MEASURING DISPROPORTIONALITY IN PR SYSTEMS

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Nispi Temsil Sistemlerinde Orantısızlığın Ölçülmesi

Özet

Nispi seçim sistemlerinde seçmenler temsili organ için belli sayıda aday gösteren ve aldıkları oy oranında sandalye elde edebilen partiler ya da ittifak blokları için oy verirler. Bu seçilmiş parlamento çeşitli seçmen gruplarının çıkarlarını ne oranda yansıtır, ya da başka bir deyişle parlamento ne ölçüde temsilidir? Bu soruyu yanıtlamak amacıyla, bir seçim sisteminin temsil oranını değerlendiren "orantısızlık göstergeleri" diye adlandırılan göstergeler kullanılmaktadır. Orantısızlık göstergeleri her partinin aldığı oy sayısı ile kazandığı sandalye sayısı arasında yapılan karşılaştırmaya dayanmaktadır.

Bu makalede nispi temsil sistemlerinin seçmenlerin tercihlerini tam yansıtmamalarının nedenleri tartışılacaktır. Çalışmada iyi bilinen çok sayıda nispi temsil sistemi betimlenecek, orantısızlık göstergeleri hakkında kısaca bilgi verilecek ve iki yeni gösterge önerilecektir: Temsil Endeksi ve Göreli Temsil Endeksi. Çalışmada bu iki gösterge çok sayıda ülkedeki parlamento seçimleri göz önünde tutularak hesaplanmaktadır.

Anahtar Kelimeler: Nispi temsil sistemleri, orantısızlık göstergeleri, parlamento seçimleri, temsil endeksi, göreli temsil endeksi.

Abstract

In proportional representation systems voters vote for parties or blocks, which run for the certain number of seats in a representative body, and get seats proportionally to the received votes. In what degree the elected parliament reflects the interests of various groups of the voters, or, in other words, to which extent it is representative? To answer this question the indices are introduced evaluating the degree of proportionality of an electoral system, called "disproportionality indices". Disproportionality indices are based on the comparison between the number of votes and number of seats each party obtains.

In the article the reasons why any electoral PR system distorts voters' preferences are discussed. Below we describe several well-known proportional representation systems, give a brief look to known disproportionality indices and introduce two new indices of that type, the Representation Index and the Index of Relative Representation, respectively. These two indices are calculated for parliamentary elections in several countries.

Keywords: Proportional representation systems, disproportionality indices, parliamentary elections, representation index, index of relative representation.

Measuring Disproportionality in PR Systems

The reasons why any electoral system distorts the voters' preferences are discussed. Several well-known disproportionality indices for proportional representation systems are analyzed. Two new disproportionality indices are introduced and calculated for parliaments in several countries.

Introduction

The idea of proportional representation (PR) systems is that in the elections the voters vote not for the separate candidates but for parties or blocks, which run for the certain number of seats in any representative body, and get seats proportionally to the received votes. An example of PR system is the elections to a parliament under party lists. In this case parties reflecting interests of various groups of the voters receive seats in the parliament according to the size of these groups: more popular parties receive more seats, less popular parties receive less seats. Thus, one can say that the purpose of proportional representation is to enable maximal number of the voters to receive the representatives in the parliament.

There is a question that frequently arises after any elections: in what degree the elected parliament reflects the interests of various groups of the voters, or, in other words, to which extent it is representative? To answer this question the indices are introduced evaluating the degree of proportionality of an electoral system, named "disproportionality indices". Disproportionality indices are based on the comparison between the quota of votes and quota of seats each party obtains. Since the end of nineteenth century many of such indices were introduced, e.g. Rae Index, Loosemore-Hanby Index, Gallagher Index and others [1 - 7]. Below we introduce two more disproportionality indices.

In Section 1 the reasons why any electoral PR system distort the voters' preferences are considered. In Section 2 we analyze several well-known disproportionality indices. In Section 3 we introduce Representation Index that takes into account absence of the voters, and Index of Relative Representation, respectively. In Section 4 these two indices are calculated for parliamentary elections in several countries.

1. Distortion of proportionality

Let n be the number of parties in the elections, N be the number of parties received seats in a parliament. Let us enumerate them according to the received votes in decreasing order. Let v_i be the percentage of the votes and r_i be the percentage of the seats received by party i, $i = \overline{1,n}$. Thus, $r_i = 0$ for all parties not represented in the parliament ($i = \overline{N+1,n}$).

Let us assume that if the voter come on the poll and vote for certain party, then this party reflects his political preferences completely, and, vice versa, if the voter does not come this means that there is no party which satisfies his interests. So, the votes v_i received by parties $i = \overline{1,n}$ can be considered as "true" preferences of the voters. If for a party i the percentage of the received votes v_i is not equal to the percentage of the seats in the parliament r_i ($v_i \neq r_i$), one can say that the voting procedure distorts preferences of the voters. Let us consider the reasons of such disproportionality.

First, electoral "threshold" can be introduced by the legislation of the country. Then parliament is formed only from the parties which overcome it (for example, in Russia parliamentary elections in 1993-2003 it was 5%). Let us assume that some parties do not get seats in the parliament ($N < n$). Then the seats are distributed between parties 1, 2, …, N; a share of seats, which they receive is equal to

$$\frac{r_i}{100} \approx \frac{v_i}{\sum_{i=1}^{N} v_i} > \frac{v_i}{100} \qquad (i = \overline{1,N}).$$

Thus, the parties passed in the parliament receive more seats at the expense of others.

Secondly, always there are voters that have no favorite party and ignore the voting process. Accordingly, the interests of these voters also are not represented in the parliament. In some countries (for example, in Russia and Ukraine) the voters have an opportunity to vote "against all parties".Thus, one can believe that these voters also are not represented in parliament.

Finally, there is one more reason that distort preferences of the voters. It stems from the fact that it is generally impossible to allocate seats between the winning parties strictly proportionally to the received votes because the number of seats that can be assigned to each party must be an integer. To solve this problem the various procedures approaching proportional distribution are invented, and each of these procedures is somewhat inaccurate.

For the analysis of such distortions the disproportionality indices are introduced. Below we analyze several indices characterizing the degree to what extent the elected parliament corresponds the preferences of the voters and introduce two new indices.

It is necessary to note that all disproportionality indices are introduced for PR systems and thus they characterize to what extent electoral systems are perfect. However, in various countries many different procedures are used at parliamentary elections (for example, one-seat and multi-seat constituencies or mixed procedures - half of MPs are elected under the party lists and other half are elected in one-seat constituencies). The disproportionality indices can be applied in these cases as well. Then they should be considered as a parameter of similarity of concrete electoral system and ideal PR, when all parties receive seats in parliament strictly proportionally to the received votes.

2. Disproportionality indices known in the literature

2.1 The Maximum Deviation index

Consider any party *i* and assume that after the seats allocation procedure the percentage of seats r_i it receives more than the percentage of the received votes v_i. But then, by an obvious ratio

$$\sum_{i=1}^{n} r_i = 100$$

(all seats in parliament, of course, should be filled) there will be at least one party for which the share of seats it receives is less than its share of votes.

Thus, for each party i the difference between the percentage of the received seats and the percentage of the received votes is equal to $|r_i - v_i|$. Note that this value diminishes as the result of seats distribution procedure in the parliament approaches to the result of elections or, in other words, as the result of this procedure becomes more proportional.

First, we consider the most simple of indices, which can be introduced for this case, the Maximum Deviation index [1, 4]:

$$MD = \max_i |r_i - v_i|.$$

It is obvious that this index measures the "fairness" of seats distribution procedure for the certain parliamentary elections: it equals to the maximal discrepancy between v_i and r_i - the top limit of distortion of proportionality.

2.2 The Rae Index

Besides the maximum deviation it is possible to find average value of deviation for all parties. The Rae Index [1, 4, 5, 7] is introduced as follows:

$$I = \frac{1}{n}\sum_{i=1}^{n} |r_i - v_i|.$$

It equals to the average value of distortion of proportionality for all parties participating in the elections.

Let us notice one disadvantage of this index: it depends on the number of parties which participates in the elections. Thus, it is hardly usable in the case of large number of parties which receive small number of votes. It is the case because they do not really influence the results of elections and make small contribution $|r_i - v_i|$ to the final value of the index, but, nevertheless, at averaging they are taken into account as well as large parties. Thus, if there are many parties with the number of votes equals to zero, the value of the index I becomes arbitrarily small.

2.3 The Grofman Index

In the analysis of electoral situations frequently there is a question: how many parties really run for seats in the parliament and how many parties are simply mentioned in the bullot?

Let us consider the following example. Let three parties participate in the elections. Consider two possible outcomes: first - all parties receive about 1/3 votes; second - two parties have received 49 % of votes, and third - 2 %. In both cases the number of parties is equal to three, but, nevertheless, the results of elections are essentially various: in the first case all parties are equally strong, in the second case the party which receive 2 % of votes is considerably weaker then the others. So, the problem is how to differ these electoral situations. To solve it the index of "effective number of parties" [4, 5] was introduced. It is written as follows:

$$E = \frac{1}{\sum_{i=1}^{n}\left(\frac{v_i}{100}\right)^2}$$

As to example above, for the first case (equality of the votes received by parties) $E = 3$, for the second case $E \approx 2{,}08$. Thus, one can see that this index is simply equal to the number of parties in the first case when all parties are equally strong. At the same time, in the second case when the third party is much more weak than the others, its contribution to the final result becomes insignificant and the value of the index decreases. So, this index shows the number of parties really run for seats in the parliament.

It is possible now to improve the property of Rae Index discussed above. Substituting the number of parties n with their effective number E, we receive one more disproportionality index named Grofman Index [1, 5]:

$$G = \frac{1}{E}\sum_{i=1}^{n}\left|r_i - v_i\right|.$$

2.4 The Gallagher Index

Two considered above indices are based on the idea of arithmetic averaging: at calculation of average value of discrepancy between percentage of seats received by a party i, r_i, and percentage of votes v_i, the final value turned

out by dividing the sum on the number of parties (common or effective). But there are many other measures that are also possible. One of these measures is the least square one.

On this idea the Gallagher Index [1 - 5] is based:

$$LS = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(r_i - v_i)^2}\ .$$

The property of this index is that the sum under a root is calculated not from deviations, but from their squares. Thus, the more deviation is, the greater contribution it brings to the value of the index. It is possible to tell that the Gallagher Index basically takes into account strong distortions of the voters' preferences and it is weakly sensitive to the small ones.

This property can be strengthened having raised a degree under the root. Let us introduce the class of indices:

$$H_s = \sqrt[s]{\frac{1}{s}\sum_{i=1}^{n}\left|r_i - v_i\right|^s}\ , \qquad s = 2, 3, 4, 5, \ldots$$

It is obvious, that as the degree s become greater, an index H_s takes into account more large discrepancies. For $s = 2$ index H_2 coincides with the Gallagher Index.

2.5 The Loosemore-Hanby Index

Consider the example from Section: let parties A, B and C receive 49 %, 49 % and 2 % of votes, respectively. Let us assume, that there is a rule of 5 %-electoral threshold. Then the parties A and B receive 50 % of seats each, and party C do not receive seats at all ($v_A = v_B = 50\%,\ \ v_C = 0\%$). Thus, for parties A and B:

$$\left|r_A - v_A\right| = \left|r_B - v_B\right| = 1\%;$$

for party C:

$$\left|r_C - v_C\right| = 2\%.$$

Thus, the parties A and B pass in parliament and receive 1% of the seats more for the expense of party C. In this case it is possible to say that the electoral procedure distorts preferences of 2% of the voters. In general, to find total number of such voters, it is necessary to summarize the discrepancies $|r_i - v_i|$ for all parties and divide the sum by two (since in summation the same voters are evaluated twice).

Loosemore-Hanby Index [1, 4 - 6] is written as follows

$$D = \frac{1}{2}\sum_{i=1}^{n}|r_i - v_i|.$$

2.6 The Equal Proportion Index

All indices considered above are based on the difference $|r_i - v_i|$ between the percentage of the received seats r_i and the percentage of the received votes v_i. Let us consider another class of indices which are based on the ratio v_i / r_i or r_i / v_i.

The Equal Proportion index [4] takes into account seat and vote shares as follows

$$EP = \sqrt{\sum_{i=1}^{n}\frac{v_i^{\,2}}{r_i} - 1}$$

The shortage of this index is that if a party obtains no seats in parliament, it will give an infinitely large contribution to the total sum.

2.7 The d'Hondt Index

The d'Hondt index [4] is introduced as follows:

$$H = \max_{i=1,n}\frac{r_i}{v_i}$$

It simply measures the seat-to-vote ratio of the most overrepresented party. Its minimum value is 1 which corresponds to the exact proportionality, when all parties have identical seat-to-vote ratios, and its maximum value, attained if a party with no votes receives some seats, is plus infinity. Another shortage of this index is that it is too sensitive to the overrepresentation of small parties.

3. Two new disproportionality indices

3.1 The index of Relative Representation

Consider situation when some get no seats in parliament. Introduce an index describing representation of elections as follows

$$R = \frac{1}{N}\sum_{i=1}^{N}\frac{r_i}{v_i}.$$

It is necessary to emphasize, that the summation is made only for first N parties, which receive seats in the parliament.

The index of Relative Representation R shows how many percentages of seats in the parliament are received on the average by each party for 1 per cent of votes. For example, if R = 1.5 it means that the party which receive 20 % votes, will receive approximately 30 % of seats. Notice, that, like d'Hondt Index, in our assumptions the parliament most satisfies to the interests of the voters when R is close to 1 (for other considered indices "optimal" value is zero).

3.2 The Representation Index Taking into Account Absence of the Voters

None of the indices above takes into account the absense of the voters and possibility of voting "against all".

Let η be the share of voters which ignore the elections, and α be the share of voters which vote "against all".

Let us introduce the index as follows

$$\rho = (1-\eta)\left[1-\frac{1}{2}\left(\sum_{i=1}^{n}\left|\frac{r_i}{100}-\frac{v_i}{100}\right|+\alpha\right)\right].$$

Using the Loosemore-Hanby Index this formula can be written down as

$$\rho = (1-\eta)\left[1-\left(\frac{D}{100}+\frac{\alpha}{2}\right)\right]$$

Index ρ shows how many voters are represented in parliament according to their interests. Really, the value *D*/100 equals the share of voters who vote for one of *n* of parties, which preferences are distorted by the method of seats distribution in the parliament. We add $\alpha/2$ to take into account the voters which vote "against all", because they are not represented in parliament, so their preferences are also distorted by the electoral procedure. Thus, the value placed in square brackets equals the share of the voters correctly represented in parliament; multiplying it to $(1-\eta)$, the share of the voters who come to the polls, we obtain the necessary value.

Let us discuss some properties of this index. It is obvious that its maximum value, $\rho = 1$, corresponds to the best situation, when all voters participate in elections ($\eta = 0$), none of them vote "against all" ($\alpha = 0$), and the electoral system do not distort the results of elections in any way that is all parties receive seats strictly proportionally to the received votes ($r_i = v_i$ for all $i = \overline{1,n}$).

Consider a situation when all parties participating in the elections get the seats in the parliament or the number of votes received by all other parties together is close to zero. In this case the value

$$\sum_{i=1}^{n}\left|\frac{r_i}{100}-\frac{v_i}{100}\right|$$

is defined only by the seats distribution procedure. Let us notice, that the more seats in parliament are, the less this discrepancy is. Thus, if the number of seats in the parliament is great enough, the index is stable with respect to a method of seats distribution.

Suppose that in the given situation the value of the discrepancy is close to zero (for example, one of the parties is much stronger than the others: its share of votes $v_i \to 100$, and it occupies all parliament – $r_i = 100$). Then at $\eta = 0$, $\rho \to 1$. With growing η the value of the index decreases as η.

The minimum value of this index is zero, but in practice it is not achieved, because some pre-conditions usually are applied by law. For instance, in Russia $\eta < 0.75$, $\alpha < v_1$.

Let us notice that this index can be considered as the parameter of stability of the parliament – the more ρ, the more the parliament is supported by the voters at the moment of the elections.

Below the indices *R* and ρ are calculated for parliamentary elections in several countries.

4. Disrpoportionality Indices *R* and ρ calculated for some parliaments

We calculate the indices *R* and ρ for several countries (the electoral data have been taken from [8 - 13]). As it is seen from Table 1, values of these indices are varied essentially. The most representative parliament is Swedish one. On the contrary, in former USSR republics, Russia, Ukraine and Lithuania, there are many parties competing with each other, and some of them get no seats in parliament at certain elections owing to electoral threshold (5% in Russia and Lithuania, 4% in the Ukraine). Poor value of indices for Turkey-2002 elections is basically explained by high value of electoral threshold in Turkey (10%).

Table 1. Indices R and ρ calculated for some parliaments in several countries

Country	Year of election	Index *R*	Index ρ
Russia	1993	1.15	0.44
	1995	1.98	0.32
	1999	1.23	0.50
	2003	1.42	0.40
Ukraine	1998	1.53	0.57
	2002	1.32	0.61
Lithuania	2000	1.31	0.50
Turkey	2002	1.75	0.43
Sweden	1991	1.05	0.82
	1994	1.02	0.84
	1998	1.02	0.78
	2002	1.04	0.76

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