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Soft Extra Strongly Semi Star Generalized Closed Sets

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Abstract

In this paper, we introduce the notion of soft extra strongly semi *g-closed sets and their properties. we showed that, soft extra strongly semi *g-closed set is strictly general than soft extra strongly g-closed set. Furthermore, the relationship between soft extra strongly semi *g-closed sets and other existing soft sets have been investigated. Also, we show that in soft extremely disconnected space, every soft extra strongly semi *g-closed set is a soft strongly g-closed set. Finally, by taking relative complements of extra strongly semi *g-closed set we study the extra strongly semi *g-open set.

1. INTRODUCTION

As associate efficacious mathematical approach to transact with doubts in fields of sciences, Molodtsov proposed [17] soft set theory. Shabir et al. [19] introduced the soft topological spaces and some of their properties, which discussed in more details in [5, 8]. After presentation of the operations of soft sets [15], the properties and applications of soft set theory have been studied. In [5, 7, 8, 10, 11, 12, 14, 15, 20], the authors introduced many studies of soft topological spaces. In [10], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kannan [14] introduced the concept of soft g-closed sets in a soft topological spaces, which is generalized in [2, 3, 4, 13, 15]. R. Hosny [18] introduced the concept of soft extra strongly g-closed sets in soft topological spaces.

In this paper, we introduce the concepts of soft extra strongly semi *g-closed sets and soft extra strongly semi *g-open. Also, their basic properties are studied in detail. Furthermore, the soft union and soft intersection of two soft supra extra strongly semi *g-closed (resp. open) sets have been obtained. The relationship between soft extra strongly semi *g-closed sets and other existing soft sets have been investigated.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory, which will be needed in this paper.

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Definition 2.1 [17] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F: A \to P(X)$. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \not\in A$, then $F(e) = \phi$ i.e

$F_A = \{(e, F(e)): e \in A \subseteq E,\ F: A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2 [16] Let $F_A,\ G_B \in SS(X)_E$. Then, F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

(1) $A \subseteq B$, and
(2) $F(e) \subseteq G(e), \forall e \in A$.

In this case, F_A is said to be a soft subset of G_B, and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3 [16] The soft complement of a soft set F_A, denoted by F_A^c, is defined by $F_A^c = (F^c, A)$, $F^c: A \to P(X)$ is a mapping given by $F^c(e) = X - F(e),\ \forall\ e \in A$ and F^c is called the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.4 [16] The soft union of two soft sets F_A and G_B over the common universe X is the soft set H_C, where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B \end{cases} .$$

Definition 2.5 [16] The soft intersection of two soft sets F_A and G_B over the common universe X is the soft set H_C, where $C = A \cap B$ and for each $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.6 [19] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X,\ \forall e \in E$,
(2) The union of any number of soft sets in τ belongs to τ,
(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.

Definition 2.7 [19] In a soft topological space (X, τ, E), the soft interior (resp., soft closure) of a soft set F_E , denoted by $int(F_E)$ (resp., $cl(F_E)$), are defined as follows:

$$int(F_E) = \tilde{\cup} \{G_E | G_E \text{ isopensoftsetand } G_E \tilde{\subseteq} F_E\},$$

$$cl(F_E) = \tilde{\cap} \{H_E | H_E \text{ isclosedsoftsetand } F_E \tilde{\subseteq} H_E\}.$$

Definition 2.8 A soft set (F, E) of a soft topological space (X, τ, E) is called

(1) [7, 10] Semi open soft, if $F_E \tilde{\subseteq} cl(int(F_E)$ (resp., semi closed soft, if $int(cl(F_E)) \tilde{\subseteq} F_E$). The set of all semi open soft sets is denoted by $SOS(X)$ and the set of all semi closed soft sets is denoted by $SCS(X)$.
(2) [12] Regular open soft, if $F_E = int(cl(F_E))$ (resp., regular closed soft, if $F_E = cl(int(F_E))$). The set of all regular open soft sets is denoted by $ROS(X)$ and the set of all regular closed soft sets is denoted by $RCS(X)$.
(3) [10] β-open soft, if $F_E \tilde{\subseteq} cl(int(cl(F_E)))$ (resp., β-closed soft, if $int(cl(int(F_E))) \tilde{\subseteq} F_E$). The set of all β-open soft sets is denoted by $\beta OS(X)$ and the set of all β-closed soft sets is denoted by $\beta CS(X)$.

Definition 2.9 [19] Let (X, τ, E) be a soft topological space, $F_E \in SS(X)_E$ and Y be a non-null subset of X. Then, the soft subset of F_E over Y denoted by (F_Y, E), is defined as follows:

$$F_Y(e)=Y \cap F(e),\ \forall e \in E. \quad \text{In other words } (F_Y, E)=\widetilde{Y} \widetilde{\cap} F_E.$$

Definition 2.10 [19] Let (X, τ, E) be a soft topological space and Y be a non-null subset of X. Then,

$\tau_Y=\{(F_Y, E): F_E \in \tau\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 2.11 A soft set F_E in a soft topological space (X, τ, E) is called

(1) [14] Soft generalized closed (soft g-closed), if $cl(F_E) \widetilde{\subseteq} G_E$ whenever $F_E \widetilde{\subseteq} G_E$ and $G_E \in \tau$.
(2) [15] Soft strongly generalized closed (soft strongly g-closed), if $cl(int(F_E)) \widetilde{\subseteq} G_E$ whenever $F_E \widetilde{\subseteq} G_E$ and $G_E \in \tau$.
(3) [18] Soft extra strongly generalized closed (soft extra strongly g-closed) if $int(cl(int(F_E))) \widetilde{\subseteq} G_E$ whenever $F_E \widetilde{\subseteq} G_E$ and $G_E \in \tau$.

3 Soft extra strongly semi *g-closed sets

Definition 3.1 A soft set F_E in a soft topological space (X, τ, E) is called soft extra strongly semi star generalized closed (soft extra strongly semi *g-closed) in a soft topological space (X, τ, E) if $int(cl(int(F_E))) \widetilde{\subseteq} G_E$ whenever $F_E \widetilde{\subseteq} G_E$ and G_E is semi open soft in X.

Example 3.2 Let $X=\{c_1, c_2, c_3\}$ be the set of cars under consideration and, $E=\{e_1(\text{costly}), e_2(\text{Model})\}$. Let A_E, B_E, C_E be three soft sets representing the attractiveness of the cars which Mr. X, Mr. Y and Mr. Z are going to buy, where:

$A(e_1) = \{c_2, c_3\} \quad A(e_2) = \{c_1, c_2\}$,

$B(e_1) = \{c_1, c_2\} \quad B(e_2) = \{c_1, c_3\}$,

$C(e_1) = \{c_2\} \quad C(e_2) = \{c_1\}$.

Then, $\tau = \{\widetilde{X}, \widetilde{\phi}, A_E, B_E, C_E\}$ defines a soft topology on X. Then, the family of all semi open soft sets are $\widetilde{X}, \widetilde{\phi}, A_E, B_E, C_E$ in addition to the following soft sets:

$D(e_1) = \{c_2\} \quad D(e_2) = \{c_1, c_2\}$,

$E(e_1) = \{c_2\} \quad E(e_2) = \{c_1, c_3\}$,

$F(e_1) = \{c_2\} \quad F(e_2) = X$,

$G(e_1) = \{c_1, c_2\} \quad G(e_2) = \{c_1\}$,

$H(e_1) = \{c_2, c_3\} \quad H(e_2) = \{c_1\}$,

$I(e_1) = X \quad I(e_2) = \{c_1\}$,

$J(e_1) = \{c_1, c_2\} \quad J(e_2) = X$,

$K(e_1) = X \quad K(e_2) = \{c_1, c_3\}$,

$M(e_1) = \{c_2, c_3\} \quad M(e_2) = X$,

$O(e_1) = X \quad O(e_2) = \{c_1, c_2\}$.

Hence, the soft sets N_E, L_E are soft extra strongly semi *g-closed, where:

$N(e_1) = \{c_1\} \quad N(e_2) = \{c_3\}$,

$L(e_1) = \{c_2\} \quad L(e_2) = \{c_2\}$.

On the other hand, the soft sets A_E, B_E, C_E are not soft extra strongly semi *g-closed.

Theorem 3.3 Every closed soft set F_E is soft extra strongly semi * g-closed in a soft topological space (X, τ, E).

Proof. Let $F_E \tilde{\subseteq} G_E$ and $G_E \in SOS(X)$. Since F_E is closed soft, $int(cl(int(F_E))) \tilde{\subseteq} cl(F_E) = F_E \tilde{\subseteq} G_E$. Therefore, F_E is soft extra strongly semi * g-closed.

Remark 3.4 The converse of the above theorem is not true in general as shall shown in the following example.

Example 3.5 In Example 3.2, the soft sets $(F, E), (G, E)$ are soft extra strongly semi * g-closed in (X, μ, E), but not closed soft ,where:

$F(e_1) = X \quad F(e_2) = \{c_1, c_2\}$,

$G(e_1) = \{c_2\} \quad G(e_2) = X$.

Theorem 3.6 In a soft topological space (X, τ, E), every soft extra strongly semi *g-closed set is soft extra strongly g-closed.

Proof. Let $F_E \tilde{\subseteq} G_E$ and $G_E \in \tau$. Then, $(G, E) \in SOS(X)$. Since F_E is soft extra strongly semi *g-closed, $int(cl(int(F_E))) \tilde{\subseteq} G_E$. Therefore, F_E is soft extra strongly g-closed.

Remark 3.7 The converse of the above theorem is not true in general. The following example supports our claim.

Example 3.8 Suppose that there are three dresses in the universe X given by $X=\{a, b, c\}$. Let $E=\{e_1(\text{cotton}), e_2(\text{woollen})\}$ be the set of parameters showing the material of the dresses. Let A_E, B_E be two soft sets over the common universe X, which describe the composition of the dresses, where

$A(e_1) = \{b\} \quad A(e_2) = \{b\}$,

$B(e_1) = \{a, b\} \quad B(e_2) = \{a, b\}$.

Then, $\tau = \{\tilde{X}, \tilde{\phi}, A_E, B_E\}$ is the soft topology over X. Then, the family of all semi open soft sets are $\tilde{X}, \tilde{\phi}, A_E, B_E$ in addition to the following soft sets:

$C(e_1) = \{b\} \quad C(e_2) = \{a, b\}$,

$D(e_1) = \{b\} \quad D(e_2) = \{b, c\}$,

$E(e_1) = \{b\} \quad E(e_2) = X$,

$F(e_1) = \{a, b\} \quad F(e_2) = \{b\}$,

$G(e_1) = \{b, c\} \quad G(e_2) = \{b\}$,

$H(e_1) = X \quad H(e_2) = \{b\}$,

$I(e_1) = \{a, b\} \quad I(e_2) = X$,

$J(e_1) = X \quad J(e_2) = \{a, b\}$.

Hence, the soft set K_E is soft extra strongly g-closed but not soft extra strongly semi *g-closed, where:

$K(e_1) = \{b, c\}$ $K(e_2) = \{b, c\}$.

Definition 3.9 A soft topological space (X, τ, E) is called soft extremely disconnected, if the soft closure of every open soft set is open soft.

Theorem 3.10 Let (X, τ, E) be a soft extremely disconnected topological space and $F_E \in SS(X)_E$. If F_E is a soft extra strongly semi *g-closed, then it is soft strongly g-closed.

Proof. Let F_E be a soft set with $F_E \tilde{\subseteq} G_E$ and $G_E \in \tau$, then $G_E \in SOS(X)$. Since F_E is soft extra strongly semi *g-closed, $int(cl(int(F_E))) \tilde{\subseteq} G_E$. Since (X, τ, E) is soft extremely disconnected topological space, $int(cl(int(F_E)))=cl(int(F_E))$. Hence, F_E is s soft strongly g-closed.

Theorem 3.11 If a soft set H_E of a soft topological space (X, τ, E) is soft extra strongly semi *g-closed, then $int(cl(int(H_E)))\backslash H_E$ contains only null semi closed soft set.

Proof. Let $F_E \tilde{\subseteq} int(cl(int(H_E)))\backslash H_E$ and F_E be a non null semi closed soft. Then, $F_E \tilde{\subseteq} H_E^{\tilde{c}}$, implies that $H_E \tilde{\subseteq} F_E^{\tilde{c}}$. Since H_E is soft extra strongly semi *g-closed and $F_E^{\tilde{c}}$ is semi open soft, $int(cl(int(H_E))) \tilde{\subseteq} F_E^{\tilde{c}}$. Hence, $F_E \tilde{\subseteq} [int(cl(int(H_E)))]^{\tilde{c}}$. Therefore, $F_E \tilde{\subseteq} [int(cl(int(H_E)))] \tilde{\cap} [int(cl(int(H_E)))]^{\tilde{c}}=\tilde{\phi}$. Thus, $F_E=\tilde{\phi}$, which is a contradiction. Thus, $int(cl(int(H_E)))\backslash H_E$ contains only null semi closed soft set.

Proposition 3.12 A soft subset H_E of a soft extremely disconnected topological space (X, τ, E) is soft extra strongly semi *g-closed if and only if $int(cl(int(H_E)))\backslash H_E$ contains only null semi closed soft set.

Proof. Immediate.

Theorem 3.13 If F_E is soft extra strongly semi *g-closed in a soft topological space (X, τ, E) and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} int(cl(int(F_E)))$, then G_E is soft extra strongly semi *g-closed.

Proof. Obvious.

Remark 3.14 The soft intersection (resp., union) of two soft extra strongly semi *g-closed sets need not be a soft extra strongly semi *g-closed as shown by the following example.

Example 3.15 In Example 3.2, the soft sets F_E, G_E are soft extra strongly semi *g-closed, where

$F(e_1) = X$ $F(e_2) = \{c_1, c_2\}$,

$G(e_1) = \{c_2\}$ $c_3\}, G(e_2) = X$.

But, their soft intersection is not soft extra strongly semi *g-closed.

Also, the soft sets V_E, U_E are soft extra strongly semi *g-closed, where:

$V(e_1) = \phi$ $V(e_2) = \{c_1\}$,

$U(e_1) = \{c_2\}$ $U(e_2) = \phi$.

But, their soft union is not soft extra strongly semi *g-closed.

Theorem 3.16 If A_E is soft extra strongly semi *g-closed and F_E is closed soft in a soft topological space (X, τ, E). Then, $A_E \tilde{\cap} F_E$ is soft extra strongly semi *g-closed.

Proof. Assume that $A_E \tilde{\cap} F_E \tilde{\subseteq} G_E$ and $G_E \in SOS(X)$. Then, $A_E \tilde{\subseteq} G_E \tilde{\cup} F_E^{c}$. Since A_E is soft extra strongly semi *g-closed, $int\Big(cl\big(int(A_E)\big)\Big) \tilde{\subseteq} (G_E \tilde{\cup} F_E^{c})$. Now,

$$int\Big(cl\Big(int\big((A_E \tilde{\cap} F_E)\big)\Big)\Big) \tilde{\subseteq} int\Big(cl\big(int(A_E)\big)\Big) \tilde{\cap} int\Big(cl\big(int(F_E)\big)\Big) \tilde{\subseteq} int\Big(cl\big(int(A_E)\big)\Big)$$

$\tilde{\cap}$ cl(F_E)=int(cl(int(A_E))) $\tilde{\cap}$ F_E $\tilde{\subseteq}$ [(G_E $\tilde{\cup}$ F_E^c) $\tilde{\cap}$ F_E]. Therefore, int(cl(int((A_E $\tilde{\cap}$ F_E)))) $\tilde{\subseteq}$ G_E. Hence, A_E $\tilde{\cap}$ F_E is soft extra strongly semi *g-closed.

Theorem 3.17 Let (X, τ, E) be a soft topological space. Then, every β-closed soft set is soft extra strongly semi *g-closed.

Proof. Let $F_E \tilde{\subseteq} G_E$ and $G_E \in SOS(X)$. Since F_E is β-closed soft, int(cl(int(F_E))) $\tilde{\subseteq} F_E \tilde{\subseteq} G_E$. Therefore, F_E is soft extra strongly semi *g-closed.

Remark 3.18 The converse of Theorem 3.17 is not true in general as shown in the following example.

Example 3.19 Suppose that there are four alternatives in the universe of houses X=$\{h_1, h_2, h_3, h_4\}$ and consider E=$\{e_1, e_2\}$ be the two parameters "quality of houses" and "price of houses" to be the a linguistic variables. Let $F_{1E}, F_{2E}, F_{3E}, F_{4E}$ be soft sets over the common universe X which describe the goodness of the houses, where:

$F_1(e_1) = \{h_1\}$ $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$ $F_2(e_2) = \{h_1, h_2\}$,

$F_3(e_1) = \{h_1, h_4\}$ $F_3(e_2) = \{h_1, h_4\}$,

$F_4(e_1) = \{h_1, h_2, h_4\}$ $F_4(e_2) = \{h_1, h_2, h_4\}$.

Then, τ=$\{\tilde{X}, \tilde{\phi}, F_{1E}, F_{2E}, F_{3E}, F_{4E}\}$ defines a soft topology on X. Clearly, the family of all semi open soft sets are the elements of τ in addition to the following soft sets:

$F_5(e_1) = \{h_1\}$ $F_5(e_2) = \{h_1, h_2\}$,

$F_6(e_1) = \{h_1\}$ $F_6(e_2) = \{h_1, h_3\}$,

$F_7(e_1) = \{h_1\}$ $F_7(e_2) = \{h_1, h_4\}$,

$F_8(e_1) = \{h_1\}$ $F_8(e_2) = \{h_1, h_2, h_4\}$.

$F_9(e_1) = \{h_1\}$ $F_9(e_2) = \{h_1, h_2, h_3\}$,

$F_{10}(e_1) = \{h_1\}$ $F_{10}(e_2) = \{h_1, h_3, h_4\}$,

$F_{11}(e_1) = \{h_1\}$ $F_{11}(e_2) = X$,

$F_{12}(e_1) = \{h_1, h_2\}$ $F_{12}(e_2) = \{h_1\}$.

$F_{13}(e_1) = \{h_1, h_3\}$ $F_{13}(e_2) = \{h_1\}$,

$F_{14}(e_1) = \{h_1, h_2, h_4\}$ $F_{14}(e_2) = \{h_1\}$,

$F_{15}(e_1) = \{h_1, h_4\}$ $F_{15}(e_2) = \{h_1\}$,

$F_{16}(e_1) = \{h_1, h_2, h_3\}$ $F_{16}(e_2) = \{h_1\}$.

$F_{17}(e_1) = \{h_1, h_3, h_4\}$ $F_{17}(e_2) = \{h_1\}$,

$F_{18}(e_1) = X$ $F_{18}(e_2) = \{h_1\}$,

$F_{19}(e_1) = \{h_1, h_2\}$ $F_{19}(e_2) = \{h_1, h_2, h_3\}$,

$F_{20}(e_1) = \{h_1, h_2\}$ $F_{20}(e_2) = \{h_1, h_2, h_4\}$.

$F_{21}(e_1) = \{h_1,, h_2\}$ $F_{21}(e_2) = X$,

$F_{22}(e_1) = \{h_1, h_2, h_3\}$ $F_{22}(e_2) = \{h_1, h_2\}$,

$F_{23}(e_1) = \{h_1, h_2, h_4\}$ $F_{23}(e_2) = \{h_1, h_2\}$,

$F_{24}(e_1) = X$ $F_{24}(e_2) = \{h_1, h_2\}$.

$F_{25}(e_1) = \{h_1, h_4\}$ $F_{25}(e_2) = \{h_1, h_2, h_4\}$,

$F_{26}(e_1) = \{h_1, h_4\}$ $F_{26}(e_2) = \{h_1, h_3, h_4\}$,

$F_{27}(e_1) = \{h_1, h_4\}$ $F_{27}(e_2) = X$,

$F_{28}(e_1) = \{h_1, h_2, h_4\}$ $F_{28}(e_2) = \{h_1, h_4\}$.

$F_{29}(e_1) = \{h_1, h_3, h_4\}$ $F_{29}(e_2) = \{h_1, h_4\}$,

$F_{30}(e_1) = X$ $F_{30}(e_2) = \{h_1, h_4\}$,

$F_{31}(e_1) = X$ $F_{31}(e_2) = \{h_1, h_2, h_4\}$,

$F_{32}(e_1) = \{h_1, h_2, h_4\}$ $F_{32}(e_2) = X$.

Hence, the soft set G_E is soft extra strongly semi *g-closed, but not β-closed soft, where: $G(e_1) = \{h_1, h_2\}$ $G(e_2) = \{h_1, h_3\}$.

Theorem 3.20 If a soft set F_E of a soft topological space (X, τ, E) is both open soft and soft extra strongly semi *g-closed, then it is a regular open soft.

Proof. Since F_E is open soft and soft extra strongly semi *g-closed, $\mathrm{int}(\mathrm{cl}(F_E)) = \mathrm{int}(\mathrm{cl}(\mathrm{int}(F_E))) \tilde{\subseteq} F_E \tilde{\subseteq} \mathrm{int}(\mathrm{cl}(F_E))$. Therefore, F_E is regular open soft.

Theorem 3.21 Let (Y, τ_Y, E) be a semi open soft subspace of a soft topological space (X, τ, E), $F_E \tilde{\subseteq} Y_E$ and F_E is soft extra strongly semi *g-closed in (X, τ, E). Then, F_E is soft extra strongly semi *g-closed in (Y, τ_Y, E).

Proof. Assume that $F_E \tilde{\subseteq} B_E \tilde{\cap} Y_E$ and $B_E \in \tau$, $B_E \in SOS(X)$. So, $B_E \tilde{\cap} Y_E \in \tau_Y$ and $F_E \tilde{\subseteq} B_E$. Since F_E is soft extra strongly semi *g-closed in (X, τ, E), $\mathrm{int}(\mathrm{cl}(\mathrm{int}(F_E))) \tilde{\subseteq} B_E$. Now, $[\mathrm{int}(\mathrm{cl}(\mathrm{int}(F_E))) \tilde{\cap} Y_E] \tilde{\subseteq} B_E \tilde{\cap} Y_E$. It follows, $[\mathrm{int}_{\tau_Y}(\mathrm{cl}_{\tau_Y}(\mathrm{int}_{\tau_Y}(F_E))) \tilde{\subseteq} B_E \tilde{\cap} Y_E$. Therefore, F_E is soft extra strongly semi *g-closed in (Y, τ_Y, E).

4 Soft extra strongly semi $^*g$-open sets

Definition 4.1 A soft set $F_E \in SS(X)_E$ is called soft extra strongly semi star generalized open (soft extra strongly semi *g-open) set in a soft topological space (X, τ, E), if its relative complement $F_E^{\tilde{c}}$ is soft extra strongly semi *g-closed.

Theorem 4.2 A soft set G_E is soft extra strongly semi *g-open set in a soft topological space (X, τ, E) if and only if $F_E \tilde{\subseteq} \mathrm{cl}(\mathrm{int}(\mathrm{cl}(G_E)))$ whenever $F_E \tilde{\subseteq} G_E$ and F_E is semi closed soft.

Proof. Let $F_E \tilde{\subseteq} G_E$ and F_E is semi closed soft in X, then $G_E^{\tilde{c}} \tilde{\subseteq} F_E^{\tilde{c}}$ and $F_E^{\tilde{c}}$ is semi open soft. Since $G_E^{\tilde{c}}$ is soft extra strongly semi *g-closed, $\mathrm{int}(\mathrm{cl}(\mathrm{int}(G_E^{\tilde{c}}))) \tilde{\subseteq} F_E^{\tilde{c}}$. Consequently, $F_E \tilde{\subseteq} [\mathrm{int}(\mathrm{cl}(\mathrm{int}(G_E^{\tilde{c}})))]^{\tilde{c}} = \mathrm{cl}(\mathrm{int}(\mathrm{cl}(G_E)))$.

On the other hand, let $G_E^{\tilde{c}} \tilde{\subseteq} H_E$ and H_E is semi open soft in X. Then, $H_E^{\tilde{c}} \tilde{\subseteq} G_E$ and $H_E^{\tilde{c}}$ is semi closed soft in X. Hence, $H_E^{\tilde{c}} \tilde{\subseteq} \mathrm{cl}(\mathrm{int}(\mathrm{cl}(G_E)))$ from the necessary condition. Thus,

$[cl(int(cl(G_E)))]^{\tilde{c}}$=$int(cl(int(G_E^{\tilde{c}}))) \tilde{\subseteq} H_E$ and H_E is semi open soft. This shows that, $G_E^{\tilde{c}}$ is soft extra strongly semi *g-closed. Therefore, G_E is soft extra strongly semi *g-open set.

Example 4.3 In Example 3.2, the soft sets N_E^c, L_E^c are soft extra strongly semi *g-open, where N_E^c, L_E^c are defined by:

$N^c(e_1) = \{c_2, c_3\} \quad N^c(e_2) = \{c_1, c_2\}$,

$L^c(e_1) = \{c_1, c_3\} \quad L^c(e_2) = \{c_1, c_3\}$.

Theorem 4.4 Every open soft set is soft extra strongly semi * g-open in a soft topological space (X, τ, E).

Proof. Follows from Theorem 3.3.

Remark 4.5 The converse of Theorem 4.4 is not true in general as shall shown in the following example.

Example 4.6 In Example 3.2, the soft sets $(P, E), (Q, E)$ are soft supra strongly semi * g-open in (X, μ, E), but not open soft ,where:

$P(e_1) = \phi \quad P(e_2) = \{c_3\}$,

$Q(e_1) = \{c_1, c_3\} \quad Q(e_2) = \phi$.

Theorem 4.7 In a soft topological space (X, τ, E), every soft extra strongly semi *g-open set is soft extra strongly g-open.

Proof. Immediate from Theorem 3.6.

Remark 4.8 The converse of the above theorem is not true in general as we shall show in the following example.

Example 4.9 In Example 3.8, the soft set $K_E^{\tilde{c}}$ is soft extra strongly g-open but not soft strongly semi *g-open, where:

$K^{\tilde{c}}(e_1) = \{a\} \quad K^{\tilde{c}}(e_2) = \{a\}$.

Theorem 4.10 If a soft set H_E of a soft topological space (X, τ, E) is soft strongly g-open, then $H_E \backslash cl(int(cl(H_E)))$ contains only null semi open soft set.

Proof. It is similar to the proof of Theorem 3.11.

Theorem 4.11 If F_E is soft extra strongly semi *g-open in a soft topological space (X, τ, E) and $int(cl(F_E)) \tilde{\subseteq} G_E \tilde{\subseteq} F_E$, then G_E is soft extra strongly semi *g-open.

Proof. Straightforward.

Remark 4.12 The soft intersection (resp. union) of two soft extra strongly semi *g-open sets need not be a soft extra strongly semi *g-open as shown by the following example.

Example 4.13 In Example 3.2, the soft sets $F_E^{\tilde{c}}, G_E^{\tilde{c}}$ are soft extra strongly semi *g-open, where:

$F^{\tilde{c}}(e_1) = \phi \quad F^{\tilde{c}}(e_2) = \{c_3\}$,

$G^{\tilde{c}}(e_1) = \{c_1\} \quad G^{\tilde{c}}(e_2) = \phi$.

But, their soft union is not soft extra strongly semi *g-open.

Also, the soft sets $V_E^{\tilde{c}}, U_E^{\tilde{c}}$ are soft extra strongly semi *g-open, where:

$V^{\tilde{c}}(e_1) = X \quad V^{\tilde{c}}(e_2) = \{c_2, c_3\}$,

$U^{\tilde{c}}(e_1) = \{c_1, c_3\} \quad U^{\tilde{c}}(e_2) = X$.

But, their soft intersection is not soft extra strongly semi *g-open.

Theorem 4.14 In a soft topological space (X, τ, E), every β-open soft set is soft extra strongly semi *g-open.

Proof. It is similar to the proof of Theorem 3.17.

Remark 4.15 The converse of Theorem 4.14 is not true in general as shown in the following example.

Example 4.16 In Example3.19, the soft set (W, E) is soft extra strongly semi *g-open, but not β-open soft, where:

$W(e_1) = \{c, d\}$ $W(e_2) = \{b, d\}$.

Theorem 4.17 If a soft subset F_E of a soft topological space (X, τ, E) is both closed soft and soft extra strongly semi *g-open, then it is regular closed soft set.

Proof. Since F_E is closed soft and soft extra strongly semi *g-open, $cl(int(F_E)) \tilde{\subseteq} F_E \tilde{\subseteq} cl(int(cl(F_E)))$=$cl(int(F_E))$. Therefore, F_E is regular closed soft.

Theorem 4.18 Let (Y, τ_Y, E) be a semi open soft subspace of a soft topological space (X, τ, E) and $F_E \tilde{\subseteq} Y_E$. If F_E is soft extra strongly semi *g-open in (X, τ, E), then it is soft extra strongly semi *g-open in (Y, τ_Y, E).

Proof. It is similar to the proof of Theorem 3.21.

5 Conclusion

In this paper, we introduce and study the notions of soft extra strongly semi *g-closed sets and soft extra strongly semi *g-open sets in soft topological spaces. Also, their basic properties are studied. Furthermore, the relationship between soft extra strongly semi *g-closed sets and other existing soft sets have been investigated, which is supported by counter examples from the real life situations. It has been pointed out in this paper that many of these parameters studied have, in fact, applications in real life situations. In future, the generalization of these concepts to supra soft topological spaces [1, 8] will be introduced and the future research will be undertaken in this direction.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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