PROBLEM POSING AND PROBLEM SOLVING ABILITY OF STUDENTS WITH DIFFERENT SOCIO ECONOMICS LEVELS

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Abstract

Many studies have demonstrated that problem posing develops creativity, improve mathematical thinking and learning. Although problem posing is accepted as important component of mathematical activities, it still has not been a major focus in mathematics education community. In the present study, it was aimed to examine how socio-economic factor play a role on problem posing and problem solving skills in terms of different districts. The participants were eighth grade students from different two schools and each group was 100 students in 2012-2013 school years. The participants were offered two semi-structured and two structured problem posing situations to pose a problem for each one. Data were collected as worksheets and then data was analyzed with non-parametric Mann-Whitney U-test. The students of high levels of socio-economic districts has been found successful than students of low levels of socio-economic districts. Also, it was determined that there were significant differences according to three questions between two districts.

Keywords: Problem Posing, Problem Solving, Eight Grade Students.

INTRODUCTION

Many researchers have explored opportunities for students to improve learning mathematics deeply. In the past several years, prominent researchers have recognized that problem posing is important component of mathematics education as problem solving. Problem posing has been examined by most researchers according to the strategies, the levels and the types (Abu-Elwan, 1999; Akay, Soybas, & Argun, 2006; Brown & Walter, 1993;Brown &Walter, 2005; Cai, 2005; Cankoy & Darbaz, 2010; Crespo, 2003; Crespo & Sinclair, 2008; Ellerton, 1986; English & Halford, 1995; English, 1998; Leung, 1996; Leung, 2012; Lowrie, 1999; Silver, 1994; Silver and Cai, 1996; Singer & Ellerton & Cai, 2013; Stoyanova & Ellerton, 1996). Silver (1994) said that problem posing could be made before, during or after the problem solving activity.

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What- if-not strategy was envisaged by Brown and Walter (1990) to develop different perspectives for problem solving and to generate new problems with changing conditions. For instance, what are some answers for $x^2+y^2=z^2$ equation. The first that come to mind is Pythagorean Theorem and numbers that satisfies equation respectively 3, 4, 5 or 5, 12, 13 or if not integer then i, 1, 0. Now, if this triangle is not right triangle? How is $x^2+y^2=z^2$ affected? This strategy was used by some researchers in their study (Moses, Bjork, & Golenberg, 1993; English, 1998; Lavy & Bershardsky, 2003).

Problem posing can be carried out by three situations:

i. free problem posing situations: students pose a problem which is particularly difficult or original or for their friends to solve,
ii. semi-structured problem posing situation : students are given an open situation to pose a problem which is given by equation, photograph, operation or figure,
iii. structured problem posing situations: student are given a specific problem to pose a problem which is reconstruction from initial problem, solution of problem or same problem. (Stoyanov & Ellerton, 1996).

Brown and Walter (2005) remarked that "*Problem posing is deeply embedded in the activity of problem solving in two very different ways First of all [...] what is this problem really asking, saying, or demanding? What if I shift my focus from what seems to be an obvious component of this problem to a part that seems remote? Second, it is frequently the case after we have supposedly solved a problem, we do not fully understand the significance of what we have done, unless we begin to generate and try to analyze a completely new set of problems"(p.2-3).*

In many studies, problem posing has been seen an activity that helps to enrich creativity, to increase mathematical understanding and to allow for autonomous learning. Also, compare to the less able students in problem solving, the more able students pose quality problem with high mathematical complexity (Ellerton, 1986; Kilpatrick, 1987; Gonzales, 1996; Silver, 1994; Cai, 1998). Contrary to these studies, Crespo (2003) stated in her study that good problem solvers may be not good at problem posing compared to less problem solvers.

Some researchers investigated to use problem posing as an assessment tool. Leung (1996) introduced problem posing to student teachers for generating problems. The student teachers posed problems and assess each other's problems. By this way, student teachers were aware of

strengths and weaknesses of problems they initially generated. Lowrie (1999) purposed to identify the children's who were different ages mathematical abilities through the problem posing. The students constructed mathematical problems for friends to solve. It was stated in conclusion that designing new problems for their friends to solve help them to understand problem solving process.

English (1997) purposed to develop the fifth grade students' problem posing ability. The students were introduced problem structures, problem types (routine, non routine and open ended) and varieties of mathematical thinking. It was emphasized that this program was very successful in developing the students' problem posing skills.

With regard to math anxiety, posed a problem has been seen as motivated activity to students (Baxter, 2005; Buerk, 1982). Brown and Walter (2005) expressed about problem posing to overcome mathophobia in their book "*There is good reason to believe that problem generation might be a critical ingredient in confronting math anxiety because the posing of problem or asking a questions is potentially less threatening than answering them. The reason is in part a logical one. That is, when you ask a question, the responses "right" or "wrong" are inappropriate, although that category is paramount for answer to questions*".

Some countries like Australia, China and America see problem posing as indispensable item of mathematics education. Although mathematics curriculum was reformed in Turkey in 2006, it is not known whether Turkish mathematics teacher use problem posing in their lessons (Arikan & Unal, 2013). According to Australian Education Council (1991) *"Students should engage in extended mathematical activities which encourage problem posing, divergent thinking, reflection and persistence. They should be expected to pursue alternative strategies, and to pose and attempt to answer their own mathematical questions"* (p.39). According to Chinese National Curriculum Standarts on Mathematics "*Students must be able to pose and understand problems mathematically, apply basic knowledge and skills to solve problems and develop application awareness"*(p.7). Also, National Council of Teachers of Mathematics suggested that teachers encourage their students to pose mathematical problems (Zakaria & Ngah, 2011).

In spite of the fact that problem posing is accepted as important component of mathematical activities, it still has not been a major

focus in mathematics education researches (Singer, Ellerton, & Cai, 2013) .

The Purpose of Study

In our study, we aimed to compare the students who live in two different locations in Istanbul. When there are high-income families in Location 1, many families migrated from east of Turkey to Location 2. While number of schools in Location 1 is 265, number of schools in Location 2 is 168. The number of students per classroom in Location 1 is 36; by contrast, the number of students per classroom in Location 2 is 73. It was questioned whether there is a difference between these students problem posing and problem solving ability because of socio-economic. Therefore we tried to investigate that what difference is and why there is.

In like manner, Harpen and Presmeg (2013) investigated of relationship between students' mathematical problem posing abilities and their mathematical content knowledge. In their study, participants had been selected from big city which is Shanghai and small city which is Jiaozhou in China and one group had been selected from USA. The Jiaozhou group had showed higher performance on mathematics content test and problem posing test than Shangai group.

METHODS

Participants

The participants of the present study were hundred each from two different district of Istanbul. While one of them is prominent district in terms of socio-economic the other one is a developing district. All of them are eighth grade students in public school.

Data Collection and Analysis Process

The framework of this study was as Stoyanova and Ellerton (1996) offered that every problem posing situation can be classified as free, semi-structured or structured. Therefore, the participants of this study were offered two semi-structured problem posing situations and two structured problem posing situations related to fractions. Besides that the students were asked to solve two problems similar to the structured problems following in Table 1and Table 2.

Table 1. *Problem Posing Situations*

Semi-Structured		Structured	
"I have 350 ₺" Pose a fraction problem such that containing the above sentence	 *Pose a fraction problem* such that appropriate to the figure.	"Ali has got 25₺. If he gives 3/5 of his money to Serhat, Serhat will have 28 ₺. How much money does Serhat have?" Pose a fraction problem adding a new data to the above problem.	"Sum of the ages of Şenay and her father is 45. Age of Şenay is 2/7 of age of her father. Then, how old is Şenay?" Pose a fraction problem changing sum of ages and fraction.

Table 2. *Problem Solving Questions*

The sum of money of Ali and Selim is 60 ₺. If Selim gives ¼ of his money to Ali, then the money are synchronized. In this case, how much money does Ali have?
"Sum of the ages of Şenay and her father is 45. Age of Şenay is 2/7 of age of her father. Then, how old is Şenay?"

In this study, students' responses were evaluated 1 point or else were 0 point to obtain statistical inference. Moreover, semi-structured interview was made with guidance teachers both of two schools for overall situation assessment. Data were analyzed quantitatively. The relationship between problem posing/problem solving ability and socio-economic factor were analyzed by Pearson Chi-square test, two independent samples non-parametric Mann-Whitney U test and Chi-square test were used for comparison.

RESULTS

To analyze the data, districts were coded as Location 1 is higher welfare level and Location 2 is less welfare level. To identify whether problem posing and solving ability deal with socio-economic factor, chi square test was used. Total score of structured problem posing and total score of problem solving were tabulated and analyzed. According to the result of this test, since the minimum expected count is 21.30, Pearson Chi-Square test was considered which was 31.121($p < 0.05$ and degree of freedom was 1). Namely, problem posing and solving ability vary in terms of socio-economic level.

Table 3. *Frequency Distribution of Problem Posing and Problem Solving*

	Semi-Structured Problem Posing 1	Semi-Structured Problem posing 2	Structured Problem Posing 1	Structured Problem Posing 2	Problem Solving 1	Problem Solving 2
Location 1	22	25	73	79	70	67
Location 2	11	22	58	56	35	48

$\chi^2 = 31.121$, $df = 1$, $p = .000$

Considering for locations separately, students responded more successfully structured problem posing situation compare to semi-structured problem posing situation according to Table 3. Stoyanova and Ellerton reflected (1996) that *"The unfinished problem structures can be given either by a Picture, equation, calculation or inequality"* (p. 523). When looking to Table 3, students could found the easy structured problem that adds new information to initial problem or changing conditions in initial problem. This could mean that the students are literal minded or do not prefer to imagine for problems. The reason of this, students may mingle with multiple choices exams.

When we look carefully to Table 3, we can see that students of location 2 were more successful in structured problem posing task than problem solving task. To put it more clearly, we applied Chi-square test for analyzing the relationship between problem posing and problem solving. While there was non significant differences for location 1 with $p = .612 > 0.05$, there was significant differences for location 2 between problem posing and problem solving ability with $p = .028 < 0.05$. Indeed, location 1 students were more successful in problem posing task than problem solving task.

Table 4. *Comparison of Locations for Problem Posing and Problem Solving*

	Problem Posing Questions				Problem Solving	
Mann-Whitney U	4450.000	4850.000	4250.000	3850.000	3250.000	4050.000
Wilcoxon W	9500.000	9900.000	9300.000	8900.000	8300.000	9100.000
Z	-2.090	-.499	-2.226	-3.464	-4.944	-2.711
Asymp. Sig. (2-tailed)	.037	.618	.026	.001	.000	.007
Significance	$\alpha < 0,05$	$\alpha > 0,05$	$\alpha < 0,05$	$\alpha < 0,05$	$\alpha < 0,05$	$\alpha < 0,05$

As seen in Table 4, it could be said that the students who are in location 1 are successful overwhelmingly exception semi-structured problem posing 2. Problem posing according to figure may be found easy for the all students. Therefore, there is no significant difference between location 1 students and location 2 students.

Prominent scientists of problem posing researchers asserted that problem posing can bolster students to solve problems, fosters analysis and thought (Cai & Cifarelli, 2005; English, 1998; Silver, 1997; Singer & Moscovici, 2008). Since structured problem posing situations were designed similar or same to the solving fraction problems, it was possible to examine the relation of problem posing and problem solving. According to Table 4, it is seen easily that there was close relationship between problem posing and problem solving for this study. This finding is accordance with Cai (1998).

In the study, marked information was obtained from interviews with the guidance teachers of locations. The guidance teacher of location 1 emphasized that "*our students' parents are at least a high school graduate, caring. These students take a course out of the school. Social activities of our school are wide range of interests."* The guidance teacher of location 2 explained that *"this place is the field of migration from east of Turkey. Therefore, some students have a problem with adapting process to a new settlement and school. Furthermore, there are issues related to students' basic knowledge. Some students have domestic problems. For instance, mother or father of some students was imprisoned. This is reflected to his or her life as well as her school life. As a result, we firstly expect from our students to solve problems in their school life. Because, we do not have homogeneous class. We have both of successful and unsuccessful students. Generating a problem aside, some students could not solve even these problems"*.

CONCLUSION

A main goal of mathematics education is to improve mathematical problem solving skill (Kilpatrick, 1987). Problem posing is accepted as an important activity for problem solving. Along the way problem posing, students make explicit their approaches of problem solving (Singer & Voica, 2012). Kilic (2013) remarked that "*problem posing is also a key indicator of good problem solving performance and creativity as well and helps to develop other mathematical abilities*

such as reasoning, connection and problem solving" (p. 152). For these reasons, we explored a correlation between problem solving and problem posing activity, we also examined whether socio-economic factor affects problem posing ability.

In our study, we have found significant difference that problem posing and problem solving ability of the students who live in two different locations which are in terms of socio-economic. However, there was no significant difference for problem posing according to figure. While problem posing according to figure 11% of students used compound fraction in location 2, this rate was only 1% in location 1. For instance, *"Ayse and her friends bought pizza. If Ayse eat 6/4 of this pizza, how many slices of pizza the others have?".* A student's problem has logic error in location 1 however problems of two students have logic error in location 2. For example, *"I gave ½ of pizza to Busra and 2/3 of pizza to Sumeyye. Then how many slices I have got?"*It is said that students did not monitored solution of their problem. Moreover, these students may have a trouble in the realization of conceptual learning.

On the other hand, there has been no student for SPP2 used compound fraction. This may indicate that students copy the fraction in which numerator is less than denominator.

As for the correlation, the strongly relationship between problem posing and problem solving was identified. It was declared that a student who can solve a certain fraction problem can modify this problem or change conditions of the problem.

Another remarkable finding is that students of location 2 were more successful in problem posing task. This case could be explained as Baxter (2005) and Brown & Walter (1993) mentioned that problem posing helps students to increase their mathematics anxiety.

It is satisfactory that the students coming from the low socio-economic areas are much more successful than the others concerning the problem posing abilities. In case of the fact that they have been provided with much more problem posing activities in their lessons, we have expected to have seen a significant progress in their attitudes towards mathematics. When the similar facilities given to the students coming from the high socio-economic areas have been delivered to those coming from the low socio-economic areas, the unbalanced situation between these two student groups at the moment might become much more balanced.

The counselor of the location 2 emphasized in review that one has to carry out problem solving process to create a problem. This means that there is a close relationship between problem posing and problem solving.

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