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# EVIDENCES OF UNDERSTANDINGS AND MISCONCEPTIONS OF GRAPHS AFTER EXPLORING USING TECHNOLOGY

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**Abstract**: Twenty-three Science, Technology, Engineering and Mathematics (STEM) preservice elementary and middle school teachers enrolled in a large public university in the southwest United States explored position/time graphs using graphing calculators and simple, inexpensive motion detectors. Using preprogrammed position/time graphs and creating their own distance/time graphs, the preservice teachers worked in groups to match, generate and explain a variety of graphs. The purpose of this study was to investigate what ways the preservice teachers interpreted position/time and distance/time graphs with an emphasis on both understandings and misconceptions. Data were primarily gathered through a pretest and posttest focused on both quantitative and qualitative instruction. The results showed that initially the participants had difficulties interpreting position/time and distance/time graphs. However, after the completion of a three-week unit, they showed increased understanding of interpreting graphs and how the slopes influenced movement in a qualitative manner. However, preservice teachers continued to struggle with quantitative interpretations and calculations. Additionally, preservice teachers had difficultly identifying errors in non-examples that mistakenly interpreted position/time graphs as elevation/time graphs and a few were unable to distinguish between position/time and distance/time graphs. Even with the increased use of technology to connect distance/time and position/time graphs, the preservice elementary and middle school teachers still struggled with understanding what the graphs represented. Guidelines will be provided that focus on how to design lessons to address the graph misconceptions including: opportunities to interpret misunderstandings, opportunities to create a story and match the story with the graph using technology and a specific and direct connection to movement, slope and y-intercept.

***Keywords***: Algebraic thinking, technology integration, graphing

## Introduction

Motion in algebra is often displayed in distance/time and position/time graphs starting at the middle school level of instruction. Rate of change is indicated through the slope of the line. And the rate of change can be demonstrated through motion when using motion detectors attached to graphing calculators. Motion detectors provide instantaneous feedback regarding the motion of an object through immediately graphing the movement across the x-axis and y-axis. When using motion detectors with graphing calculators observations of motions can be made in real time and movement data is stored and displayed for easy access and analysis (MacDonald, Vásquez, & Caverly, 2002). Students can directly see how their movement influences the slope of the line and can quickly and easily test and adjust conjectures (Kutzler, 2000). Movement can be easily adjusted and changed; physical experiences are directly connected to mathematical meaning with the use of motion detectors (MacDonald et al. 2002). Using motion detectors can benefit student understanding because through their actions, an authentic learning experience occurs that allows insight into how movement can influence graphs (Kersaint, 2007). Physical models supported with technology can have the potential to improve students' understanding of relationships (Kaput,1994; Vahey, Tatar, & Roschelle, 2004). There is a connection to qualitative and quantitative reasoning through movement with an emphasis on the meaning of the variables (Leinhardt, Zaslavsky, & Stein, 1990).



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In this research, we explored how motion detectors supported preservice teachers' development of distance/time and position/time graphs. We first surveyed how many Science, Technology, Engineering and Mathematics (STEM) preservice elementary and middle school teachers had experience with motion detectors as well as their initial understanding of distance/time and position/time graphs. Then, the preservice teachers explored for three weeks using graphing calculators and motion detectors. Our research question was, how do motion detectors support STEM preservice teachers' understanding and changes in thinking of distance/time and position/time graphs? Additionally, what misconceptions do preservice teachers still have regarding these graphs after the explorations using motion detectors?

## Methods

### Participants

STEM preservice teachers were enrolled in an upper division mathematics content course that emphasized algebraic thinking at a large public university in the southwest United States. The course was recently developed to provide an opportunity to revisit algebraic thinking moving beyond computation and variable manipulation. Instead, the course focused on conceptual understanding of algebraic ideas (Stump, Roebuck, & Bishop, 2009) with an emphasis on elementary and middle school curricular concepts with some high school concepts explored as well (discovering features of quadratic functions using manipulatives). Twenty-three preservice teachers partook in the exploration; four were males.

### Course Structure

Calculator-Based Rangers™ (CBRs) are sonic motion detectors that connect to graphing calculators (TI-83 Plus and TI-84) and collect information on distance, velocity and acceleration (Texas Instruments, 2006). Horton, Storm and Leonard (2004) indicate that graphing calculators are supportive in developing algebraic thinking in that students can quickly analyze and adjust data with the immediate feedback possible with the technology.

Because the course focused on a variety of topics in algebra, three weeks could be devoted to motion graphs. Preservice teachers met twice a week for one hour and fifteen minutes per session. Participants explored all graphs using technology with an emphasis on distance/time, position/time and velocity/time graphs; data were analyzed for this study on the first two topics. Table 1 provides a structure to the activities and exploration descriptions.

Table 1. Structure of motion unit

| Week | Content | Description |
|---|---|---|
| One | Pretest<br>Distance/time | Preservice teachers completed the pretest prior to any instruction<br>Graphing calculator and motion detector use and operation<br>Preservice teachers were provided with a meter stick and explored how speed influences distance/time graphs using motion detectors |
| Two | Position/time<br>Velocity/time | Using preprogrammed content, graphs were presented and participants were asked to match a variety of position/time and velocity/time graphs through movement<br>There was an emphasis on both qualitative and quantitative interpretations of the graphs |
| Three | Review of graphs<br>Posttest | The content of all three graphs were reviewed with an emphasis a stronger emphasis on quantitative meaning based on preservice teachers' misunderstandings from the previous weeks<br>A complete posttest of the unit was administered |

### Data Collection

In order to explore the changes in thinking and understanding in relation to distance/time and position/time graphs, a pretest and posttest were given to the preservice teachers. Prior to this study, a pilot study was conducted that informed our data collection. In that pilot study, preservice teachers indicated very little experience with motion detectors (two out of 29 from the pilot study had ever used motion detectors). Additionally, the preservice teachers in this pilot study had very little background knowledge in relation to distance/time and positon/time graphs. The pilot study results informed the creation and administration of the pretest. We limited the questions asked because we had surmised from our pilot study that preservice teachers would have a limited understanding of these graphs at the start of data collection. We did not want the participants to feel inadequate by being asked questions in which they did not understand; class time was also very limited. So, the pretest consisted of questions regarding CBR use along with interpretations of position/time and distance/time graphs. However, the posttest contained more comprehensive questions that examined position/time and distance/time graphs. All of the pretest content questions were also on the posttest along with additional, comprehensive questions.

## Data Analysis

All participants were assigned a number 1-23; then a random number generator app on the graphing calculator was used to generate numbers; these numbers were used to select the participants for analysis. The pretest was administered and examined for prior experience with CBRs, correctness and misconceptions. Then, the posttest was analyzed with an emphasis on both changes in thinking (growth) and misconceptions. Two specific content questions on both the pretest and posttest were analyzed: 1) a position/time graph and 2) a distance/time graph. Additionally, two posttest only questions were analyzed for this study: 1) quantitative calculations on a given graph and 2) the identification of an imaginary student's misconception of a position/time graph.

### *Position/Time Graph*

In both the pretest and posttest, participants were asked to interpret the graph below (Figure 1).

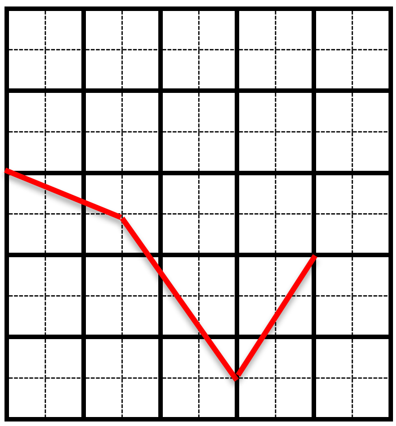
Distance (meters)

4

3

2

1



0 1 2 3 4 5 Time (seconds)

Figure 1. A position/time graph provided in the pretest and posttest

### *Distance/Time Graph*

In both the pretest and posttest, the participants were asked to interpret the distance/time graph below (Figure 2).

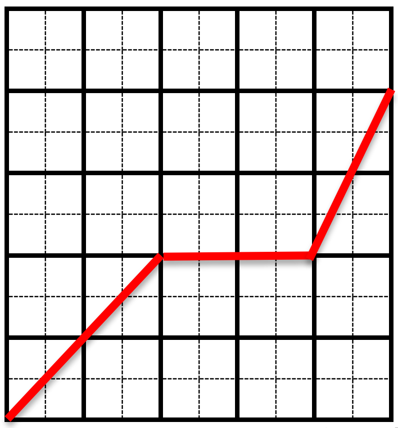
Distance (meters)

4

3

2

1



0 1 2 3 4 5 Time (seconds)

Figure 2. A distance/time graph provided in the pretest and posttest

### *Quantitative Calculations*

In the posttest, the participants were asked to determine an equation and when the equation is valid for the three different movements represented in the graph below (Figure 3). The content was discussed in class and explored when preservice teachers used the CBRs. Preservice teachers were asked to calculate slopes and their meaning as well as to determine when specific equations were valid in relation to time by finding inequalities.

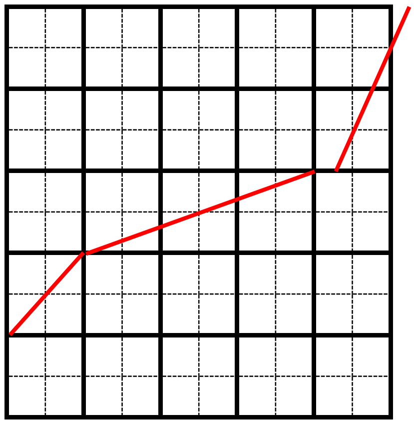
Distance (meters)

4

3

2

1



0 1 2 3 4 5 Time (seconds)

Figure 3. A given distance/time graph used to determine equations

### *Student Misconceptions*

Participants were given the graph (Figure 4) below along with an accompanying story: A fifth grade student in your class says, “There is a very small hill only 3 meters high. I started at the bottom of the hill. I walked up the hill. Then I walked across the top and started coming down the hill. I did not make it all the way down the hill. The entire journey took 5 seconds.” Justify (explain) why the student is correct (or incorrect).

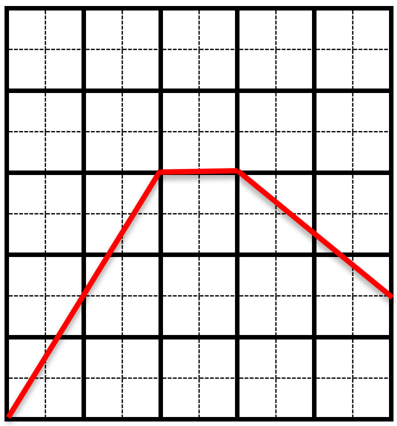
Distance (meters)

4

3

2

1



0 1 2 3 4 5 Time (seconds)

Figure 4. The graph provided along with an incorrect narrative

The course was intentionally designed to include this kind of student misconception only on the posttest rather than as part of the discussed/integrated class curriculum. The purpose was to explore whether or not the preservice teachers could transition from the creating the graphs to critically analyzing errors in graphs in relation to student thinking.

## Findings

### Abby

Abby had no experience with CBRs prior to the course. On the pretest, she wrote “No clue on any of it” and most of the pretest was left blank.

### *Position/Time Graph*

On the pretest, Abby stated, "graphing distance in meters over how many seconds" when asked to interpret the position/time graph. On the posttest (see Fig. 1), she stated, "A person starts three meters from home, and heads toward home [at] .5 meters/ 1.5 seconds. Next goes 2 meters/1.5 seconds towards home. Then goes 1.5 meters/1 second away from home."

### *Distance/Time Graph*

The distance/time graph was left blank on the pretest. On the posttest (see Fig. 2), Abby stated, "A person goes 4 meters in 5 seconds. They travel quick for 2 seconds. Rest for 2 seconds then run like heck for 1 second."

### *Quantitative Calculations*

The qualitative calculations were left blank on the posttest.

### *Student Misconceptions*

When asked to determine whether the student's reasoning was correct, Abby stated "The above map [graph] is a position map [graph]. You cannot go a negative distance (yet…time travel) [meaning that negative distances are possible only with time travel]."

## **Betsy**

Betsy stated that she had no experience with CBRs at the start of the unit. She answered most of the questions on the pretest.

### *Position/Time Graph*

On the pretest, Betsy stated "It is comparing how far in meters something is traveling per second." On the posttest, Betsy stated "This graph is impossible because you cannot be in two different places at one time."

### *Distance/Time Graph*

On the pretest, Betsy stated, "As the distance in meters increases, the length of time in seconds increases." On the posttest, Betsy stated "In this graph, you walk to the store at a rate of 1 meter/second, you get to the store and do your shopping (time is passing still but you aren't walking further), and then you leave the store, walking back home a little faster, at a rate of 2 meters per second because you want to get home in time for your favorite show."

### *Quantitative Calculations*

Betsy misread the instructions on the posttest and only calculated the slopes and not the equations. She was able to create inequalities using the x-axis values to determine when the slopes were valid. For example, she calculated the correct slope of 1 for the first movement in the graph and wrote: $0 < x \leq 1$. All components (but the missing equation) were correct for the three movements in the graph.

### *Student Misconceptions*

Betsy stated, "This student is incorrect. First of all, distance can never go backwards. Secondly, the first segment of the graph shows a pretty steep slope, meaning the student probably ran up to the top of the hill, not just walked."

**Canan**

Canan too had no experience with CBRs. She completed the entire pretest.

***Position/Time Graph***

On the pretest, Canan stated, "An object is declining in speed until a certain point and then increasing speed again." On the posttest, Canan stated, "An object is starting 3 meters away and coming toward [the] CBR until 1.5 seconds when it speeds up. At 3 seconds it starts moving away from [the] CBR at a quick pace."

***Distance/Time Graph***

On the pretest, Canan stated, "An object is increasing speed until it reaches a plateau and then increasing in speed again." On the posttest, Canan stated, "An object stars moving away at a constant rate for two seconds then stops. It says in the same place for 2 more seconds, then starts moving away again at a faster rate."

***Quantitative Calculations***

Canan correctly calculated the formulas and the accompanying inequalities for all movements in the graph. She stated: "$y= x + 1$ for $0 < x \leq 1$ [and] $y = 1/3\ x + 1\ 2/3$ for $1 < x \leq 4$ [and] $y = 2x – 5$ for $4 < x \leq 5$."

***Student Misconceptions***

Canan stated, "The student is correct. From 0-2 second mark is when the student is going up the hill. The interval from 2-3 seconds is when the student is walking on top assuming the top is level and flat. The interval from 3-5 seconds is when the student is coming down. The student is correct if distance is distance from [the] ground."

## Discussion

All three of the randomly selected participants started out with limited knowledge in regards to distance/time and position/time graphs, and none of the participants had any experience with the motion detectors. As indicated by the two questions from the pretest and posttest, all showed some growth in relation to these graphs. Both Abby and Canan included richer detail when explaining the meaning of the graphs. Both used specific numbers in the posttest to support their descriptions as compared to the pretest. Betsy showed less growth and was incorrect in her interpretations. When she used the store example, she connected incorrect distances to her motion; a home would be much further away from a store than just a few meters. As well, she indicated that the position graph shows negative time travel.

Abby had no knowledge of how to calculate an equation and accompanying inequalities indicated by leaving this portion of the posttest blank. Betsy was able to calculate slope and the inequalities, but forgot to include equations for the three movements. While Canan was able to correctly find the equations and inequalities.

And with the question focused on student misconceptions, all three participants were unable to correctly explain what was wrong with the student's thinking. Abby made the same error that Betsy made with the posttest position/time question. She mistakenly thought that moving down in distance means that time is becoming negative. Betsy's response first seems like it is correct, as she states the student is incorrect. However, her reasoning is wrong. She states that distance cannot be negative (it is not negative in the graph) and then states that the student misinterpreted the slopes. Canan said that the student was correct but notes that the student is only correct if distance means "distance from [the] ground." Canan has some insight but was unable to articulate that the student is describing elevation while the graph does not show elevation, but position in relation to time (as indicted by the graph and the x- and y-axes).

## Conclusion

Learning with motion detectors appeared to help the preservice teachers in terms of qualitatively describing the movement indicated by the graphs. There was still some confusion regarding the meaning of movements, what constitutes an impossible graph (negative or backwards time) and what constitutes validity in a graph. As well, they still struggled quantitatively explaining meaning of graphs. Generally, preservice teachers define algebra in relation to symbolic manipulation (Nathan & Petrosino, 2003; Stephens, 2008). However, with the technology, the preservice teaches were able to move beyond symbolic representation in algebra and were able to focus on meaning of graphs; an area where they showed some growth in their development after using the motion detectors. The motion detector activities described provided an opportunity for preservice teachers to explore algebraic thinking while focusing on modeling and alignment of multiple representations. Using such devises that support the learning of mathematical concepts can be beneficial to student understanding and development (Vahey et al., 2004).

## Recommendations

While the motion detector seemed to support development in relation to position/time graphs and distance/time graphs, the success was related to qualitative descriptions of the features of the graphs in relation to movement. Describing the movement using data was one of the fruitful benefits of using the motion detectors. We recommend using motion detectors to help learners understand the meaning of graphs. In relation to quantitative data, there needs to be more of a focus on how to find determine the equations of the movement and when the equations are valid. When using motion detectors, we recommend a focused analysis of supported the qualitative descriptions with quantitative data. And finally, while we intentionally did not discuss student misconceptions whole class to see if the participants could transition from their experiences to errors in student thinking, our results showed that the preservice teachers could not. We recommend specific activities that provide opportunities to model graphs with the motion detectors that are impossible or indicate errors in thinking. In providing specific opportunities to discover errors in thinking using motion detectors, perhaps there will be a richer understanding of graphs and what they specifically mean.

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