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Research Article

Benchmark Study of The Exchange-Corrected Density Functionals: Application to Strained Boron Nitride Clusters

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Abstract: We present a quantum chemical study of three small boron nitride clusters B₂N₂, B₃N₃ and B₄N₄. Their structure and electronic characteristics are calculated by means of the coupled cluster (CC) and density functional theory (DFT) techniques. In order to find the best match with the coupled cluster data the twenty-four DFT exchange-corrected functionals are analyzed. According to our results, B3P86V5 and B97 functionals reproduce well the geometry of small boron-nitrides, whereas for the electronic characteristics OP and VWN functionals give the closest to CC results. Note that prevalent B3LYP and PBE0 DFT-functionals demonstrate lower accuracy.

Keywords: *coupled cluster, density functional theory, exchange-corrected functionals, boron nitride clusters, boron nitride cubane.*

1. Introduction

After discoveries of fullerenes, nanotubes and graphene, many novel carbon architectures were proposed and investigated. They include peapods [1], fullerites [2], diamonds [3,4], and many others [5-8]. In these structures, carbon atoms form k-membered cycles, in which k value varies mostly from 4 to 8. Note, that only a limited number of high-strained structures contains triangle cycles with k = 3 (for example, tetrahedrane derivatives [9-10] and Ladenburg's benzene [11]). The values of k = 4÷8 are prevalent, because it provides more energetically favorable valence angles. Larger cycles with k > 8 often tend to split into two smaller

ones via forming of additional carbon-carbon bond between the opposite atoms.

The square cycles (k = 4) are quite strain. The angle between C-C bonds of about 90° is far from the typical values of 109.5° (as in diamond) or 120° (as in graphite). Nevertheless, a number of structures with the square cycles were found to be stable, for example, cubane [12-13] and its derivatives [14-16], prismanes [17] and hypercubane [18]. Moreover, some “non-classical” fullerenes with square cycles on its surfaces are even more stable than the “classical” ones [19-22]. Four-membered rings are also contained in many recently proposed carbon structures [23-24].

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Boron nitrides form the other promising class of new materials. Both boron and nitrogen atoms are the nearest neighbors to carbon in the Mendeleev's table. For this reason, many carbon architectures have boron nitride analogues, consisted of alternated boron and nitrogen atoms instead of carbon ones. So, boron nitride fullerenes [25], nanotubes [26,27] and other structures [28-30] are actively investigated. Construction of boron nitrides based on already known carbon cages was an efficient way of searching new BN materials. Similar to carbon compounds, some boron nitrides also include four-membered B-N-B-N cycles [31-32].

Most computational studies of novel boron nitrides are based on density functional theory. The commonly used exchange-correlation functionals are B3LYP [33-34] and PBE [35], because they provide high accuracy for many systems [36-37], including boron nitride species [38]. Nevertheless, these functionals were not tested on high-strained boron nitrides with square cycles. For this reason, their application to such untypical systems remains questionable.

In this study, we perform a benchmark study of 24 exchange-correlation functionals on a set of small boron nitride clusters including those with the square cycles. The results, obtained with the density functional theory, were compared with the more accurate data derived from coupled clusters calculations [39-41].

2. Materials and Methods

To test different density functional methods, we chose three boron nitride clusters C_2N_2 , C_3N_3 and C_4N_4 with alternated boron and nitrogen atoms. Their structures are presented at Figure 1. Geometries of all three systems were optimized within the density functional and coupled clusters methods until the forces acting on atoms become smaller than 10^{-4} Ha/Bohr. No symmetry constrains were introduces. To confirm that the obtained geometries are true minima on the potential energy hypersurface, we calculate the Hessian matrix at the same level of theory. All considered structures have not any imaginary frequencies and therefore correspond to metastable states.

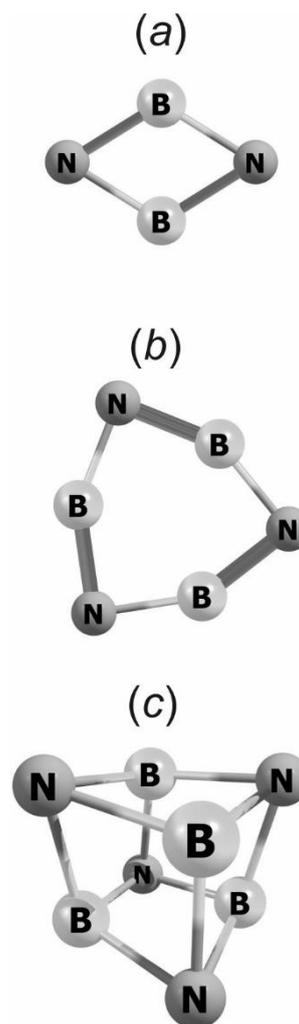


Figure 1. Structures of small boron nitride clusters B_2N_2 (a), B_3N_3 (b) and B_4N_4 (c).

The values of l_{BN} , a_{BNB} and a_{NBN} for each cluster are calculated as the arithmetic means of all B-N bonds lengths, B-N-B and N-B-N angles, respectively (note, that all averaged numbers, corresponding to the same molecule, are almost the same due to the symmetries of the considered systems). The chemical potentials μ are evaluated according to the Koopmans theorem [42] as $\mu = (E_{HOMO} + E_{LUMO})/2$, where E_{HOMO} and E_{LUMO} are the energies of the highest occupied molecular orbital (HOMO) and the lowest unoccupied molecular orbital (LUMO), respectively.

In our density functional calculations, we compare the follow exchange-correlation functionals: B3LYP [33-34], B3LYPV1R [43], B3P86V5 [34], B3PW91 [34], B97 [44], B97-2 [45], B97-3 [46], B97-K [47], CAMB3LYP [48],

LYP [49], M11 [50], OP [49], PBE0 [51], PW91C [52], VWN [53], VWN1RPA [54], wB97 [55-56], wB97X [55-56], wB97X-D [55-56], X3LYP [57], BMK [47], dLDF [58], M05 [59], M06 [60]. The results are compared with the data obtained with the coupled clusters CCSD(T) method [61]. For all calculations, we use the same 6-311G(2d,2p) basic set [62]. Since we restrict our study by the only singlet configurations, the restricted Hartree-Fock method of self-consistent field calculation is applied. All calculations are performed with the GAMESS software [63].

As a measure of difference between geometries, obtained with coupling clusters and density functional methods, we use the value

$$\varepsilon_g = \frac{1}{N} \sum |x_{CC} - x_{DFT}|/x_{CC}. \quad (1)$$

Here x is one of the geometric parameters (l_{BN} , a_{BNB} or a_{NBN}), $N = 9$ is the number of terms, summation is performed over all parameters of all considered clusters, indices “CC” and “DFT” correspond to coupled clusters and density functional methods, respectively. A very similar value ε_e is used as a measure of difference of electronic properties. In this case, x is the chemical potential μ or it means partial Mulliken q_M or Lowdin q_L charge of boron atoms.

3. Results and Discussion

In all three considered boron nitride structures, optimized with the coupled clusters method, all obtained B-N bonds lengths are the same. They are equal to 1.409, 1.363 and 1.511 Å for B_2N_2 , B_3N_3 and B_4N_4 clusters, respectively. The value for cubane B_4N_4 (see Figure 1c) slightly differs from the earlier reported coupled clusters results (1.492 [64] and 1.505 Å [65]). However, the authors of Ref. [64] performed only single point calculation of pre-optimized structure, whereas the authors of Ref. [65] applied the CCSD method with symmetry constrains. So, reported here CCSD(T) results are obtained with the higher level of theory and should be regarded as the most accurate.

In Table 1, we present all geometry parameters of considered boron nitride clusters. We can see that the bonds lengths and valence angles, obtained with the density functional methods, differ from the coupled clusters data by ~ 0.01 Å and $\sim 1^\circ$, respectively. The calculated electronic parameters are listed at Table 2. Chemical potentials are compared well with the each other, whereas partial charges demonstrate huge dispersions.

Table 1. Geometric parameters l_{BN} (Å), a_{BNB} and a_{NBN} (degree) of clusters B_2N_2 , B_3N_3 and B_4N_4 , obtained with the CCSD(T) method. The differences between l_{BN} , a_{BNB} and a_{NBN} values, calculated with the CCSD(T) and DFT approaches, are also listed.

Method	B_2N_2			B_3N_3			B_4N_4		
	l_{BN}	a_{BNB}	a_{NBN}	l_{BN}	a_{BNB}	a_{NBN}	l_{BN}	a_{BNB}	a_{NBN}
CCSD(T)	1.409	63.81	116.19	1.363	88.32	151.68	1.511	75.37	102.91
B3LYP	-0.017	0.46	-0.46	-0.010	0.87	-0.87	-0.010	0.18	-0.14
B3LYPV1R	-0.017	0.46	-0.46	-0.010	0.88	-0.88	-0.010	0.18	-0.14
B3P86V5	-0.015	0.31	-0.31	-0.008	0.03	-0.04	-0.011	-0.13	0.10
B3PW91	-0.016	0.27	-0.27	-0.009	-0.22	0.23	-0.012	-0.26	0.20
B97	-0.012	0.56	-0.57	-0.005	-0.09	0.09	-0.006	0.19	-0.15
B97-2	-0.014	0.15	-0.15	-0.008	-0.36	0.36	-0.012	-0.60	0.46
B97-3	-0.017	0.49	-0.49	-0.010	0.03	-0.03	-0.013	0.01	-0.01
B97-K	-0.013	0.68	-0.68	-0.007	0.37	-0.37	-0.007	0.00	0.00
CAMB3LYP	-0.025	0.71	-0.71	-0.016	1.50	-1.50	-0.018	0.18	-0.14
LYP	-0.045	1.42	-1.42	-0.036	2.87	-2.87	-0.042	0.15	-0.12
M11	-0.019	0.84	-0.84	-0.010	0.86	-0.86	-0.011	-0.78	0.59
OP	-0.042	1.36	-1.36	-0.033	2.58	-2.58	-0.039	0.00	0.00
PBE0	-0.017	0.25	-0.25	-0.228	-0.37	0.37	-0.014	-0.40	0.31
PW91C	-0.044	1.23	-1.23	-0.035	1.94	-1.93	-0.043	-0.25	0.19
VWN	-0.045	1.43	-1.43	-0.036	2.87	-2.86	-0.018	0.19	-0.14
VWN1RPA	-0.047	1.44	-1.44	-0.037	2.80	-2.86	-0.043	0.06	-0.04
wB97	-0.017	0.42	-0.42	-0.008	-0.30	0.30	-0.013	-1.13	0.86
wB97X	-0.020	0.59	-0.59	-0.011	0.27	-0.27	-0.015	-0.53	0.41
wB97X-D	-0.020	0.78	-0.78	-0.011	0.19	-0.18	-0.015	-0.02	0.01

Table 1 continued

X3LYP	-0.018	0.50	-0.50	-0.011	0.98	-0.98	-0.011	0.19	-0.15
BMK	-0.026	1.32	-1.32	-0.015	-0.97	0.97	-0.019	0.15	-0.11
dLDF	-0.014	0.38	-0.38	-0.009	1.62	-1.62	-0.006	-0.95	0.72
M05	-0.017	-0.13	0.13	-0.007	-3.23	3.23	-0.014	-1.89	1.42
M06	-0.022	0.19	-0.19	-0.015	-0.24	0.24	-0.018	-0.67	0.51

Table 2. Chemical potentials μ (eV), mean Mulliken q_M or Lowdin q_L charges of boron atoms ($|e|$, where e is the elementary charge) of clusters B_2N_2 , B_3N_3 and B_4N_4 , obtained with the CCSD(T) method. The differences between the values, calculated with the CCSD(T) and DFT approaches, are also listed.

Method	B_2N_2			B_3N_3			B_4N_4		
	μ	q_M	q_L	μ	q_M	q_L	μ	q_M	q_L
CCSD(T)	-5.73	-0.437	0.040	-5.60	-0.275	0.128	-6.37	-0.513	0.146
B3LYP	-0.20	0.144	0.083	-0.21	0.166	0.066	-0.04	0.178	0.081
B3LYPV1R	-0.29	0.143	0.083	-0.30	0.165	0.066	-0.12	0.177	0.081
B3P86V5	-0.39	0.128	0.082	-0.43	0.142	0.069	-0.25	0.157	0.087
B3PW91	-0.29	0.129	0.081	-0.33	0.140	0.069	-0.16	0.155	0.087
B97	-0.18	0.125	0.072	-0.22	0.150	0.064	-0.04	0.158	0.075
B97-2	-0.18	0.139	0.074	-0.19	0.134	0.061	-0.04	0.169	0.083
B97-3	-0.21	0.116	0.070	-0.26	0.141	0.061	-0.10	0.138	0.075
B97-K	-0.17	0.104	0.061	-0.21	0.138	0.057	-0.10	0.135	0.067
CAMB3LYP	-0.39	0.147	0.084	-0.41	0.174	0.066	-0.24	0.185	0.084
LYP	-0.74	0.018	0.024	-0.75	0.032	0.013	-0.73	0.053	0.038
M11	-0.45	-0.014	0.075	-0.56	-0.055	0.059	-0.40	0.043	0.085
OP	-0.62	0.001	0.016	-0.63	0.001	0.006	-0.62	0.027	0.031
PBE0	-0.28	0.141	0.081	-0.34	0.152	0.072	-0.17	0.189	0.092
PW91C	-0.82	0.003	0.021	-0.87	-0.002	0.015	-0.85	0.025	0.045
VWN	-1.13	-0.006	0.010	-1.12	-0.003	-0.004	-1.12	0.021	0.022
VWN1RPA	-1.60	-0.009	0.009	-1.59	-0.004	-0.004	-1.59	0.020	0.022
wB97	-0.28	0.134	0.069	-0.37	0.145	0.059	-0.21	0.165	0.082
wB97X	-0.29	0.130	0.070	-0.36	0.148	0.060	-0.24	0.184	0.088
wB97X-D	-0.26	0.103	0.066	-0.32	0.130	0.059	-0.14	0.118	0.073
X3LYP	-0.25	0.146	0.084	-0.27	0.170	0.067	-0.10	0.845	0.082
BMK	-0.30	0.024	0.054	-0.39	0.135	0.069	-0.20	0.094	0.071
dLDF	-0.66	0.081	0.033	-0.66	0.069	0.008	-0.62	0.099	0.033
M05	-0.13	0.120	0.066	-0.28	0.171	0.066	-0.04	0.124	0.086
M06	-0.27	0.074	0.068	-0.33	0.095	0.053	-0.13	0.038	0.074

The mean geometry and electronic errors (ε_g and ε_e), calculated with different exchange-corrected functionals using formula (1), are collected at Table 3. We conclude that B3P86V5 and B97 functionals reproduce well the geometries,

whereas the OP and VWN provide the best matches of electronic properties. OP and VWN functionals also provide the minimal values of $\varepsilon_g + \varepsilon_e$. Using of popular B3LYP and PBE0 functionals results in higher errors ε_g and ε_e .

Table 3. The values of mean errors ε_g and ε_e for different exchange-corrected functionals (see formula (1) for details). The sums of two errors are also presented.

functional	ε_g	ε_e	$\varepsilon_g + \varepsilon_e$
B3LYP	0.0063	0.5003	0.5066
B3LYPV1R	0.0063	0.5045	0.5108
B3P86V5	0.0038	0.4981	0.5019
B3PW91	0.0047	0.4892	0.4939
B97	0.0039	0.4476	0.4515
B97-2	0.0051	0.4558	0.4609
B97-3	0.0045	0.4319	0.4364
B97-K	0.0047	0.3905	0.3952
CAMB3LYP	0.0099	0.5224	0.5323
LYP	0.0195	0.1777	0.1972
M11	0.0089	0.3862	0.3951
OP	0.0177	0.1151	0.1328
PBE0	0.0233	0.5108	0.5341
PW91C	0.0172	0.1605	0.1777
VWN	0.0178	0.1190	0.1368
VWN1RPA	0.0196	0.1438	0.1634
wB97	0.0073	0.4501	0.4574
wB97X	0.0069	0.4631	0.4700
wB97X-D	0.0061	0.4082	0.4143
X3LYP	0.0069	0.6547	0.6616
BMK	0.0105	0.3620	0.3725
dLDF	0.0087	0.2304	0.2391
M05	0.0140	0.4413	0.4553
M06	0.0068	0.3708	0.3776

4. Conclusion

In the study presented, we perform a comparable analysis of density functional approaches applied to the small boron nitride clusters. The data obtained provide a reasonable choice of the most suitable exchange-corrected DFT-functional for strained BN-systems numerical simulation. We consider that the reported results stimulate further density functional studies not only of pristine boron nitrides, but also of their strained analogues such as prismanes, non-classical fullerenes, and silicic cages both in molecular and crystalline forms.

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