu keşfedebilirler. Özellikle, programların özellikleri olan sürükleme ve ölçme özelliklerinin kullanılması, öğrencilerin geometrik çevirilerin özelliklerini belirlemelerine, hipotezler oluşturmalarına, farklı stratejiler kullanmalarına ve yeni anlayışlar oluşturmalarına yardımcı olur.

Matematik Ders Kitaplarında Geometrik Dönüşüm Etkinlikleri Nasıl Yer Alıyor?

Matematik ders kitaplarında geometrik dönüşüm etkinliklerinin nasıl yer aldığına dair yapılan incelemeler, öğrenci öğrenimini etkileyen temel faktörler arasındadır. Bu incelemeler, içerik analizi yoluyla ders kitaplarının standartlarla olan ilişkilerini ve materyallerin etkililiğini anlamak için gereklidir. Özellikle geometrik dönüşüm etkinlikleri, hareket ve eşleme perspektifleri gözetilerek öğrencilerin kavramsal anlamda düşünmelerini teşvik etmek adına tasarlanmalıdır. Ancak yapılan analizler, ders kitaplarının genellikle hareket perspektifine odaklandığını ve öğrencilerin kavramsal anlamda düşünme becerilerini destekleme konusunda yetersiz kaldığını ortaya koymaktadır. Bu nedenle, ders kitaplarının geometrik dönüşümlerle ilgili içeriğinin, öğrencilerin problem çözme yetenekleri, eleştirel düşünme becerileri ve geometrik dönüşümler konusundaki yetkinliklerini destekleyip desteklemediğini değerlendirmek önemlidir.

Sonuç ve Tartışma

Geometrik dönüşümün detaylı bir şekilde incelenmesi, gelecekteki araştırmalar için geniş bir temel oluşturur. Öncelikli olarak, öğretmenlerin, öğretmen adaylarının ve öğrencilerin geometrik dönüşüm kavramını nasıl tanımladıklarına dair cevapsız kalan soruların ele alınması, kavramsal anlayışı derinleştirmek ve genişletmek için önemlidir. Tanımların ve karşılaşılan zorlukların incelenmesi, kavramsal anlayıştaki eksiklikleri belirleyebilir. Geometrik dönüşümü vurgulayan yenilikçi öğretim yöntemlerinin keşfi, öğrencilerin öğrenme deneyimlerini geliştirebilir. Çalışma, geometrik dönüşümün lise geometri derslerine ve ilkokul matematik müfredatına entegre edilmesinin önemini açıkça ortaya koymaktadır. Gelecekteki araştırmalar, bu entegrasyonun öğrencilerin matematik anlayışı, uzamsal düşünme becerileri ve disiplinler arası çalışma becerileri üzerindeki etkisini değerlendirmelidir.

A Research on Transformation Geometry: Answered and Unanswered Questions

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Abstract

This study synthesizes the current state of knowledge in geometric transformation research and suggests directions for future study. Specifically, we address the following five questions: (1) What is geometric transformation?, (2) Why is geometric transformation important in school mathematics?, (3) What do we know about understanding geometric transformation?, (4) How can technology be used in geometric transformation activities?, (5) How are geometric transformation activities included in mathematics textbooks? Each of these questions presents a fertile research area within the realm of geometric transformations. Following an overview of the existing knowledge in this domain, we proceed to examine, for each question, related unanswered issues that warrant additional consideration from the research community.

Keywords: Geometric Transformation, Translation, Reflection, Rotation, Technology.

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1. Introduction

The National Council of Teachers of Mathematics (NCTM) emphasized the significance of geometric transformation and the creation of learning environments that facilitate its incorporation into mathematics education (NCTM, 1989; 2000). According to the NCTM (2000), instructional programs across pre-kindergarten to grade 12 should empower students to apply transformations and employ symmetry for analyzing mathematical situations. In acknowledgment of the role of geometric transformation in the curriculum, research on this subject has been ongoing since the 1990s. This systematic review of research literature in mathematics education since 1990 aims to provide a summary and identify gaps in geometric transformation research that future investigations can address. In the ensuing discussion, we provide a summary and synthesis of the present state of research in geometric transformations while proposing potential avenues for future studies. Our emphasis lies in delineating research questions to lay the groundwork for indicating which questions still require answers. The questions guiding this systematic review are as follows:

(1) What is geometric transformation?

(2) Why is geometric transformation important in school mathematics?

(3) What do we know about understanding geometric transformations?

(4) How can technology be used in geometric transformation activities?

(5) How are geometric transformation activities included in mathematics textbooks?

2. Method

The study commenced with a comprehensive literature review conducted across prominent scientific databases housing the most widely read journals and thesis repositories in mathematics education. Given the extensive research on geometric transformation over the past twenty-five years, efforts were directed towards accessing relevant works within this timeframe from the databases. This involved gathering articles, master's and doctoral theses, conference papers, and reports pertinent to the topic. Keywords such as "transformation," "transformation geometry," "translation," "reflection," and "rotation" were utilized in both Turkish and English versions to ensure inclusivity in the literature review. Reference lists from studies focusing on transformation geometry were extracted and organized for further analysis. The review encompassed studies involving teachers, teacher candidates, and students, without distinction. All identified studies underwent thorough examination to ascertain various aspects, including research questions, study type and design, participant selection and description. Abstracts containing pertinent information regarding the research questions and study details were compiled to facilitate subsequent data analysis. During the data analysis phase, researchers endeavored to address the research questions central to the study by systematically collating relevant insights gleaned from the reviewed studies. Subsequently, literature was synthesized based on the compiled data, incorporating responses to the research questions. Moreover, unanswered questions identified during the analysis were delineated as "unanswered questions," with the aim of offering guidance for future studies.

3. Findings

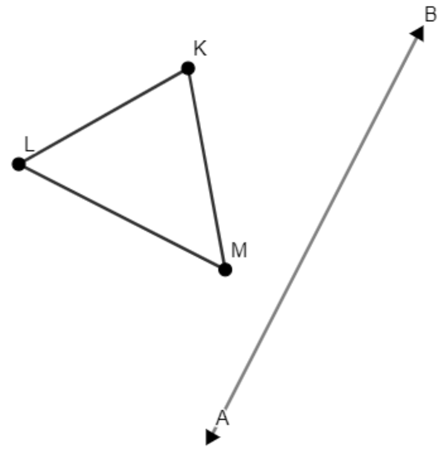
3.1. What is Geometric Transformation?

In the literature, various definitions of geometric transformation can be observed. Klein (1870) provided one of the earliest definitions, describing geometric transformation as the fundamental subject in the field of learning geometry. Karakuş (2008) and Pleet (1990) explained that geometric transformation is a subfield of learning geometry, consisting of translation, reflection, and rotation movements. While these definitions emphasize the general structure of geometric transformation, they lack focus on its conceptual dimensions. In contrast, Martin (1982) offered a conceptual definition, describing geometric transformation as 'the one-to-one correspondence of a set of points in a plane with itself' (p. 1). This definition highlights two crucial concepts: understanding the domain and codomain of geometric transformation and grasping the meaning of the one-to-one and onto concepts in a geometric transformation.

In geometric transformation, 'one-to-one' implies that distinct elements in the domain (denoted as K ≠ L for points K and L) result in different images under the transformation (T(K) ≠ T(L), where T represents a transformation). This expression signifies that in geometric transformations, all points in the plane have a unique corresponding point, ensuring comprehensive coverage of all points. Consequently, geometric transformation is not only applied to the object but also extends to all points on the plane. For instance, when reflecting a triangle KLM over the line AB (Figure 1), the transformation affects not only the triangle but also all points on the plane, including the line AB. Understanding geometric transformation conceptually through its definition becomes a crucial criterion for learning geometric transformations. Dodge (2012) reiterated the importance of the concepts of one-to-one and onto (correspondence) in achieving a conceptual understanding of geometric transformation.

Figure 1.

Reflecting a triangle KLM over the line AB



On the other hand, studies examining the subtopics of geometric transformations, such as reflection, rotation, and translation transformations, can be found in the literature (Akarsu, 2018; 2022; Akarsu & İler, 2022; Akarsu & Öçal, 2022; Aktaş & Gürefe, 2021; Demir & Kurtuluş, 2019; Edwards, 2003; Flanagan, 2001; Glass, 2001; Gülkılık & Uğurlu & Yürük, 2015; Gürbüz & Durmuş, 2009; Hacısalihoğlu Karadeniz, Baran, Bozkuş & Gündüz, 2015; Harper, 2002; Hollebrands, 2003, 2004; Son, 2006; Yanik, 2006; Yanik & Flores, 2009; Zembat, 2007, 2013, 2015). These studies have examined the understanding of three fundamental geometric transformations among teachers, preservice teachers, and students. Furthermore, analyses involving the creation of definitions for the three fundamental geometric transformations have been conducted. As a result of these studies, it has been revealed that, even though teachers (Akarsu & İler, 2022; Gürbüz & Durmuş, 2009; Son, 2006), teacher candidates (Akarsu, 2018, 2022; Aktaş & Gürefe, 2021; Yanik, 2006; Harper, 2002; Yanik & Flores, 2009; Hacısalihoğlu Karadeniz, et al., 2015), and students (Flanagan, 2001; Glass, 2001; Gülkılık & Uğurlu & Yürük, 2015; Edwards, 2003; Demir & Kurtuluş, 2019) can provide definitions for the three geometric transformations, they often do not address the conceptual dimensions in their definitions or encounter difficulties in defining them. Teachers, preservice teachers, and students have difficulties in explaining that reflection, rotation, and translation transformations are functions, each with their domain and codomain, and that they are one-to-one, onto, and distance-preserving transformations. These difficulties hinder conceptual understanding and learning. To deepen conceptual understanding, it is necessary to learn the definition of geometric transformations conceptually. In the following sections, the challenges and conceptual misconceptions that teachers, teacher candidates, and students face in defining reflection, rotation, and translation transformations will be examined in detail. However, a detailed elaboration on these difficulties and conceptual misconceptions is not provided in this question.

When examining the literature, it was found that there are studies providing the definition of geometric transformation; however, no studies investigating how teachers, teacher candidates, and students make the definition of geometric transformation were found. Nevertheless, analyzing the definitions and discourse of teachers, teacher candidates, and students is a crucial criterion for examining whether geometric transformation is conceptually understood correctly and, if not, for revealing the reasons behind it. Therefore, the following questions, which were not addressed in this section, have been identified:

3.1.1. Unanswered Question 1

How do teachers, preservice teachers, and students define reflection, rotation, and translation transformations, and to what extent do their definitions address conceptual dimensions?

3.1.2. Unanswered Question 2

What conceptual difficulties do teachers, preservice teachers, and students encounter when defining reflection, rotation, and translation transformations as functions with their domain and codomain, and as one-to-one, onto, and distance-preserving transformations?

3.1.3. Unanswered Question 3

How do these conceptual difficulties hinder the overall conceptual understanding and learning of geometric transformations?

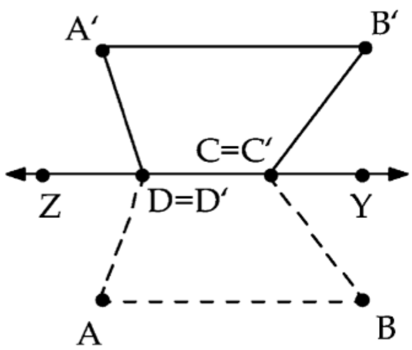
3.2. Why is geometric transformation important in school mathematics?

In today's context, the subject of geometric transformation has evolved into a crucial point in terms of education and teaching, standing out as a concept intensely studied in recent years. It is observed in the literature that the application areas of geometric transformation are explored, emphasizing its significance in teaching. Therefore, the subject of geometric transformation is encountered in various fields today. According to Knuchel (2004), patterns originating from Islamic civilization and introduced to Europe through Arab conquests in the 13th century can be associated with the concepts of rotating, reflecting, and translating objects in the plane. This allows achieving a smooth order without gaps or overlaps. Pumfrey and Beardon (2002) stated that tessellations are common in decorative art and can be observed in the natural world around us. They also explained that the use of translation, rotation, and reflection transformations contributes to the creation of tessellations and a better understanding of the movement of objects in space. In his study, Flanagan (2001) demonstrated that geometric transformation helps students perceive mathematics as an interdisciplinary discipline connected to other branches of science.

When reviewing the literature, there are studies emphasizing and revealing the importance of geometric transformation. Kort (1971) investigated the impact of taking a geometric transformation course in the tenth grade on the preparedness, skills, and conceptual learning of two groups of students in the eleventh grade. The results indicated that learning concepts and properties of transformations in the tenth grade increased students' ability to recall and use prior knowledge while learning topics such as congruence, similarity, and symmetry, contributing positively to the permanence of conceptual learning. According to Hollebrands (2003), geometric transformation (a) provides opportunities for students to think about important mathematical concepts such as functions and symmetry, (b) creates a context for students to see mathematics as an interconnected discipline, and (c) enables students to engage in higher-order reasoning activities using various representations. For example, in an application of reflection transformations on a trapezoid shape in a technological environment, students were asked to specify which points they reflected after applying the reflection transformation (See Figure 2). Participating students mentioned that they only reflected the corner points of the trapezoid. When asked by the researcher if any other points were reflected, students stated that they did not reflect any other points

Figure 2.

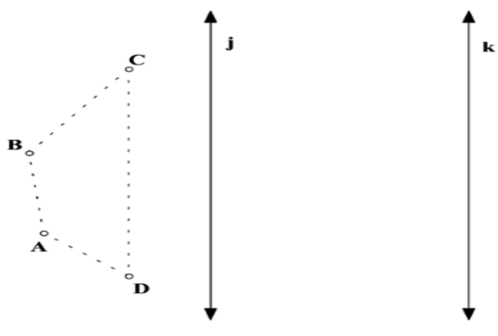
Trapezoid given to students to practice reflection transformation (Flanagan, 2001) (p.194)



In another implemented activity, students were asked to reflect a trapezoid shape created from disjointed points (see Figure 3). In this case, students expressed the belief that the shape was actually composed of points and, during reflection, all points forming the image should be reflected. Some students stated that the plane is also composed of points and that all visible and invisible points are reflected during reflection. From this moment on, students started to think about the definition and properties of the function and discovered that the one-to-one and commutative properties of the function are also valid for the reflection transformation. At the same time, in this question, students started to make connections between different mathematical topics. This was a result of the realization that when they applied the reflection transformation to the trapezoid according to the lines j and k, the last image formed was actually a translated version of the first image. This played a role in students' perception of mathematics as an interconnected discipline.

Figure 3.

Trapezoid given to students to practice reflection transformation (Flanagan, 2001, p.195)



On the other hand, numerous studies in the literature have clearly emphasized the importance of geometric transformation by demonstrating that when students learn geometric transformation, they enhance their visualization skills and analytical strategies (Boulter & Kirby, 1994; Edwards, 1997; Edwards & Zazkis, 1993; Jung, 2002), as well as cognitive skills such as critical thinking, intuitive perspectives, problem-solving, making assumptions, deductive reasoning, logical argumentation, and judgment (Jones, 2002). They have also highlighted the development of mathematical skills such as pattern recognition, exploring the fundamental properties of isometries, making generalizations, and enhancing spatial competencies (Clements, Battista, Sarama & Swaminathan, 1997; Portnoy, Grundmeier & Graham, 2006).

Jung (2002) investigated how students' use of representations in geometric transformations influenced their conceptual understanding. At the beginning of the study, students predominantly used verbal representations. As the process advanced, their ways of understanding basic concepts evolved. With an improved understanding of fundamental concepts and the application of these concepts, students gradually shifted away from verbal representations towards using visual representations. The use of visual representations supported the development of various skills such as analysis and problem-solving.

Boulter and Kirby (1994) examined the solution strategies employed by students in a geometric transformation test they conducted. The results indicated that as the content of the geometric transformation test questions became more complex, students tended to use analytical solution strategies rather than procedural ones. It was observed that as students utilized analytical solution strategies, their accuracy in answering the questions increased.

In their study, Clements et al. (1997) aimed to uncover students' competence in spatial skills on the plane and the development of these skills over time. The study incorporated activities using a Tetris game environment, where students were tasked with exploring patterns on a plane and covering the plane without leaving any gaps by applying geometric transformations (translation, rotation, and reflection) using multiple copies of a single shape. In these activities, students demonstrated competence in discovering and creating patterns on the plane, positioning a shape in various ways using geometric transformations, and covering the plane without leaving any gaps. The study revealed that, through the topic of geometric transformations, students not only explored congruence and similarity but also developed the ability to quickly visualize shapes in their minds and make decisions while positioning them on the plane, thereby enhancing spatial competency.

Therefore, integrating geometric transformation into high school geometry classes emerges as a highly significant mathematical topic, having a positive impact on students' mathematical understanding (Hollebrands, 2003). However, integrating geometric transformation into the elementary mathematics curriculum is crucial for successful advanced mathematical studies and contributes significantly to developing a strong background knowledge and readiness in geometry (Boulter & Kirby, 1994; Dixon, 1997; NCTM, 2000). Additionally, geometric transformation plays an important role in teaching by associating geometric transformations with other mathematical concepts, aiding in the conceptual understanding and interpretation of mathematical relationships. It also helps students effectively organize knowledge (Sünker & Zembat, 2012).

From various curriculum studies, it is evident that geometric transformation is an important topic that should be taught at the primary and secondary education levels. Indeed, the National Council of Teachers of Mathematics (NCTM) in 2000 emphasized the inclusion of geometric transformation and symmetry topics in the curriculum, highlighting the significance of geometric transformation for students to understand and analyze certain mathematical concepts. Furthermore, the incorporation and teaching of geometric transformation in the curriculum provide students with the opportunity to develop and utilize high-level reasoning skills (Edwards, 2003; Flanagan, 2001; Hollebrands, 2003; Jung, 2002; Yanik, 2006). Yanik (2014) asserted, based on inference, that geometric transformation contributes to students in various ways by enabling them to explore patterns and pattern rules, engage in deductive reasoning and generalization, and enhance spatial and critical thinking skills. The Ministry of National Education (MoNE) included geometric transformation in the mathematics curriculum implemented in 2005, stating that it plays a significant role in daily life mathematics and supports the development of students' creative thinking skills. Indeed, in their study, Ersoy and Duatepe (2003) expressed that applying translations and rotations to recurring shapes in a carpet pattern helped students gain a different perspective on their surroundings.

Integrating the topic of geometric transformation into the curriculum is crucial not only for students to grasp this concept conceptually but also for enhancing their understanding of various mathematical topics. Transformation geometry also enables the exploration of mathematical connections with other sciences. Consequently, students have the opportunity to develop interdisciplinary study skills. Therefore, integrating geometric transformation into curricula plays a vital role in nurturing diverse skills among students and fostering their active application of these skills.

The study underscores the evolving importance of geometric transformation in education and teaching, emphasizing its multi-faceted significance in various fields. Investigating historical patterns and its application in tessellations, the research demonstrates its role in creating smooth order and fostering interdisciplinary connections in mathematics. Studies reveal its positive impact on students' conceptual learning, problem-solving skills, and cognitive abilities, emphasizing the importance of integrating geometric transformation into high school geometry and elementary mathematics curricula. The findings underscore the role of geometric transformation in fostering spatial competency, visualization skills, and analytical strategies. Therefore, the following questions, which were not addressed in this section, have been identified:

3.2.1. Unanswered Question 4

How can future research explore innovative teaching methods to enhance students' understanding of geometric transformation?

3.2.2. Unanswered Question 5

What are the potential interdisciplinary connections that future research could explore in relation to geometric transformation?

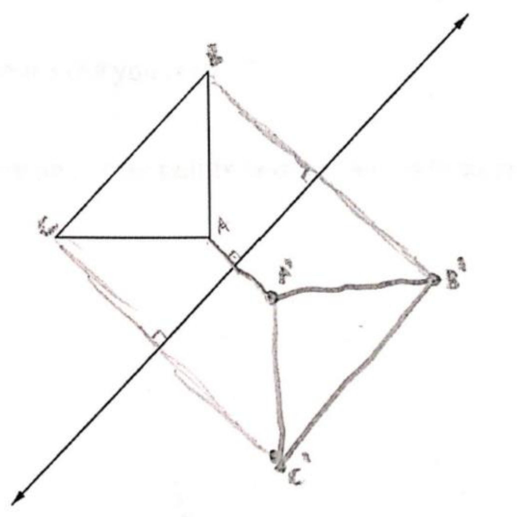
3.3. What Do We Know About Understanding Geometric Transformations?

In the last several decades, there has been increasing attention to understanding geometric transformations. The study of geometric transformations provides foundational knowledge for various mathematical concepts like functions, symmetry, and congruence, contributing to the development of both mathematical and cognitive skills. These outcomes highlight the crucial role of geometric transformations in geometry. Studies have shown that learners have two perspectives for understanding geometric transformations including translations, reflections and rotations: motion and mapping perspective (Akarsu, 2018; Flanagan, 2001; Yanik, 2006).

Researchers found that learners have a motion perspective of geometric transformations rather than a mapping perspective (Akarsu, 2018; Yanik & Flores, 2009). According to the findings of Flanagan (2001) and Yanik (2006), three crucial sub-concepts within the motion view and mapping view are parameters, domain, and plane. Regardless of their understanding of parameters (reflection line, vector, and center of rotation), domain, and plane, learners tend to perceive geometric transformations from a motion perspective. In their understanding of geometric transformations within the motion view, learners utilize the reflection line without incorporating the properties of perpendicularity and equidistance. In contrast, in the mapping view, prospective teachers (PTs) comprehend the role of the reflection line by integrating the properties of perpendicularity and equidistance. For instance, Akarsu (2018) examined how four pre-service teachers understand geometric reflections in terms of both motion and mapping views. One pre-service teacher executed a reflection of a triangle over a slanted reflection line (see Figure 4). Initially, he selected the triangle's nearest point (point A) to the reflection line and drew a perpendicular line from point A to the reflection line. Subsequently, he reflected point A over the reflection line, maintaining the equal distance between A and A' using the equidistance property. This process was then repeated for the remaining vertices (points B and C), and finally, he connected all three vertices to form a triangle.

Figure 4.

Pre-service teachers' drawing of a reflection (Akarsu, 2018, p.64)



In executing the triangle task, the pre-service teacher demonstrated an awareness of the associations between pre-image and image points of the figures, utilizing the geometric reflection properties of equidistance and perpendicularity, as seen in the figure 4. His act of mapping points and applying the properties of equidistance and perpendicularity serves as an indication that he possessed a mapping view of the reflection line for understanding of geometric reflections.

The results indicated that grasping the concept of the domain as comprising all points in the plane was both novel and challenging for learners in their comprehension of geometric transformations (Akarsu & Öçal, 2022; Yanik & Flores, 2009). Initially, learners tended to perceive the domain as a single object or point rather than encompassing all points in the plane. For instance, when presented with a task to translate a triangle, a pre-service teacher contemplated opting for the vertices and sides of the triangle exclusively, rather than selecting all the points in the plane, including those within and outside the triangle (Yanik, 2006). A potential explanation for this tendency could be that learners may perceive the plane as an empty background rather than recognizing it as comprising an infinite number of points.

In Akarsu's investigation (2018), a pre-service teacher received a triangle with an oblique reflection line and, he was tasked with identifying the figure after conducting a reflection across the line. The teacher mirrored the three vertices of the triangle by gauging the distance from each vertex to the reflection line using an index card. Subsequently, the teacher connected the mirrored vertices to reconstruct the triangle in the image. When prompted to elucidate which points underwent reflection, the teacher specified that only the triangle, including visible and labeled points, was reflected. His explanation and illustration reveal that he focused solely on the provided points or figures, rather than considering all points in the plane—a manifestation of a motion perspective in understanding geometric reflections.

The study's results suggested that translations, reflections, and rotations could be characterized as the movement of all points in the plane, rather than mapping of the plane onto itself (Akarsu, 2018; Hollebrands, 2003; Yanik, 2006). This perspective arose from learners considering points as distinct entities rather than integral components of the plane. For instance, Yanik (2006) discovered that preservice teachers, viewing the outcome of a transformation as the motion of geometric figures or points, believed that there could be no translation, reflection, or rotation without some form of movement. Similarly, in his study with prospective mathematics teachers, Akarsu (2018) observed that some of them mentioned executing geometric reflections by moving points or geometric shapes in the plane. Therefore, learners' challenge in comprehending the relationship between the plane and geometric figures remained a significant obstacle in grasping the essence of mapping the plane. This finding indicates that learners face challenges in comprehending the significance of the parameters in defining geometric transformations. These challenges include difficulties in understanding the connection between corresponding preimage and image points and the parameters, as well as applying some properties in geometric transformations.

In her study on high school students' understanding of translations, reflections, and rotations using The Geometer's Sketchpad, Flanagan (2001) found that students tend to adopt a motion perspective in understanding these transformations. For instance, most students described translation as the movement of a point, figure, or object, struggling with the use of vectors in the process. They encountered difficulties considering the direction and magnitude of the vector when performing translations. In the case of reflection, the majority envisioned it as a mirror effect applied to a single point or object across the reflection line, rather than encompassing all points in the plane. Additionally, they demonstrated a limited understanding of the role of the reflection line. Regarding rotation, all students depicted it as a pivot around another point using a specific angle, yet they exhibited a restricted comprehension of the center of rotation and the angle of rotation.

In summary, students tend to perceive geometric transformations from a motion perspective, influenced by the roles of parameters (vector, reflection line, center of rotation). They often consider the domain as a single point or object and observe the movement of points or objects within the plane rather than as integral parts of the plane. One reason for students thinking in this way may be the lack of understanding or insufficient knowledge about the relationship between the given figure and the figure resulting from the transformation and parameters.

An improved grasp of geometric transformations can provide valuable insights for teaching. In an ideal scenario, the greater understanding teachers have of their students' thought processes, the more effectively they can support their development. Nevertheless, there is considerable effort required to establish a connection between research-based insights into student cognition and actual teaching practices. Therefore, the following questions, which were not addressed in this section, have been identified:

3.3.1. Unanswered Question 5:

How does the progression from a motion perspective of geometric reflections evolve into a mapping view for learners?

3.3.2. Unanswered Question6:

Which elements contribute to the transition of learners from developing a motion perspective of geometric transformations to adopting a mapping perspective?

3.3.3. Unanswered Question 7:

What role do parameters like reflection lines, vectors, and centers of rotation play in learners' understanding of geometric transformations?

3.3.4. Unanswered Question 8:

How do learners' conceptualizations of the domain evolve from perceiving it as individual objects to understanding it as comprising all points in the plane?

3.4. How Can Technology be Used in Geometric Transformation Activities?

The utilization of technology in mathematics education has drawn the attention of researchers, primarily due to its potential to enhance the teaching and learning of math. The National Council of Teachers of Mathematics (NCTM) (2000) stated that “technology is essential in teaching and learning mathematics; it influences the mathematics taught and enhances students’ learning” (p. 11). Fisher and Leitzel (1996) also claimed, “Programs preparing teachers of mathematics must assure that students have the opportunity to enroll in mathematics courses that make use of graphing calculators, computers and other technology” (p. 5). Specifically, computer-based technologies such as dynamic geometry software (DGS) are seen as highly adaptable tools for encouraging exploration and experimentation, particularly in the context of geometric transformations.

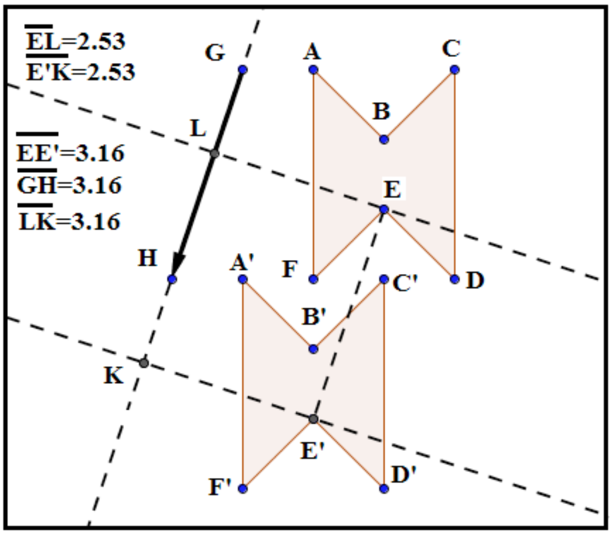
NCTM (2000) described the role of dynamic geometry software (DGS) in teaching and learning for students to gain a deeper understanding of transformations by allowing them to manipulate shapes and observe how each manipulation affects the shape's image. This approach encourages middle-grade students to explore congruence by closely examining the original and transformed figures' positions, side lengths, and angle measures. Additionally, transformations can be studied as a standalone topic. Teachers can prompt students to visualize and articulate the connections between lines of reflection, centers of rotation, and the positions of pre-images and images. Through interactive figures, students may discover that a reflection results in the transformed shape being equidistant from the line of reflection, compared to the original shape.

Dynamic geometry software (DGS) have proven highly effective in offering students the chance to engage with interactive visual representations of dynamic geometric scenarios. DGS provide visual feedback (Yanik, 2013; Harper, 2003) that helps learners to understand geometric transformation, perceive mathematical relationships, identify basic features of isometries (Hollebrands, 2007; Yanik, 2009; Harper, 2003), and show their activities (Hollebrands, 2007). In particular, the use of dragging and measurement features of the programs help learners to identify properties of geometric translations, make conjectures (Marrades & Gutierrez, 2000; Hollebrands, 2007), use different strategies, and construct new understandings (Yanik, 2013). For instance, by dragging the tail or the head of a vector, participants in Yanik’s (2013) study could identify how changes in the direction and the magnitude of the vector impacted the location, size, and shape of the translation.

Learners often use the measurement capabilities of GeoGebra for testing and verifying conjectures (Chazan, 1993, Hollebrands, 2007; Yanik, 2013). One of Yanik’s participants (2013) tested the conjecture that when the whole vector was dragged, the distances between the pre-image and image points and the length of the vector would be the same. Additionally, the measurement features of GeoGebra are helpful for learners to examine properties of geometric transformations. Another participant conjectured that the distances between pre-image and image points would be equal to the length of the vector (e.g., EE’= GH) (see Figure 5). To test and verify this conjecture, the participant drew a line that crossed the end points of the vector and then drew perpendicular lines to that line from the points E and E’. As a result, the participant measured with DGS and confirmed that his or her conjecture was correct (Yanik, 2013). Therefore, by expanding opportunities for learners to examine the properties of geometric transformations, DGS technologies can potentially assist learners in testing and verifying conjectures (Harper, 2003).

Figure 5.

(Yanik, 2013, p. 281)



Jung (2002) conducted research on the comprehension of translation, reflection and rotation by two pre-service secondary school math teachers in a technology-centered college math class. The results revealed that at the beginning, pre-service teachers employed informal mathematical language (e.g., using terms like "moving" and "flipping") and predominantly utilized visual and verbal explanations rather than symbolic notations to explain transformations. Their engagement with the Geometer's Sketchpad, facilitated them in formulating hypotheses about the characteristics of geometric transformations and gave them the opportunity to validate these hypotheses. By the end of the study, prospective teachers were able to offer more precise and lucid explanations of transformations using visual, verbal, and symbolic representations.

On the other hand, the utilization of DGS to comprehend the geometric transformations noted to have disadvantages as well as advantages. Flanagan's study in 2001 involved high school students using the Geometer's Sketchpad (GSP) to understand translation, rotation, reflection, and dilation. The research revealed that students tended to view the domain as individual points or shapes separate from the overall plane. For example, when a student was asked to predict the effects of moving a reflection line using the GSP, they assumed both pre-image and image points would move equally. However, upon testing, the student discovered only the image points moved, not the pre-image points. This highlighted that students tended to see geometric reflection as affecting specific given points rather than the entire plane and considered points as movable elements rather than integral parts of the plane. The use of GSP's features limited the students in perceiving the domain and plane as dynamic elements rather than as a fixed mapping of the domain and plane.

Akarsu and Öçal (2022) analyzed four prospective secondary teachers' (PSTs) understanding of geometric reflections by using GeoGebra. PSTs encountered challenges in mentioning or applying the principles of equidistance and perpendicularity (Yanik, 2013). This difficulty could be attributed to the automatic provision of equidistance and perpendicularity by GeoGebra's reflect-about-line tool, eliminating the need for PSTs to consider these properties when conducting geometric reflection. These indicate that although PSTs were aware of the equidistance and perpendicularity aspects for conducting geometric reflection without technology, they did not apply this knowledge when GeoGebra executed similar operations on their behalf.

Additionally, looking more specifically at the interviews of PSTs, the researchers provided a triangle with labeled points both inside and outside of the triangle. After performing the reflection, following the conclusion of each task, the researchers asked the PSTs if there were any other points or figures remaining to reflect. However, none of the PSTs indicated or hinted at comprehending that there were infinitely numerous points in the pre-image plane, and these points should also be reflected. This lack of understanding might be attributed to their consistent reliance on the reflect-about-line and segment tools, which directed their attention towards reflecting the labeled points and figures of the pre-images, without feeling the need to apply their understanding of the plane to the geometric reflection because GeoGebra was handling the entire process.

In conclusion, the study's exploration into the use of Dynamic Geometry Software reveals a predominant emphasis on a limited comprehension of geometric transformations. DGS tools, particularly the reflect-about-line and segment tools, automatically handle equidistance and perpendicularity, potentially hindering learners from actively engaging with these fundamental properties during geometric transformations. Moreover, the polygon tool encourages perceiving shapes as singular entities rather than compositions of individual points. The operational dynamics of software programs, requiring specific object selection for reflection, suggest a discrete application of reflection to these chosen elements, diverting attention from a holistic understanding. Additionally, the dragging tool leads learners to view their actions as point movements rather than reflections within the context of the plane.

3.4.1. Unanswered Question 9

How can Dynamic Geometry Software (DGS) be optimized to enhance learners' conceptual understanding of geometric transformations, particularly in facilitating comprehension and teaching of specific concepts such as geometric reflection, translation, and rotation?

3.4.2. Unanswered Question 10

How does the automatic handling of equidistance and perpendicularity by DGS tools influence learners' engagement with these fundamental properties during geometric transformations?

3.4.3. Unanswered Question 11

What strategies can be employed to mitigate the potential limitations of DGS in promoting a holistic understanding of geometric transformations among learners?

3.4.4. Unanswered Question 12

What advantages and disadvantages are associated with the use of DGS in teaching geometric transformations?

3.5. How are Geometric Transformation Activities Included in Mathematics Textbooks?

Key factors influencing student learning include textbooks (Fan, Zhu & Miao, 2013; Son & Diletti, 2017; Valverde, Bianchi, & Wolfe, 2002), which play a crucial role in shaping the content of a subject and how it is presented to students. As per the National Research Council's (NRC, 2004) recommendations, evaluating textbooks should initiate with content analysis to discern the connections between standards and the efficacy of the materials. Content analysis specifically examines the cognitive demand levels, the structure of lesson presentations, and the types of tasks and activities provided for students in each lesson (Thaqi & Gimenez, 2016).

Numerous educators have expressed dissatisfaction with the content emphasis in textbooks (Jones, 2004; Ma, 1999; Sun, Kulm & Capraro, 2023). Given that teachers rely on textbooks as instructional guides for teaching mathematical concepts, and students depend on them for valuable exercises and examples to enhance their understanding of mathematical concepts, insufficient coverage of these concepts in textbooks becomes a significant issue. Insufficient content poses challenges for teachers who must seek supplementary materials to compensate for the inadequacies.

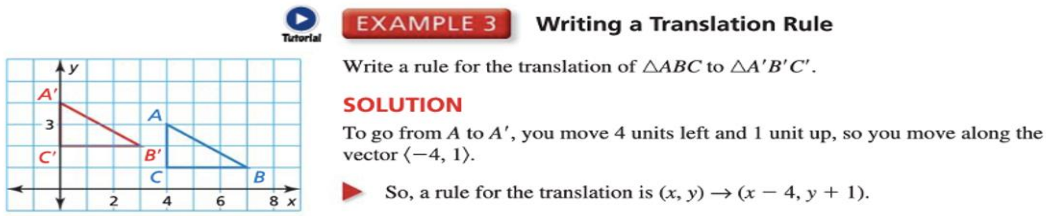
The NCTM (2000) has recommended the provision of tasks that aid students in developing problem-solving skills, critical thinking abilities, and proficiency in reasoning and proofs, as these tasks communicate fundamental messages about the nature and practice of mathematics. To achieve these objectives, it is crucial to present intricate tasks that challenge students to think conceptually, incorporating elements like communication, explanation of mathematical ideas, conjecture, generalization, and justification of strategies, while also involving the interpretation and framing of mathematical problems and drawing conclusions. In order to grasp mathematical concepts thoroughly, students must engage with high-level complex tasks. Consequently, the analysis of textbook content is of utmost importance, as it significantly shapes the curriculum within the classroom.

Research indicates that both learners tend to adopt a motion perspective rather than a mapping perspective one in their understanding of geometric reflections, particularly concerning the roles of the reflection line, domain, and plane (Flanagan, 2001; Yanik, 2006). This issue may be attributed, in part, to a deficiency of relevant content in textbooks. In this section, we explore how geometric reflections are addressed in middle-grade (6, 7, 8) textbooks from the National Science Foundation (NSF) funded Curriculum of Mathematics Project (CMP) Textbook series, and Turkish Mathematıcs Textbooks encompassing both teacher's guides and student editions. The selection of these textbooks is based on their widespread use in the United States and Turkey.

When the Big Idea book, which is frequently used in the American curriculum, was examined, it was observed that there were definitions and activities for 3 basic geometric transformations which are reflection, rotation and translation. Firstly, when the translation transformation is examined in the Big Idea book, its definition is "A translation moves every point of a figure the same distance in the same direction. More specifically, a translation maps, or moves, the points P and Q of a plane figure along a vector 〈a, b〉 to the points P′ and Q′." (p.174). When the given definition is examined, it states that in the translation transformation, the points match or move with a corresponding point along a certain vector. Explaining the translation transformation as the movement of points is an approach to the perspective of motion. At the same time, the definition states that all points belonging to a shape must undergo a translational transformation. However, according to the matching perspective, all points on the plane, that is, all points forming the plane, must match the corresponding points along a vector. It can be interpreted that this definition includes expressions for both matching and motion perspectives. However, when the activities in the Big Idea book were analyzed together with the definitions of translational transformation, it was understood that the activities were prepared for the motion perspective (see Figure 6).

Figure 6.

Activity of translation in Bıg Idea Textbook (p.175)


EXAMPLE 3 **Writing a Translation Rule**

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$.

SOLUTION

To go from A to A', you move 4 units left and 1 unit up, so you move along the vector $\langle -4, 1 \rangle$.

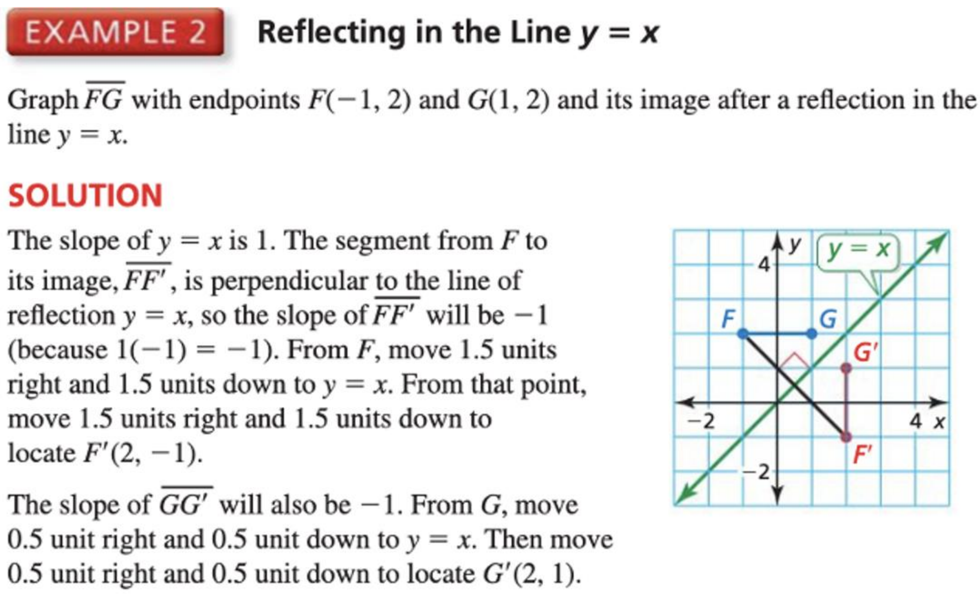
So, a rule for the translation is $(x, y) \rightarrow (x - 4, y + 1)$.

When the activity given in Figure 1 is examined, it is seen that only the points forming the shape are translated, and this is emphasized, but it is not mentioned that the other points on the plane undergo any transformation. However, according to the matching perspective of the translation transformation, it is explained that all points on the plane must match the corresponding point when they are translated along a vector. The definition of matching perspective and the activity do not match with each other. In addition, all activities in the book are explained with their operational properties. A conceptual explanation language is not used. Coordinate information during translation is tried to be taught by formulating. It was observed that the rest of the examples in the book were the same. In this respect, the translation transformation activities in the book are prepared for the motion perspective.

Secondly, the definition of reflection transformation is "a transformation that uses a line like a mirror to reflect a figure. A reflection in a line m maps every point P in the planet to a point P'...." (p. 182). At the same time, the definition emphasizes that the points should be perpendicular and equidistant from the axis of reflection. When the definition is analyzed, it is seen that the perpendicular, equal distance and the one-to-one correspondence of all points in the plane, which are the properties of reflection transformation, are mentioned. From this point of view, it can be interpreted that the definition of reflection in the Big Idea book is oriented towards the matching perspective. When the activities were analyzed together with the definitions of reflection transformation in the Big Idea book, it was understood that the activities were prepared for the motion perspective (see Figure 7).

Figure 7.

Activity of reflection in Bıg Idea Textbook (p.183)


EXAMPLE 2 **Reflecting in the Line $y = x$**

Graph $\overline{FG}$ with endpoints $F(-1, 2)$ and $G(1, 2)$ and its image after a reflection in the line $y = x$.

SOLUTION

The slope of $y = x$ is 1. The segment from F to its image, $\overline{FF'}$, is perpendicular to the line of reflection $y = x$, so the slope of $\overline{FF'}$ will be -1 (because $1(-1) = -1$). From F, move 1.5 units right and 1.5 units down to $y = x$. From that point, move 1.5 units right and 1.5 units down to locate $F'(2, -1)$.

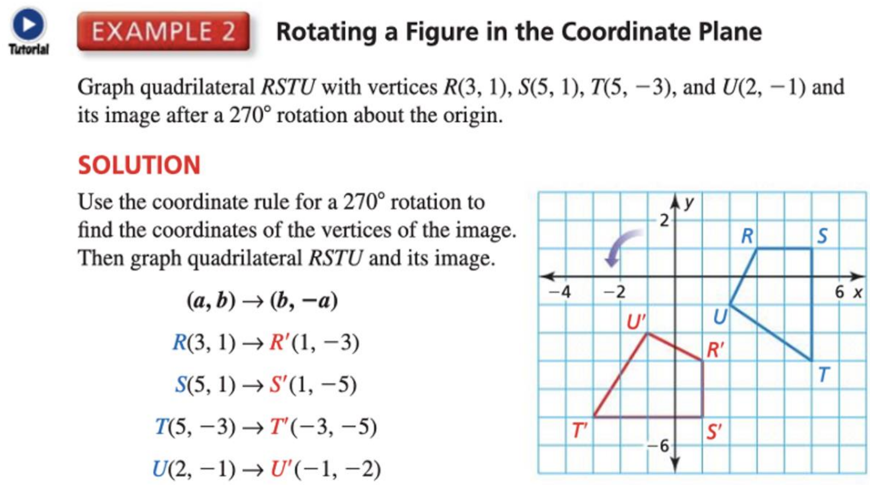
The slope of $\overline{GG'}$ will also be -1. From G, move 0.5 unit right and 0.5 unit down to $y = x$. Then move 0.5 unit right and 0.5 unit down to locate $G'(2, 1)$.

When the given activity is examined, the reflection transformation is performed in the coordinate plane. Performing all reflection transformation activities in the book in the coordinate plane may cause students to have misconceptions. In addition, as seen in the example, it is emphasized that the beginning and end points of the shape should be reflected when reflection is performed. However, no comment was made that all the points that make up the shape and the points outside the shape should be reflected. It should be inferred from this that the activities in the book are oriented towards the motion perspective.

Secondly, when the definition of rotation transformation is examined in the Big Idea book, it is explained as the rotation of all points on the plane around a point called the center of rotation by the specified degree. It is also stated that with a rotation of x degrees around any point P on the plane, every point Q on the plane must match Q'. When this definition is examined, it is emphasized that the plane should turn into a plane again, and each point should match with a corresponding point, so this definition of rotation transformation in the Big Idea book is a definition for the matching perspective. On the other hand, when the activities provided with the definition were examined, it was observed that the activities were mostly oriented towards the procedural process rather than the conceptual process and were designed to encourage the motion perspective rather than the matching perspective (see Figure 8).

Figure 8.

Activity of rotation in Bıg Idea Textbook (p.191)


Tutorial

EXAMPLE 2 **Rotating a Figure in the Coordinate Plane**

Graph quadrilateral $RSTU$ with vertices $R(3, 1)$, $S(5, 1)$, $T(5, -3)$, and $U(2, -1)$ and its image after a 270° rotation about the origin.

SOLUTION

Use the coordinate rule for a 270° rotation to find the coordinates of the vertices of the image. Then graph quadrilateral $RSTU$ and its image.

$$(a, b) \rightarrow (b, -a)$$

$$R(3, 1) \rightarrow R'(1, -3)$$

$$S(5, 1) \rightarrow S'(1, -5)$$

$$T(5, -3) \rightarrow T'(-3, -5)$$

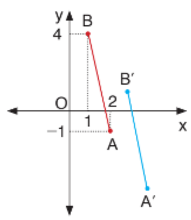
$$U(2, -1) \rightarrow U'(-1, -2)$$

When the activity given in Figure 8 is examined, it is seen that only the points forming the shape are rotated, and this is emphasized, but it is not mentioned that the other points on the plane undergo any transformation. However, in the description of the rotation transformation for the matching perspective, it is explained that all points on the plane should match the corresponding point when rotated by x degrees. The definition and the activity do not match with each other. In addition, the activities are explained with their procedural properties. During rotation, coordinate information is tried to be taught by formulating. It was observed that the rest of the examples in the book were the same. In this respect, the activities in the book are prepared for the motion perspective.

In addition to the Big Idea books, the Ministry of National Education (MoNE) curriculum high school mathematics books examined within the scope of the study were also examined according to the motion and matching perspectives of the three basic geometric transformations. In this context, firstly when the translation transformation is examined in the Science High School book, its definition is mentioned as "A'(x'+y') = A (x, y) + (a,b) = A' (x'+a, y'+b), where A'(x'+a, y'+b) is the point obtained by moving the point A(x, y) a unit in the x-axis direction and b units in the y-axis direction" (p.189). When the given definition is examined, it states that in the translation transformation, the points move to a point corresponding to it along a certain vector. Explaining the translation transformation as the movement of points is an approach to the perspective of motion. At the same time, the definition does not specify that all points belonging to a shape must undergo translation transformation. However, according to the matching perspective, all points on the plane, that is, all points forming the plane, must match the corresponding points along a vector. It can be interpreted that this definition includes expressions for the motion perspective. However, when the definition of translation transformation and the activities in the science high school book were examined, it was understood that the activities were prepared for the motion perspective (see Figure 9).

Figure 9.

Activity of translation in MoNE Curriculum Textbook (p. 190)



ÖRNEK 1

A(2, −1) ve B(1, 4) olmak üzere AB doğru parçasının x ekseni doğrultusunda 2 birim sağa, y ekseni doğrultusunda 3 birim aşağı ötelenmesi ile elde edilen A′B′ doğru parçasının uç noktalarının koordinatlarını bulunuz.

ÇÖZÜM

$A'(x_2, y_2) = A(2, -1) + (2, -3)$

$x_2 = 2 + 2 = 4$

$y_2 = -1 - 3 = -4$ olur.

Buradan $A'(4, -4)$ bulunur.

$B'(x_2, y_2) = B(1, 4) + (2, -3)$

$x_2 = 1 + 2 = 3$

$y_2 = 4 - 3 = 1$ olur.

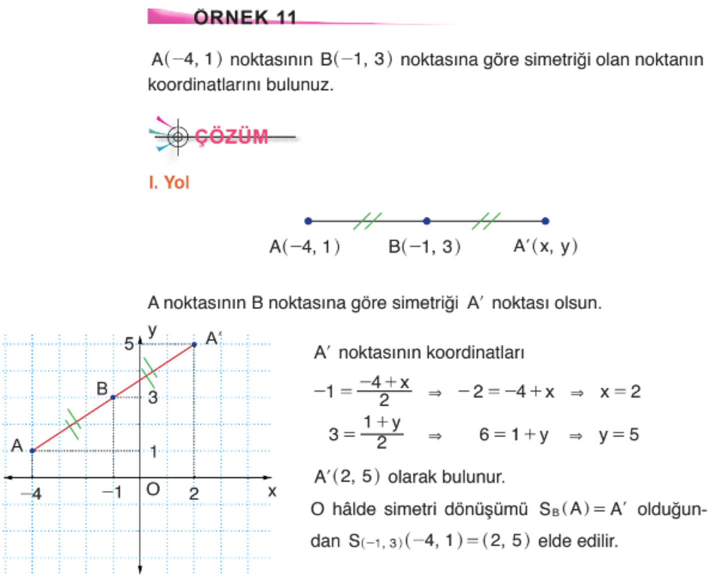
Buradan $B'(3, 1)$ bulunur.

When the activity given in Figure 9 is examined, it is seen that only the points forming the shape are translated, and this is emphasized, but it is not mentioned that the other points on the plane undergo any transformation. However, according to the matching perspective of the translation transformation, it is explained that all points on the plane must match the corresponding point when they are translated along a vector. The definition of matching perspective and the activity do not match with each other. In addition, all activities in the book are explained with their operational properties. A conceptual explanation language is not used. Coordinate information during translation is tried to be taught by formulating. It was observed that the rest of the examples in the book were the same. In this respect, the translation transformation activities in the book are prepared for the motion perspective.

Secondly, reflection transformation in the 12th grade science high school mathematics textbook of the Ministry of National Education, the definition of reflection transformation is given as "a transformation formed by taking the images of all points of a figure equidistant from a point or a line" (p. 201). When the definition is examined, it is emphasized that only the shape is the part on the plane to which reflection transformation should be applied. However, according to the matching perspective of reflection transformation, all points on the plane should match one-to-one with each point that is equidistant and perpendicular to the reflection axis. Therefore, this definition given in the science high school mathematics textbook is a definition for the motion perspective. As a result of examining the activities for reflection transformation in the science high school mathematics book along with the definition, it was revealed that these activities were presented in accordance with the procedural process rather than the conceptual process and that the activities were oriented towards the motion perspective (see Figure 10)

Figure 10.

Activity of reflection in MoNE Curriculum Textbook (p.202)


ÖRNEK 11

A(−4, 1) noktasının B(−1, 3) noktasına göre simetriği olan noktanın koordinatlarını bulunuz.

ÇÖZÜM

I. Yol

A(−4, 1) B(−1, 3) A′(x, y)

A noktasının B noktasına göre simetriği A′ noktası olsun.

A′ noktasının koordinatları

$-1 = \frac{-4+x}{2} \Rightarrow -2 = -4 + x \Rightarrow x = 2$

$3 = \frac{1+y}{2} \Rightarrow 6 = 1 + y \Rightarrow y = 5$

A′(2, 5) olarak bulunur.

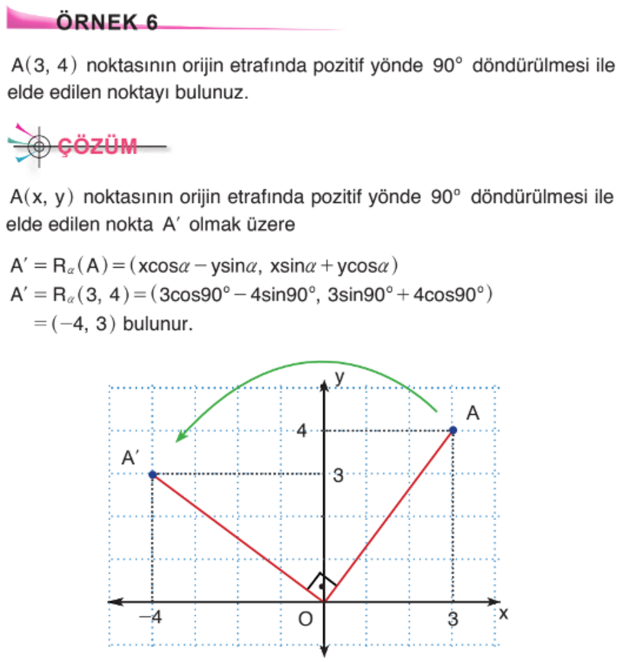
O hâlde simetri dönüşümü $S_B(A) = A'$ olduğundan $S_{(-1,\,3)}(-4, 1) = (2, 5)$ elde edilir.

When the activity given in the high school science book is examined, the reflection transformation is performed on a line segment in the coordinate plane. The reflection transformation was applied only to this line segment and the reflection transformation was explained with the formula. Performing all reflection transformation activities in the coordinate plane may encourage students to have misconceptions. In addition, as seen in the example, it is emphasized that the beginning and end points of the shape should be reflected when reflection is performed. However, no comment was made that all the points forming the shape and points outside the shape should be reflected. It should be inferred from this that the activities in the book are oriented towards the motion perspective.

Finally, when the definition of rotation transformation is examined in the Science High School book, it is seen that rotation is explained as "P' rotation transformation obtained by rotating the point P(x,y) around the origin in the positive direction by the angle α" (p. 194). In other words, it is stated that with a rotation of x degrees around any point on the plane, every point P on the plane must match P'. When this definition is examined, since it is emphasized that the plane should be transformed into a plane again and each point should match with a corresponding point, this definition of rotation transformation in the science high school mathematics book is a definition for the matching perspective. On the other hand, when the activities provided with the definition were examined, it was observed that the activities were mostly oriented towards the procedural process rather than the conceptual process and were designed to encourage the motion perspective rather than the matching perspective (see Figure 11).

Figure 11.

Activity of rotation in MoNE Curriculum Textbook (p.196)



ÖRNEK 6

A(3, 4) noktasının orijin etrafında pozitif yönde 90° döndürülmesi ile elde edilen noktayı bulunuz.

ÇÖZÜM

A(x, y) noktasının orijin etrafında pozitif yönde 90° döndürülmesi ile elde edilen nokta A′ olmak üzere

$A' = R_\alpha(A) = (x\cos\alpha - y\sin\alpha,\ x\sin\alpha + y\cos\alpha)$

$A' = R_\alpha(3, 4) = (3\cos 90° - 4\sin 90°,\ 3\sin 90° + 4\cos 90°)$

$= (-4, 3)$ bulunur.

When the activity given in Figure 11 is examined, it is seen that only the endpoints of the line segment are transformed and this is emphasized, but it is not mentioned that the other points on the plane undergo any transformation. However, in the description of the rotation transformation for the matching perspective, it is explained that all points on the plane should match the corresponding point when rotated by x degrees. The definition and the activity do not match with each other. In addition, the activities are explained with their operational properties. During rotation, coordinate information is tried to be taught by formulating. It was observed that the rest of the examples in the book were the same. In this respect, the activities in the science high school book were prepared for the motion perspective.

3.5.1. Unanswered Question 13

The insufficient research in this field emphasizes the need to focus more on the textbooks utilized by students and teachers rather than solely on the underlying curriculum frameworks. The approach textbooks take in incorporating geometric transformations is unclear, and understanding the choices made by textbook writers and curriculum developers in this regard is essential. With the current emphasis on geometric transformations in curriculum frameworks, it is crucial to explore the role they might play in comprehending other mathematical topics, such as functions and congruence, within textbooks.

3.5.1. Unanswered Question 14

If curriculum designers aim to incorporate geometric transformations into textbooks and educational materials, what would be the most effective approaches? In their examination of Turkish and USA mathematics textbooks, Akarsu (2018) specifically focused on tasks related to motion and mapping perspectives, emphasizing parameters, domain, and plane.

3.5.1. Unanswered Question 15

An examination of the relationship between textbook content and student learning outcomes, addressing the crucial question of whether the content in textbooks adequately supports students in developing problem-solving skills, critical thinking abilities, and proficiency in geometric transformations.

4. Results and Looking to the Future

This comprehensive exploration of geometric transformation provides a foundation for future studies to delve deeper into several key areas. The initial focus should involve refining and expanding the conceptual understanding of geometric transformation, particularly by addressing the unanswered questions regarding how teachers, teacher candidates, and students define this concept. Investigating the nuances of their definitions and the challenges faced can shed light on potential gaps in conceptual understanding. As a matter of fact, Hollebrands (2003) and Yanik (2006) revealed the difficulties and misconceptions of students and pre-service teachers, and examined the strategies and mental structures they used. However, as a result of the updates made in the curricula, there is a need for up-to-date studies. Furthermore, future research could extend into the practical implications of teaching geometric transformation, specifically examining the effectiveness of various instructional strategies and interventions. This could include exploring innovative teaching methods that emphasize the conceptual dimensions of geometric transformation, ultimately enhancing the learning experience for students.

The study highlights the importance of integrating geometric transformation into both high school geometry classes and elementary mathematics curricula. Future research should evaluate the impact of such integration on students' mathematical understanding, spatial competencies, and interdisciplinary study skills. Comparative studies across different educational levels and settings could provide valuable insights into the transferability of these benefits. Additionally, considering the evolving landscape of education, investigating the role of technology in teaching geometric transformation could be a fruitful avenue for research. Analyzing the impact of technological tools and virtual environments on students' engagement and comprehension of geometric transformation may contribute to the ongoing evolution of teaching methods. Hollebrands (2003) investigated the contribution of technology integration by examining students' conceptual understanding of geometric transformation and shared the results.

Future studies should explore the nuanced integration of technology, specifically Dynamic Geometry Software (DGS), in transformation geometry activities. While existing research emphasizes the advantages of DGS, such as GeoGebra and The Geometer's Sketchpad, in fostering a deeper understanding of geometric transformations, there is a need to address potential drawbacks and challenges associated with these tools. Understanding how learners transition from perceiving transformations as movements of individual points to adopting a holistic mapping perspective can provide insights for optimizing the design and implementation of technology-enhanced learning environments.

Overall, future studies should build upon the foundation laid by this research, addressing the identified gaps and expanding our understanding of geometric transformation's conceptual dimensions, instructional strategies, and broader educational implications.

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Yazar(lar), bu makalenin araştırılması, yazarlığı ve/veya yayınlanmasına ilişkin herhangi bir potansiyel çıkar çatışması beyan etmemiştir.

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