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Confidence Intervals for Ratios of the Coefficients of Variation of the Delta-Birnbaum-Saunders Distributions

Usanee JANTHASUWAN, Sa-Aat NIWITPONG, Suparat NIWITPONG *

Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, 10800, Bangkok, Thailand

Highlights

- This paper focuses on methods used in constructing confidence intervals.
- The evaluation and comparison are conducted using coverage probability and average width.
- Wind speed data from Thailand has been analyzed using the proposed methods.

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Abstract

The Delta-Birnbaum-Saunders distribution is a combination of positive values that follow the Birnbaum-Saunders distribution and zeros that follow the binomial distribution, making it a relatively new distribution. The coefficient of variation is calculated as the ratio of the standard deviation to the mean. It is important for comparing the dispersion of datasets. Therefore, this paper aims to generate confidence intervals for ratios of coefficients of variation under the Delta-Birnbaum-Saunders distributions. We have proposed four methods for constructing confidence intervals, namely, the method of variance estimates recovery, the bootstrap confidence interval, the generalized confidence interval based on the variance stabilized transformation, and the generalized confidence interval based on the Wilson score method. The assessment of their performance relies on coverage probabilities and average widths obtained through Monte Carlo simulations. The overall study results reveal that the generalized confidence interval based on the variance stabilized transformation and the generalized confidence interval based on the Wilson score methods provide similar values in both the coverage probabilities and average widths, making them the two most efficient methods. Furthermore, it was found that the method of variance estimates recovery performs well when the shape parameters are small. Finally, all the proposed methods will be applied to wind speed data in Thailand.

1. INTRODUCTION

The Birnbaum-Saunders distribution originated from a study on vibrations in commercial aircraft leading to material fatigue. Fatigue, characterized by structural deterioration under fluctuating stress, prompted Birnbaum and Saunders [1] to introduce the fatigue life distribution (commonly referred to as the Birnbaum-Saunders distribution). This distribution is utilized to model the failure time of materials and equipment, where the failure results from the initiation and progression of a predominant fracture. Moreover, the Birnbaum-Saunders distribution is very effective for fitting data that is all positive. Despite its origins in materials science, the Birnbaum-Saunders distribution has recently been applied to various other fields, including the medical sciences, business, finance, industry, and environment [2–5]. In some situations, various random variables, assumed to be continuous and nonnegative, are often characterized using probability distributions. The probability density functions associated with these variables tend to exhibit asymmetrical and positive skewness, making neither the normal distribution nor symmetrical distributions suitable for their description. To address this, the Birnbaum-Saunders distribution, a positively skewed distribution, has gained significant attention as an appropriate model for representing such random variables. Additionally, several researchers have developed and extended the Birnbaum-Saunders distribution to enhance its versatility for various applications. For example, Cordeiro et al. [6]

*Corresponding author, e-mail: suparat.n@sci.kmutnb.ac.th

introduced the odd log-logistic Birnbaum-Saunders-Poisson distribution, which is an extended fatigue lifetime model. They developed a regression model based on the logarithm of this distribution. Martínez-Flórez et al. [7] presented the flexible Birnbaum-Saunders distribution, a bimodal extension of the Birnbaum-Saunders model that includes an extra parameter. They also studied the skew Birnbaum-Saunders model. Benkhelifa [8] proposed the Weibull-Birnbaum-Saunders distribution, which is a mixture of the Weibull and Birnbaum-Saunders distributions. This distribution extends the Birnbaum-Saunders distribution and provides significant flexibility in practical data modeling.

In practical situations, there are scenarios where we encounter data that is distorted and has a relatively high proportion of zero values. Examples include rainfall data, wind speed data, medical data, fisheries survey data, and many others. This renders the standard Birnbaum-Saunders distribution unsuitable for application. Therefore, the Delta-Birnbaum-Saunders distribution is utilized, which is a mixture distribution combining a binomial distribution and the Birnbaum-Saunders distribution. The concept of the Delta-Birnbaum-Saunders distribution originated from Aitchison [9] The Delta-Birnbaum-Saunders distribution represents an extended version of the Birnbaum-Saunders distribution. Suppose that random variable H follows the Delta-Birnbaum-Saunders distribution with parameters λ, α, and β. Therefore, based on the concept proposed by De la Mare [10], the probability density function (PDF) for the Delta- Birnbaum-Saunders population is expressed as

$$f\left(h;\lambda,\alpha,\beta\right)=\lambda \mathrm{I}_0\left[h\right]+\left(1-\lambda\right)\frac{1}{2\alpha\beta\sqrt{2\pi}}\left[\left(\frac{\beta}{h}\right)^{1/2}+\left(\frac{\beta}{h}\right)^{3/2}\right]\exp\left[-\frac{1}{2\alpha^2}\left(\frac{h}{\beta}+\frac{\beta}{h}-2\right)\right]\mathrm{I}_{(0,\infty)}\left[h\right],$$

where I is an indicator function, in which $\mathrm{I}_0\left[h\right]$ takes the value 1 when $h=0$ and 0 otherwise, and $\mathrm{I}_{(0,\infty)}\left[h\right]$ takes the value 0 when $h=0$ and 1 when $h>0$. The graph of the PDF of the Delta-Birnbaum-Saunders distribution is shown in Figure 1.

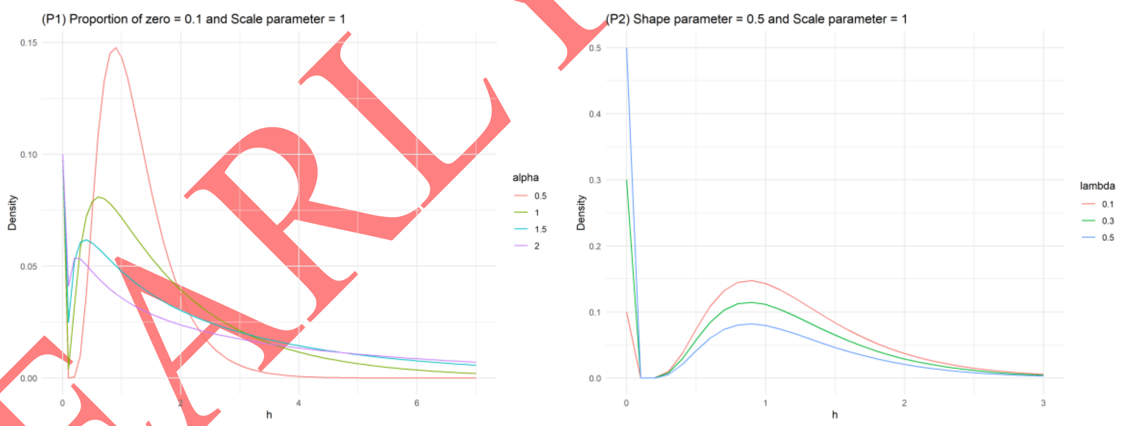

Figure 1. *(P1) The graph of the PDF of the Delta-Birnbaum-Saunders distribution with different shape parameters (alpha); (P2) The graph of the PDF of the Delta-Birnbaum-Saunders distribution with different proportions of zero (lambda)*

The first term denotes the discrete probability mass at the origin, while the subsequent term signifies the probability density. Since the Delta-Birnbaum-Saunders distribution is a new probability distribution, no researchers have utilized it before. However, the concepts introduced by Aitchison [9] and De la Mare [10], involving the use of delta distributions, have been applied to other distributions such as lognormal distributions [11,12], gamma distributions [13,14], and two-parameter exponential distributions [15].

The coefficient of variation (CV) is a widely used statistical measure that expresses the relative variability of a dataset about its mean. It is calculated as the ratio of the standard deviation to the mean, often presented as a percentage. The CV is particularly valuable for comparing the dispersion of datasets, allowing for meaningful comparisons even when the datasets have different measurement units. Researchers commonly utilize the coefficient of variation to analyze and compare variability in diverse fields such as meteorology, medical science, economics, and agriculture [16–19]. Its popularity arises from its ability to provide insights into the proportional variability of data concerning the mean, making it a valuable tool for assessing and comparing datasets with differing scales. Furthermore, in statistical inference, considering various approaches to establish confidence intervals for the ratio of coefficients of variation between populations has attracted the interest of many researchers. For example, Buntao and Niwitpong [20] introduced the generalized variable approach (GPA) and the method of variance estimates recovery (MOVER) to construct confidence intervals in the delta-lognormal distribution. Then, Sangnawakij et al. [21] presented confidence intervals in gamma distributions using the MOVER method, along with the Score and Wald interval methods. In the following year, they utilized two-parameter exponential distributions to construct confidence intervals and compare two methods: the MOVER and the generalized confidence interval (GCI) [22]. Next, Nam and Kwon [19] employed Wald-type, Fieller-type, log methods, and MOVER to construct confidence intervals in the lognormal distribution. Hasan and Krishnamoorthy [23] suggested confidence intervals for two lognormal distributions using the MOVER method and a fiducial approach for lognormal distributions. After that, Puggard et al. [24] studied the GCI with the biased-corrected percentile bootstrap method and the biased-corrected and accelerated method for constructing confidence intervals for Birnbaum-Saunders distributions. Yosboonruang and Niwitpong [25] introduced approaches that incorporate the GCI concept and the MOVER. These methods are utilized in conjunction with three techniques: variance stabilizing transformation, the Wilson score method, and the Jeffreys method. Subsequently, Yosboonruang et al. [26] created confidence intervals for lognormal distributions with excess zeros by proposing the fiducial GCI, Bayesian methods relying on left-invariant Jeffreys, Jeffreys rule, and uniform priors, as well as the Wald and Fieller log-likelihood methods. Meanwhile, Thangjai et al. [27] developed a Bayesian method for creating confidence intervals for two normal distributions. The effectiveness of this Bayesian approach is assessed by comparing it to two conventional methods: the GCI and the MOVER. Chankham et al. [28] presented techniques including the fiducial confidence interval, the fiducial-highest posterior density confidence interval, and the MOVER. These methods were compared with the GCI methods for constructing confidence intervals for the inverse Gaussian distribution. Recently, La-ongkaew et al. [29] constructed confidence intervals for Weibull distributions using various methods, including the GCI, the MOVER, and Bayesian methods based on gamma and uniform priors.

A comprehensive review of research studies reveals that the coefficient of variation is an important parameter in analysis and that no research has yet investigated the Delta-Birnbaum-Saunders distribution. Additionally, using statistical estimation to create confidence intervals for the ratio of coefficients of variation is an important method that can effectively analyze data. However, there are currently no researchers examining the ratio of coefficients of variation in the Delta-Birnbaum-Saunders distributions. Consequently, this research investigated confidence intervals for ratios of the coefficients of variation for the Delta-Birnbaum-Saunders distributions. The methods employed for comparison include MOVER, Bootstrap confidence interval, GCI based on the variance-stabilized transformation, and GCI based on the Wilson score method. The efficiency of these methods is compared using coverage probabilities and average widths. Finally, all these methods will be applied to wind speed data from Thailand.

2. MATERIAL METHOD

Suppose that $H_{ij} = \left(H_{i1}, H_{i2}, \dots, H_{im_i}\right)$; $i = 1, 2$ and $j = 1, 2, \dots, m_i$ be a non-negative random sample from the Delta- Birnbaum-Saunders distribution with the proportion of zero values λ_i, shape parameter α_i, and scale parameter β_i, denoted as $H_{ij} \sim DBS\left(\alpha_i, \beta_i, \lambda_i\right)$. The instances with zero observed values $m_{i(0)}$ follow the binomial distribution denoted as $m_{i(0)} \sim Binomial\left(m_i, \lambda_i\right)$, while the positive observed values $m_{i(1)}$ adhere to the Birnbaum-Saunders distribution, with m_i being the sum of $m_{i(0)}$ and $m_{i(1)}$. The cumulative distribution function (CDF) of H_{ij} can be expressed as

$$G\left(h_{ij};\lambda_i,\alpha_i,\beta_i\right)=\begin{cases}\lambda_i & ;h_{ij}=0\\ \lambda_i+(1-\lambda_i)F\left(h_{ij};\alpha_i,\beta_i\right) & ;h_{ij}>0\end{cases},$$

where $F\left(h_{ij};\alpha_i,\beta_i\right)$ is the Birnbaum-Saunders cumulative distribution function. The graph of the CDF of the Delta-Birnbaum-Saunders distribution is shown in Figure 2.

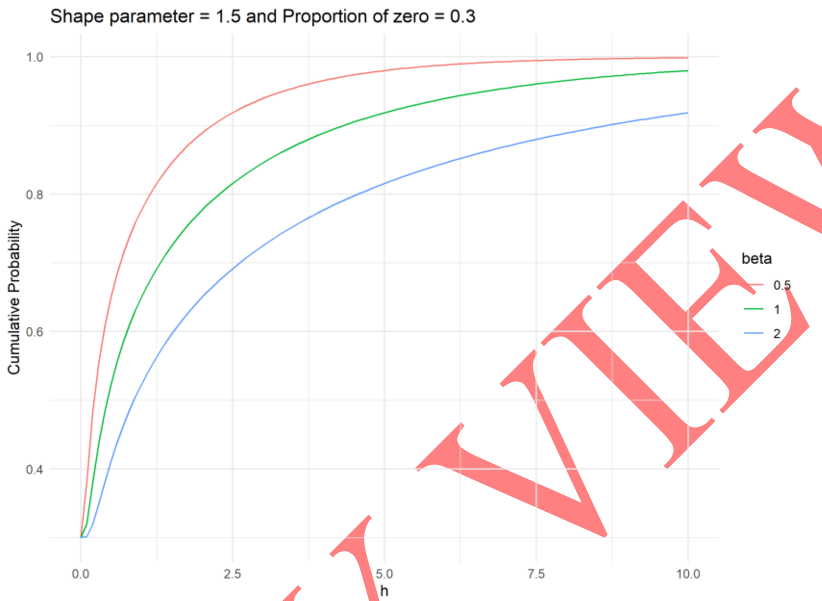

Figure 2. *The graph of the CDF of the Delta-Birnbaum-Saunders distribution with different* scale parameters (bata)

According to the concept proposed by Aitchison [9], the expected value of H_{ij} is given by

$$E\left(H_{ij}\right)=(1-\lambda_i)\beta_i\left(1+\frac{\alpha_i^2}{2}\right), \tag{1}$$

and the variance of H_{ij} is given by

$$V\left(H_{ij}\right)=(1-\lambda_i)(\alpha_i\beta_i)^2\left(1+\frac{5\alpha_i^2}{4}\right)+\lambda_i(1-\lambda_i)\beta_i^2\left(1+\frac{\alpha_i^2}{2}\right)^2. \tag{2}$$

From Equations (1) and (2), the coefficient of variation of H_{ij} is defined as

$$CV\left(H_{ij}\right)=\kappa_i=\frac{\sqrt{Var\left(H_{ij}\right)}}{E\left(H_{ij}\right)}=\frac{1}{2+\alpha_i^2}\sqrt{\frac{\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2}{1-\lambda_i}}. \tag{3}$$

Therefore, the ratios of the coefficient of variation for the Delta- Birnbaum-Saunders distributions can be written as

$$\delta=\frac{\kappa_1}{\kappa_2}=\frac{1}{2+\alpha_1^2}\sqrt{\frac{\alpha_1^2\left(4+5\alpha_1^2\right)+\lambda_1\left(2+\alpha_1^2\right)^2}{1-\lambda_1}}\Bigg/\frac{1}{2+\alpha_2^2}\sqrt{\frac{\alpha_2^2\left(4+5\alpha_2^2\right)+\lambda_2\left(2+\alpha_2^2\right)^2}{1-\lambda_2}}.$$

The method for constructing confidence intervals for the ratios of the coefficient of variation for the Delta-Birnbaum-Saunders distributions will be presented in the subsection.

2.1. Method of Variance Estimates Recovery

Zou and Donner [30] introduced the concept of the method of variance estimates recovery (MOVER) in their work, and further details were provided by Zou et al. [31]. Additionally, Donner and Zou [32] put forward a confidence interval method for a parameter ratio, denoted as $\frac{\kappa_1}{\kappa_2}$.

Let $G_{ij}=\left(G_{i1},G_{i2},...,G_{im_{i(1)}}\right)$; $i=1,2$ and $j=1,2,...,m_{i(1)}$ be positive random variables from the Birnbaum-Saunders distributions. According to Ng et al. [33], the modified moment estimators (MMEs) for α_i are

$$\hat{\alpha}_i=\left\{2\left[\left(g_i^{\#}\sum_{j=1}^{m_{i(1)}}\frac{g_{ij}^{-1}}{m_{i(1)}}\right)^{1/2}-1\right]\right\}^{1/2},$$

where $g_i^{\#}=\sum_{j=1}^{m_{i(1)}}\frac{g_{ij}}{m_{i(1)}}$. Hence, from Equation (3), the estimates for κ_i can be written as

$$\hat{\kappa}_i=\frac{1}{2+\hat{\alpha}_i^2}\sqrt{\frac{\hat{\alpha}_i^2\left(4+5\hat{\alpha}_i^2\right)+\hat{\lambda}_i\left(2+\hat{\alpha}_i^2\right)^2}{1-\hat{\lambda}_i}}, \tag{4}$$

where the maximum likelihood estimates of λ_i are $\hat{\lambda}_i=m_{i(0)}/m_i$. For the asymptotic variance of the estimator κ_i, we applied the delta method to obtain an asymptotically normal distribution based on the Taylor series, as follows:

$$g\left(\hat{\alpha}_i,\hat{\lambda}_i\right)=g\left(\alpha_i,\lambda_i\right)+\frac{\partial g\left(\alpha_i,\lambda_i\right)}{\partial\alpha_i}\left(\hat{\alpha}_i-\alpha_i\right)+\frac{\partial g\left(\alpha_i,\lambda_i\right)}{\partial\lambda_i}\left(\hat{\lambda}_i-\lambda_i\right)+Remainder,$$

where $g\left(\alpha_i,\lambda_i\right)=\frac{1}{2+\alpha_i^2}\sqrt{\frac{\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2}{1-\lambda_i}}$. In the process of computing the partial derivatives, the results are as follows:

$$\frac{\partial g\left(\alpha_i,\lambda_i\right)}{\partial\alpha_i}\approx\frac{8\alpha_i\left(1+2\alpha_i^2\right)}{\left(2+\alpha_i^2\right)^2\sqrt{\left(1-\lambda_i\right)\left[\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2\right]}}$$

and

$$\frac{\partial g(\alpha_i,\lambda_i)}{\partial \lambda_i} \approx \frac{2+\alpha_i^2\left(4+3\alpha_i^2\right)}{\left(2+\alpha_i^2\right)\sqrt{\left(1-\lambda_i\right)^3\left[\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2\right]}}.$$

From the Taylor series, we can express that

$$\begin{aligned} g\left(\hat{\alpha}_i,\hat{\lambda}_i\right) \approx & \frac{1}{2+\alpha_i^2}\sqrt{\frac{\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2}{1-\lambda_i}} \\ & + \frac{8\alpha_i\left(1+2\alpha_i^2\right)}{\left(2+\alpha_i^2\right)^2\sqrt{\left(1-\lambda_i\right)\left[\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2\right]}}\left(\hat{\alpha}_i-\alpha_i\right) \\ & + \frac{2+\alpha_i^2\left(4+3\alpha_i^2\right)}{\left(2+\alpha_i^2\right)\sqrt{\left(1-\lambda_i\right)^3\left[\alpha_i^2\left(4+5\alpha_i^2\right)+\lambda_i\left(2+\alpha_i^2\right)^2\right]}}\left(\hat{\lambda}_i-\lambda_i\right), \text{ as } m_i\to\infty. \end{aligned}$$

It is well known that the asymptotic distribution of $\hat{\alpha}_i$ and $\hat{\lambda}_i$ is represented as $\hat{\alpha}_i \sim N\left(\alpha_i, \frac{\alpha_i^2}{2m_{i(1)}}\right)$ and $\hat{\lambda}_i \sim N\left(\lambda_i, \frac{\lambda_i(1-\lambda_i)}{m_i}\right)$. Thus, the asymptotic variance of $\hat{\kappa}_i$, is defined as

$$V\left(\hat{\kappa}_i\right)=V\left[g\left(\hat{\alpha}_i,\hat{\lambda}_i\right)\right]\approx\frac{1}{O_i\left(1-\lambda_i\right)\left[\alpha_i^2\left(4+5\alpha_i^2\right)+O_i\lambda_i\right]}\left\{\frac{32\alpha_i^4\left(1+2\alpha_i^2\right)^2}{m_{i(1)}O_i}+\frac{\lambda_i\left[2+\alpha_i^2\left(4+3\alpha_i^2\right)\right]^2}{m_i\left(1-\lambda_i\right)}\right\},$$

where $O_i=\left(2+\alpha_i^2\right)^2$; $i=1,2$. Due to the unknown values of parameters α_i and λ_i in this context, we will estimate the parametric function κ_i with the sample. Consequentially, we use the plug-in estimators of $V\left(\hat{\kappa}_i\right)$, which can be expressed as

$$\hat{V}\left(\hat{\kappa}_i\right)\approx\frac{1}{O_i^*\left(1-\hat{\lambda}_i\right)\left[\hat{\alpha}_i^2\left(4+5\hat{\alpha}_i^2\right)+O_i^*\hat{\lambda}_i\right]}\left\{\frac{32\hat{\alpha}_i^4\left(1+2\hat{\alpha}_i^2\right)^2}{m_{i(1)}O_i^*}+\frac{\hat{\lambda}_i\left[2+\hat{\alpha}_i^2\left(4+3\hat{\alpha}_i^2\right)\right]^2}{m_i\left(1-\hat{\lambda}_i\right)}\right\}, \tag{5}$$

where $O_i^*=\left(2+\hat{\alpha}_i^2\right)^2$; $i=1,2$. Suppose that l_i and u_i are the lower and upper limits of the interval for $\hat{\kappa}_i$, respectively. Then, the $(1-\upsilon)100\%$ asymptotic confidence interval for $\hat{\kappa}_i$ can be expressed as

$$l_i=\hat{\kappa}_i-z_{1-\upsilon/2}\sqrt{\hat{V}\left(\hat{\kappa}_i\right)}$$

and

$$u_i=\hat{\kappa}_i+z_{1-\upsilon/2}\sqrt{\hat{V}\left(\hat{\kappa}_i\right)}.$$

Therefore, the $(1-\upsilon)100\%$ confidence interval for $\frac{\kappa_1}{\kappa_2}$ using the MOVER method is obtained as

$$CI_{MOVER} = [L_{MOVER}, U_{MOVER}], \tag{6}$$

where $L_{MOVER} = \frac{\hat{\kappa}_1\hat{\kappa}_2 - \sqrt{(\hat{\kappa}_1\hat{\kappa}_2)^2 - l_1u_2(2\hat{\kappa}_1 - l_1)(2\hat{\kappa}_2 - u_2)}}{u_2(2\hat{\kappa}_2 - u_2)}$

and $U_{MOVER} = \frac{\hat{\kappa}_1\hat{\kappa}_2 + \sqrt{(\hat{\kappa}_1\hat{\kappa}_2)^2 - u_1l_2(2\hat{\kappa}_1 - u_1)(2\hat{\kappa}_2 - l_2)}}{l_2(2\hat{\kappa}_2 - l_2)}$.

Algorithm 1: For the MOVER

Step I: Generate dataset $h_{i1}, h_{i2}, \ldots, h_{im_i}$ from the Delta-Birnbaum-Saunders distribution.
Step II: Compute $\hat{\alpha}_i$ and $\hat{\lambda}_i$.
Step III: Compute $\hat{\kappa}_i$ by employing Equation (4).
Step IV: Compute $\hat{V}(\hat{\kappa}_i)$ by employing Equation (5).
Step V: Compute L_{MOVER} and U_{MOVER} by employing Equation (6).
Step VI: Repeat steps I. – V. 3,000 times.

2.2. Bootstrap Confidence Interval

The bootstrap method, introduced by Efron [34], involves repeatedly resampling to estimate the sampling distribution of a statistic. Let $\hat{\alpha}_i^{/}$ and $\hat{\lambda}_i^{/}$ be observed values of $\hat{\alpha}_i$ and $\hat{\lambda}_i$ based on bootstrap samples. Assume we have P bootstrap samples that are available. The estimator of the bias is defined as $P(\hat{\alpha}_i, \alpha_i) = E(\hat{\alpha}_i) - \alpha_i$. Suppose that $\hat{\alpha}_{il}^{/}$ is the sequence of the bootstrap maximum likelihood estimates of α_i; $i = 1, 2$ and $l = 1, 2, \ldots, P$, then the bootstrap expectation $E(\hat{\alpha}_i)$ can be approximated using the mean $\hat{\alpha}_{i(.)}^{/} = \frac{1}{P}\sum_{l=1}^{P}\hat{\alpha}_{il}^{/}$. The bootstrap bias estimate base on P replications of $\hat{\alpha}_i$ is $\hat{P}(\hat{\alpha}_i, \alpha_i) = \hat{\alpha}_{i(.)}^{/} - \hat{\alpha}_i$. Following this, the constant-bias-correcting estimates, as Mackinnon and Smith [35] define them, are utilized to generate the bias-corrected estimator denoted as

$$\alpha_i^* = \hat{\alpha}_i^{/} - 2\hat{P}(\hat{\alpha}_i, \alpha_i). \tag{7}$$

Subsequently, the bootstrap estimator of κ_i, can be expressed as

$$\hat{\kappa}_i^{(Boot)} = \frac{1}{2 + (\alpha_i^*)^2}\sqrt{\frac{(\alpha_i^*)^2\left(4 + 5(\alpha_i^*)^2\right) + \hat{\lambda}_i^*\left(2 + (\alpha_i^*)^2\right)^2}{1 - \hat{\lambda}_i^*}}, \tag{8}$$

where $\hat{\lambda}_i^* \sim Beta\left(m_i\hat{\lambda}_i^{/} + \frac{1}{2},\ m_i\left(1 - \hat{\lambda}_i^{/}\right) + \frac{1}{2}\right)$, was presented by Brown et al. [36].

Furthermore, we can establish the coefficient of variation ratio $\left(\frac{\kappa_1}{\kappa_2}\right)$ from Equation (9), resulting in

$$\hat{\delta}^{(Boot)} = \frac{\hat{\kappa}_1^{(Boot)}}{\hat{\kappa}_2^{(Boot)}} = \frac{\frac{1}{2+\left(\alpha_1^*\right)^2}\sqrt{\frac{\left(\alpha_1^*\right)^2\left(4+5\left(\alpha_1^*\right)^2\right)+\hat{\lambda}_1^*\left(2+\left(\alpha_1^*\right)^2\right)^2}{1-\hat{\lambda}_1^*}}}{\frac{1}{2+\left(\alpha_2^*\right)^2}\sqrt{\frac{\left(\alpha_2^*\right)^2\left(4+5\left(\alpha_2^*\right)^2\right)+\hat{\lambda}_2^*\left(2+\left(\alpha_2^*\right)^2\right)^2}{1-\hat{\lambda}_2^*}}}. \tag{9}$$

Hence, the $(1-\upsilon)100\%$ confidence interval for $\frac{\kappa_1}{\kappa_2}$ using the BCI method can be written as

$$\left[L_{BCI}, U_{BCI}\right] = \left[\hat{\delta}^{(Boot)}(\upsilon/2), \hat{\delta}^{(Boot)}(1-\upsilon/2)\right], \tag{10}$$

where $\hat{\delta}^{(Boot)}(\upsilon)$ as the $100\upsilon th$ percentile of $\hat{\delta}^{(Boot)}$.

Algorithm 2: For the BCI

Step I: Generate dataset $h_{i1}, h_{i2}, ..., h_{im_i}$ from the Delta-Birnbaum-Saunders distribution.

Step II: At the *k*th step

a) Compute $h_{i1}^*, h_{i2}^*, ..., h_{im_i}^*$ with replacement from $h_{i1}, h_{i2}, ..., h_{im_i}$.

b) Compute $\hat{\alpha}_i'$ and $\hat{P}(\hat{\alpha}_i, \alpha_i)$.

c) Compute α_i^* by employing Equation (7), and compute $\hat{\lambda}_i^*$.

d) Compute $\hat{\kappa}_i^{(Boot)}$ by employing Equation (8).

e) Compute $\hat{\delta}^{(Boot)}$ by employing Equation (9).

Step III: Repeat step II. 500 times.

Step IV: Compute L_{BCI} and U_{BCI} by employing Equation (10).

Step V: Repeat steps I. – IV. 3,000 times.

2.3. Generalized Confidence Interval

Weerahandi [37] introduced the generalized confidence interval (GCI), which relies on the generalized pivotal quantity (GPQ) concept. This is a general characteristic of the usual pivotal quantity. The GPQ features an unknown parameter that is distribution-free and an observed pivotal independent of the nuisance parameter. Let $Q_{ij} = \left(Q_{i1}, Q_{i2}, ..., Q_{im_{i(1)}}\right)$; $i = 1, 2$ and $j = 1, 2, ..., m_{i(1)}$ be positive random variables from the Birnbaum-Saunders distributions. When generating confidence intervals for parameter δ using GCI, consideration is given to the GPQs for parameters β_i and α_i recommended by Sun [38] and Wang [39], respectively. According to Sun [38], it is suggested that the GPQ is calculated using

$$R_{\beta_i}\left(q_{ij}; \Lambda_i\right) = \begin{cases} \max\left(\beta_{i1}, \beta_{i2}\right) & ; \Lambda_i \le 0 \\ \min\left(\beta_{i1}, \beta_{i2}\right) & ; \Lambda_i > 0 \end{cases}, \tag{11}$$

where Λ_i follows the *t*-distribution with $m_{i(1)}-1$ degrees of freedom, which is represented by $\Lambda_i \sim t\left(m_{i(1)}-1\right)$, and this distribution is independent of the unknown parameters α_i and β_i. From Equation (11), we get algebraic rearrangement and the two solutions for β_i denoted as β_{i1} and β_{i2} can be derived by solving the following equation:

$$\left[\left(m_{i(1)}-1\right)A_i^2-\frac{1}{m_{i(1)}}B_i\Lambda_i^2\right]\beta_i^2-2\left[\left(m_{i(1)}-1\right)A_iC_i-\left(1-A_iC_i\right)\Lambda_i^2\right]\beta_i+\left(m_{i(1)}-1\right)C_i^2-\frac{1}{m_{i(1)}}D_i\Lambda_i^2=0,$$

where $A_i=\frac{1}{m_{i(1)}}\sum_{j=1}^{m_{i(1)}}\frac{1}{\sqrt{Q_{ij}}}$, $B_i=\sum_{j=1}^{m_{i(1)}}\left(\frac{1}{\sqrt{Q_{ij}}}-A_i\right)^2$, $C_i=\frac{1}{m_{i(1)}}\sum_{j=1}^{m_{i(1)}}\sqrt{Q_{ij}}$, and $D_i=\sum_{j=1}^{m_{i(1)}}\left(\sqrt{Q_{ij}}-C_i\right)^2$.

Meanwhile, the GPQ of the α_i can be calculated using

$$R_{\alpha_i}\left(q_{ij};\Upsilon_i,\Lambda_i\right)=\sqrt{\frac{E_{i1}+E_{i2}R_{\beta_i}^2-2m_{i(1)}R_{\beta_i}}{R_{\beta_i}\Upsilon_i}}, \tag{12}$$

where $E_{i1}=\sum_{j=1}^{m_{i(1)}}Q_{ij}$, $E_{i2}=\sum_{j=1}^{m_{i(1)}}\frac{1}{Q_{ij}}$, and Υ_i follows the Chi-squared distribution with $m_{i(1)}$ degrees of freedom, which is represented by $\Upsilon_i \sim \chi^2_{m_{i(1)}}$.

Subsequently, the GPQs for λ_i will be employed, focusing on two concepts: the variance stabilized transformation (VST) and the Wilson score method (WS). The details will be explained in the following subsections.

2.3.1. GCI based on the VST (G.VST)

DasGupta [40] employed the delta method to construct the VST. Subsequently, Wu and Hsieh [41] explained the application of the GPQs based on the VST to establish a confidence interval. Therefore, the GPQ of λ_i can be expressed as

$$R_{\lambda_i}^{(VST)}=\sin^2\left[\arcsin\sqrt{\hat{\lambda}_i}-\frac{W_i}{2\sqrt{m_i}}\right], \tag{13}$$

where $W_i=2\sqrt{m_i}\left(\arcsin\sqrt{\hat{\lambda}_i}-\arcsin\sqrt{\lambda_i}\right)\sim N(0,1); i=1,2$. Therefore, we can use Equations (12) and (13) to calculate the GPQs for κ_i, and the result is

$$R_{\kappa_i}^{(VST)}=\frac{1}{2+R_{\alpha_i}^2}\sqrt{\frac{R_{\alpha_i}^2\left(4+5R_{\alpha_i}^2\right)+R_{\lambda_i}^{(VST)}\left(2+R_{\alpha_i}^2\right)^2}{1-R_{\lambda_i}^{(VST)}}}. \tag{14}$$

Furthermore, Equation (14) will be used to calculate the ratio of the coefficient of variation, which is

$$R_{\delta}^{(VST)}=\frac{R_{\kappa_1}^{(VST)}}{R_{\kappa_2}^{(VST)}}=\frac{\frac{1}{2+R_{\alpha_1}^2}\sqrt{\frac{R_{\alpha_1}^2\left(4+5R_{\alpha_1}^2\right)+R_{\lambda_1}^{(VST)}\left(2+R_{\alpha_1}^2\right)^2}{1-R_{\lambda_1}^{(VST)}}}}{\frac{1}{2+R_{\alpha_2}^2}\sqrt{\frac{R_{\alpha_2}^2\left(4+5R_{\alpha_2}^2\right)+R_{\lambda_2}^{(VST)}\left(2+R_{\alpha_2}^2\right)^2}{1-R_{\lambda_2}^{(VST)}}}}. \tag{15}$$

Consequently, the $(1-\upsilon)100\%$ confidence interval for $\frac{\kappa_1}{\kappa_2}$ using the G.VST method can be written as

$$[L_{G.VST}, U_{G.VST}]=\left[R_{\delta}^{(VST)}(\upsilon/2), R_{\delta}^{(VST)}(1-\upsilon/2)\right], \tag{16}$$

where $R_{\delta}^{(VST)}(\upsilon)$ as the $100\upsilon th$ percentile of $R_{\delta}^{(VST)}$.

2.3.2. GCI based on WS (G.WS)

Li et al. [42] presented the GPQs for parameter λ_i in the binomial distribution by employing the score interval, as outlined by Wilson [43], with a calculated value of

$$R_{\lambda_i}^{(WS)}=\frac{m_{i(0)}+\left(\Psi_i^2/2\right)}{m_i+\Psi_i^2}-\frac{\Psi_i}{m_i+\Psi_i^2}\sqrt{m_{i(0)}\left(1-\frac{m_{i(0)}}{m_i}\right)+\frac{\Psi_i^2}{4}}, \tag{17}$$

where $\Psi_i=\frac{m_{i(0)}-m_i\lambda_i}{\sqrt{m_i\lambda_i(1-\lambda_i)}}$ follows a standard normal distribution. From Equations (12) and (17), we can consequently calculate the GPQs for κ_i and $\frac{\kappa_1}{\kappa_2}$ as

$$R_{\kappa_i}^{(WS)}=\frac{1}{2+R_{\alpha_i}^2}\sqrt{\frac{R_{\alpha_i}^2\left(4+5R_{\alpha_i}^2\right)+R_{\lambda_i}^{(WS)}\left(2+R_{\alpha_i}^2\right)^2}{1-R_{\lambda_i}^{(WS)}}} \tag{18}$$

and

$$R_{\delta}^{(WS)}=\frac{R_{\kappa_1}^{(WS)}}{R_{\kappa_2}^{(WS)}}=\frac{\frac{1}{2+R_{\alpha_1}^2}\sqrt{\frac{R_{\alpha_1}^2\left(4+5R_{\alpha_1}^2\right)+R_{\lambda_1}^{(WS)}\left(2+R_{\alpha_1}^2\right)^2}{1-R_{\lambda_1}^{(VST)}}}}{\frac{1}{2+R_{\alpha_2}^2}\sqrt{\frac{R_{\alpha_2}^2\left(4+5R_{\alpha_2}^2\right)+R_{\lambda_2}^{(WS)}\left(2+R_{\alpha_2}^2\right)^2}{1-R_{\lambda_2}^{(WS)}}}}, \tag{19}$$

respectively. Finally, the $(1-\upsilon)100\%$ confidence interval for $\frac{\kappa_1}{\kappa_2}$ using the G.WS method can be created as follows:

$$[L_{G.WS}, U_{G.WS}]=\left[R_{\delta}^{(WS)}(\upsilon/2), R_{\delta}^{(WS)}(1-\upsilon/2)\right], \tag{20}$$

where $R_{\delta}^{(WS)}(\upsilon)$ as the $100\upsilon th$ percentile of $R_{\delta}^{(WS)}$.

Algorithm 3: For the G.VST and G.WS

Step I: Generate dataset $h_{i1}, h_{i2}, ..., h_{im_i}$ from the Delta-Birnbaum-Saunders distribution.

Step II: Compute $A_i, B_i, C_i, D_i, E_{i1}$ and E_{i2}, respectively.

Step III: At the *k*th step

a) Generate $\Lambda_i \sim t\left(m_{i(1)} - 1\right)$, and then compute $R_{\beta_i}\left(q_{ij}; \Lambda_i\right)$ by employing Equation (13).

b) If $R_{\beta_i}\left(q_{ij}; \Lambda_i\right) < 0$, regenerate $\Lambda_i \sim t\left(m_{i(1)} - 1\right)$.

c) Generate $\Upsilon_i \sim \chi^2_{m_{i(1)}}$, and then compute $R_{\alpha_i}\left(q_{ij}; \Upsilon_i, \Lambda_i\right)$ by employing Equation (12).

d) For the *G.VST* method, compute $R^{(VST)}_{\lambda_i}$, $R^{(VST)}_{\kappa_i}$, and $R^{(VST)}_{\delta}$ employing Equations (13), (14), and (15), respectively.

e) For the *G.WS* method, compute $R^{(WS)}_{\lambda_i}$, $R^{(WS)}_{\kappa_i}$, and $R^{(WS)}_{\delta}$ employing Equations (17), (18), and (19), respectively.

Step IV: Repeat step III. 1,000 times.

Step V: Compute $L_{G.VST}$ and $U_{G.VST}$ by employing Equation (16).

Step VI: Compute $L_{G.WS}$ and $U_{G.WS}$ by employing Equation (20).

Step VII: Repeat steps I. – VI. 3,000 times.

3. THE RESEARCH FINDINGS AND DISCUSSION

The simulation study will be conducted using different sample sizes and parameter values to cover a range of possible scenarios. Since β_i is the scale parameter, $\left(\beta_1, \beta_2\right)$ = (1.0, 1.0) was fixed to avoid loss of generality. The shape parameters $\left(\alpha_1, \alpha_2\right)$ are (0.5, 0.5), (1.0, 1.0), and (2.0, 2.0), and the sample sizes $\left(m_1, m_2\right)$ are (30, 30), (30, 50), (30, 100), (50, 50), (50, 100), and (100, 100) as recommended by Puggard et al. [44]. Additionally, we selected proportions of zero values $\left(\lambda_1, \lambda_2\right)$ as (0.1, 0.1), (0.1, 0.3), (0.1, 0.5), (0.3, 0.3), (0.3, 0.5), and (0.5, 0.5). A total of M = 3,000 simulation runs were performed, containing 1,000 iterations for the GCI and 500 iterations for the BCI. We will use coverage probabilities greater than or equal to the nominal confidence level of 0.95, along with the shortest average widths from Monte Carlo simulations in the statistical software R, to compare the performance of the MOVER, BCI, G.VST, and G.WS methods used to construct confidence intervals. Algorithm 4 shows the computational steps to estimate the coverage probability and average width performances of all the methods. Figure 3 depicts a flowchart that illustrates the process of studying the simulated scenario.

The results in Table 1 indicate that the MOVER method provides coverage probabilities greater than the nominal confidence level of 0.95 in almost every case. However, it consistently yields the widest average widths when compared to other methods. For the BCI method, although the average widths are shorter than those of the MOVER method in all cases, it provides coverage probabilities that are mostly below 0.95. Considering the G.VST and G.WS methods, they provide coverage probabilities close to the target. Additionally, both methods yield the shortest average widths and are very close to each other. In Figure 4, the graphs compare the efficiency of the sample sizes versus coverage probabilities and average widths for the shape parameter = (0.5, 0.5). It was found that as the sample size increases, the average width of all methods decrease, resulting in improved efficiency. In Figure 5, the graphs compare the efficiency of the proportion of zeros versus coverage probabilities and average widths for the shape parameter = (0.5, 0.5). When considering equal proportions of zeros, it was found that as the proportion of zeros increases, the average widths of the MOVER and BCI methods increase, while those of the G.VST and G.WS methods decrease. For unequal proportions of zeros, it was found that as the proportion of zeros increases, the average widths for all methods tend to decrease.

The results in Table 2 clearly show that the MOVER method provides coverage probabilities greater than 0.95, except for the case where $(\lambda_1, \lambda_2) = (0.1, 0.1)$ and $(m_1, m_2) = (30, 100)$. On the other hand, the BCI method generally yields coverage probabilities below 0.95. Furthermore, the BCI method provides average widths that are shorter than the MOVER method. As for the G.VST and G.WS methods, both show similar and highly efficient performance, demonstrating consistent coverage probabilities and the shortest average widths. In Figure 6, the graphs compare the efficiency of the sample sizes versus coverage probabilities and average widths for the shape parameter = (1.0, 1.0). It was found that as the sample size increases, the average widths for all methods tend to decrease. In Figure 7, the graphs compare the efficiency of the proportion of zeros versus coverage probabilities and average widths for the shape parameter = (1.0, 1.0). When considering equal proportions of zeros, it was found that as the proportion of zeros increases, the average widths for all methods increase. For unequal proportions of zeros, it was found that as the proportion of zeros increases, the average widths for all methods decrease.

The results in Table 3 show that, considering coverage probabilities, it is evident that both MOVER and BCI methods have probabilities lower than the specified confidence level. Meanwhile, the G.VST and G.WS methods provide probabilities close to the target. Furthermore, in terms of average widths, the G.VST and G.WS methods are shorter than the MOVER and BCI methods. In Figure 8, the graphs compare the efficiency of the sample sizes versus coverage probabilities and average widths for the shape parameter = (2.0, 2.0). It is evident that as the sample size increases, the average widths for all methods tend to decrease. In Figure 9, the graphs compare the efficiency of the proportion of zeros versus coverage probabilities and average widths for the shape parameter = (2.0, 2.0). When considering equal proportions of zeros, it was found that as the proportion of zeros increases, the average widths for all methods increase. For unequal proportions of zeros, it was found that as the proportion of zeros increases, the average widths for all methods tend to decrease.

Algorithm 4: Comparison of coverage probabilities and average widths for all confidence intervals

Step I: For $k = 1$ to M.

Step II: Generate h_{ij} from the Delta-Birnbaum-Saunders distribution by:

i) Generate h_0 where the number of zero values is $m_{i(0)} \sim Binomial(m_i, \lambda_i)$.

The number of positive values will be $m_{i(1)} = m_i - m_{i(0)}$.

ii) Set parameters for the BS distribution (α_i, β_i), then generate random values (h_1) from the BS distribution using the package \`rbs($m_{i(1)}, \alpha_i, \beta_i$)\` in the R program.

iii) Combine h_0 and h_1 together to obtain h_{ij}.

Step III: Compute the unbiased estimates $\hat{\alpha}_i$ and $\hat{\lambda}_i$.

Step IV: Compute the 95% confidence intervals for δ based on the MOVER, BCI, G.VST, and G.WS via Algorithms 1, 2, and 3, respectively.

Step V: Let $C_k = 1$ if δ falls within the intervals of MOVER, BCI, G.VST, or G.WS, else $C_k = 0$.

Step VI: The coverage probability and average width for each method are obtained by $CP = \frac{1}{M}\sum_{k=1}^{M} C_k$ and $AW = \frac{U - L}{M}$, respectively, where U and L are the upper and lower confidence limits, respectively. (end k loop)

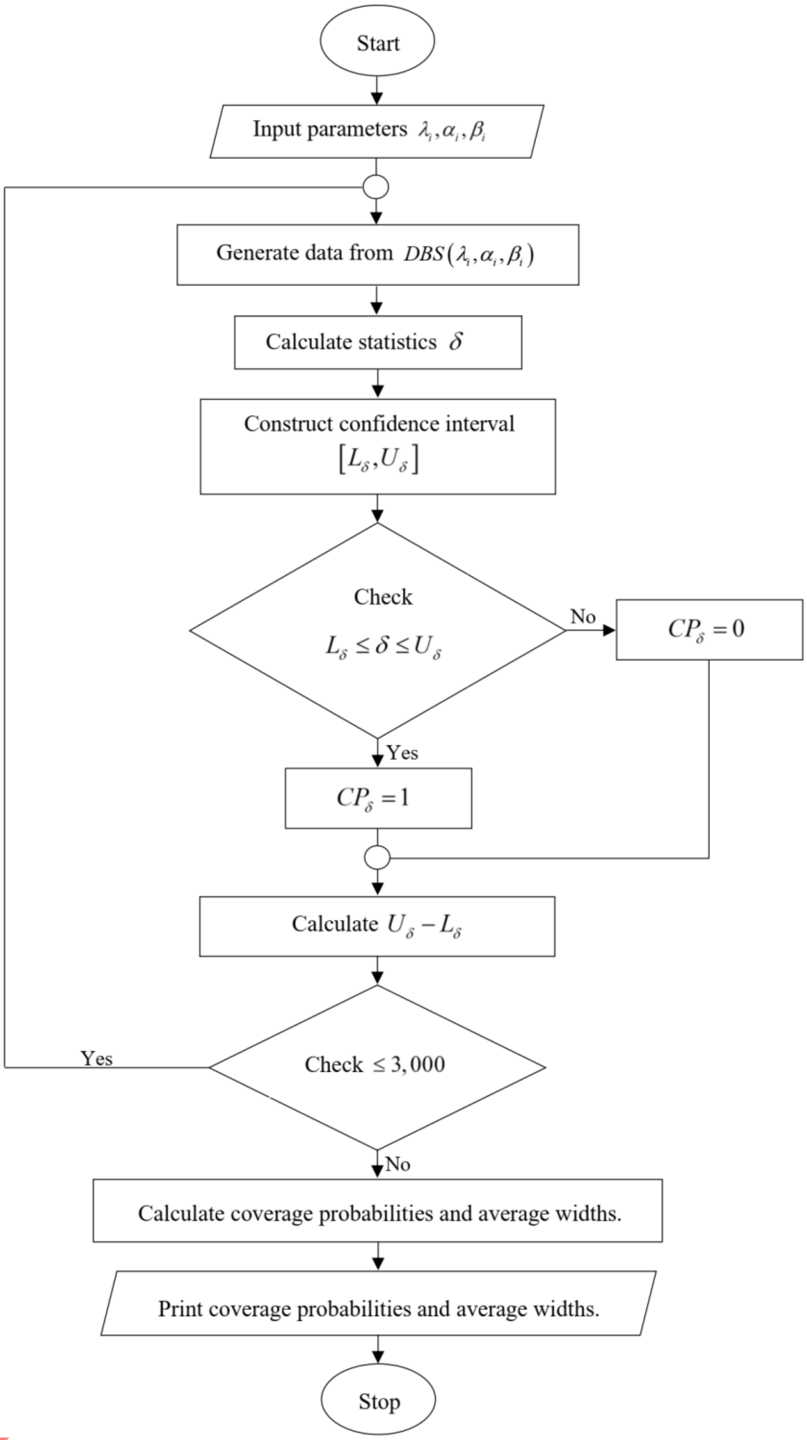

Figure 3. *A flowchart of the simulation study*

Table 1. *The coverage probabilities and average widths for the 95% CIs for* δ*;* $\alpha_1, \alpha_2 = 0.5, 0.5$

m_1, m_2	λ_1, λ_2	Coverage probabilities				Average widths			
		MOVER	BCI	G.VST	G.WS	MOVER	BCI	G.VST	G.WS
30,30	0.1,0.1	**0.9593**	0.9470	**0.9537**	**0.9550**	0.9272	0.8370	**0.5909**	0.5912
	0.1,0.3	**0.9580**	0.9413	0.9466	0.9483	**0.6767**	0.6090	0.3725	0.3726
	0.1,0.5	**0.9690**	0.9467	**0.9510**	0.9493	0.5151	0.4587	**0.2656**	0.2657
	0.3,0.3	**0.9753**	**0.9537**	**0.9554**	**0.9547**	0.9951	0.8867	**0.4564**	0.4567
	0.3,0.5	**0.9750**	0.9490	**0.9540**	**0.9550**	0.7529	0.6637	**0.3226**	0.3228
	0.5,0.5	**0.9797**	0.9407	**0.9563**	**0.9543**	1.0897	0.9695	**0.4397**	0.4398
30,50	0.1,0.1	**0.9583**	**0.9537**	**0.9527**	**0.9503**	0.7844	0.7417	**0.5321**	0.5322
	0.1,0.3	**0.9563**	0.9487	**0.9533**	**0.9533**	0.5670	0.5349	0.3428	**0.3426**
	0.1,0.5	**0.9597**	0.9447	0.9491	0.9480	**0.4268**	0.4001	0.2444	0.2439
	0.3,0.3	**0.9663**	0.9490	0.9470	0.9480	**0.8453**	0.7866	0.4062	0.4063
	0.3,0.5	**0.9640**	0.9470	**0.9522**	**0.9513**	0.6308	0.5848	0.2870	**0.2863**
	0.5,0.5	**0.9717**	0.9453	**0.9514**	0.9470	0.9230	0.8623	**0.3876**	0.3876
30,100	0.1,0.1	0.9440	0.9437	**0.9580**	**0.9553**	0.6836	0.6648	0.4821	**0.4815**
	0.1,0.3	0.9417	0.9417	0.9460	0.9467	0.4912	0.4773	0.3224	0.3225
	0.1,0.5	0.9490	0.9470	0.9487	0.9490	0.3621	0.3508	0.2292	0.2285
	0.3,0.3	**0.9623**	**0.9510**	0.9493	0.9489	0.7405	**0.7108**	0.3681	0.3685
	0.3,0.5	**0.9613**	0.9483	**0.9520**	0.9457	0.5463	0.5228	**0.2618**	0.2618
	0.5,0.5	**0.9640**	0.9463	0.9478	0.9483	**0.8116**	0.7837	0.3492	0.3492
50,50	0.1,0.1	**0.9580**	0.9487	0.9437	0.9480	**0.6865**	0.6368	0.4457	0.4464
	0.1,0.3	**0.9610**	0.9470	0.9480	0.9480	**0.4990**	0.4621	0.2778	0.2780
	0.1,0.5	**0.9653**	0.9480	0.9463	0.9470	**0.3787**	0.3480	0.1962	0.1960
	0.3,0.3	**0.9637**	0.9487	**0.9537**	**0.9503**	0.7366	0.6761	0.3360	**0.3357**
	0.3,0.5	**0.9680**	0.9403	0.9493	0.9490	**0.5551**	0.5056	0.2321	0.2319
	0.5,0.5	**0.9743**	0.9493	**0.9517**	**0.9507**	0.7967	0.7297	**0.3105**	0.3106
50,100	0.1,0.1	**0.9550**	**0.9500**	**0.9527**	**0.9526**	0.5729	0.5492	**0.3867**	0.3871
	0.1,0.3	**0.9573**	0.9481	**0.9503**	0.9489	0.4140	0.3965	**0.2521**	0.2523
	0.1,0.5	**0.9607**	0.9440	**0.9520**	**0.9531**	0.3107	0.2961	0.1790	**0.1788**
	0.3,0.3	**0.9587**	0.9437	0.9490	**0.9520**	0.6133	0.5812	0.2898	**0.2894**
	0.3,0.5	**0.9643**	0.9480	0.9487	**0.9500**	0.4598	0.4340	0.2043	**0.2043**
	0.5,0.5	**0.9723**	**0.9543**	**0.9502**	0.9490	0.6668	0.6318	**0.2667**	0.2669
100,100	0.1,0.1	**0.9603**	0.9490	0.9472	0.9487	**0.4711**	0.4468	0.3091	0.3090
	0.1,0.3	**0.9587**	0.9470	**0.9530**	**0.9513**	0.3447	0.3257	**0.1921**	0.1922
	0.1,0.5	**0.9617**	0.9423	**0.9527**	**0.9550**	0.2591	0.2434	0.1345	**0.1343**
	0.3,0.3	**0.9600**	0.9447	0.9493	0.9493	**0.5029**	0.4711	0.2290	0.2293
	0.3,0.5	**0.9659**	0.9480	0.9450	0.9450	**0.3783**	0.3531	0.1574	0.1572
	0.5,0.5	**0.9686**	0.9487	0.9450	0.9467	**0.5454**	0.5092	0.2063	0.2065

Note: Bold text indicates coverage probabilities greater than or equal to 0.95 and the most appropriate average widths.

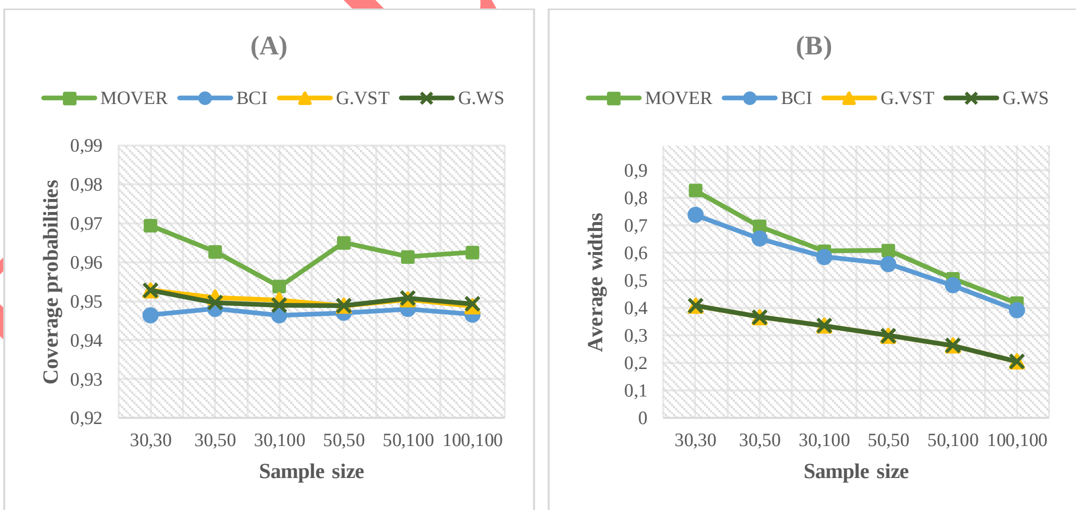

Figure 4*. The graphs compare the efficiency of the sample sizes versus (A) coverage probabilities and (B) average widths for shape parameter = 0.5,0.5*

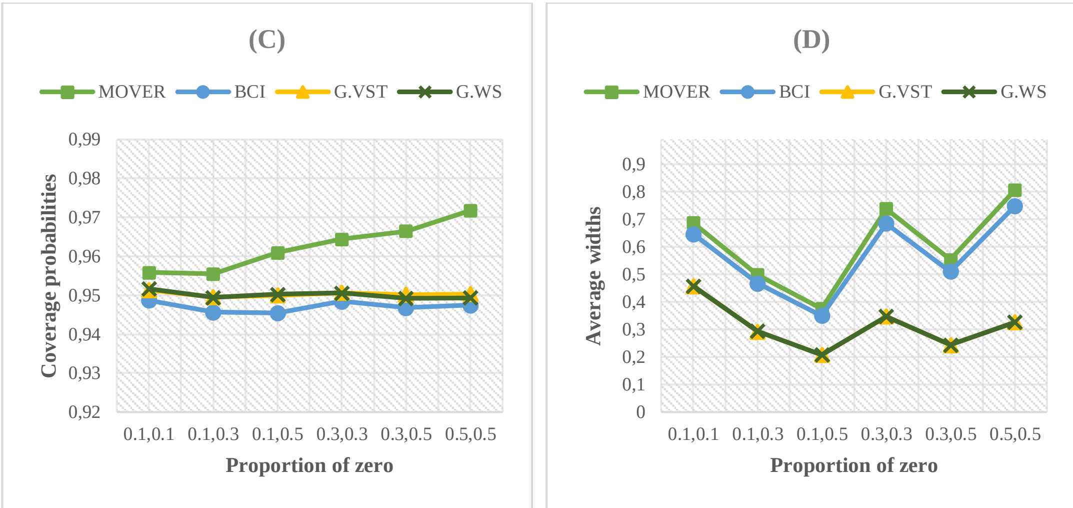

Figure 5. *The graphs compare the efficiency of the proportion of zero versus (C) coverage probabilities and (D) average widths for shape parameter = 0.5,0.5*

Table 2. *The coverage probabilities and average widths for the 95% CIs for* δ; α_1, α_2 *= 1.0,1.0*

m_1, m_2	λ_1, λ_2	Coverage probabilities				Average widths			
		MOVER	BCI	G.VST	G.WS	MOVER	BCI	G.VST	G.WS
30,30	0.1,0.1	0.9443	0.9397	0.9496	0.9483	0.7358	0.6946	0.6381	0.6383
	0.1,0.3	**0.9520**	0.9425	0.9497	**0.9503**	0.6509	0.6021	0.5031	**0.5038**
	0.1,0.5	**0.9597**	0.9396	**0.9500**	**0.9510**	0.5766	0.5149	0.4018	**0.4015**
	0.3,0.3	**0.9563**	0.9403	**0.9512**	**0.9527**	0.8556	0.7961	**0.6152**	**0.6152**
	0.3,0.5	**0.9560**	0.9439	**0.9507**	**0.9500**	0.7546	0.6782	0.4928	**0.4927**
	0.5,0.5	**0.9683**	0.9427	0.9443	0.9433	**1.0270**	0.9374	0.6455	0.6459
30,50	0.1,0.1	0.9478	0.9483	**0.9567**	**0.9563**	0.6284	0.6109	0.5650	**0.5646**
	0.1,0.3	0.9483	0.9416	0.9438	0.9467	0.5451	0.5244	0.4518	0.4513
	0.1,0.5	**0.9587**	**0.9500**	0.9493	**0.9503**	0.4683	0.4430	0.3587	**0.3586**
	0.3,0.3	**0.9500**	0.9427	0.9462	0.9490	**0.7292**	0.7061	0.5498	0.5503
	0.3,0.5	**0.9573**	0.9428	0.9440	0.9437	**0.6219**	0.5916	0.4359	0.4359
	0.5,0.5	**0.9627**	0.9444	**0.9507**	**0.9523**	0.8602	0.8287	0.5744	**0.5739**
30,100	0.1,0.1	0.9367	0.9403	0.9490	0.9490	0.5517	0.5461	0.5075	0.5080
	0.1,0.3	0.9350	0.9357	0.9433	0.9450	0.4667	0.4601	0.4091	0.4088
	0.1,0.5	0.9437	0.9453	**0.9523**	**0.9533**	0.3850	0.3775	0.3227	**0.3226**
	0.3,0.3	0.9390	0.9420	0.9477	0.9487	0.6308	0.6250	0.4924	0.4924
	0.3,0.5	0.9443	0.9393	**0.9508**	**0.9513**	0.5221	0.5137	0.3915	**0.3911**
	0.5,0.5	0.9453	0.9387	**0.9500**	0.9493	0.7512	0.7472	**0.5202**	0.5206
50,50	0.1,0.1	**0.9514**	0.9450	**0.9513**	**0.9513**	0.5495	0.5320	**0.4834**	0.4842
	0.1,0.3	0.9483	0.9410	0.9392	0.9370	0.4836	0.4620	0.3841	0.3842
	0.1,0.5	**0.9558**	0.9450	0.9483	0.9476	**0.4246**	0.3978	0.3062	0.3060
	0.3,0.3	**0.9603**	0.9491	**0.9540**	**0.9520**	0.6358	0.6084	**0.4658**	0.4661
	0.3,0.5	**0.9600**	0.9467	**0.9520**	**0.9523**	0.5539	0.5202	0.3729	**0.3728**
	0.5,0.5	**0.9633**	0.9486	**0.9503**	**0.9503**	0.7547	0.7150	**0.4839**	0.4840
50,100	0.1,0.1	0.9457	0.9433	0.9470	0.9460	0.4634	0.4575	0.4173	0.4173
	0.1,0.3	0.9472	0.9450	0.9480	0.9467	0.3954	0.3885	0.3332	0.3336
	0.1,0.5	**0.9520**	0.9450	0.9473	0.9470	**0.3347**	0.3266	0.2647	0.2647
	0.3,0.3	**0.9503**	0.9470	**0.9533**	**0.9563**	0.5323	0.5246	0.4031	**0.4028**
	0.3,0.5	**0.9540**	0.9477	0.9480	0.9486	**0.4454**	0.4353	0.3195	0.3193
	0.5,0.5	**0.9557**	0.9483	**0.9505**	0.9463	0.6276	0.6171	**0.4199**	0.4200
100,100	0.1,0.1	0.9470	0.9460	**0.9520**	**0.9507**	0.3809	0.3743	**0.3373**	0.3376
	0.1,0.3	**0.9524**	0.9470	**0.9520**	**0.9533**	0.3328	0.3253	0.2678	**0.2677**
	0.1,0.5	**0.9530**	0.9467	0.9492	0.9477	**0.2904**	0.2809	0.2139	0.2139
	0.3,0.3	**0.9593**	**0.9510**	**0.9530**	**0.9576**	0.4380	0.4283	0.3257	**0.3256**
	0.3,0.5	**0.9593**	**0.9557**	**0.9508**	**0.9502**	0.3775	0.3660	0.2597	**0.2596**
	0.5,0.5	**0.9579**	0.9490	0.9473	0.9473	**0.5126**	0.4992	0.3353	0.3355

Note: Bold text indicates coverage probabilities greater than or equal to 0.95 and the most appropriate average widths.

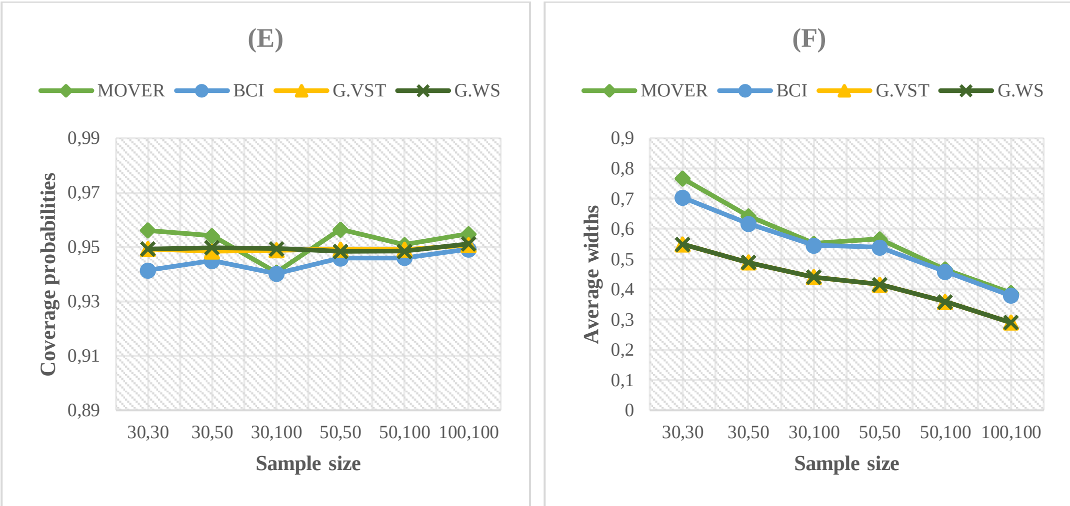

Figure 6*. The graphs compare the efficiency of the sample sizes versus (E) coverage probabilities and (F) average widths for shape parameter = 1.0,1.0*

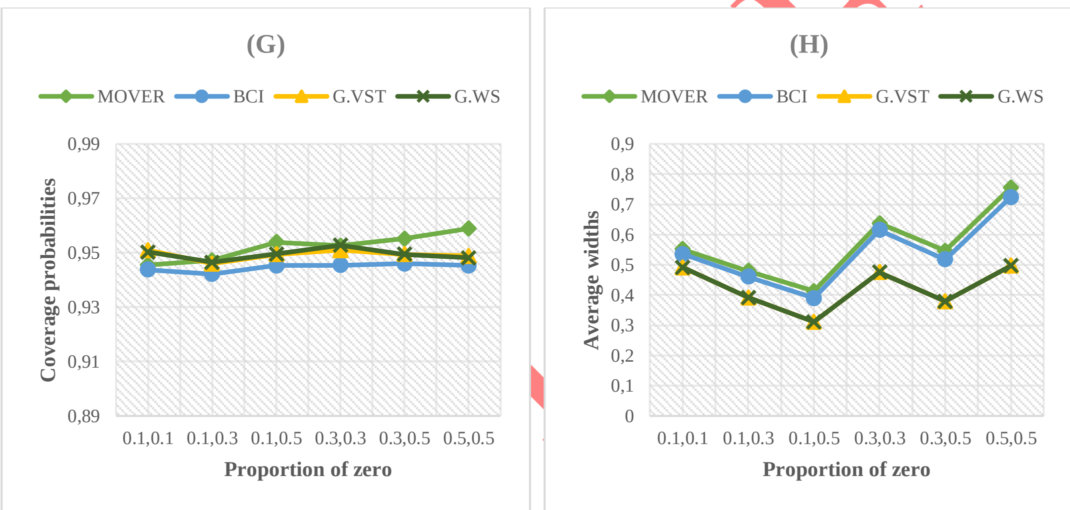

Figure 7*. The graphs compare the efficiency of the proportion of zero versus (G) coverage probabilities and (H) average widths for shape parameter = 1.0,1.0*

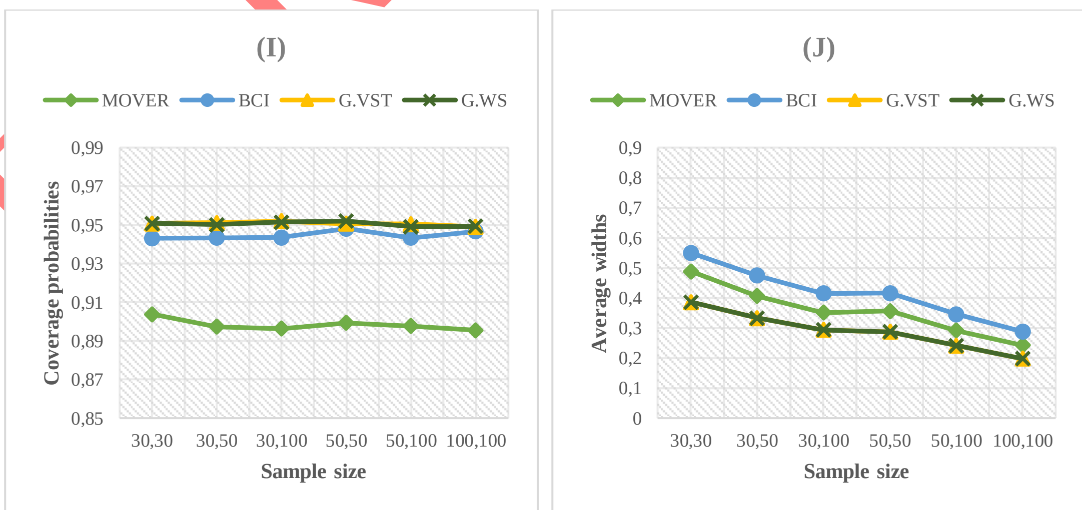

Figure 8*. The graphs compare the efficiency of the sample sizes versus (I) coverage probabilities and (J) average widths for shape parameter = 2.0,2.0*

Table 3. *The coverage probabilities and average widths for the 95% CIs for* δ; α_1, α_2 *= 2.0,2.0*

m_1, m_2	λ_1, λ_2	Coverage probabilities				Average widths			
		MOVER	BCI	G.VST	G.WS	MOVER	BCI	G.VST	G.WS
30,30	0.1,0.1	0.9193	0.9393	0.9467	0.9470	0.4270	0.4582	0.3914	0.3918
	0.1,0.3	0.9150	0.9493	**0.9517**	**0.9507**	0.4116	0.4542	**0.3495**	0.3496
	0.1,0.5	0.8990	0.9410	**0.9510**	**0.9513**	0.3998	0.4405	0.3080	**0.3078**
	0.3,0.3	0.8980	0.9400	0.9493	0.9493	0.5224	0.5984	0.4200	0.4204
	0.3,0.5	0.8977	0.9450	**0.9527**	**0.9520**	0.4984	0.5658	**0.3687**	0.3689
	0.5,0.5	0.8940	0.9440	**0.9537**	**0.9543**	0.6720	0.7831	**0.4809**	**0.4809**
30,50	0.1,0.1	0.9220	0.9383	**0.9517**	**0.9510**	0.3685	0.4011	**0.3423**	0.3424
	0.1,0.3	0.9043	0.9447	0.9467	0.9473	0.3398	0.3824	0.2989	0.2990
	0.1,0.5	0.8873	0.9420	0.9483	0.9487	0.3149	0.3628	0.2579	0.2585
	0.3,0.3	0.8920	0.9410	**0.9523**	**0.9500**	0.4448	0.5251	**0.3647**	0.3648
	0.3,0.5	0.8883	0.9480	**0.9507**	0.9490	0.4075	0.4847	**0.3162**	0.3161
	0.5,0.5	0.8903	0.9460	**0.9573**	**0.9550**	0.5631	0.6882	**0.4153**	0.4156
30,100	0.1,0.1	0.9167	0.9387	**0.9503**	0.9490	0.3249	0.3569	**0.3046**	0.3048
	0.1,0.3	0.9127	0.9430	0.9490	0.9493	0.2910	0.3280	0.2633	0.2635
	0.1,0.5	0.9040	**0.9500**	**0.9575**	**0.9570**	0.2560	0.2959	0.2228	**0.2225**
	0.3,0.3	0.8903	0.9473	**0.9557**	**0.9553**	0.3946	0.4735	0.3260	**0.3259**
	0.3,0.5	0.8830	0.9403	0.9470	0.9460	0.3417	0.4136	0.2750	0.2750
	0.5,0.5	0.8713	0.9417	**0.9517**	**0.9517**	0.4959	0.6186	0.3697	**0.3696**
50,50	0.1,0.1	0.9240	0.9500	**0.9500**	**0.9527**	0.3187	0.3452	**0.2943**	**0.2943**
	0.1,0.3	0.9130	0.9480	**0.9523**	**0.9550**	0.3021	0.3416	**0.2592**	**0.2592**
	0.1,0.5	0.8893	0.9453	**0.9513**	**0.9517**	0.2884	0.3341	0.2283	**0.2282**
	0.3,0.3	0.8960	0.9507	**0.9537**	**0.9547**	0.3841	0.4520	0.3128	**0.3126**
	0.3,0.5	0.8930	0.9493	0.9463	0.9470	0.3629	0.4302	0.2767	0.2767
	0.5,0.5	0.8810	0.9453	**0.9500**	**0.9507**	0.4846	0.5900	0.3546	**0.3545**
50,100	0.1,0.1	0.9200	0.9403	0.9493	**0.9523**	0.2683	0.2934	0.2503	**0.2497**
	0.1,0.3	0.9060	0.9437	**0.9537**	**0.9513**	0.2457	0.2792	**0.2179**	**0.2179**
	0.1,0.5	0.8990	0.9423	**0.9500**	**0.9503**	0.2230	0.2612	0.1868	**0.1867**
	0.3,0.3	0.8903	0.9410	0.9483	0.9470	0.3234	0.3865	0.2666	0.2670
	0.3,0.5	0.8930	0.9487	**0.9500**	0.9477	0.2876	0.3502	**0.2273**	0.2271
	0.5,0.5	0.8773	0.9437	**0.9510**	0.9463	0.4005	0.5019	**0.2999**	0.2997
100,100	0.1,0.1	0.9207	0.9427	0.9477	0.9497	0.2200	0.2397	0.2037	0.2040
	0.1,0.3	0.9060	0.9483	0.9447	0.9430	0.2069	0.2370	0.1792	0.1791
	0.1,0.5	0.8903	0.9460	**0.9510**	**0.9520**	0.1943	0.2318	0.1565	**0.1564**
	0.3,0.3	0.8950	0.9440	0.9487	0.9473	0.2638	0.3147	0.2165	0.2165
	0.3,0.5	0.8790	0.9463	**0.9503**	**0.9517**	0.2430	0.2972	**0.1881**	0.1882
	0.5,0.5	0.8820	0.9527	**0.9517**	**0.9523**	0.3247	0.4063	**0.2415**	0.2416

Note: Bold text indicates coverage probabilities greater than or equal to 0.95 and the most appropriate average widths.

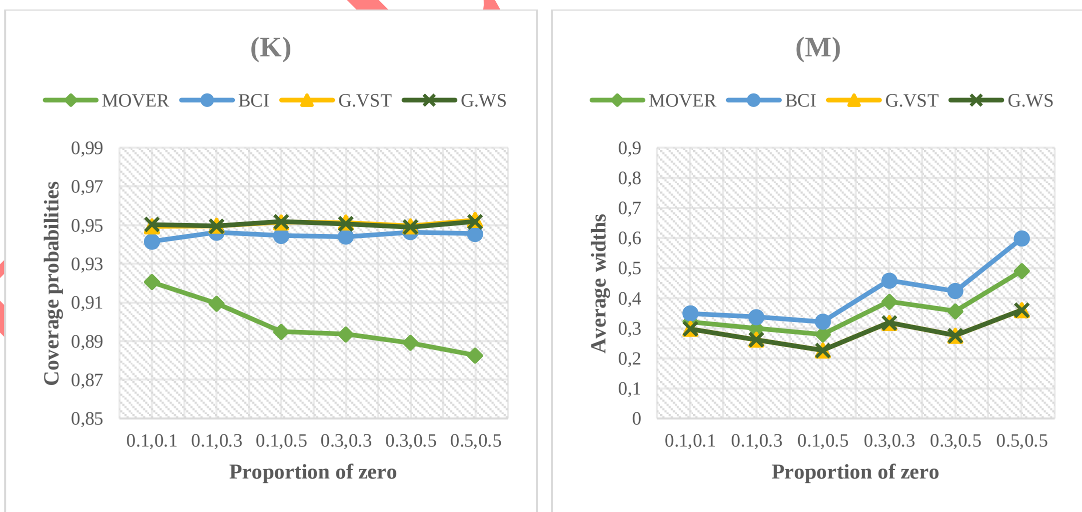

Figure 9. *The graphs compare the efficiency of the proportion of zero versus (K) coverage probabilities and (M) average widths for shape parameter = 2.0,2.0*

4. APPLICATION

Wind speed has various impacts on different aspects of life and the economy. There are many interesting benefits, such as Renewable Energy Production: Wind speed significantly influences the production of renewable energy. Wind turbines use the wind's rotation of blades to generate electricity, reducing the reliance on energy sources that emit pollutants and minimizing the use of essential natural resources. Aircraft Operations: Wind speed affects the takeoff and landing of aircraft. It can help reduce energy consumption on flights and enhance travel efficiency. Crop Production: Wind plays a role in dispersing plant seeds, aiding in the pollination of crops, and facilitating the dispersion of agricultural chemicals. Appropriate wind speeds can increase agricultural productivity. Enhancing Maritime Efficiency: In maritime activities, utilizing wind as an energy source helps reduce the use of fossil fuels and lowers ship emissions during travel. Weather Preparedness: Monitoring and predicting wind speed contributes to preparedness and disaster prevention. This is crucial for anticipating and managing the impacts of severe weather conditions. By harnessing the benefits of wind speed, we can reduce reliance on energy produced from environmentally polluting sources, mitigate natural resource depletion, and establish sustainable and balanced energy systems. Since wind speed affects many factors and is highly significant, we have considered wind speed data in our analysis. Therefore, we incorporate wind speed in the application of this research, using the wind speed for the hourly periods of October 2-3, 2022, and 2023 from the Khao Kheow Weather Observing Station, Thailand [44], as presented in Table 4. We plotted histograms of the wind speed data from the Khao Kheow Weather Observing Station to visualize the distribution, as shown in Figure 8. Additionally, we provided statistical information for the wind speed data in Table 5. Since the wind speed data includes zero values (no wind) and positive values, to assess the suitability of data distribution for positive values, we employed the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) criteria, calculated as $\text{AIC} = -2\ln(L) + 2p$ and $\text{BIC} = -2\ln(L) + 2p\ln(o)$, respectively, where p is the number of parameters estimated, o is the number of observations, and L is the likelihood function [45,46]. This was done to evaluate the appropriateness of data distribution by comparing it with different distributions, namely the Normal, Weibull, Exponential, Gamma, Birnbaum-Saunders, Cauchy, and Logistic distributions. It is apparent from Table 6 that the Birnbaum-Saunders distribution has the lowest AIC and BIC values in comparison to the other distributions. This indicates that the Birnbaum-Saunders distribution is most appropriate for the wind speed data that exhibits the positive value. Hence, the wind speed data, which comprises both zero and positive values, is consequently represented by the Delta-Birnbaum-Saunders distribution. This distribution has thus been utilized to compute confidence intervals for the ratios of the coefficients of variation of the wind speed data. From the presented data in Table 7, it is evident that the G.VST method is the most suitable approach for calculating confidence intervals for the ratios of the coefficients of variation of wind speed data at the Khao Kheow Weather Observing Station. This is due to its shortest interval width compared to other methods.

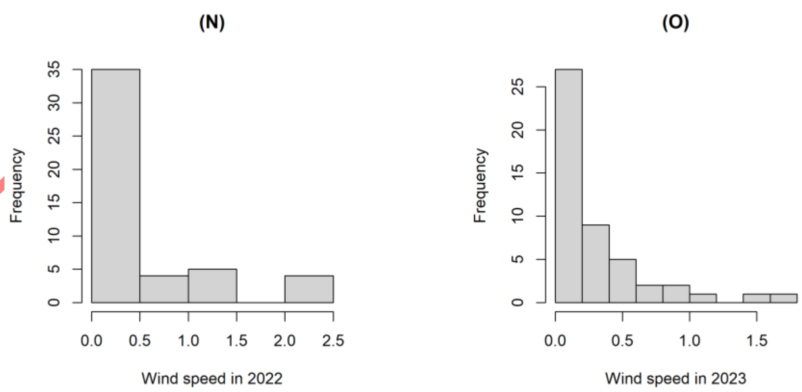

Figure 8. *Histograms of wind speed data for (N) October 2–3, 2022, and (O) October 2–3, 2023*

Table 4. *Data on the wind speed (knot) from the Khao Kheow Weather Observing Station, Thailand*

October 2–3, 2022				October 2–3, 2023			
0.0	0.2	0.2	0.0	0.7	0.0	0.0	0.4
0.0	0.0	1.4	0.0	0.5	0.1	0.0	0.2
0.2	0.4	0.7	0.0	0.5	0.3	0.1	0.1
0.0	0.5	1.1	0.2	0.0	0.1	0.4	0.0
0.0	0.2	2.2	0.2	0.2	0.0	0.5	0.0
0.0	0.0	2.4	0.2	0.3	0.1	0.7	0.3
0.0	0.6	2.5	0.0	0.4	0.2	0.5	1.0
0.5	0.4	1.2	0.0	0.3	0.1	0.0	1.5
1.5	0.1	0.3	0.3	0.2	0.0	0.1	0.5
2.3	0.6	0.0	0.3	0.0	0.1	0.4	1.7
0.5	0.5	0.0	1.1	0.0	0.0	0.3	0.9
0.1	0.1	0.0	0.7	0.0	0.1	0.0	1.1

Table 5. *Summary statistics for the wind speed data*

Data	m_i	$m_{i(0)}$	$m_{i(1)}$	$\hat{\lambda}_i$	$\hat{\alpha}_i$	$\hat{\beta}_i$	$\hat{\kappa}_i$
In 2022	48	16	32	0.333	1.021	0.487	1.434
In 2023	48	14	34	0.292	0.903	0.311	1.259

Table 6. *The AIC and BIC values of each distribution for the wind speed data*

Distributions	Data	Maximum likelihood estimates		AIC	BIC
		Shape parameter	Scale parameter		
Normal	In 2022	0.741	0.713	73.142	76.073
	In 2023	0.438	0.391	36.648	39.701
Weibull	In 2022	1.107	0.772	48.227	51.159
	In 2023	1.227	0.472	13.594	16.647
Exponential	In 2022	-	0.741	46.783	48.249
	In 2023	-	0.438	13.900	15.426
Gamma	In 2022	1.285	0.576	47.622	50.554
	In 2023	1.557	0.282	12.351	15.404
Birnbaum-Saunders	In 2022	1.021	0.487	**44.015**	**48.413**
	In 2023	0.903	0.311	**9.414**	**13.993**
Cauchy	In 2022	0.372	0.241	66.495	69.427
	In 2023	0.291	0.164	31.462	34.514
Logistic	In 2022	0.610	0.376	71.207	74.138
	In 2023	0.372	0.196	31.900	34.953

Table 7. *The 95% confidence intervals for the coefficients of variation of the wind speed data*

Point estimation	Methods	Interval [L, U]	Width
$\frac{\hat{\kappa}_1}{\hat{\kappa}_2} = 1.1385$	MOVER	[0.8273, 1.5689]	0.7416
	BCI	[0.8326, 1.5077]	0.6751
	G.VST	[0.8991, 1.4441]	0.5450
	G.WS	[0.8717, 1.4316]	0.5599

5. CONCLUSIONS

In this conclusion, our research has presented four methods, namely MOVER, BCI, G.VST, and G.WS, for constructing confidence intervals for ratios of the coefficients of variation under the Delta-Birnbaum-Saunders distributions. To assess the performance of all four methods, we use coverage probabilities along with average widths obtained from Monte Carlo simulations. The simulation results indicate that as the sample size increases, all the presented methods exhibit improved performance. Additionally, when considering each method individually, the MOVER method performs well when the shape parameter is small. However, this method provides average widths that are wider than those of other methods in almost every case studied. For the BCI method, although it generally provides shorter average widths compared to MOVER, except for large shape parameters, the BCI method still provides coverage probabilities below the specified criterion in almost every case, resulting in the lowest overall performance. The G.VST and G.WS methods provide similar values in terms of coverage probabilities and average widths. Importantly, the G.VST and G.WS offer the shortest average widths, resulting in better performance compared to other methods and demonstrating the highest overall performance. Moreover, the application of these methods to wind speed data aligns with simulation results. Consequently, this research recommends the use of the generalized confidence interval method for constructing confidence intervals for the ratios of coefficients of variation in the Delta-Birnbaum-Saunders distributions.

As a final remark, our research findings demonstrate that the MOVER performs well for cases with small shape parameters. However, the variance estimator used in the MOVER may have some weaknesses affecting α_i, resulting in wider confidence intervals compared to other methods, even though it meets the specified coverage probability criteria. In contrast, the GCI provides the shortest confidence intervals, leading to the best overall performance. This is because the GPQ of λ_i influences the GPQ of α_i, allowing the GCI to perform well in almost all cases. These results are consistent with the research of Yosboonruang and Niwitpong [25]. In future research, we will investigate alternative methods for constructing confidence intervals, such as Bayesian estimation or Highest Posterior Density (HPD) intervals, which may lead to improved performance. Additionally, we plan to incorporate other real-world datasets beyond wind speed data to broaden the scope of analysis and applicability.

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CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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