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Neutrosophic ideals of neutrosophic KU-algebras

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Abstract

In this paper, the concept of a neutrosophic KU-algebra is introduced and some related properties are investigated. Also, neutrosophic KU-ideals of a neutrosophic KU-algebra are studied and a few properties are obtained. Furthermore, a few results of neutrosophic KU-ideals of a neutrosophic KU-algebra under homomorphism are discussed

Keywords

KU-algebra
Neutrosophic KU-algebra
Neutrosophic KU-ideal

1. Introduction

Prabpayak and Leerawat [8, 9] introduced a new algebraic structure which is called KU-algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU-algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU-algebras and isomorphism.

Neutrosophy is a new branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set and neutrosophic logic were introduced in1998 by Smarandache as generalizations of fuzzy set and respectively intuitionistic fuzzy logic. In neutrosophic logic, each proposition has a degree of truth (T), a degree of indeterminancy (I) and a degree of falsity (F),where T,I, F are standard or non-standard subsets of $]-0, 1+[$, see [10, 11, 12]. Neutrosophic logic has wide applications in science, engineering, Information Technology, law, politics, economics, finance, econometrics, operations research, optimization theory, game theory and simulation etc.

The notion of neutrosophic algebraic structures was introduced by Kandasamy and Smarandache in 2006, see [5, 6]. Since then, several researchers have studied the concepts and a great deal of literature has been produced. For example, Agboola et al. in [1, 2] continued the study of some types of neutrosophic algebraic structures. Agboola and Davvaz introduced the concept of neutrosophic BCI/BCK-algebras in [3, 4]. In this paper, we introduce a neutrosophic KU-ideal of a neutrosophic KU-algebra and investigated some related properties. Also, we study a neutrosophic homomorphism of a neutrosophic KU-algebra and some results are obtained.

2. Preliminaries

Now, we will recall some known concepts related to KU-algebra from the literature which will be helpful in further study of this article.

Definition 2.1 [8, 9]. Let X be a set with a binary operation $*$ and a constant 0. Then $(X,*,0)$ is called

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KU-algebra if the following axioms hold: for all $x, y, z \in X$,

(KU_1) $(x*y)*[(y*z)*(x*z)]=0$,

(KU_2) $x*0=0$,

(KU_3) $0*x=x$,

(KU_4) $if\ \ x*y=0=y*x\ implies\ \ x=y$.

Define a binary relation $\leq$ by : $x \leq y \Leftrightarrow y*x=0$, we can prove that $(X,\leq)$ is poset. By the binary relation$\leq$, we can write the previous axioms in another form as follows:

$(KU_1^{'})$ $(y*z)*(x*z) \leq x*y,$

$(KU_2^{'})$ $0 \leq x,$

$(KU_3^{'})$ $x \leq y \Leftrightarrow y*x=0,$

$(KU_4^{'})$ $if\ \ x \leq y\ and\ \ y \leq x \rightarrow x=y.$

Example 2.2 [7]. Let X = {0, 1, 2, 3, 4} be a set with a binary operation $*$ defined by the following table.

$*$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Using the algorithms in Appendix, we can prove that (X, *, 0) is a KU-algebra.

Corollary 2.3 [7]. In a KU-algebra the following identities are true, for all, $x, y, z \in X$,

(i) $z*z=0$,

(ii) $z*(x*z)=0$,

(iii) If $x \leq y$ $implies\ that$ $y*z \leq x*z$,

(v) $z*(y*x)=y*(z*x)$,

(vi) $y*[(y*x)*x]=0$.

Definition 2.4 [9]. A subset S of a KU-algebra X is called a sub algebra of X if $x*y \in S$, whenever $x, y \in S$.

Definition 2.5 [9]. A non empty subset A of a KU-algebra X is called an ideal of X if it is satisfied the following conditions.

(i) $0 \in A$,

(ii) $y*z \in A, y \in A\ implies\ \ z \in A, \forall y, z \in X$.

Definition2.6 [13] . Let A be a non empty subset of a KU-algebra X . Then, A is said to be a KU-ideal of X if the following conditions hold:

(I_1) $0 \in A$,

(I_2) $\forall x, y, z \in X$, $x*(y*z) \in A$ and $y \in A$ imply $x*z \in A$.

Example 2.7[13]. Let $X=\{0,a,b,c,d,e\}$ be a set with the operation $*$ defined by the following table.

$*$	0	a	b	c	d	e
0	0	a	b	c	d	e
a	0	0	b	b	d	e
b	0	0	0	a	d	e
c	0	0	0	0	d	e
d	0	0	0	a	0	e
e	0	0	0	0	0	0

Then $(X,*,0)$ is a KU-algebra and it is easy to show that $\{0,a\}$, $\{0,a,b,c,d\}$ are KU-ideals of X .

Definition 2.8. Let $(X,*,0)$ be a KU-algebra, then for every $x,y\in X$ we denote $x\wedge y=(x*y)*y$.

3. Neutrosophic KU-algebra

Let X be a nonempty set and let I be an indeterminate. The set $X(I)=\langle X,I\rangle=\{(x,yI):x,y\in X\}$ is called a neutrosophic set generated by X and I . If $+$ and $\bullet$ are ordinary addition and multiplication, then I has the following properties:

(1) $I + I + \ldots + I = \mathrm{n}I$,
(2) I + (−I) = 0,
(3) $I\bullet I\bullet \ldots\bullet I = I^n = I$ for all positive integer n,
(4) $0\bullet I = 0$,
(5) $-I$ is undefined and therefore does not exist.

If $*: X(I)\times X(I)\to X(I)$ is a binary operation defined on $X(I)$, then the couple ($X(I)$, $*$) is called a neutrosophic algebraic structure and it is named according the axioms satisfied by $*$. Let ($X(I)$, $*$) and $(Y(I),\ *')$ be two neutrosophic algebraic structures, then a mapping $Q:(X(I),*)\to(Y(I),*')$ is called a neutrosophic homomorphism if the following conditions hold .

(1) $Q\{(w,xI)*(y,zI)\}=Q(w,xI)*'Q(y,zI)$,
(2) $Q(I)=I \quad \forall\ (w,xI)\ ,(y,zI)\in X(I)$.

Definition3.1. Let $(X,*,0)$ be any KU-algebra and $X(I)=\langle X,I\rangle$ be a set generated by X and I . The triple $(X(I),*,(0,0))$ is called a neutrosophic KU-algebra, if (a,bI) *and* (c,dI) are any two elements of $X(I)$ with $a,b,c,d\in X$, we define the following

$$(a,bI)\ *\ (c,dI)=(a*c,(a*d\wedge b*c\wedge b*d)I).........(1)$$

An element $x\in X$ is represented by $(x,0)\in X(I)$ and $(0,0)$ represents the constant element in $X(I)$. For all $(x,0)$, $(y,0)\in X$, we define

$$(x,0)*(y,0)=(x*y,0)=(y\wedge\neg x)...........(2)$$

where $\neg x$ is the negation of x in X .

Theorem3.2. Every neutrosophic KU-algebra $(X(I),*,(0,0))$ with condition

$((0,0I)*(a,bI)=(a,(a\wedge b)I)$ is a KU-algebra.

Proof . Suppose that $(X(I),*,(0,0))$ is neutrosophic KU-algebra. Let $x=(a,bI)$, $y=(c,dI)$, $z=(f,gI)$ be an arbitrary elements of $X(I)$. Then we have

$(KU_1)\quad (x*y)*[(y*z)*(x*z)]$

$=[(a,bI)*(c,dI)]*\lfloor((c,dI)*(f,gI))*((a,bI)*(f,gI))\rfloor\equiv$

$=[(a*c,(a*d\wedge b*c\wedge b*d)I)]*\{([(c*f,(c*g\wedge d*f\wedge d*g)I)*$

$((a*f),(a*g\wedge b*f\wedge b*g))\}$

$=(r,sI)*[(p,qI)*(u,vI)],$

$(r,sI)=(a*c,(a*d\wedge b*c\wedge b*d)I)=(c\wedge\neg a,(d\wedge\neg a\wedge c\wedge\neg b\wedge d\wedge\neg b)I)$,

$(p,qI)=(c*f,(c*g\wedge d*f\wedge d*g)I)=(f\wedge\neg c,(g\wedge\neg c\wedge f\wedge\neg d\wedge g\wedge\neg d)I$,

$(u,vI)=((a*f),(a*g\wedge b*f\wedge b*g))=(f\wedge\neg a),(g\wedge\neg a\wedge f\wedge\neg b\wedge g\wedge\neg b))$,

Hence,

$(p,qI)*(u,vI)=(p*u,(p*v\wedge q*u\wedge q*v)I$

$=(u\wedge\neg p,(v\wedge\neg p\wedge u\wedge\neg q\wedge v\wedge\neg q)I)=(m,kI),$

$(r,sI)*(m,kI)=(r*m,(r*k\wedge s*m\wedge s*k)I)$

$=(m\wedge\neg r,(k\wedge\neg r\wedge m\wedge\neg s\wedge k\wedge\neg s)I)=(\Gamma,hI).$

Now, we obtain

$$\Gamma=m\wedge\neg r=u\wedge\neg p\wedge\neg r=(f\wedge\neg a)(\neg c\vee a)=0$$

Also, we have

$$\mathrm{h}=(k\wedge\neg r\wedge m\wedge\neg s\wedge k\wedge\neg s)=k\wedge\neg r\wedge u\wedge\neg p\wedge\neg s\wedge k\wedge\neg s$$

$$=v\wedge\neg p\wedge\neg q\wedge u\wedge\neg r\wedge\neg s=\overbrace{g\wedge\neg a\wedge f\wedge\neg b\wedge g\wedge\neg b}^{v}\wedge\overbrace{(\neg f\vee c)}^{\neg p}\wedge$$

$$\overbrace{(\neg g\vee c\vee\neg f\vee d\vee\neg g\vee d)}^{\neg q}\wedge\overbrace{(f\wedge\neg a)}^{u}\wedge\overbrace{(\neg c\vee a)}^{\neg r}\wedge\overbrace{(\neg d\vee a\vee\neg c\vee b\vee\neg d\vee b)}^{\neg s}=0$$

This shows that $(\Gamma,hI)=(0,0I)$ and consequently $(x*y)*[(y*z)*(x*z)]=0$.

(2) We have

$$x*0=(a,bI)*(0,0I)=(a*0,(a*0\wedge b*0)I)=(0,(0\wedge 0)I)=(0,0I).$$

(3) We have

$$0*x=((0,0I)*(a,bI)=(0*a,(0*b\wedge 0*a)I)=(a,(b\wedge a)I)=(a,bI).$$

(4) If $x*y=0=y*x$, then we have

$$(a,bI)*(c,dI)=(a*c,(a*d\wedge b*c\wedge b*d)I)=(0,0I),$$
$$(c,dI)*(a,bI)=(c*a,(c*b\wedge d*a\wedge d*b)I)=(0,0I).$$

These imply that

$$(a*c,(a*d\wedge b*c\wedge b*d)I)=(0,0I)$$
$$(c\wedge\neg a,(d\wedge\neg a\wedge c\wedge\neg b\wedge d\wedge\neg b))=(0,0I)\ \textit{and}$$
$$(a\wedge\neg c,(b\wedge\neg c\wedge a\wedge\neg d\wedge b\wedge\neg d))=(0,0I).$$

Therefore,

$$c\wedge\neg a=0\ ,\ (d\wedge\neg a\wedge c\wedge\neg b\wedge d\wedge\neg b)=0\ \textit{and}$$
$$a\wedge\neg c=0\ ,\ (b\wedge\neg c\wedge a\wedge\neg d\wedge b\wedge\neg d)=0$$

From which we obtain $a=c$, $b=d$. Hence, $(a,bI)=(c,dI)$ that is $x=y$.

From (1) – (4), we have $(X(I),*,(0,0))$ is a KU-algebra.

Lemma3.3. Let $(X(I),*,(0,0))$ be a neutrosophic KU-algebra. Then

$$(0,0I)*(a,bI)=(a,bI)\Leftrightarrow a=b.$$

Proof. Suppose that $(0,0I)*(a,bI)=(a,bI)$. Then $(0,0I)*(a,bI)=(0*a,(0*a\wedge 0*b)I)=(a,bI)$ which implies $(a,(a\wedge b)I)=(a,bI)$ from which we obtain a = b. The converse is obvious.

Lemma3.4. Let $(X(I),*,(0,0))$ be a neutrosophic KU-algebra. Then for all (a,bI), (c,dI), $(e,fI)\in X(I)$:

(1) If $(a,bI)*(c,dI)=(0,0)$ *implies that* $[(e,fI)*(a,bI)]*[(e,fI)*(c,dI)]=(0,0)$ and $[(c,dI)*(e,fI)]*[(a,bI)*(e,fI)]=(0,0)$.

(2) $(a,bI)*[(c,dI)*(e,fI)]=(c,dI)*[(a,bI)*(e,fI)]$.

(3) $[(a,bI)*(c,dI)]*\{[(e,fI)*(a,bI)]*[(e,fI)*(c,dI)]\}=(0,0)$.

Proof . (1) Suppose that $(a,bI)*(c,dI)=(0,0)$. Then $(a*c,(a*d\wedge b*c\wedge b*d))=(0,0)$ from which we have that $c\wedge\neg a=0$, $(d\wedge\neg a\wedge c\wedge\neg b\wedge d\wedge\neg b)=0$. Now,

$(e,fI)*(a,bI)=(a\wedge\neg e,(b\wedge\neg e\wedge a\wedge\neg f)I)=(x,yI)$

and

$(e,fI)*(c,dI)=(c\wedge\neg e,(d\wedge\neg e\wedge c\wedge\neg f)I)=(p,qI)$.

Hence, $(x,yI)*(p,qI)=(p\wedge\neg x,(q\wedge\neg x\wedge p\wedge\neg y)I)=(u,vI)$, where

$u=p\wedge\neg x=c\wedge\neg e\wedge\neg x=c\wedge\neg e\wedge(\neg a\vee e)=c\wedge\neg e\wedge\neg a=0$ and

$$\begin{aligned}v=q\wedge\neg x\wedge p\wedge\neg y&=(d\wedge\neg e\wedge c\wedge\neg f)\wedge(\neg a\vee e)\wedge(c\wedge\neg e)\wedge(\neg b\vee e\vee\neg a\vee f)\\&=(d\wedge\neg e\wedge c\wedge\neg f\wedge\neg a)\wedge(\neg b\vee e\vee\neg a\vee f)=0\end{aligned}$$

This show that $(u,vI)=(0,0)$ and so $[(e,fI)*(a,bI)]*[(e,fI)*(c,dI)]=(0,0)$.

A similar computation show that $[(c,dI)*(e,fI)]*[(a,bI)*(e,fI)]=(0,0)$.

(2) LHS $(a,bI)*[(c,dI)*(e,fI)]=(a,bI)*(n,mI)$, where $n=e\wedge\neg c$, $m=f\wedge\neg c\wedge e\wedge\neg d$,

$$\begin{aligned}x*(y*z)&=(a,bI)*[(c,dI)*(e,fI)]\\&=(a,bI)*(n,mI)\\&=(n\wedge\neg a,(m\wedge\neg a\wedge n\wedge\neg b)I)\\&=[e\wedge\neg c\wedge\neg a,(f\wedge\neg c\wedge e\wedge\neg d\wedge\neg a\wedge e\wedge\neg c\wedge\neg b)I]\\&=[e\wedge\neg c\wedge\neg a,(f\wedge\neg c\wedge e\wedge\neg d\wedge\neg a\wedge\neg b)I]\text{..........................}(i)\end{aligned}$$

RHS

$(c,dI)*[(a,bI)*(e,fI)]=(c,dI)*(u,vI)$, where $u=e\wedge\neg a$, $v=f\wedge\neg a\wedge e\wedge\neg b$. Hence

$$\begin{aligned}(c,dI)*(u,vI)&=(u\wedge\neg c,(v\wedge\neg c\wedge u\wedge\neg d)I)\\&=(e\wedge\neg a\wedge\neg c,f\wedge\neg a\wedge e\wedge\neg b\wedge\neg c\wedge e\wedge\neg a\wedge\neg d)I)\\&=(e\wedge\neg a\wedge\neg c,f\wedge\neg a\wedge e\wedge\neg b\wedge\neg c\wedge\neg d)I)\text{.....................}(ii)\end{aligned}$$

From (i) and (ii), we get $(a,bI)*[(c,dI)*(e,fI)]=(c,dI)*[(a,bI)*(e,fI)]$.

(3) Put

$[(a,bI)*(c,dI)]*\{[(e,fI)*(a,bI)]*[(e,fI)*(c,dI)]\}=(u,vI)*\{(x,yI)*(p,qI)\}$, where

$(u,vI)=(a,bI)*(c,dI)=(c\wedge\neg a,(d\wedge\neg a\wedge c\wedge\neg b)I)$,

$(x,yI)=(e,fI)*(a,bI)=(a\wedge\neg e,(b\wedge\neg e\wedge a\wedge\neg f)I)$,

$(p,qI)=(e,fI)*(c,dI)=(c\wedge\neg e,(d\wedge\neg e\wedge c\wedge\neg f)I)$.

Thus, we have that

$$(x,yI)*(p,qI)=(p\wedge\neg x,(q\wedge\neg x\wedge p\wedge\neg y)I)$$
$$=(g,hI).$$

Now,

$$(u,vI)*(g,hI)=(g\wedge\neg u,(h\wedge\neg u\wedge g\wedge\neg v)I)$$
$$=(m,kI),$$

where,

$$\begin{aligned}m&=g\wedge\neg u\\&=p\wedge\neg x\wedge(\neg c\vee a)\\&=(c\wedge\neg e)\wedge(\neg a\vee e)\wedge(\neg c\vee a)\\&=(c\wedge\neg e\wedge\neg a\wedge\neg c)\vee(c\wedge\neg e\wedge\neg a\wedge a)\\&=(0\vee 0)=0,\end{aligned}$$

$$\begin{aligned}k&=h\wedge\neg u\wedge g\wedge\neg v\\&=(q\wedge\neg x\wedge p\wedge\neg y)\wedge(\neg c\vee a)\wedge(p\wedge\neg x)\wedge(\neg d\vee a\vee\neg c\vee b)\\&=(q\wedge\neg x\wedge p\wedge\neg y)\wedge(\neg d\vee a\vee\neg c\vee b)\\&=[(d\wedge\neg e\wedge c\wedge\neg f)\wedge(\neg a\vee e)\wedge(c\wedge\neg e)\wedge(\neg b\vee e\vee\neg a\vee f)]\wedge(\neg d\vee a\vee\neg c\vee b)\\&=[(d\wedge\neg e\wedge c\wedge\neg f\wedge\neg b)\vee(d\wedge\neg e\wedge c\wedge\neg f\wedge\neg a)]\wedge(\neg d\vee a\vee\neg c\vee b)=0.\end{aligned}$$

It follows that $(m,kI)=(0,0)$. Hence the proof is complete.

Definition 3.5. Let $(X(I),*,(0,0))$ be a neutrosophic KU-algebra. A non-empty subset $A(I)$ is called neutrosophic subalgebra of $X(I)$ if the following conditions hold:

(i) $(0,0)\in A(I)$,

(ii) $(a,bI)*(c,dI)\in A(I)$ for all (a,bI), $(c,dI)\in A(I)$,

(iii) $A(I)$ contains a proper subset which is a KU-algebra.

If $A(I)$ does not contain a proper subset which is a KU-algebra, then $A(I)$ is called a pseudo neutrosophic subalgebra of $X(I)$.

Theorem 3.6. Let $(X(I),*,(0,0))$ be a neutrosophic KU-algebra and $A_{(a,aI)}(I)$ be a subset of $X(I)$ defined by $A_{(a,aI)}(I)=\{(x,yI)\in X(I):(a,aI)*(x,yI)=(0,0)$, for $a\neq 0$. Then,

(1) $A_{(a,aI)}(I)$ is a neutrosophic subalgebra of , $X(I)$,

(2) $A_{(a,aI)}(I)\subseteq A_{(0,0I)}(I)$.

Proof. (1) Obviously, (0, 0) ∈ $A_{(a,aI)}(I)$ and $A_{(a,aI)}(I)$ contain a proper subset which is a KU-algebra. Let $(x,yI),(p,qI)\in A_{(a,aI)}(I)$. Then $(a,aI)*(x,yI)=(0,0)$ and $(a,aI)*(p,qI)=(0,0)$, it follows that $a*x=0, a*x\wedge a*y=0$, $a*p=0, a*p\wedge a*q=0$. Since $a\neq 0$, we have $x=y=p=q=a$. Now, we have

$$\begin{aligned}(a,aI)*[(x,yI)*(p,qI)]&=(a,aI)*[(x*p,(x*q\wedge y*p\wedge y*q)I]\\&=[a*(x*p),\{[a*(x*q\wedge y*p\wedge y*q)]\wedge a*(x*p)\}I]\\&=[a*0,\{a*a\wedge a*0\}I]=[0,(0\wedge 0)I]=(0,0).\end{aligned}$$

This shows that $(x,yI)*(p,qI)\in A_{(a,aI)}(I)$ and the required result follows.

(2) It is clear .

Theorem 3.7. Let $(X(I),*,(0,0))$ be a neutrosophic KU-algebra and $X_{\mathrm{T}}(I)$ be a subset of $X(I)$

defined by $X_T(I) = \{(x, xI); x \in X\}$. Then $X_T(I)$ is a neutrosophic subalgebra of $X(I)$.

Proof. Obviously $(0,0) \in X_T(I)$. Let (a, aI) , $(b, bI) \in X_T(I)$. Then, we have

$$(a, aI) * (b, bI) = (a * b, (a * b)I) = (b \wedge \neg a, (b \wedge \neg a)I) \in X_T(I).$$

The proof is complete.

Remark 3.8. Since $(X_T(I), *, (0,0))$ is a neutrosophic subalgebra, then $X_T(I)$ is a neutrosophic commutative KU-algebra in its own right.

Example 3.9. Let $X_T(I) = \{(0,0I), (a, aI), (b, bI), (c, cI)\}$ be a set with the operation $*$ defined by the following table

*	(0,0***I***)	(a,a***I***)	(b,b***I***)	(c,c***I***)
(0,0***I***)	(0,0***I***)	(a,a***I***)	(b,b***I***)	(c,c***I***)
(a,a***I***)	(0,0***I***)	(0,0***I***)	(a,a***I***)	(c,b***I***)
(b,b***I***)	(0,0***I***)	(0,0***I***)	(0,0***I***)	(c,a***I***)
(c,c***I***)	(0,0***I***)	(a,a***I***)	(b,b***I***)	(0,0***I***)

Then $X_T(I)$ is a neutrosophic subalgebra of $X(I)$.

Definition 3.10. Let $(X(I), *, (0,0))$ be a neutrosophic KU-algebra. A non-empty subset $A(I)$ is called a neutrosophic ideal of $X(I)$ if the following conditions hold:

(i) $(0,0) \in A(I)$,

(ii) If $(a, bI) * (c, dI) \in A(I)$ and $(a, bI) \in A(I)$ implies $(c, dI) \in A(I)$, for all (a, bI), $(c, dI) \in A(I)$.

Definition3.11. A non-empty subset $A_T(I)$ is called a neutrosophic KU-ideal of $X_T(I)$ if the following conditions hold:

(I_1) $(0,0) \in A_T(I)$,

(I_2) If $(x, xI) * [(y, yI) * (z, zI)] \in A_T(I)$ *and* $(y, yI) \in A_T(I)$ implies $(x, xI) * (z, zI) \in A_T$, for all (x, xI), $(y, yI), (x, xI) \in A_T(I)$.

Theorem 3.12. Every neutrosophic KU-ideal of $X_T(I)$ is a neutrosophic ideal of $X_T(I)$

Proof. Putting $(x, xI) = (0,0I)$ in (I_2), the result follows.

Definition 3.13. Let $(X(I), *, (0,0))$ and $(X(I), \bullet, (0',0'))$ be two neutrosophic KU-algebras.

A mapping $f : X(I) \to X'(I)$ is called a neutrosophic homomorphism if the following conditions hold:

(1) $f[(x, yI) * (z, mI)] = f(x, yI) \bullet f(z, mI)$, $\forall (x, yI), (z, mI) \in X(I)$.

(2) $f(0,0I) = (0', 0'I)$.

Also,

(3) If f is injective, then f is called a neutrosophic monomorphism.

(4) If f is surjective, then f is called a neutrosophic epimorphism.

(5) If f is a bijection, then f is called a neutrosophic isomorphism.

A bijective neutrosophic homomorphism from $X(I)$ onto $X'(I)$ is called a neutrosophic automorphism.

Theorem 3.14. Let $f: X(I) \rightarrow X'(I)$ be a neutrosophic homomorphism of neutrosophic KU-algebras. Then,

(i) If $(0,0I)$ is the identity in $X(I)$, then $f(0,0I)$ is the identity in $X'(I)$.

(ii) If S is a neutrosophic subalgebra of $X(I)$, then $f(S)$ is a neutrosophic subalgrbra of $X'(I)$.

(iii) If S is a neutrosophic subalgebra of $X'(I)$, then $f^{-1}(S)$ is a neutrosophic subalgebra of $X(I)$.

Proof. It is straightforward.

Definition 3.15. Let $f: X(I) \rightarrow X'(I)$ be a neutrosophic homomorphism of neutrosophic KU-algebras. Then the kernel of f denoted by $\ker f$, is defined to be the set

$$\ker f = \{(x, yI) \in X(I) : f(x, yI) = (0', 0'I)\}.$$

Theorem 3.16. Let $f: X(I) \rightarrow X'(I)$ be a neutrosophic homomorphism of neutrosophic KU-algebras. Then, f is a neutrosophic monomorphism if and only if $\ker f = \{(0,0)\}$

Proof. It is straightforward.

Theorem 3.17. Let $f: X(I) \rightarrow X'(I)$ be a neutrosophic homomorphism from a neutrosophic KU-algebra $X(I)$ into a neutrosophic KU-algebra $X'(I)$. Then the kernel f is a neutrosophic KU-ideal of $X(I)$.

Proof . Since $f(0,0I) = (0',0'I)$, then $(0,0I) \in \ker f$. Let $(a,bI)*[(c,dI)*(p,qI)] \in \ker f$ *and* $(c,dI) \in \ker f$, then $f\{(a,bI)*[(c,dI)*(p,qI)]\} = (0',0'I)$ *and* $f(c,dI) = (0',0'I)$, since

$$\begin{aligned}(0',0'I) = f\{(a,bI)*[(c,dI)*(p,qI)]\} &= f(a,bI)* f[(c,dI)*(p,qI)] \\ &= f(a,bI)*[f(c,dI)* f(p,qI)] \\ &= f(c,dI)*[f(a,bI)* f(p,qI)] \\ &= (0',0'I)*[f(a,bI)* f(p,qI)] \\ &= [f(a,bI)* f(p,qI)] \\ &= f[(a,bI)*(p,qI)].\end{aligned}$$

We get $[(a,bI)*(p,qI)] \in \ker f$, so $\ker f$ is neutrosophic KU-ideal of $X(I)$.

4. Appendix-Algorithms

This appendix contains all necessary algorithms

<u>Algorithm for KU-algebras</u>

Input (X :set, $*$:binary operation)

Output (“ X is a KU-algebra or not”)

Begin

If $X = \phi$ then go to (1.);

End If

If $0 \notin X$ then go to (1.);

End If

Stop: =false;

$i := 1;$

While $i \le |X|$ and not (Stop) do

If $x_i * x_i \ne 0$ then

Stop: = true;

End If

$j := 1$

While $j \le |X|$ and not (Stop) do

If $((y_j * x_i) * x_i) \ne 0$ then

Stop: = true;

End If

End If

$k := 1$

While $k \le |X|$ and not (Stop) do

If $(x_i * y_j) * ((y_j * z_k) * (x_i * z_k)) \ne 0$ then

Stop: = true;

End If

End If While

End If While

End If While

If Stop then

(1.) Output (" X is not a KU-algebra")

Else

Output (" X is a KU-algebra")

End If

End

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