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# Bijective soft rings with applications

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**Abstract —** In recent years, soft sets have been widely used in many important decision-making real-life problems. In this paper, observing the usage of soft sets in such kind of vital problems, we have introduced the bijective-unitary bijective soft rings. Firstly, we have defined and exemplified a bijective soft ring and a unitary bijective soft ring. Moreover, we have presented some applications of bijective soft rings. We have shown the usage of bijective soft rings in coding theory. In this context, we have observed that by obtaining a bijective soft ring over a finite ring, we have a coding matrix to encode a given set of messages. Besides these applicable results, we have also obtained some relations between bijective soft and classical rings.

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## 1. Introduction

Some real-life problems do not have certain suitable solution methods and may involve some inaccurate data. Dealing with such problems and making a decision for them is a vital step in many areas of real life e.g. economics, engineering, environmental science, etc. Generally, such uncertain conditions cannot be dealt with by classical methods successfully since they have some inherited restrictions.

In order to overcome uncertainties in such kind of mentioned problems above, soft set theory was initiated by Molodtsov [1]. It has been used in many important applications recently. After the introduction of soft sets, many authors defined new subjects related to soft sets, and many different usage aspects of soft sets were given in different areas. Soft set theory has improved rapidly since 1999. Maji et al. [2] used soft sets to solve a decision-making problem by using rough sets and studied some properties of soft sets by giving new definitions on soft sets in [3]. Chen et al. [4] studied on the parameterization reduction of soft sets and some of its applications.

Many authors widely studied on applications of soft sets and extended soft set theory to different soft algebraic structures. Aktaş and Çağman [5] introduced soft groups. Sun et al. [6] defined soft modules. Feng et al. [7] defined soft semirings and examined their properties. Jun et al. [8] introduced soft ideals and idealistic soft BCK/BCI-algebras and provided some relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras. Ali et al. [9] defined some new operations in soft set theory.

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Aygünoğlu et al. [11] introduced fuzzy soft groups and studied some of their properties. Kazancı et al. [12] introduced soft BCH-algebras and studied on their structural characteristics. Çağman et al. [13] defined the product of soft sets and uni-int decision function. Babitha et al. [14] introduced the concept of soft set relations as a sub soft set of the Cartesian product of the soft sets. Xiao et al. [15] initiated the exclusive disjunctive soft sets.

Gong et al. [16] introduced bijective soft sets. Acar et al. [17] introduced the initial concepts of soft rings. Feng et al. [18] obtained an important relation between rough sets and soft sets. Atagün et al. [19] study on soft substructures of rings, fields, and modules. Deli et al. [20] defined intuitionistic fuzzy parameterized soft sets and studied some of their properties. Aktaş [21] introduced bijective soft groups and mentioned their applications. Koyuncu et al. [22] defined soft fields, some kinds of ideals of soft rings and soft quotient rings. Chaipunya et al. [23] investigated the finite soft intersection property of a family of soft sets indexed by another soft set. Ali et al. [24] designed a new class of linear algebraic codes using soft sets. Goldar et al. [25] gave a new definition of soft rings and soft ideals using soft elements. Öztunç et al. [26] constructed the category of soft groups and soft group homomorphisms. Anjum et al. [27] discussed two different types of soft hemirings, soft intersection and soft union. Aygün et al. [28] introduced the notions of soft intersection near-field and soft union near-field. Nagarajan et al. [29] defined soft intersection action (SI) on M-N module structures on a soft set.

In this paper, inspired by [16,17,21] we have defined bijective soft rings and unitary bijective soft rings and concentrated on some of their properties and applications. In the sequel, we present definitions of soft sets, bijective soft sets, and soft rings in Section 2. In Section 3, we define bijective soft rings and unitary bijective soft rings, the bijective soft subring of a bijective soft ring, and the bijective soft ideal of a bijective soft ring, and study some of their properties. In this section, we also present an important application of bijective soft rings to coding theory. We define idealistic bijective soft rings and study some of their properties in Section 4. Finally, we obtain some relations between bijective soft rings and rings in Section 5.

## 2. Preliminaries

Throughout the study, $U$ denotes an initial universal set and the power set of $U$ is denoted by $\mathcal{P}(U)$, $E$ is a set of parameters, $A \subseteq E$. Moreover, $R$ denotes a ring with zero $0_R$ unless it's specified, $R \neq \{0_R\}$ and $R^* = R - \{0_R\}$. For a set $S$, $|S|$ denotes the cardinality of $S$.

**Definition 2.1.** [1] A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by

$$F : A \longrightarrow \mathcal{P}(U)$$

Generally, $(F, A)$ is represented by $(F, A) = \{(a, F(a)) : a \in A\}$.

**Definition 2.2.** [3] Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$. The union of $(F, A)$ and $(G, B)$ is defined as the soft set $(H, C)$ satisfying

*i.* $C = A \cup B$

*ii.* For all $x \in C$,

$$H(x) = \begin{cases} F(x) & \text{if } x \in A - B \\ G(x) & \text{if } x \in B - A \\ F(x) \cup G(x) & \text{if } x \in A \cap B \end{cases}$$

This is denoted by $(H, C) = (F, A) \,\tilde{\cup}\, (G, B)$.

**Definition 2.3.** [3] Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$. Then, AND-product of $(F, A)$ and $(G, B)$, denoted by $(F, A) \wedge (G, B)$, is defined as the soft set

$$(H, C) = (F, A) \wedge (G, B)$$

where $C = A \times B$ and $H(x, y) = F(x) \cap G(y),$ for all $(x, y) \in C$.

**Definition 2.4.** [3] Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $U$. Then, OR-product of $(F, A)$ and $(G, B)$, denoted by $(F, A) \vee (G, B)$, is defined as the soft set

$$(H, C) = (F, A) \vee (G, B)$$

where $C = A \times B$ and $H(x, y) = F(x) \cup G(y),$ for all $(x, y) \in C$.

**Definition 2.5.** [5] Let $(F, A)$ be a soft set over a group $G$. Then, $(F, A)$ is called a soft group over $G$ if $F(x)$ is a subgroup of $G$ for all $x \in A$.

**Definition 2.6.** [7] Let $(F, A)$ be a soft set. The support of $(F, A)$ is defined as the set

$$\text{Supp}(F, A) = \{x \in A | F(x) \neq \emptyset\}$$

A soft set is said to be non-null if its support is not equal to the empty set.

In this study, all considered soft sets are supposed to be non-null.

**Definition 2.7.** [10] Let $(F, A)$ and $(G, B)$ be two soft sets over the sane universe $U$. The restricted intersection of $(F, A)$ and $(G, B)$ is denoted by $(F, A) \cap_{\mathcal{R}} (F, B)$ and is defined as

*i.* $(F, A) \cap_{\mathcal{R}} (G, B) = (H, A \cap B)$ where $H(x) = F(x) \cap G(x)$ for all $x \in A \cap B$ if $A \cap B \neq \emptyset$,

*ii.* $(F, A) \cap_{\mathcal{R}} (G, B) = \emptyset_{\emptyset}$ if $A \cap B = \emptyset$.

**Definition 2.8.** [1, 14] Let $(F, A)$ and $(G, B)$ be two nonempty soft sets over the same universe $U$. Then, the Cartesian product of $(F, A)$ and $(G, B)$ is defined as the soft set

$$(H, A \times B) = (F, A) \times (G, B)$$

where $H : A \times B \longrightarrow \mathcal{P}(U \times U)$ is the mapping given by $H(x, y) = F(x) \times G(y),$ for all $(x, y) \in A \times B$.

**Definition 2.9.** [16] Let $(F, A)$ be a soft set over a universe $U$. Then, $(F, A)$ is called a bijective soft set if it satisfies the following conditions:

*i.* $\bigcup\limits_{a \in A} F(a) = U$

*ii.* For any parameters $a_i$ and $a_j$ with $a_i \neq a_j$, $F(a_i) \cap F(a_j) = \emptyset$.

**Definition 2.10.** [17] Let $(F, A)$ be a soft set over a ring $R$. Then, $(F, A)$ is called a soft ring over $R$ if $F(x)$ is a subring of $R$ for all $x \in A$.

**Definition 2.11.** Let $(F, A)$ be a soft ring over a ring $R$. Then,

*i.* $(F, A)$ is called an identity soft ring over $R$ if $F(x) = \{0_R\}$ for all $x \in A$.

*ii.* $(F, A)$ is called an absolute soft ring over $R$ if $F(x) = R$ for all $x \in A$.

**Definition 2.12.** [17] Let $(F, A)$ and $(G, B)$ be soft rings over the rings $R$ and $R'$, respectively. Let $f : R \longrightarrow R'$ and $g : A \longrightarrow B$ be two mappings. The pair $(f, g)$ is called a soft ring homomorphism if the following conditions are satisfied:

*i.* $f$ is a ring epimorphism.

*ii.* $g$ is surjective.

*iii.* $f(F(x)) = G(g(x))$, for all $x \in A$.

If we have a soft ring homomorphism between $(F, A)$ and $(G, B)$, $(F, A)$ is said to be homomorphic to $(G, B)$, which is denoted by $(F, A) \sim (G, B)$. In addition, if $f$ is a ring isomorphism and $g$ is bijective, then $(f, g)$ is called a soft ring isomorphism. In this case, we say that $(F, A)$ is soft isomorphic to $(G, B)$, which is denoted by $(F, A) \simeq (G, B)$.

## 3. Introduction to Bijective Soft Rings and Unitary Bijective Soft Rings

In this section, we define bijective soft rings and unitary bijective soft rings. Moreover, we introduce an important algebraic application of bijective soft rings to cryptography.

**Definition 3.1.** Let $(F, A)$ be a soft ring over $R$. Then, $(F, A)$ is called a bijective soft ring if it satisfies the following conditions:

*i.* $\bigcup_{a \in A} F(a) = R$

*ii.* For any parameters $a_i$ and $a_j$ with $a_i \neq a_j$, $F(a_i) \cap F(a_j) = \{0_R\}$.

When we have a bijective soft ring $(F, A)$ over $R$, using the image $F[A] = \{F(a) \,|\, a \in A\}$ of $F$ and choosing a suitable subset $B$ of $A$, we can obtain a bijective function $F : B \longrightarrow F[A]$.

**Example 3.2.** Let $S = \{1, 2\}$ be a subset of $\mathbb{Z}^+$ and

$$R = \mathcal{P}(S) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

be the power set of $S$. We know that $R$ is a ring with respect to addition " $+$ " and multiplication " $\cdot$ " operations defined by

$$A + B = A \Delta B = \{x \,|\, x \in A \text{ or } x \in B, \text{but not both}\} \text{ (symmetric difference)}$$

$$A \cdot B = A \cap B \text{ (intersection)}$$

on $R$. Here, $\emptyset$ is the zero elements of, $R$ and for any $r \in R$, $r$ is the additive inverse of itself. Moreover, $R$ is a finite commutative ring with unity $S$. We observe that the sets

$$S_0 = \{\emptyset\}, S_1 = \{\emptyset, \{1\}\}, S_2 = \{\emptyset, \{2\}\}, \text{ and } S_3 = \{\emptyset, \{1, 2\}\}$$

are subrings of $R$. Let $A = \{e_0, e_1, e_2, e_3\}$ be a set of parameters. We define the function

$$F : A \longrightarrow \mathcal{P}(R)$$

given by $F(e_i) = S_i, i \in \{0, 1, 2, 3\}$. Then, $(F, A)$ is a bijective soft ring over $R$ since

*i.* $\bigcup_{e_i \in A} F(e_i) = \bigcup_{i=0}^{3} S_i = S_0 \cup S_1 \cup S_2 \cup S_3 = R$

*ii.* $F(e_i) \cap F(e_j) = S_i \cap S_j = \{\emptyset\}$ for $i \neq j$, where $\emptyset$ is the zero element of $R$.

As implied by the previous example, we can give the following more general example.

**Example 3.3.** Let $n \in \mathbb{Z}^+$ and $S = \{1, 2, 3, ..., n\}$ be a subset of $\mathbb{Z}^+$. We observe that the power set

$$R = \mathcal{P}(S)$$

of $S$ is a ring under addition and multiplication operations defined in the previous example. Moreover, the subsets

$$S_0 = \{\emptyset\} \text{ and } S_i = \{\emptyset, r_i\}, i \in \{1, 2, 3, ..., 2^n - 1\}$$

are subrings of $R$ for all nonzero elements $r_i \in R$ i.e. elements different from the empty set. Hence, for any parameter set

$$A = \{e_i \,|\, i \in \{0, 1, 2, ..., 2^n - 1\}\}$$

the map

$$F : A \longrightarrow \mathcal{P}(R)$$

given by $F(e_i) = S_i, i \in \{0, 1, 2, ..., 2^n - 1\}$ is one-to-one. Consequently, $(F, A)$ is a bijective soft ring over $R$ since

*i.* $\bigcup_{e_i \in A} F(e_i) = \bigcup_{i=0}^{2^n-1} S_i = R$

*ii.* $F(e_i) \cap F(e_j) = S_i \cap S_j = \{\emptyset\}$ for $i \neq j$, where $\emptyset$ is the zero element of $R$.

**Example 3.4.** Let us consider the ring

$$R = \left\{ \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} |x, y \in \mathbb{Z}_2 \right\} = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

under the usual operations of matrix addition and matrix multiplication, which is a subring of $M_2(\mathbb{Z}_2)$. $R$ has the following subrings

$$S_1 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}, S_2 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}, S_3 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}, \text{ and } S_4 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

We observe that for any parameter set $A = \{e_1, e_2, e_3, e_4\}$, the function

$$F : A \longrightarrow \mathcal{P}(R)$$

given by $F(e_i) = S_i, i \in \{1, 2, 3, 4\}$ is one-to-one. Therefore, $(F, A)$ is a bijective soft ring over $R$ since

*i.* $\bigcup_{e_i \in A} F(e_i) = \bigcup_{i=1}^{4} S_i = R$

*ii.* $F(e_i) \cap F(e_j) = S_i \cap S_j = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$, for $i \neq j$, where $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is the zero element of $R$.

We know that generally, the union of two subrings of a ring $R$ need not be a subring of itself. Hence, it is not so direct to find examples of bijective soft rings. Therefore, to be more usable, we extend the concept of bijective soft rings to unitary bijective soft rings.

**Definition 3.5.** Let $R$ be a ring with unity $1_R \neq 0_R$ and let $(F, A)$ be a soft ring over $R$. Then, $(F, A)$ is called a unitary bijective soft ring over $R$ if it satisfies the following conditions:

*i.* $\bigcup_{a \in A} F(a) = R$

*ii.* For any parameters $a_i$ and $a_j$ with $a_i \neq a_j$, $F(a_i) \cap F(a_j) = \{0_R, 1_R\}$.

**Example 3.6.** Consider the subring

$$R = \left\{ \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} |x, y, z \in \mathbb{Z}_2 \right\}$$

of $M_3(\mathbb{Z}_2)$ under the usual operations of matrix addition and matrix multiplication. $R$ is a commutative ring of order 8 with unity. It has three subrings of order 4 containing the unity of $R$ as

$$S_1 = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$$S_2 = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

and

$$S_3 = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

As is seen, for any parameter set $A = \{e_1, e_2, e_3\}$, the map

$$F : A \longrightarrow \mathcal{P}(R)$$

defined by $F(e_i) = S_i, i \in \{1, 2, 3\}$ is one-to-one. Hence, $(F, A)$ is a unitary bijective soft ring over $R$ since

*i.* $\bigcup_{e_i \in A} F(e_i) = \bigcup_{i=1}^{3} S_i = R$

*ii.* $F(e_i) \cap F(e_j) = S_i \cap S_j = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} = \{0_R, 1_R\}$, for $i \neq j$.

Note that according to the suitably considered subrings of $R$ in this example, we can also obtain a bijective soft ring over $R$.

Tabular representation of a bijective soft group is introduced in [21]. Similarly, we can form the tabular representation of a bijective (or unitary bijective) soft ring $(F, A)$ over a ring $R$. Actually, this can be applied by relabeling the elements of the parameter set $A$ suitably. That is, if $e_i \in A$ and $F(e_i) = S_i$, which is a subring of $R$, then we can choose an element $r_i \in S_i$ and relabel $e_i$ as $r_i$. For $r \in R$, if $r \in F(x)$ then we can indicate the tuple $(F(x), r)$ by 1 in the representation, otherwise we indicate it by 0. In any row of this representation, collecting the elements $r$ of $R$ for which $(F(x), r)$ are indicated by 1 into a set, we can obtain a subring of $R$ easily.

For example, in Example 3.2 we can take the parameter set as $A = \{r_0, r_1, r_2, r_3\}$, where the elements $e_i$ are relabeled as $r_i$ such that

$$r_0 = \emptyset, r_1 = \{1\}, r_2 = \{2\}, \text{ and } r_3 = \{1, 2\}$$

Thus, we can form the tabular representation of the bijective soft ring $(F, A)$ over $R$ as in Table 1.

**Table 1.** Representation of $(F, A)$ over $R$

| $R$ | $r_0$ | $r_1$ | $r_2$ | $r_3$ |
|---|---|---|---|---|
| $F(r_0)$ | 1 | 0 | 0 | 0 |
| $F(r_1)$ | 1 | 1 | 0 | 0 |
| $F(r_2)$ | 1 | 0 | 1 | 0 |
| $F(r_3)$ | 1 | 0 | 0 | 1 |

**Definition 3.7.** If $(F, A)$ is a bijective soft ring over $R$ and $F(a) = R$ for some $a \in A$, then $(F, A)$ is called an absolute bijective soft ring over $R$.

**Remark 3.8.** For a parameter set $A$ having at least two elements, if $(F, A)$ is an absolute bijective soft ring over $R$, then the image of $F$ must have only two elements, one is $\{0_R\}$, the other is $R$, since we require that

$$F(a_i) \cap F(a_j) = \{0_R\}$$

for any parameters $a_i$ and $a_j$ in $A$ with $a_i \neq a_j$. Similarly, if $(F, A)$ is a unitary bijective soft ring over a ring $R$ with unity and $F(a) = R$ for some $a \in A$, then $(F, A)$ is called an absolute unitary bijective soft ring over $R$. As is seen, if $(F, A)$ is an absolute unitary bijective soft ring over a ring $R$ with unity $1_R \neq 0_R$, then the image of $F$ have only two elements $\{0_R, 1_R\}$ and $R$ since for $a_i \neq a_j$ we must have

$$F(a_i) \cap F(a_j) = \{0_R, 1_R\}$$

For example, if $p$ is a prime number, $R = \mathbb{Z}_p$, $A$ is a parameter set having at least two elements and $(F, A)$ is a bijective soft ring over $R$, then $(F, A)$ is an absolute bijective soft ring over $R$.

We provide a sequence of properties of bijective soft rings. Similar results can also be given for unitary bijective soft rings.

**Theorem 3.9.** Let $(F, A)$ and $(G, B)$ be two bijective soft rings over a common ring $R$. Then, $(F, A) \wedge (G, B)$ is a bijective soft ring over $R$.

**Proof.** Similar to the proof of [21, Theorem 3.3]. □

**Proposition 3.10.** Let $(F, A)$ be a bijective soft ring over $R$ and $(G, B)$ be an identity soft ring over $R$. Then, $(F, A) \mathbin{\tilde{\cup}} (G, B)$ is a bijective soft ring over $R$.

**Proof.** This is an obvious result while $(F, A)$ is a bijective soft ring. The proof is similar to the proof of [21, Theorem 3.4]. □

Logically, preserving the requirements of [21, Definition 3.5], we define a bijective soft subring of a bijective soft ring as follows:

**Definition 3.11.** Let $(F, A)$ be a bijective soft ring over $R$ and $(G, B)$ be a soft subring of $(F, A)$. Then, $(G, B)$ is called a bijective soft subring of $(F, A)$ if the following conditions are satisfied.

*i.* $\bigcup_{b \in B} G(b)$ is a subring of $R$.

*ii.* For any parameters $b_i$ and $b_j$ in $B$ with $b_i \neq b_j$, $G(b_i) \cap G(b_j) = \{0_R\}$.

**Example 3.12.** Let $S = \{1, 2, 3\}$ be a subset of $\mathbb{Z}^+$. Consider the ring

$$R = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

under the binary operations given in Example 3.2, where elements of $R$ are labeled as

$$r_0 = \emptyset, r_1 = \{1\}, r_2 = \{2\}, r_3 = \{3\}, r_4 = \{1, 2\}, r_5 = \{1, 3\}, r_6 = \{2, 3\}, \text{ and } r_7 = \{1, 2, 3\}$$

Let $A = \{r_0, r_2, r_3, r_5, r_6, r_7\}$ and $B = \{r_0, r_2, r_3, r_6\} \subseteq A$ be two sets of parameters. We can define the functions $F : A \longrightarrow \mathcal{P}(R)$ and $G : B \longrightarrow \mathcal{P}(R)$ as $F(r_2) = S_8, F(r_i) = S_i,$ for $i \in \{0, 3, 5, 6, 7\}$ and $G(r_i) = S_i,$ for all $r_i \in B$, where $S_i$ are the subrings of $R$ given by

$$S_0 = \{\emptyset\}, S_1 = \{\emptyset, \{1\}\}, S_2 = \{\emptyset, \{2\}\}, S_3 = \{\emptyset, \{3\}\},$$

$$S_4 = \{\emptyset, \{1, 2\}\}, S_5 = \{\emptyset, \{1, 3\}\}, S_6 = \{\emptyset, \{2, 3\}\}, S_7 = \{\emptyset, \{1, 2, 3\}\},$$

$$S_8 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}, S_9 = \{\emptyset, \{1\}, \{3\}, \{1, 3\}\}, \text{ and } S_{10} = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$$

Then, $(F, A)$ is a bijective soft ring over $R$, and $(G, B)$ is a soft subring of $(F, A)$. Moreover,

$$G(r_i) \cap G(r_j) = S_i \cap S_j = \{0_R\}$$

for elements $r_i$ and $r_j$ in $B$ with $r_i \neq r_j$ and

$$S_{10} = S_0 \cup S_2 \cup S_3 \cup S_6 = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$$

is a subring of $R$. Consequently, $(G, B)$ is a bijective soft subring of $(F, A)$.

Using the tabular representation of the bijective soft ring $(F, A)$ given in Example 3.12, we get the following matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

We can easily check that rows of $M$ are linearly independent. Hence, $M$ generates a binary linear $(8, 6)$ code. As is observed, by obtaining a bijective soft ring over a finite ring, it seems that we can obtain many binary linear codes to generate new codes that can be used to encode several messages.

**Theorem 3.13.** Let $(F, A)$ be a bijective soft ring over $R$, $(G, B)$ and $(H, C)$ be two bijective soft subrings of $(F, A)$. Then, $(G, B) \wedge (H, C)$ is a bijective soft subring of $(F, A)$.

**Proof.** A direct consequence of Definition 3.11. □

**Theorem 3.14.** Let $(F, A)$ be a bijective soft ring over $R$, $(G, B)$ be a soft ring over a ring $S$ and $(f, g)$ be a soft ring homomorphism from $(F, A)$ to $(G, B)$. If $f$ is an isomorphism, then $(G, B)$ is a bijective soft ring over $S$.

**Proof.** Obvious, similar to the proof of [21, Theorem 3.8]. □

**Theorem 3.15.** Let $(F, A)$ and $(G, B)$ be two bijective soft rings over the rings $R$ and $S$, respectively. Then, the Cartesian product $(F, A) \times (G, B)$ is a bijective soft ring over $R \times S$.

**Proof.** Clear, similar to the proof of [21, Theorem 3.9]. □

Depending on Definition 3.11, we define the concept of bijective soft ideal of a bijective soft ring as follows.

**Definition 3.16.** Let $(F, A)$ be a bijective soft ring over $R$ and $(G, B)$ be a soft ideal of $(F, A)$. Then, $(G, B)$ is called a bijective soft ideal of $(F, A)$ if

*i.* $\bigcup_{b \in B} G(b)$ is an ideal of $R$.

*ii.* For any parameters $b_i$ and $b_j$ in $B$ with $b_i \neq b_j$, $G(b_i) \cap G(b_j) = \{0_R\}$.

In Example 3.12, we observe that $(G, B)$ is a bijective soft ideal of $(F, A)$.

## 4. Idealistic Bijective Soft Rings and Their Applications

In this section, we focus on idealistic bijective soft rings which are special bijective soft rings. Similar results can be given for unitary bijective soft rings. Naturally, using [17, Definition 5.1] and Definition 3.1, we define an idealistic bijective soft ring as follows.

**Definition 4.1.** Let $(F, A)$ be an idealistic soft ring over $R$. Then, $(F, A)$ is called an idealistic bijective soft ring if it satisfies the following conditions:

*i.* $\bigcup_{a \in A} F(a) = R$.

*ii.* For any parameters $a_i$ and $a_j$ with $a_i \neq a_j$, $F(a_i) \cap F(a_j) = \{0_R\}$.

We observe that bijective soft rings given in Examples 3.2 and 3.3 are idealistic bijective soft rings.

**Theorem 4.2.** Let $(F, A)$ and $(G, B)$ be two idealistic bijective soft rings over a common ring $R$. Then, $(F, A) \wedge (G, B)$ is an idealistic bijective soft ring over $R$.

**Proof.** Straightforward from Theorem 3.9. □

In the following theorem, $F|_C$ and $G|_C$ denotes the restrictions of $F$ and $G$ to $C$, respectively.

**Theorem 4.3.** Let $(F, A)$ and $(G, B)$ be two idealistic bijective soft rings over a common ring $R$, $C = A \cap B \neq \emptyset$, $R_1$ and $R_2$ be subrings of $R$. If $(F|_C, C)$ and $(G|_C, C)$ are idealistic bijective soft rings over $R_1$ and $R_2$, respectively, then $(H, C) = (F, A) \cap_{\mathcal{R}} (G, A)$ is an idealistic bijective soft ring over the ring $R' = R_1 \cap R_2$.

**Proof.** Suppose that $(F, A)$ and $(G, B)$ are two idealistic bijective soft rings over a common ring $R$, $C = A \cap B \neq \emptyset$, $R_1$ and $R_2$ are subrings of $R$. For any $x \in C$, $H(x) = F(x) \cap G(x)$ is an ideal of $R'$. So, $(H, C) = (F, A) \cap_{\mathcal{R}} (G, A)$ is an idealistic soft ring over $R'$. Therefore, it is enough to show that $(H, C)$ is an idealistic bijective soft ring over $R'$. Let $(F|_C, C)$ and $(G|_C, C)$ be idealistic bijective soft rings over $R_1$ and $R_2$, respectively. Then, we have

$$\bigcup_{x \in C} H(x) = \bigcup_{x \in C} \Big(F(x) \cap G(x)\Big) = \left(\bigcup_{x \in C} F(x)\right) \cap \left(\bigcup_{x \in C} G(x)\right) = R_1 \cap R_2 = R'$$

In addition, for parameters $x_i$ and $x_j$ in $C$ with $x_i \neq x_j$, we get

$$\begin{aligned} H(x_i) \cap H(x_j) &= \Big(F(x_i) \cap G(x_i)\Big) \cap \Big(F(x_j) \cap G(x_j)\Big) \\ &= \Big(F(x_i) \cap F(x_j)\Big) \cap \Big(G(x_i) \cap G(x_j)\Big) \\ &= \{0_R\} \cap \{0_R\} = \{0_R\} \end{aligned}$$

□

**Example 4.4.** Consider the same ring

$$R = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

given in Example 3.12, where

$$r_0 = \emptyset, r_1 = \{1\}, r_2 = \{2\}, r_3 = \{3\}, r_4 = \{1, 2\}, r_5 = \{1, 3\}, r_6 = \{2, 3\}, \text{ and } r_7 = \{1, 2, 3\}$$

Let $A = \{a_0, a_1, a_2, a_3, a_4, a_5, r_6, a_7\}$ and $B = \{a_0, a_1, a_2, a_3, b_4, b_5, b_6, b_7\}$ be sets of parameters, where $C = A \cap B = \{a_0, a_1, a_2, a_3\}$. We define the maps $F : A \longrightarrow \mathcal{P}(R)$ and $G : B \longrightarrow \mathcal{P}(R)$ given by

$$F(a_0) = S_0, F(a_1) = S_1, F(a_2) = S_2, F(a_3) = S_4, F(a_4) = S_3, F(a_5) = S_5, F(a_6) = S_6, \text{ and } F(a_7) = S_7$$

and

$$G(a_0) = S_0, G(a_1) = S_3, G(a_2) = S_2, G(a_3) = S_6, G(b_4) = S_1, G(b_5) = S_4, G(b_6) = S_7, \text{ and } G(a_7) = S_5$$

where $S_i$ are the subrings of $R$ given by

$$S_0 = \{\emptyset\}, S_1 = \{\emptyset, \{1\}\}, S_2 = \{\emptyset, \{2\}\}, S_3 = \{\emptyset, \{3\}\}, S_4 = \{\emptyset, \{1, 2\}\},$$
$$S_5 = \{\emptyset, \{1, 3\}\}, S_6 = \{\emptyset, \{2, 3\}\}, \text{ and } S_7 = \{\emptyset, \{1, 2, 3\}\}$$

As mentioned before, $(F, A)$ and $(G, B)$ are idealistic bijective soft rings over $R$. We observe that $(F|_C, C)$ is an idealistic bijective soft ring over $R_1 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ which is a subring of $R$, and $(G|_C, C)$ is an idealistic bijective soft ring over $R_2 = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$ which is a subring of $R$.

Moreover, we have

$$H(a_0) = F(a_0) \cap G(a_0) = S_0 = \{\emptyset\}$$

$$H(a_1) = F(a_1) \cap G(a_1) = S_0 = \{\emptyset\}$$

$$H(a_2) = F(a_2) \cap G(a_2) = S_2 = \{\emptyset, \{2\}\}$$

and

$$H(a_3) = F(a_3) \cap G(a_3) = S_0 = \{\emptyset\}$$

Consequently, $(H, C)$ is an idealistic bijective soft ring over the ring $R' = R_1 \cap R_2 = \{\emptyset, \{2\}\}$.

As indicated in [17], when we have a soft set $(F, A)$ over $R$ and a function $f : R \longrightarrow S$ from $R$ to a ring $S$, then we can define a new soft set $(f(F), A)$ over $S$, where $f(F)(x) = f(F(x)),$ for all $x \in A$.

**Theorem 4.5.** Let $f : R \longrightarrow S$ be a ring isomorphism. If $(F, A)$ is an idealistic bijective soft ring over $R$, then $(f(F), A)$ is an idealistic bijective soft ring over $S$. In particular, if $(F, A)$ is a bijective soft ring over $R$, then $(f(F), A)$ is a bijective soft ring over $S$.

**Proof.** Let $f : R \longrightarrow S$ be a ring isomorphism and $(F, A)$ be an idealistic bijective soft ring over $R$. Then, since $f$ is a ring epimorphism, $(f(F), A)$ is an idealistic soft ring over $S$ by [17, Proposition 5.11]. Hence, it is enough to prove that $(f(F), A)$ is an idealistic bijective soft ring over $S$. Since $f$ is an isomorphism and $(F, A)$ is an idealistic bijective soft ring over $R$, we have

$$\bigcup_{a \in A} f(F)(a) = \bigcup_{a \in A} f(F(a)) = f\left(\bigcup_{a \in A} F(a)\right) = f(R) = S$$

Moreover, for parameters $a_i$ and $a_j$ in $A$ with $a_i \neq a_j$, we have

$$f(F)(a_i) \cap f(F)(a_j) = f(F(a_i)) \cap f(F(a_j)) = f(F(a_i) \cap F(a_j)) = f(\{0_R\}) = \{0_S\}$$

where $0_R$ and $0_S$ are the zeros of $R$ and $S$, respectively. □

Note that all the presented results in this section are valid for bijective soft rings since any ideal of a ring $R$ is a subring of itself. That is, all theorems in this section hold for bijective soft rings.

## 5. Relations Between Bijective Soft Rings and Rings

If $(F, A)$ is a bijective soft ring over $R$, then using some properties of $(F, A)$ we can obtain some properties of $R$ and conversely, using some properties of $R$ we can deduce some properties of $(F, A)$. In this section, we finally concentrate on some of such properties. Similar suitable properties should hold for unitary bijective soft rings.

**Theorem 5.1.** Let $(F, A)$ be a bijective soft ring over a finite ring $R$ such that $|R|$ is a prime number. Then, we have the following:

*i.* $(F, A)$ is an absolute bijective soft ring over $R$.

*ii.* The number of different bijective soft rings over $R$ is equal to $|A|$.

**Proof.** i. Follows from [21, Theorem 4.1] the group structure of $R$.
ii. Let $(F, A)$ be a bijective soft ring over $R$, where $|R|$ is a prime number. Since $R$ has only two subrings as $\{0_R\}$ and $R$, $(F, A)$ must be of the form

$$F(a) = R, \text{ for } a \in A \text{ and } F(x) = \{0_R\}, \text{ for all } x \in A - \{a\}$$

As we observe, for each $a \in A$, we can obtain a bijective soft ring over $R$, which completes the proof. □

Motivated by [21, Definition 4.2], we can give the following definition.

**Definition 5.2.** Let $(R, +, \cdot)$ be a ring and $a$, $b$ be two nonzero elements of $R$. If

$$a = \underbrace{b \cdot b \cdot ... \cdot b}_{k \text{ times}} = b^k$$

for some positive integer $k$, then $a$ and $b$ are called dependent elements of $R$, otherwise they are called independent elements of $R$.

**Example 5.3.** Let $R = \mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6\}$. Here, 1 and 2 are dependent elements of $R$ since $1 = 2^3$, whereas 4 and 6 are independent elements of $R$ as

$$6^k = 1 \text{ or } 6$$

for any positive integer $k$.

**Definition 5.4.** Let $(F, A)$ be a soft ring over $R$, $r_1$ and $r_2$ be dependent elements of $R$. If there exist different elements $a_1$ and $a_2$ in $A$ such that $r_1 \in F(a_1)$ and $r_2 \in F(a_2)$, then $F(a_1)$ and $F(a_2)$ are called dependent elements of $(F, A)$, otherwise they are called independent elements of $(F, A)$. If all elements of $(F, A)$ are dependent, then it is called a dependent soft ring.

**Example 5.5.** Let $R = \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ and $A = \{1, 2, 3\}$. Consider the soft ring

$$(F, A) = \{(1, \{0\}), (2, \{0, 3\}), (3, \{0, 2, 4\}), (6, R)\}$$

over $R$. As we see, 2 and 4 are dependent elements of $R$ since $2^2 = 2 \cdot 2 = 4$ and $4 \in F(3) = \{0, 2, 4\}$ while $2 \in F(6) = R$. Hence, $F(3)$ and $F(6)$ are dependent elements of $(F, A)$. But, $(F, A)$ is not a dependent soft ring since $F(2)$ and $F(3)$ are independent elements of $(F, A)$.

**Example 5.6.** Let $A = \mathbb{Z}^+$. Consider the soft ring $(F, A)$ over $\mathbb{Z}$ defined by

$$F(x) = x\mathbb{Z}$$

We see that that $(F, A)$ is a dependent soft ring. Because, for positive integers $x$, $y$ and subrings $x\mathbb{Z}$ and $y\mathbb{Z}$ of $\mathbb{Z}$, we have $l \in l\mathbb{Z}$ and $l\mathbb{Z} = x\mathbb{Z} \cap y\mathbb{Z}$ is a subring of both $x\mathbb{Z}$ and $y\mathbb{Z}$, where $l$ is the least common multiple of $x$ and $y$. For example, $6\mathbb{Z}$ and $4\mathbb{Z}$ are dependent elements of $(F, A)$ since we have

$$144 = 12^2, 12 \in 12\mathbb{Z} = 6\mathbb{Z} \cap 4\mathbb{Z}, 144 \in 6\mathbb{Z}, \text{ and } 12 \in 4\mathbb{Z}$$

**Proposition 5.7.** Let $(F, A)$ be a bijective soft ring over $R$. Then, $(F, A)$ has no dependent elements.

**Proof.** Let $(F, A)$ be a bijective soft ring over $R$ which has two dependent elements $F(a_1)$ and $F(a_2)$. Then, we must have $F(a_1) \cap F(a_2) \neq \{0_R\}$, which is a contradiction. □

**Proposition 5.8.** Let $(F, A)$ be a soft ring over a finite ring $R$. If all elements of $(F, A)$ are independent and $\bigcup_{a \in A} F(a) = R$, then $(F, A)$ is a bijective soft ring over $R$.

**Proof.** A direct result of Definitions 3.1 and 5.4. □

**Proposition 5.9.** Let $(F, A)$ be a dependent soft ring over $R$, $(G, B)$ be a soft ring over a ring $S$, and $(f, g)$ be a soft ring homomorphism from $(F, A)$ to $(G, B)$. If $f$ is an isomorphism, then $(G, B)$ is a dependent soft ring over $S$.

**Proof.** This is an obvious result while $f$ is an isomorphism. □

**Theorem 5.10.** Let $(F, A)$ be a bijective soft ring over $R$. If $(F, A)$ is not an absolute bijective soft ring, then $(R^*, \cdot)$ cannot be a cyclic group.

**Proof.** Suppose that $(F, A)$ is a bijective soft ring different from an absolute bijective soft ring over $R$. Let us assume the contrary that $(R^*, \cdot)$ is a cyclic group generated by $r$. Then, any element of $R^*$ is a power of $r$. Hence, all elements of $R^*$ are dependent on $r$ and there exist dependent elements $F(a_1)$ and $F(a_2)$ of $(F, A)$ such that $s \in F(a_1)$, $r \in F(a_2)$ and $s = r^m$ for some $s \in R^*$ and $m \in \mathbb{Z}^+$, which is a contradiction to Proposition 5.7. Consequently, $(R^*, \cdot)$ cannot be a cyclic group. □

**Corollary 5.11.** Let $(F, A)$ be a bijective soft ring over a finite field $K$. Then, $(F, A)$ is not an absolute bijective soft ring,

**Proof.** An immediate consequence of Theorem 5.10 since $(K^*, \cdot)$ is a cyclic group for any finite field $K$. □

**Theorem 5.12.** Let $(F, A)$ be a bijective soft ring over $R$. If all subrings $F(a)$ of $R$, different from $\{0_R\}$ has only two elements, then $R$ has characteristic 2.

**Proof.** Let $F(a_1) = \{0_R\}$ and $F(a) = \{0, r_a\}$, for $a \in A - \{a_1\}$ and $r_a \in R$. Then, $F(a)$ is a subring of $R$, especially it is a subgroup of $(R, +)$. Therefore, we must have

$$r_a + r_a = 2r_a = 0_R$$

which indicates that $x + x = 2x = 0_R$ for all $x \in R$, i.e. $R$ has characteristic 2. □

It can be observed that the ring given in Example 3.3 has characteristic 2.

## 6. Conclusion

Soft set theory was first initiated by Molodtsov [1] as a tool to deal with uncertainty and it was used to solve some kinds of uncertain problems practically. Gong et al. [16] introduced the bijective soft sets and Aktaş [21] introduced bijective soft groups. Here, motivated by [21], we have defined bijective soft rings and unitary bijective soft rings. Moreover, we have defined idealistic bijective soft rings and examined several properties of them. We have also determined some relations between bijective soft rings and rings. Finally, we have defined a dependent soft ring. Beyond these concepts, one can study other similar properties of bijective soft rings and unitary bijective soft rings in future studies.

## Author Contributions

The author read and approved the final version of the paper.

## Conflicts of Interest

The author declares no conflict of interest.

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