Turkish Journal of Forecasting vol. 8 no. 2 (2024) pp 45-53

Content list available at JournalPark

Turkish Journal of Forecasting

Journal Homepage: tjforecasting.com

ANALYZING COAL CONSUMPTION IN CHINA: FORECASTING WITH THE ECFGM(1,1) MODEL AND A PERSPECTIVE ON THE FUTURE

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Abstract

Recent studies have highlighted the efficacy and applicability of grey modeling in time series forecasting. Fractional grey models have become preferable despite computational challenges due to their ability to yield more effective results compared to standard models. This study introduces the definitions and theorems of the conformable fractional calculus, a fractional derivative of which is utilized. Subsequently, an exponential conformable fractional grey model is given. The model is then applied to analyze coal consumption using time series data, with model parameters determined using the Brute Force Algorithm to identify the optimal α value. The prediction performance of the model is evaluated using data from 2020-2022, and Mean Absolute Percentage Error (MAPE) is computed to assess the prediction accuracy. This analysis could serve as a significant guiding framework in energy policy and resource utilization domains.

Keywords: Coal Consumption, ECFGM(1,1), Brute Force Algorithm.

1. Introduction

The Grey System Theory is an analytical approach used for the analysis and modeling of systems involving uncertainty. This theory is developed by Chinese scientist Prof. Deng Julong (1982) in 1982. Grey systems are designed to cope with situations characterized by uncertainty, incomplete information, and complexity. The fundamental principle of Grey system Theory is to develop models that incorporate uncertainty to better understand and manage complex and uncertain systems. These models aim to work with limited or incomplete datasets and cope with uncertainty (David, 1994). Grey systems are commonly employed in addressing real-world problems across various fields such as industry, economics, environment, health, agriculture, among others. They are particularly effective in analyzing and predicting systems characterized by complex structures and uncertainties. Grey System Theory comes into play when traditional statistical and mathematical methods prove inadequate. It is regarded as a powerful tool when dealing with data characterized by information scarcity and uncertainty, contributing to the understanding and management of complex systems.

The EXGM(1, 1) model developed by Bilgil (2020) is used to predict new cases, deaths, and recovered COVID-19 cases in Turkey. Zeng et al. (2024) utilized the proposed ODGM(1, 1, k^{θ}) model to forecast China's shale gas production. Li et al. (2023) proposed a structural adaptive fractional time-delayed nonlinear systematic grey prediction model to distinguish nonlinear relationships among the internal structure and variables of sea and land economy-energy-environment systems. Ozturk et al.

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(2022) conducted predictions using the OCCFGM(1, 1) model for China's energy consumption, Russia's CO_2 emission, and total drinking water drawn from sources in Turkey. Ma et al. (2019) developed the fractional time-delayed grey model and applied it to four real-world case studies. Ozturk and Bilgil (2019) investigated the estimation of future healthcare expenditures in Turkey using Grey Modeling. Wang and Li (2019) elucidated the relationship between carbon dioxide emissions and economic growth by proposing a novel grey model. Wu et al. (2020) conducted a study on the prediction of countries' natural gas consumption by proposing a new Grey Bernoulli model. Ma et al. (2019) proposed the utilization of fractional order accumulation, an effective tool for improving the accuracy of grey modeling, by developing a fractional discrete multivariate grey system model. Javed and Liu (2018) implemented the Even GM(1, 1) and NDGM models. Ikram et al. (2019) conducted the forecasting of ISO 14001 certification numbers for certain countries using three grey models found in the literature. Wang et al. (2018) developed a new hybrid forecasting model. Yuxiao et al. (2021) introduced the variable order fractional grey model and discussed its applications. Liu et al. (2021) developed a new fractional grey model, abbreviated as OFAGM(1, 1). Wu et al. (2019) developed a novel multivariate fractional grey model. Wang et al. (2022) proposed a fractional time-delayed grey Bernoulli model. Kang et al. (2022) developed a fractional grey viscoelastic traffic flow model. Zhou et al. (2020) developed a novel discrete grey model. Duan et al. (2018) proposed a new model based on the fractional order accumulation generation. Wang et al. (2023) conducted an application of a novel fractional grey model in energy consumption prediction. Wang et al. (2023) implemented a new exponential time-delayed fractional grey model. Thike et al. (2023) proposed an approach to enhance the efficiency of the Brute Force Algorithm in solving discrete optimization problems. Morton et al. (2000) propose an approach aiming to enhance the efficiency of the Brute Force Algorithm in network reconfiguration by addressing the effective method used to determine a minimal-loss radial configuration for power distribution networks.

This research aims to predict China's coal consumption using the exponential conformable fractional grey model (ECFGM(1, 1)) developed by Erdinc et al. (2024). In the first section, a review of the literature is provided. In the second section, fundamental concepts and theorems are given. In the third section, comprehensive information about the structure of the ECFGM(1, 1) model is presented. The fourth section utilizes China's coal consumption data for the implementation of the model. The fifth and final section focuses on the obtained results, providing assessments.

2. Definitions and Theorems of Conformable Fractional Calculus

In this section, the introduced definition of conformable fractional derivative by Khalil et al. (2014) and its properties are examined. Subsequently, the definitions of conformable fractional difference and conformable fractional accumulation, introduced using the conformable fractional derivative definition by Ma et al. (2020), are detailed.

Definition 1: (See (Khalil et al., 2014).) f: $[0,\infty)\longrightarrow$ R be a differentiable function and then the conformable fractional derivative of f with $\alpha\in(n,n+1]$ order is defined as

$$T_\alpha(f)(t)=\lim_{\epsilon\to 0}\frac{f(t+\epsilon t^{\lceil\alpha\rceil-\alpha})-f(t)}{\epsilon}=t^{\lceil\alpha\rceil-\alpha}\frac{df(t)}{dt}, \quad (1)$$

where $\lceil . \rceil$ is the ceil function, i. e. the $\lceil\alpha\rceil$ is the smallest integer no larger than α. It is clear that $\lceil\alpha\rceil=1$ for $\alpha\in(0,1]$. Thence, Equation (1) can be written as

$$T_\alpha(f)(t)=\lim_{\epsilon\to 0}\frac{f(t+\epsilon t^{1-\alpha})-f(t)}{\epsilon}=t^{1-\alpha}\frac{df(t)}{dt},\quad \forall t>0.$$

Theorem 1: (See (Khalil et al., 2014).) If the function f and g are differentiable, $\alpha\in(0,1]$, then we have

1. $T_\alpha(f)(t)=t^{1-\alpha}\frac{df(t)}{dt}$

2. $T_\alpha(kf+hg)=kT_\alpha(f)+hT_\alpha(g); \forall k,h\in\mathbb{R}$

3. $T_\alpha(f.g)=fT_\alpha(g)+gT_\alpha(f)$

4. $T_\alpha\left(\frac{f}{g}\right)=\frac{gT_\alpha(f)-fT_\alpha(g)}{g^2}$

5. $T_\alpha(k) = 0;\ \forall k \in \mathbb{R}$

6. $T_\alpha\left(t^k\right) = kt^{k-\alpha};\ \forall k \in \mathbb{R}$

7. $T_\alpha\left(e^{kx}\right) = kx^{1-\alpha}e^{kx};\ \forall k \in \mathbb{R}$

Definition 2: (See (Ma et al., 2020).) The conformable fractional difference (CFD) of f with α order is defined as

$$\Delta^\alpha f(k) = k^{1-\alpha}\Delta f(k) = k^{1-\alpha}[f(k) - f(k-1)] \tag{2}$$

for all $k \in N^+$, $\alpha \in (0,1]$ and

$$\Delta^\alpha f(k) = k^{\lceil\alpha\rceil-\alpha}\Delta^{n+1} f(k) \tag{3}$$

for all $k \in N^+$, $\alpha \in (n, n+1]$.

Definition 3: (See (Ma et al., 2020).) The conformable fractional accumulation (CFA) of f with α order is defined as

$$\nabla^\alpha f(k) = \nabla\left(\frac{f(k)}{k^{1-\alpha}}\right) = \sum_{j=1}^{k} \frac{f(j)}{j^{1-\alpha}} \tag{4}$$

for all $k \in N^+$, $\alpha \in (0,1]$ and

$$\nabla^\alpha f(k) = \nabla^{n+1}\left(\frac{f(k)}{k^{\lceil\alpha\rceil-\alpha}}\right) \tag{5}$$

for all $k \in N^+$, $\alpha \in (n, n+1]$.

3. Exponential Conformable Fractional Grey Model

This section provides the definition and theorems of the exponential conformable fractional grey model developed by Erdinc et al. (2024). Following that, mathematical formulas to be utilized in the calculation of prediction error rates are presented. Finally, information about the general structure of the Brute Force algorithm is given.

Initially, the raw dataset

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right).$$

is provided. Let α be a positive number; the conformable fractional accumulation series (CFA) with α order is calculated as follows:

$$X^{(\alpha)} = \left(x^{(\alpha)}(1), x^{(\alpha)}(2), \ldots, x^{(\alpha)}(n)\right), \tag{7}$$

where

$$x^{(\alpha)}(k) = \nabla^\alpha x^{(0)}(k) = \sum_{i=1}^{k} \begin{bmatrix} \lceil\alpha\rceil \\ k-i \end{bmatrix} \frac{x^{(0)}(i)}{i^{\lceil\alpha\rceil-\alpha}}, \quad \alpha \in R^+ \tag{8}$$

where $\lceil\cdot\rceil$ is the ceil function and $\begin{bmatrix} \lceil\alpha\rceil \\ k-i \end{bmatrix} = \frac{\Gamma(k-i+\lceil\alpha\rceil)}{\Gamma(k-i+1)\Gamma(\lceil\alpha\rceil} = \frac{(k-i+\lceil\alpha\rceil-1)!}{(k-i)!(\lceil\alpha\rceil-1)!}$ (Wu et al., 2020).

Definition 4: (See (Erdinc et al., 2024).) The first-order whitening differential equation of the ECFGM(1,1) is defined as,

$$\frac{dx^{(\alpha)}(t)}{dt} + ax^{(\alpha)}(t) = b + ce^{-t} \tag{9}$$

where a is a development coefficient, b is called driving coefficient and ce^{-t} is an exponential grey action quantity.

Theorem 2: (See (Erdinc et al., 2024).) For the computed CFA and the value of fractional order,the system parameters a, b and c of the ECFGM(1, 1) satisfy the following equation

$$[a, b, c]^T = (B^T B)^{-1} B^T Y, \tag{10}$$

where the matrix B and Y are

$$B=\begin{bmatrix}-0.5(x^{(\alpha)}(2)+x^{(\alpha)}(1)) & 1 & (e-1)e^{-2}\\ -0.5(x^{(\alpha)}(3)+x^{(\alpha)}(2)) & 1 & (e-1)e^{-3}\\ . & . & .\\ . & . & .\\ -0.5(x^{(\alpha)}(n)+x^{(\alpha)}(n-1)) & 1 & (e-1)e^{-n}\end{bmatrix}, Y=\begin{bmatrix}x^{(\alpha)}(2)-x^{(\alpha)}(1)\\ x^{(\alpha)}(3)-x^{(\alpha)}(2)\\ .\\ .\\ x^{(\alpha)}(n)-x^{(\alpha)}(n-1)\end{bmatrix}. \tag{11}$$

Theorem 3: (See (Erdinc et al., 2024).) The discrete form of the response function of ECFGM(1,1) model is given as

$$\hat{x}^{(\alpha)}(k)=\left(x^{(0)}(1)-\frac{b}{a}-\frac{c}{a-1}e^{-1}\right)e^{a(1-k)}+\frac{b}{a}+\frac{c}{a-1}e^{-k} \tag{12}$$

where $k=2,3,\dots,n$.

Theorem 4: (See (Erdinc et al., 2024).) The restored values can be given as

$$\hat{x}^{(0)}(k)=\Delta^{\alpha}\hat{x}^{(\alpha)}(k)=k^{[\alpha]-\alpha}\Delta^{n+1}\hat{x}^{(\alpha)}(k),\quad \alpha\in(n,n+1] \tag{13}$$

where $k=2,3,\dots,n$.

3.1. Error Analysis of the Model

While conducting error analysis of the model, the data is divided into two groups. The first data group is used to run the model, and the data generated by the model is compared with the original data. Thus, the MAPE $_{fit}$ value, representing the error due to data fitting, is calculated.

Subsequently, the second data group is used to calculate the MAPE $_{pre}$ value, representing the prediction error between the generated data by the model and the actual data. This enabled the separate evaluation of the model's performance in data fitting and prediction generation. Finally, the mean absolute percentage error (MAPE) value is computed as the average of these performance metrics. The mathematical expression for prediction errors is as follows:

$$RPE(k)=\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%$$

$$MAPE_{fit}=\frac{1}{h}\sum_{k=1}^{h}RPE(k)$$

$$MAPE_{pre}=\frac{1}{n-h}\sum_{k=h+1}^{n}RPE(k)$$

$$MAPE=\frac{1}{n}\sum_{k=1}^{n}RPE(k)$$

Here, h represents the sample size in the first data group. The elements $x^{(0)}(k)$ represent the elements of the raw data set, while $\hat{x}^{(0)}(k)$ denote the elements of the data set generated by the model.

3.1.1. Brute Force Algorithm for Optimal α

The Brute Force methodology, also referred to as the Naive algorithm, represents a direct and uncomplicated approach utilized in addressing optimization challenges. This method relies on raw computational power by exhaustively testing every possible solution rather than employing sophisticated techniques to enhance efficiency. Unlike several other prevalent swarm intelligence algorithms, Brute Force can be applied to a broad spectrum of problems. It stands out as an effective and straightforward means of determining the optimal value within the solution space.

Although other algorithms may offer faster convergence speeds, a drawback is that may focus on local extremum points over global extremum. In contrast, the Brute Force algorithm systematically scans the entire domain, evaluates each point, computes the Mean Absolute Percentage Errors (MAPEs) at these points, and consequently identifies optimal parameters without any confusion or ambiguity.

The aim of this section is to determine the optimal parameter α that minimizes the Mean Absolute Percentage Error (MAPE) of the model. To achieve this, we systematically evaluate α over a defined interval with a step size of 0.01. For convenience, α will be generated within the interval (0, 1] in the subsequent section. This approach allows us to calculate the MAPE for each α value throughout the entire interval. The optimal parameter α is then identified as the one that yields the lowest calculated MAPE. These computational steps are repeated for all α values considered in the analysis. The detailed algorithm is given in Algorithm 1.

Algorithm 1. Brute Force method to compute the fractional order α of ECFGM(1, 1)

Input: Original series $\left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right)$
Output: Optimal α^*
Initialize $\alpha^* = 0$, $MAPE_{min}$=+inf;
For $\alpha = 0$ to 1; step 0.01 do
Construct B and Y using Eq. (11)
    Compute $\hat{a}, \hat{b}$ using Eq. (10)
    For k=1 to n; step 1 do
        Compute $\hat{x}^{(\alpha)}(k)$ using Eq. (12)
        Compute $\hat{x}^{(0)}(k)$ using Eq. (13)
    End
    Compute MAPE using the objective function in Eq. (11), Eq. (10), Eq. (12), Eq. (13)
    If MAPE < $MAPE_{min}$ then
        $MAPE_{min} \leftarrow$ MAPE;
        $\alpha^* \leftarrow \alpha$;
    End
End
Return α^*;

3. Prediction of Coal Consumption in China

While emerging as a prominent player in the global economy, China has drawn significant attention on a global scale due to its energy consumption, particularly in the realm of coal usage. This article aims to investigate the coal consumption of China, shedding light on how the energy policies of this major economy have evolved at both the national and global levels and how they might impact future predictions.

In this section, the ECFGM(1,1) model is used to predict China's coal consumption. Time series data from the years 2016 to 2019 are obtained from a study conducted by the Statistical Review of World Energy (https://www.energyinst.org). The data from these years are then applied to the ECFGM(1,1) model. Subsequently, the Brute Force Algorithm is utilized to determine the optimal α value. The interval (0,1] is scanned with a step size of 0.01, and the model is run for each point, calculating the corresponding $MAPE_{fit}$ values. The $MAPE_{fit}$ values for the (0,1] interval are presented in Figure 1. The optimal α value is determined to be 0.87 for a $MAPE fit$ value of 0.0000615. The parameters of the model for the determined α value are then calculated as $a = -0.000286354, b = 68.3330502, c = 16.0638425$, completing the process of creating the solution function for the model. The data from the years 2020-2022, which are not included in the model, are used to assess the predictive power of the model, yielding a $MAPE_{pre}$ value of 0.6097. Finally, the MAPE value of the model is obtained as 0.2613 (Table 1). This low MAPE value indicates excellent forecast accuracy for the given dataset, instilling confidence in the model's ability to forecast subsequent elements of the series. The forecasted China coal consumption for the years 2023-2028 is provided in Table 2. Additionally, Figure 2 shows the simulation of China's coal consumption.

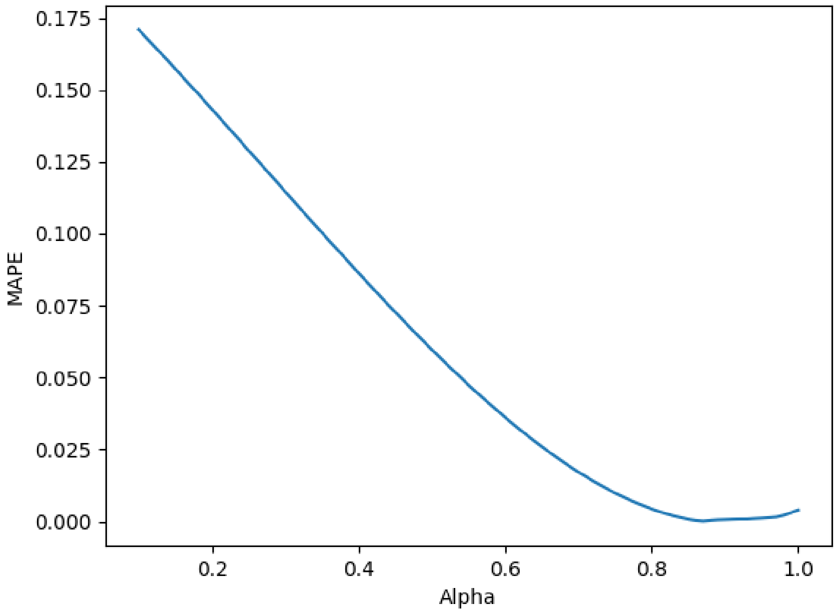

Figure 1. MAPE $_{fit}$ values at each point in the interval (0, 1].

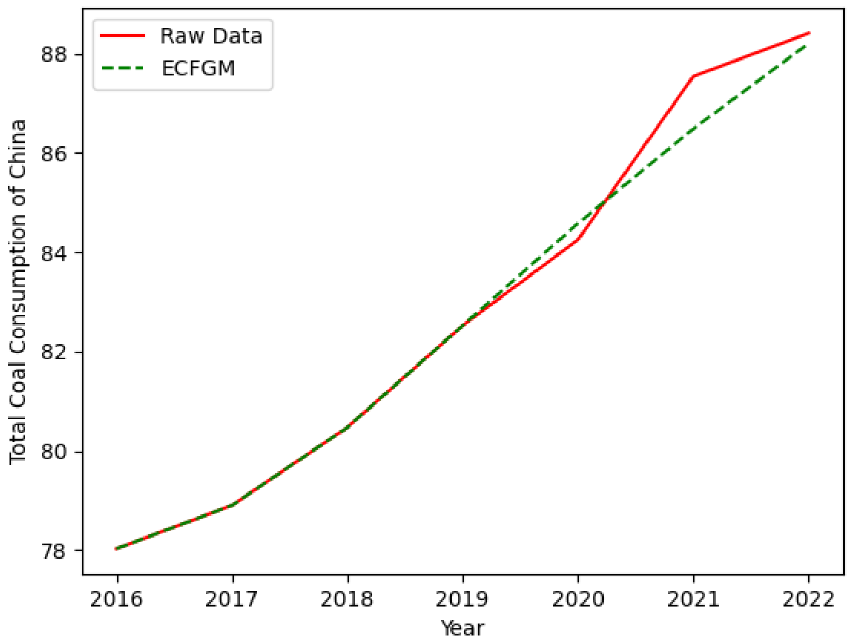

Figure 2. Real, fitting and predicting values of China's coal consumption.

Table 1. China's coal consumption predict values and MAPE values using the ECFGM(1,1) model.

Year	Raw Data	ECFGM
2016	78.03	78.0300
2017	78.90	78.9000
2018	80.47	80.4699
2019	82.52	82.5199
$MAPE_{fit}$		0.0000615
2020	84.25	84.5791
2021	87.54	86.4841
2022	88.41	88.2045
$MAPE_{pre}$		0.6097
MAPE		0.2613

Table 2. China's coal consumption forecast values

Year	Forecasting Values
2023	89.7535
2024	91.1570
2025	92.4376
2026	93.6156
2027	94.7073
2028	95.7254

4. Conclusions

The conclusions of this study aim to define the definitions and properties of conformable fractional derivatives and to make predictions using real-world data with the exponential conformable fractional grey model. The model has been successfully applied to predict China's coal consumption. Additionally, the parameters of the model are calculated using the optimal α value determined by the Brute Force Algorithm, and the predictive capability of the model is evaluated. The analyses conducted revealed that the model exhibits high predictive accuracy with low MAPE values. Consequently, it is concluded that the proposed model assists in forecasting future trends more accurately by better fitting past data. These findings highlight the model's significance as a crucial reference point for the development of energy policies and strategies related to energy consumption.

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