

Interactive Fuzzy Decision Making Algorithm for Two Level Linear Fractional Programming Problems

Hasan Dalman*

*Department of Computer Engineering, Faculty of Engineering and Architecture, Istanbul Gelişim University, Avcilar, 34510, Istanbul, Turkey

(hdalman@gelisim.edu.tr)

‡Corresponding Author; Hasan Dalman, Department of Computer Engineering, Faculty of Engineering and Architecture, Istanbul Gelişim University, Avcilar, 34510, Istanbul, Turkey, Tel: +90 507 723 3421,

hdalman@gelisim.edu.tr

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Abstract- This paper offers an interactive fuzzy decision-making algorithm for solving two-level linear fractional programming (TLLFP) problem which contains a single decision maker at the upper level and multiple decision makers at the lower level. In the presented interactive mechanism, the fuzzy goals and associated weight of the objective at all levels are first determined and the satisfactory solution is attained by renewing the satisfactory degrees of decision makers including the overall satisfactory balance among all levels. Moreover, the value of distance function is used in order to verify the satisfaction grades. Finally, a numerical example is given to illustrate the performance of the presented algorithm.

Keywords Two level linear fractional programming problem; fuzzy programming; fuzzy goals; Interactive methods.

1. Introduction

Multilevel programming problems usually occur in a much hierarchical system of large organizations such as government offices, profit or non-profit organizations, manufacturing plants, logistic companies, etc. Solution procedures show that all Decision Makers (DMs) has a single objective, a set of decision variables and a set of general constraints that affect all DMs. Each unit individually searches itself earnings. But each of them is affected via the actions of other units.

Multilevel programming proposed by Bracken and McGill [1] to model a decentralized noncooperative decision system with one leader and multiple followers in 1973. Multilevel programming is an NP-hard problem [5]. The Stackelberg method has been employed to solve the multilevel programming problems. It has much applicability in practical such as strategic planning (Bracken and McGill, [2]), resource allocation (Aiyoshi and Shimizu, [3]), and water management (Anandalingam and Apprey, [4]). In order to establish mathematical model of multilevel programming, many methods and algorithms have been proposed such as extreme point algorithm (Candler and Towersley, [6]), k.th best algorithm (Bialas and Karwan, [7]), branch and bound algorithm (Bard and Falk, [8]),

descent method (Savard and Gauvin, [9]). and genetic algorithm (Liu, [10]). A fuzzy multilevel programming model is presented by Gao and Liu [11]. They defined a Stackelberg–Nash equilibrium. These classical methods are based on Karush–Kuhn–Tucker conditions and/or penalty functions [12]. Furthermore, the Stackelberg method does not provide Pareto optimality because of its non-cooperative nature [13]. These solution procedures are related to Karush–Kuhn–Tucker conditions and/or penalty functions [12]. Besides, solution procedure of the Stackelberg method does not give Pareto optimality because of its noncooperative structure [13]. In such hierarchical decisions, it has been concluded that each DM should have a difficulty of motivation to cooperate with the other, and a minimum level of satisfaction of the DM at a lower level must be subject to the overall profit of the organization. In order to satisfactory solutions, fuzzy set theory to multilevel programming problems was first applied by Lai [12] in 1996. By utilizing a search procedure and fuzzy set theory, this procedure of satisfactory solution was improved by Shih et al. [14, 15]. Moreover, fuzzy programming approaches were employed by many authors for solving multiple level linear programming problems [16, 17], bilevel quadratic fractional programming problem [13, 17, 18], two-level non-convex programming problems with fuzzy parameters [18],

decentralized two-level linear programming problems [19, 20] and so on.

Recently, Baky [21] presented two fuzzy goal programming algorithm for multi-level multi-objective linear programming problems. Arora and Gupta [22] presented an interactive fuzzy goal programming algorithm connecting bilevel programming problems with the theory of dynamic programming. Wang et al. [23] introduced a concept to deal with the bilevel multilevel programming problem. A fuzzy TOPSIS algorithm is introduced in [24]. The distance function, which was introduced by Yu [24], has been widely employed to obtain compromise solutions of multi objective programming problems. Moitra and Pal [25] applied fuzzy goal programming method with the theory of distance function and produced a satisfactory balance by lessening the deviations of the leader and follower as far as for bilevel programming. However, some interesting interactive fuzzy decision making algorithms have widely been employed to obtain the efficient results of bilevel and multilevel programming problems [27, 28, 29, 30, 32]. Toksari and Bilim [31] introduced an interactive fuzzy goal method based on the Jacobian matrix for solving the multilevel fractional programming problem.

An interactive fuzzy decision making algorithm in this paper is presented for two-level linear fractional programming problems (TLLFPP) a single DM at the first level as well as multiple decision makers at the second level. Objective functions and constraint functions for DMs at both levels are fractional and linear functions, respectively. In order to solve the problem, the fuzzy goal of each of objective function is defined by getting individual optimal solutions. Thereafter, the membership function of each fractional objective for TLLFPP is constructed. Then the overall satisfactory balance between the leader and the follower is defined by introducing a new balance function. Finally, numerical examples are presented to demonstrate the feasibility of the presented interactive algorithm.

The remaining of this paper is arranged as follows. A mathematical model of bilevel fractional programming problem is given in Section 2 and an interactive fuzzy decision making algorithm is presented in Section 3. At least, two comparative examples are implemented in Section 4 and the paper is concluded in Section 5.

2. Problem Formulation

In (TLLFPP), two DMs are located at two diverse hierarchical levels including multiple objectives. Moreover, each DM independently controls a set of decision variables. The first level decision maker (DM) is known as the leader, which executes its decision in the scope of the second level DMs known as the follower. Here, each DM tries to optimize

its objective function and is affected by the activities of the other DMs.

A mathematical model of two level linear fractional programming problem is formulated as follows (see; Ahlacioglu and Tiryaki [13]):

$$\begin{aligned} \text{Upper Level: } \max_{x_0} f_0(x) &= \frac{c_0x + \alpha_0}{d_0x + \beta_0} \\ \text{Lower Level: } \max_{x_i} f_i(x) &= \frac{c_ix + \alpha_i}{d_ix + \beta_i}, i=1,2,\dots,k \end{aligned} \quad (1)$$

Subject to

$$x \in S = \{x \in \mathbb{R}^n \mid Ax \leq b, x \geq 0\}$$

where; $c_0 = (c_{00}, c_{01}, c_{02}, \dots, c_{0k}), c_i = (c_{i0}, c_{i1}, c_{i2}, \dots, c_{ik}),$
 $d_0 = (d_{00}, d_{01}, d_{02}, \dots, d_{0k})$ and
 $d_i = (d_{i0}, d_{i1}, d_{i2}, \dots, d_{ik}), i=1,2,\dots,k$ are n -dimensional fixed row vectors; $\alpha_0, \beta_0, \alpha_i$ and $\beta_i, i=1,2,\dots,k$ are reel numbers; b is m -dimensional constant column vector; A is an $m \times n$ constant matrix with full rank r . S is a non-empty, convex and compact set in \mathbb{R}^n ; $d_0x + \beta_0$ and $d_ix + \beta_i$ are greater than zero.

2.1. Construction of Membership Function

Each of the decision makers aims to minimize its own objective over the feasible region. The optimal solutions of them are found, individually and these solutions can be chosen as the best solution. Besides, the achieved value of each of objective can be admitted as the aspiration levels for the corresponding fuzzy goals. For convenience, the method given in paper [28] is used to determine membership functions. Let us $\mu(f_j), j=0,1,2,\dots,k$ to define the fuzzy goals of the leader and the follower, respectively.

$$\mu(f_j) = \begin{cases} 0 & \text{if } f_j < f_j^L \\ \frac{f_j - f_j^L}{f_j^U - f_j^L} & \text{if } f_j^L \leq f_j \leq f_j^U \\ 1 & \text{if } f_j \geq f_j^U \end{cases} \quad (2)$$

where f_j^U is called an ideal value and f_j^L is tolerance limit of j -the fuzzy goal. f_j^U and f_j^L denote the values of the objective $f_j(x), j=0,1,2,\dots,k$ such that the degrees of the membership function are 1 and 0, respectively. For the sake of simplicity, we suppose that f_j^U and f_j^L are the optimal solutions of the following fractional problems, respectively. For instance,

$$f_j^U = \max f_j(x), j=0,1,2,\dots,k \quad (3)$$

and

$$f_j^L = \min f_j(x), j=0,1,2,\dots,k \quad (4)$$

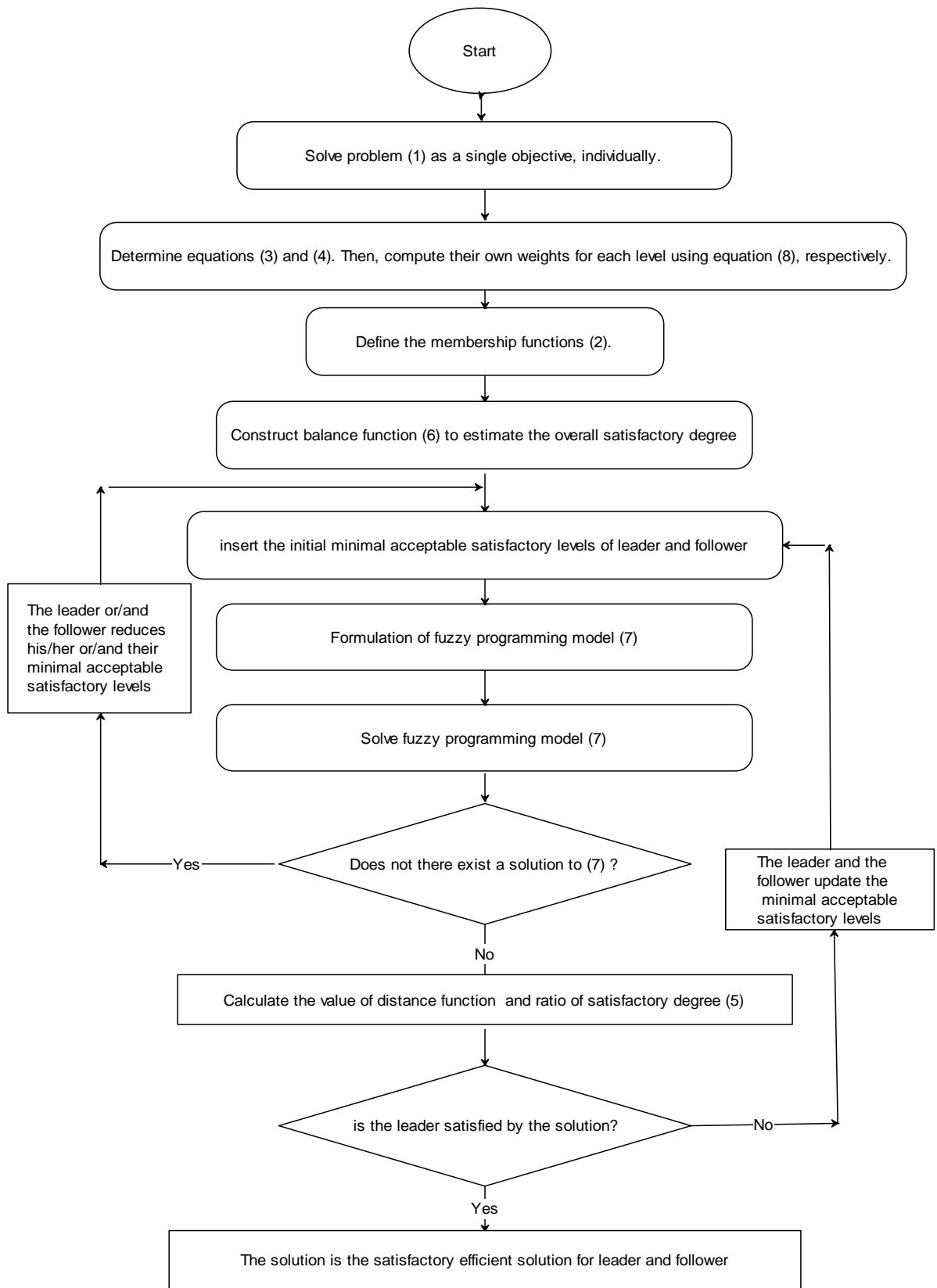


Fig. 1. Application framework of the interactive fuzzy decision making algorithm

3. Interactive Fuzzy Decision Making Method

In decision making process, achievement value of highest

membership function for a fuzzy goal is forever desired by a DM. But it is difficult to obtain the highest degree for all membership function values. Therefore, we need the theory

of the general satisfactory degree between the leader and follower. To do this, the following concept is given [12]:

$$\Delta = \frac{\sum_{i=1}^k w_i \mu(f_i)}{w_0 \mu(f_0)} \tag{5}$$

Note that, many authors implemented the notation given in above to achieve the satisfactory degree between the leader and follower at the decision making process [12, 22]. Nevertheless, this condition may result in some efficient computation. But, it can be inadequate for complex and large-scale calculations. Therefore, the following balance function is presented to estimate the overall satisfactory degree which can be characterized as the ratio of two functions:

$$d(x) = \frac{\sqrt{\sum_{j=0}^k (f_j(x) - f_j^L)^2}}{\sqrt{\sum_{j=0}^k (f_j(x) - f_j^U)^2 + \sum_{j=0}^k (f_j^U - f_j^L)^2}} \tag{6}$$

Clearly, $0 \leq d(x) \leq 1$ for all $x \in S$.

If each decision maker achieves the ideal value, $d(x)$ is equal to 1. In addition, $d(x)$ grows as the objective function values of the leader and the follower are regenerated. Therefore, we can employ the value of $d(x)$ to balance the overall satisfactory degree between the leader and follower at the two level decision making process.

Now, the formulation of the proposed method can be stated as:

$$\begin{aligned} &\max d(x) \\ &\text{subject to} \\ &\frac{\mu(f_0)}{w_0} \geq \mu_0 \\ &\sum_{i=1}^k \frac{\mu(f_i)}{w_i} \geq \mu_1 \\ &x \in S \end{aligned} \tag{7}$$

where μ_0 and μ_1 are the minimal acceptable satisfactory levels specified by the leader and the follower, respectively. w_0 and $w_i, i=1,2,\dots,k$ are importance weight of each objective. Here, lower level functions are combined using their own weights.

Furthermore, w_0 is always 1 for single decision at the upper level. Therefore, the weights are not considered and by taking into account the minimal satisfactory level of the objective at the upper level and by determining a proportional satisfaction balance among all objectives and their importance weights, we aim to achieve a satisfying solution from a Pareto optimal solution set for TLLFP problem such that the satisfactory levels of all objectives are proportional to their own weights. Here, $w_i, i=1,2,\dots,k$ is calculated as follows: (see: Kassem [33]):

$$w_i = \frac{f_i^L - f_i^U}{\sum_{k=1}^l |f_i^L - f_i^U|}, \quad i = 1, 2, \dots, l. \tag{8}$$

Theorem: If (x^*, y^*) is an optimal solution to problem (7), then it is also an efficient solution to problem (1).

Proof: If (x^*, y^*) is not an efficient solution, then there exists $(\tilde{x}, \tilde{y}) \in S$ such that $f_j(\tilde{x}, \tilde{y}) \geq f_j(x^*, y^*)$ for all $j = 0, 1, 2, \dots, k$ and $f_k(\tilde{x}, \tilde{y}) > f_k(x^*, y^*)$, $j \neq k$ for at least one index k . This contradicts that (x^*, y^*) is an optimal solution of (6).

When the leader achieves the solution of problem (7) as a satisfactory solution, the iterative process finishes. Now, we consider the following idea for refreshing the minimal acceptable satisfactory level μ_0 (see page 92 of [34]):

If the leader is not satisfied with the achieved solution and experts that it is desirable to increase the satisfactory degree of the leader at the expense of the satisfactory degree of the follower, then he/she increases the minimal acceptable satisfactory level μ_0 . Otherwise, if the leader experts that it is desirable to increase the satisfactory degree of the follower at the expense of the satisfactory degree of the leader, then he/she decreases the minimal acceptable satisfactory level μ_0 .

3.1. The proposed Interactive Fuzzy Decision Algorithm to Solve TLLFP

➤ Step 1 Solve the problem (1) as in equation (3) and (4) by taking single objective function at a time and neglecting all others.

➤ Step 2 Determine the ideal values f_j^U ($j = 0, 1, 2, \dots, k$) and tolerance limits f_j^L ($j = 0, 1, 2, \dots, k$) and weights w_i for all objective.

➤ Step 3 Construct the membership functions (2) and then combine all of objective with their own weights, respectively.

➤ Step 4 Construct balance function (6).

➤ Step 5 The leader and follower insert the initial minimal acceptable satisfactory levels μ_0, μ_1 .

➤ Step 6 Formulate the fuzzy decision making programming model (7). Then, solve problem (7) to obtain the optimal solutions.

➤ Step 7 If there does not exist a solution to (7), the leader or/and the follower reduces his/her or/and their minimal acceptable satisfactory levels, until a solution (x^*, y^*) is obtained for (7).

➤ Step 8 If the leader is satisfied by the solution in Step 7, go to Step 9, else go to Step 10.

➤ Step 9 The solution is the satisfactory efficient solution for leader and follower in problem (1).

➤ Step 10 The leader and the follower update the minimal acceptable satisfactory levels μ_0 and μ_1 , go to step (6).

A comparison of results based on linearization procedures given above is shown in Fig. 1.

In order to evaluate the satisfaction, we not only use the value of the overall satisfactory degree $d(x)$, but also the

value of distance function $D = \left\{ \sum_{j=0}^k [1 - \mu(f_j)]^2 \right\}^2$ (for details, see; [35, 36]) where $\mu(f_j)$ represents the achieved membership value of the j -th decision maker.

4. Numerical Example

The suggested interactive fuzzy decision making method will be used to a known numerical example. The following numerical example was given by Ahlatcioglu and Tiryaki [13]. They used the decentralized method to solve the following problem.

Example

Upper level:

$$\max_{x_0} f_0 = \frac{3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32}}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1}$$

Lower level:

$$\max_{x_1} f_1 = \frac{x_{11} + x_{12}}{2x_{11} + x_{12} + 1}, \max_{x_2} f_2 = \frac{x_{21} - x_{22}}{x_{21} + x_{22} + 1}, \max_{x_3} f_3 = \frac{x_{31} + 2x_{32} + 3}{x_{32} + 1}$$

$$s.t. \begin{cases} g_1 = 2x_{01} + x_{02} + x_{11} - x_{12} + 2x_{21} - 2x_{22} - x_{31} + 3x_{32} \leq 12, \\ g_2 = -x_{01} + x_{02} + 2x_{11} + 4x_{12} + 3x_{21} - x_{22} + 2x_{31} - x_{32} \leq 24, \\ g_3 = 3x_{01} - x_{02} + 3x_{11} - x_{21} + x_{31} + 2x_{32} \leq 9, \\ g_4 = x_{01} + 2x_{02} + 5x_{12} + x_{22} - 2x_{31} - x_{32} \leq 10, \\ g_5 = x_{01} - x_{02} + x_{11} + 4x_{21} + 5x_{22} + x_{32} \leq 20, \\ g_6 = 4x_{01} + 3x_{02} + 2x_{11} - x_{12} + x_{21} + x_{22} \leq 36, \\ g_7 = 5x_{01} + 2x_{02} + x_{12} + 3x_{22} + 4x_{32} \leq 30, \\ g_8 = -2x_{01} - x_{02} + 5x_{11} + x_{21} + 2x_{32} \leq 20, \\ g_9 = x_{01} + x_{02} + 2x_{11} + x_{22} + 4x_{31} + x_{32} \leq 8, \\ g_{10} = -x_{01} - 3x_{11} + 2x_{12} + x_{21} + 4x_{31} + 2x_{32} \leq 48, \\ g_{11} = 3x_{01} - 5x_{11} + 2x_{21} + 5x_{22} - 2x_{32} \leq 15, \\ g_{12} = x_{01} + 2x_{02} + x_{11} - x_{12} + 3x_{21} + 5x_{31} - x_{32} \leq 60 \\ x_{01}, x_{02}, x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32} \geq 0 \end{cases}$$

where $x_0 = (x_{01}, x_{02}), x_1 = (x_{11}, x_{12}), x_2 = (x_{21}, x_{22})$ and

$x_3 = (x_{31}, x_{32})$.

Table 1 presents the individual minimum and maximum values (Step 1), the ideal values, tolerance limits and weights (Step 2) of all the objective functions in both the levels.

Step 3:

Upper level membership function:

$$\mu(f_0) = \begin{cases} 0 & \text{if } f_0 < -5 \\ \frac{f_0 - 5}{2.455 + 5} & \text{if } -5 \leq f_0 \leq 2.455 \\ 1 & \text{if } f_0 \geq 2.455 \end{cases}$$

Table 1: The individual minimum and maximum values, the ideal value and tolerance limits and weights

	f_0	f_1	f_2	f_3
max f_j	2.455	0.765	0.849	5
min f_j	-5	0	-804	2.160
f_j^U	2.455	0.765	0.849	5
f_j^L	-5	0	-804	2.160

$$= \frac{0.134(3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32})}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1} + 0.671$$

Lower level membership functions:

$$\mu(f_1) = \begin{cases} 0 & \text{if } f_1 < 0 \\ \frac{f_1 - 0}{0.765 - 0} & \text{if } 0 \leq f_1 \leq 0.765 \\ 1 & \text{if } f_1 \geq 0.765 \end{cases}$$

$$\Rightarrow \mu(f_1) = \frac{1.307(x_{11} + x_{12})}{2x_{11} + x_{12} + 1}$$

$$\mu(f_2) = \begin{cases} 0 & \text{if } f_2 < 0.804 \\ \frac{f_2 + 0.804}{0.849 + 0.804} & \text{if } 0.804 \leq f_2 \leq 0.849 \\ 1 & \text{if } f_2 \geq 0.849 \end{cases}$$

$$\Rightarrow \mu(f_2) = \frac{0.605(x_{21} - x_{22})}{x_{21} + x_{22} + 1} + 0.486$$

$$\mu(f_3) = \begin{cases} 0 & \text{if } f_3 < 2.16 \\ \frac{f_3 - 2.16}{5 - 2.16} & \text{if } 2.16 \leq f_3 \leq 5 \\ 1 & \text{if } f_3 \geq 5 \end{cases}$$

$$\Rightarrow \mu(f_3) = \frac{0.375(x_{31} + 2x_{32} + 3)}{x_{32} + 1} - 0.873$$

and all of them are combined using their weights as:

$$\sum_{i=1}^3 \frac{\mu(f_i)}{w_i} = \frac{1.307(x_{11} + x_{12})}{0.172} + \frac{0.605(x_{21} - x_{22})}{0.371} + 0.486 + \frac{0.375(x_{31} + 2x_{32} + 3)}{0.600} - 0.873$$

Step 4:

Balance function (6) is determined as

$$d(x) = \frac{\sqrt{\left(\frac{3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32} + 5}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1} \right)^2 + \left(\frac{x_{11} + x_{12}}{2x_{11} + x_{12} + 1} \right)^2 + \left(\frac{x_{21} - x_{22}}{x_{21} + x_{22} + 1} + 0.804 \right)^2 + \left(\frac{x_{31} + 2x_{32} + 3}{x_{32} + 1} - 2.16 \right)^2}}{\sqrt{\left(\frac{3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32} - 2.455}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1} \right)^2 + \left(\frac{x_{11} + x_{12}}{2x_{11} + x_{12} + 1} - 0.765 \right)^2 + \left(\frac{x_{21} - x_{22}}{x_{21} + x_{22} + 1} - 0.849 \right)^2 + \left(\frac{x_{31} + 2x_{32} + 3}{x_{32} + 1} - 5 \right)^2}} + 66.024$$

Step 5: Let $\mu_0 = 1$ and $\mu_1 = 0.9$.

Step 6: Then, the corresponding problem (7) can be formulated as:

$$\max d(x) = \frac{\sqrt{\left(\frac{3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32} + 5}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1}\right)^2 + \left(\frac{x_{11} + x_{12}}{2x_{11} + x_{12} + 1}\right)^2 + \left(\frac{x_{21} - x_{22}}{x_{21} + x_{22} + 1} + 0.804\right)^2 + \left(\frac{x_{31} + 2x_{32} + 3}{x_{32} + 1} - 2.16\right)^2}}{\sqrt{\left(\frac{3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32} - 2.455}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1}\right)^2 + \left(\frac{x_{11} + x_{12}}{2x_{11} + x_{12} + 1} - 0.765\right)^2 + \left(\frac{x_{21} - x_{22}}{x_{21} + x_{22} + 1} - 0.849\right)^2 + \left(\frac{x_{31} + 2x_{32} + 3}{x_{32} + 1} - 5\right)^2}} + 66.024$$

Subject to

$$\frac{0.134(3x_{01} + 5x_{02} + x_{11} + 3x_{12} - x_{21} + 2x_{22} + x_{31} + 2x_{32})}{x_{01} + 2x_{02} + 2x_{11} + x_{12} + x_{22} + x_{31} + 2x_{32} + 1} + 0.671 \geq 1,$$

$$\frac{1.307(x_{11} + x_{12})}{2x_{11} + x_{12} + 1} + \frac{0.605(x_{21} - x_{22})}{x_{21} + x_{22} + 1} + 0.486 \frac{0.375(x_{31} + 2x_{32} + 3)}{x_{32} + 1} - 0.873 \geq 0.9,$$

$$\frac{0.172}{0.371} + \frac{0.371}{0.600}$$

$$g_1 = 2x_{01} + x_{02} + x_{11} - x_{12} + 2x_{21} - 2x_{22} - x_{31} + 3x_{32} \leq 12,$$

$$g_2 = -x_{01} + x_{02} + 2x_{11} + 4x_{12} + 3x_{21} - x_{22} + 2x_{31} - x_{32} \leq 24,$$

$$g_3 = 3x_{01} - x_{02} + 3x_{11} - x_{21} + x_{31} + 2x_{32} \leq 9,$$

$$g_4 = x_{01} + 2x_{02} + 5x_{12} + x_{22} - 2x_{31} - x_{32} \leq 10,$$

$$g_5 = x_{01} - x_{02} + x_{11} + 4x_{21} + 5x_{22} + x_{32} \leq 20,$$

$$g_6 = 4x_{01} + 3x_{02} + 2x_{11} - x_{12} + x_{21} + x_{22} \leq 36,$$

$$g_7 = 5x_{01} + 2x_{02} + x_{12} + 3x_{22} + 4x_{32} \leq 30,$$

$$g_8 = -2x_{01} - x_{02} + 5x_{11} + x_{21} + 2x_{32} \leq 20,$$

$$g_9 = x_{01} + x_{02} + 2x_{11} + x_{22} + 4x_{31} + x_{32} \leq 8,$$

$$g_{10} = -x_{01} - 3x_{11} + 2x_{12} + x_{21} + 4x_{31} + 2x_{32} \leq 48,$$

$$g_{11} = 3x_{01} - 5x_{11} + 2x_{21} + 5x_{22} - 2x_{32} \leq 15,$$

$$g_{12} = x_{01} + 2x_{02} + x_{11} - x_{12} + 3x_{21} + 5x_{31} - x_{32} \leq 60,$$

$$x_{01}, x_{02}, x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32} \geq 0$$

Step 7: There does not exist feasible solution for the above problem with $\mu_0 = 1$ and $\mu_1 = 0.9$. So the leader adjusts the minimal acceptable level $\mu_0 = 1$ by reducing $\mu_0 = 1$ to $\mu_0 = 0.9$.

The above problem is solved with $\mu_0 = 0.9$ and $\mu_1 = 0.9$ using the Maple 18.02 software program, the optimal solutions for the above problem are $x_{01} = 2.640, x_{02} = 0., x_{11} = 0., x_{12} = 2.008,$
 $x_{21} = 0.260, x_{22} = 0., x_{31} = 1.340, x_{32} = 0.$

For this optimal solution, membership functions values are $\mu(f_0) = 0.959, \mu(f_1) = 0.873, \mu(f_2) = 0.611, \mu(f_3) = 0.753$ and objective values are $f_0 = 2.150, f_1 = 0.668, f_2 = 0.207, f_3 = 4.340.$

The ratio of satisfactory degrees is $\Delta = 0.934$.

We execute a comparison with the obtained solutions from [13] in Table 2. From the obtained solutions of $d(x)$ and D , the obtained solution of the suggested method in this paper is better than the method of Tiryaki and Ahlatcioglu [13]. Furthermore, all of the sum of the leader's values and the follower's values generated by our suggested method is greater than that generated by Ahlatcioglu and Tiryaki [13]. So, these solutions indicate that the suggested method in this paper is practicable.

Numerical results prove that the suggested method in this paper has the following interesting features.

- According to Table 2, we can observe that the value of D by the suggested method is smaller than that of other method.

It should be noted that the larger value of Δ in (5) is not the more satisfactory the solution. We can see from the distance function D .

5. Conclusion

In this paper, a new interactive fuzzy decision making method based on the idea of the membership function is suggested for solving the two-level fractional programming problem. We use the overall satisfactory balance between the leader and the follower into consideration by introducing a new balance function. Then, a satisfactory solution is achieved. This solution involves knowledge concerning importance weights of lower level objectives and the minimal satisfactory level of all objectives. Furthermore, this method has an interactive structure as it provides leader to provide the opportunity of exchange the data presented that the leader is not satisfied from this solution. Consequently, application of the suggested method is discussed with a numerical model and the effectiveness of the solutions obtained by the suggested method is verified. Moreover, from table 2, our suggested approach gives a more efficient solution comparing to the approaches of Ahlatcioglu and Tiryaki [13].

Hence, our suggested algorithm can be easily extended both the lower level and upper level with multiple objectives (for example, [31]).

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Table 2: Comparison of results of Example.

	f_0	f_1	f_2	f_3	$\mu(f_0)$	$\mu(f_1)$	$\mu(f_2)$	$\mu(f_3)$	$d(x)$	Δ	D
The proposed method	2.150	0.668	0.207	4.340	0.959	0.873	0.611	0.753	0.919	0.934	0.480
Method in [13]	2.082	0.655	0.510	4.22	0.950	0.856	0.601	0.709	–	0.738	0.517

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