On Contra $\pi gs$-Continuity

Nebiye Korkmaz*

Abstract
In this work, a novel form of contra continuity entitled as contra $\pi gs$-continuity is examined, which has connections to $\pi gs$-closed sets. Furthermore, correlations between contra $\pi gs$-continuity and several previously established forms of contra continuous functions are further explored, as well as basic features of contra $\pi gs$-continuous functions are disclosed.

Keywords: $\pi gs$-closed sets, Contra $\pi gs$-continuity, Contra continuity
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1. Introduction

The idea of LC-continuous functions was first introduced and analyzed by Ganster and Reilly [5] in 1989. Dontchev [6] produced contra-continuity, as a more robust variant of LC-continuity in 1996. As a very interesting subject, contra continuous functions have continued to attract the attention of many researchers over the years. After Ekici gave the definition of contra $\pi g$-continuous functions [7] in 2008, contra $\pi gs$-continuous [8] functions were also defined in Caldas et al.’s studies in 2010, which essentially introduced and examined contra $\pi gp$-continuous functions [8].

The requirement that every open set in the codomain possesses a preimage that is $\pi gs$-closed in the domain identifies contra $\pi gs$-continuous functions [8]. A milder version of contra-continuity [6] and contra $gs$-continuity [9] is contra $\pi gs$-continuity. Crucial characteristics of contra $\pi gs$-continuous functions are also examined.

2. Preliminaries

Unless otherwise specified, topological spaces in this work always refer to on which no separation axioms are required; $\Psi$ will stand for the topological space $(\Psi, \tau)$ and $\Phi$ will stand for the topological space $(\Phi, \perp)$; $\mathbb{N}$ will...
stand for any subset of the space $\Psi$. The interior of $\aleph$ is indicated as $int(\aleph)$ and the closure of $\aleph$ in indicated as $cl(\aleph)$. Whenever $\aleph = int(cl(\aleph))$ (correspondingly, $\aleph = cl(int(\aleph))$), afterwards $\aleph$ is a regular closed set (correspondingly, regular open set) [10]. Whenever $\aleph \subset cl(int(\aleph))$, afterwards $\aleph$ is considered as a semi-open set [1]. Whenever $\aleph$ could be expressed as union of regular open sets, afterwards it is accepted as a $\delta$-open set [11]. Complementarity of semi-open set (correspondingly $\delta$-open set) is introduced as semi-closed (correspondingly $\delta$-closed).

The intersection of whole semi-closed sets involving $\aleph$ is known as semi-closure [12] of $\aleph$ which is expressed by $scl(\aleph)$. Dually the semi-interior [12] of $\aleph$ is characterized as union of whole semi-open sets involved in $\aleph$ and indicated by $sint(\aleph)$.

$\nu \in \Psi$ is termed $\delta$-cluster point [11] of $\aleph$, when $int(cl(F)) \cap \aleph \neq \emptyset$ for every $F \in O(\nu, \Psi)$, where $O(\nu, \Psi)$ stands for all open subsets of $\Psi$ containing the point $\nu$. Whole $\delta$-cluster points of $\aleph$ comprises $\delta$-closure [11] of $\aleph$ that is shown with $cl_{\delta}(\aleph)$.

When $\aleph \subset cl(int(cl_{\delta}(\aleph)))$, then $\aleph$ is named as an $e^{*}$-open set [13]. We speak of an $e^{*}$-closed [13] set as complementarity of an $e^{*}$-open. The $e^{*}$-closure [13] of $\aleph$ is the intersection of whole $e^{*}$-closed sets involving subset $\aleph$ and it is symbolized by $e^{*}cl(\aleph)$.

Whenever $e^{*}cl(F) \cap \aleph \neq \emptyset$ for each $e^{*}$-open set $F$ involving point $\nu$, afterwards $\nu$ is identified as $e^{*}$-$\theta$-cluster point [14] of $\aleph$. The $e^{*}$-$\theta$-closure [14] of $\aleph$ is the set of whole $e^{*}$-$\theta$-closed points of $\aleph$, and it is expressed by $e^{*}$-$cl_{\theta}(\aleph)$. For $\aleph = e^{*}cl_{\delta}(\aleph)$, then $\aleph$ is $e^{*}$-$\theta$-closed [15]. $e^{*}$-$BC(\Psi)$ is the notion for the collection of whole $e^{*}$-$\theta$-closed subsets of space $\Psi$.

When for every $\nu$ in $\aleph$, if there exists an $e^{*}$-open set $F$ comprising $\nu$ such that $F \setminus \aleph$ is countable, then $\aleph$ is termed $we^{*}$-open [16]. A $we^{*}$-closed [16] set is the complementarity of an $we^{*}$-open.

When $\aleph \subset cl(int(\aleph)) \cup int(cl(\aleph))$, subsequently $\aleph$ is named as $b$-open [17] (or $sp$-open [18] or $\gamma$-open [19]). A $b$-closed [17] (or $\gamma$-closed [20, 21]) set is the complementarity of a $b$-open (or $\gamma$-open). The $b$-closure [17] (or $\gamma$-closure [20]) of $\aleph$ is expressed as $cl(b) (\aleph)$ or $cl_{b}(\aleph)$ and it is the intersection of whole $b$-closed (or $\gamma$-closed) sets comprising $\aleph$. The set $\aleph$ is said to be pre-closed [22] if $cl(int(\aleph)) \subset \aleph$. The intersection of all pre-closed sets containing $\aleph$ is called pre-closure [20] of $\aleph$ and denoted by $pcl(\aleph)$.

A subset $\aleph$ of a space $\Psi$ is characterized as a $\hat{g}$-closed [23] set, if $cl(\aleph) \subset F$, whenever $F$ is a semi-open set satisfying the condition $\aleph \subset F$. $\hat{g}$-open sets [23] are the complement of $\hat{g}$-closed sets. When $cl(b) (\aleph) \subset F$ whenever $\aleph \subset F$ and $F$ is a $\hat{g}$-open set in $\Psi$, $\aleph$ is a $\hat{g}$-closed [24] set. A $\hat{g}$-open [25] is the complementarity of a $\hat{g}$-closed set. When $\aleph \subset cl(\aleph) \subset F$ whenever $\aleph \subset F$ and $F$ is a $\hat{g}$-open set in $\Psi$, $\aleph$ is called as a $sb\hat{g}$-closed [26] set.

$\pi$-open [27] corresponds to the finite union of regular open sets. $\pi$-closed represents the complementarity of a $\pi$-open. When $\aleph \subset F$ and $F$ is open (correspondingly, $\pi$-open), afterwards $\aleph$ is regarded as a generalized closed (briefly, $g$-closed) [2] (correspondingly, $\pi g$-closed [17]) if $cl(\aleph) \subset F$. $g$-open [24] (correspondingly, $\pi g$-open [7]) is the complementarity of $g$-closed (correspondingly, $\pi g$-closed). While $\aleph \subset F$ and $F$ is open (correspondingly, $\pi$-open), afterwards $\aleph$ is regarded to be generalized semi-closed (briefly, $gs$-closed) [28] (correspondingly, $\pi gs$-closed [4]) if $scl(\aleph) \subset F$. $gs$-open [24] (correspondingly, $\pi gs$-open) constitutes the complementarity of a $gs$-closed (correspondingly, $\pi gs$-closed) set. If $pcl(\aleph) \subset F$ for all $F$ which are $\pi$-open sets containing $\aleph$, then $\aleph$ is called as $\pi g p$-closed [29]. The set $\aleph$ is called as $\pi g s$-closed [20], if $\gamma cl(\aleph) \subset F$ for all $\pi$-open sets $F$ containing $\aleph$.

The entire $\pi gs$-closed (correspondingly, $\pi gs$-open, $\pi gp$-closed, $\pi g\gamma$-closed, $gs$-closed, $gs$-open, closed, semi-closed, semi-open, $\pi g$-open, regular open, regular closed, $g$-closed, $\pi g$-closed, $we^{*}$-closed, $e^{*}$-$\theta$-closed, $\hat{g}$-$\theta$-closed, $sb\hat{g}$-closed) subsets of $\Psi$ are expressed by $\pi G\pi(\Psi)$ (correspondingly, $\pi GSO(\Psi)$, $\pi GPC(\Psi)$, $\pi G\gamma C(\Psi)$, $GSC(\Psi)$, $GSO(\Psi)$, $C(\Psi)$, $SO(\Psi)$, $\gamma O(\Psi)$, $\pi O(\Psi)$, $\pi GO(\Psi)$, $RO(\Psi)$, $RC(\Psi)$, $GC(\Psi)$, $\pi Gc(\Psi)$, $wc^{*} C(\Psi)$, $e^{*} C(\Psi)$, $e^{*} \theta C(\Psi)$, $\hat{g} \theta C(\Psi)$, $sb\hat{g} \theta C(\Psi)$).

$\pi G(\nu, \Psi)$ (correspondingly, $\pi GSO(\nu, \Psi)$, $RO(\nu, \Psi)$, $C(\nu, \Psi)$, $SO(\nu, \Psi)$, $O(\nu, \Psi)$) means the collection of whole $\pi gs$-closed (correspondingly, $\pi gs$-open, regular open, closed, semi-open, open) sets of $\Psi$ comprising point $\nu \in \Psi$.

$\pi g s$-closure of the set $\aleph$ is denoted by $Cl_{\pi g s}(\aleph)$, which is the intersection of whole $\pi gs$-closed sets involving $\aleph$. On the other hand, $\pi gs$-interior of a set $\aleph$ is expressed by $int_{\pi g s}(\aleph)$, which corresponds to the union of whole $\pi gs$-open sets included in $\aleph$.

**Definition 2.1.** A topological space $\Psi$ is said to be:
1. $(i)$ strongly $S$-closed [6] while a finite subcover matching could found for each closed cover of $\Psi$,
2. $(ii)$ strongly countably $S$-closed [7] when a finite subcover matching found for each countable cover of $\Psi$ consisting of closed sets,
3. $(iii)$ strongly $S$-Lindelöf [7] when a countable subcover matching could found for each closed cover of $\Psi$,
4. $(iv)$ ultra normal [30] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets,
5. $(v)$ ultra Hausdorff [30] if for each couple of distinct points, $v_1$ and $v_2$ in $\Psi$ there exist clopen sets $\aleph_1$ and $\aleph_2$ comprising $v_1$ and $v_2$ correspondingly, providing $\aleph_1 \cap \aleph_2 = \emptyset$ equality.

**Definition 2.2.** When $\aleph \in \Psi$ is strongly $S$-closed as a subspace, then $\aleph$ is named strongly $S$-closed [6].
Definition 2.3. \( \mathfrak{N} \) in \( \Psi \) is called:

(i) \( \alpha \)-open [31] whenever \( \mathfrak{N} \subset int(cl(int(\mathbb{R}))) \),

(ii) preopen [22] or nearly open [5] whenever \( \mathfrak{N} \subset int(cl(\mathfrak{N})) \),

(iii) \( \beta \)-open [32] or semi-preopen [33] whenever \( \mathfrak{N} \subset cl(int(cl(\mathfrak{N}))) \).

Complement of an \( \alpha \)-open (correspondingly, preopen, \( \beta \)-open) set is introduced as \( \alpha \)-closed (correspondingly, preclosed, \( \beta \)-closed) set [7]. \( \alpha O(\Psi) \) (correspondingly, \( PO(\Psi), \beta O(\Psi) \)) stands for the collection of whole \( \alpha \)-open (correspondingly, preopen, \( \beta \)-open) subsets of \( \Psi \).

Lemma 2.1. Whenever \( \mathfrak{N} \subset \Psi \),

\( (i) \ cl_{\pi gs}(\Psi \setminus \mathfrak{N}) = \Psi \setminus int_{\pi gs}(\mathfrak{N}) \);

\( (ii) \ \nu \in cl_{\pi gs}(\mathfrak{N}) \Leftrightarrow \forall F \in \pi GS(\nu, \Psi), \mathfrak{N} \cap F \neq \emptyset \).

Proof. Before starting the proof, let’s remind the definitions of \( \pi gs \)-interior and \( \pi gs \)-closure of a set in a topological space. Let \( (\Psi, \tau) \) be a topological space, \( \mathfrak{N} \subset \Psi \). Then, \( \pi gs \)-closure of \( \mathfrak{N} \) is \( cl_{\pi gs}(\mathfrak{N}) = \bigcap \{ \Theta : \mathfrak{N} \subset \Theta, \Theta \in \pi GSC(\Psi) \} \) and \( \pi gs \)-interior of \( \mathfrak{N} \) is \( int_{\pi gs}(\mathfrak{N}) = \bigcup \{ \Theta : \mathfrak{N} \subset \Theta, \Theta \in \pi GSO(\Psi) \} \). Now we can start the proof.

\( (i) \): We will complete the proof by showing that the sets claimed to be equal include each other. Let \( (\Psi, \tau) \) be a topological space and \( \mathfrak{N} \subset \Psi \).

\( (\Rightarrow) \): Let \( \nu \in cl_{\pi gs}(\Psi \setminus \mathfrak{N}) \). Assume that \( \nu \notin \Psi \setminus int_{\pi gs}(\mathfrak{N}) \). Since \( \nu \in int_{\pi gs}(\mathfrak{N}) \) then \( \nu \notin \pi GS(\nu, \Psi) \), it can be said that there exists a set \( F \subset \pi GS(\nu, \Psi) \) and \( F \subset \mathfrak{N} \). Then using \( \Theta \) we can conclude that \( \nu \notin cl_{\pi gs}(\Psi \setminus \mathfrak{N}) \) contrary to our assumption. Hence as a result \( \nu \in cl_{\pi gs}(\Psi \setminus \mathfrak{N}) \).

\( (\Leftarrow) \): Let \( \nu \in \Psi \setminus int_{\pi gs}(\mathfrak{N}) \). So it can be clearly seen that \( \nu \notin int_{\pi gs}(\mathfrak{N}) \). Then for all the sets \( \Theta \subset \pi GS(\nu, \Psi) \), we have a contradiction \( \nu \notin cl_{\pi gs}(\mathfrak{N}) \). Thus the proof is completed.

Example 2.1. Consider the subset \( \mathfrak{N} = \{ \nu_1, \nu_2 \} \) of the set \( \Psi = \{ \nu_1, \nu_2, \nu_3, \nu_4, \nu_5 \} \) and the topological space \( (\Psi, \tau) \), where \( \tau = \{ \emptyset, \{ \nu_1 \}, \{ \nu_2 \}, \{ \nu_1, \nu_2 \}, \{ \nu_1, \nu_3, \nu_4, \nu_5 \}, \Psi \} \). Then the set \( \mathfrak{N} \) is an acceptable sample that fits the given situation just above, since \( \mathfrak{N} = int_{\pi gs}(\mathfrak{N}) \), while \( \mathfrak{N} \notin \pi GSC(\Psi) \).

\( ker(\mathfrak{U}) \) [34] means \( \bigcap \{ F : F \subset \Psi \} \) which is known as the kernel of \( \mathfrak{U} \).

Lemma 2.2. [35] The subsequent characteristics apply to subsets \( F \) and \( \mathfrak{U} \) of \( \Psi \):

\( (i) \nu \in ker(F) \Leftrightarrow (\forall \Theta \subset C(\nu, \Psi))(F \cap \mathfrak{U} \neq \emptyset) \);

\( (ii) \ F \subset ker(F) \);

\( (ii) \ F \subset ker(F) \);

\( (iv) \ F \subset \mathfrak{U} \Rightarrow ker(F) \subset ker(\mathfrak{U}) \).

3. Contra \( \pi gs \)-continuous functions

In this section, first the characterization of contra \( \pi gs \)-continuous functions is presented. Afterwards, the relationships between some types of contra continuous functions and contra \( \pi gs \)-continuous functions were examined. In addition, some new definitions in relation with \( \pi gs \)-open sets are given in order to examine various properties of contra \( \pi gs \)-continuous functions, and these properties are presented through theorems and results.

Definition 3.1. \( \Delta : (\Psi, \tau) \to (\Phi, \downarrow) \) is referred as contra \( \pi gs \)-continuous [8], whenever \( \Delta^{-1}(\mathfrak{U}) \in \pi GS(\Psi) \) for each \( \mathfrak{U} \in \downarrow \).
Theorem 3.1. Under the assumption $\pi GSO(\Psi)$ is closed under arbitrary unions (or likewise $\pi GSC(\Psi)$ is closed under arbitrary intersections), subsequent statements are coequal for $\Delta: (\Psi, \top) \to (\Phi, \bot)$.

$\alpha \Delta$ is contra $\pi g$s-continuous:

$(\alpha_i) \exists \in C(\Phi) \implies \Delta^{-1}(\exists) \in \pi GSO(\Psi)$;

$(\alpha_{ii}) (\forall \nu \in \Psi)(\forall \Theta \in C(\Delta(\nu), \Phi))(\exists F \in \pi GSO(\nu, \Psi))(\Delta(F) \subset \Theta)$;

$(\alpha_{iv}) \forall \Theta \in C(\Delta(\nu), \Phi) \exists F \in \pi GSO(\nu, \Psi)$ such that $\Delta(F) \subset \Theta$.

$(\alpha_{v}) \exists \in C(\Phi) \implies \Delta^{-1}(\exists) \subset \pi GSO(\Psi)$.$\Phi$.

Proof. Let $\Delta: (\Psi, \top) \to (\Phi, \bot)$ be a function, where $(\Psi, \top)$ and $(\Phi, \bot)$ are two topological spaces and let $\pi GSO(\Psi)$ be closed under arbitrary unions (or likewise $\pi GSC(\Psi)$ be closed under arbitrary intersections).

$(\alpha_i) \implies (\alpha_{ii})$: Let $\Theta \in C(\Phi)$. Then $\Phi \setminus \Theta$ is open in $\Phi$. Since $\Delta$ is contra $\pi g$s-continuous, $\Psi \setminus \Delta^{-1}(\Theta) = \Delta^{-1}(\Phi \setminus \Theta)$ is $\pi g$s-closed in $\Psi$. Therefore, $\Delta^{-1}(\Theta)$ is $\pi g$s-open in $\Psi$.

$(\alpha_{ii}) \implies (\alpha_i)$: Obvious.

$(\alpha_{iv}) \implies (\alpha_{ii})$: Let $\nu \in \Psi$ and $\Theta \in C(\Delta(\nu), \Phi)$. Then by $(\alpha_i)$, we have $\Delta^{-1}(\Theta) \in \pi GSO(\Psi)$. Choosing $F = \Delta^{-1}(\Theta)$ we obtain that $F \in \pi GSO(\nu, \Psi)$ and $\Delta(F) \subset \Theta$.

$(\alpha_{v}) \implies (\alpha_{ii})$: Let $\Theta \in C(\Phi)$ and $\nu \in \Delta^{-1}(\Theta)$. Since $\Delta(\nu) \in \Theta$, by $(\alpha_{v})$ there exist a $\pi g$s-open set $F_\nu \in \pi GSO(\nu, \Psi)$ such that $\Delta(F_\nu) \subset \Theta$. So we have $\nu \in F_\nu \subset \Delta^{-1}(\Theta)$ and hence $\Delta^{-1}(\Theta) = \bigcup \{F_\nu : \nu \in \Delta^{-1}(\Theta)\}$ is $\pi g$s-open in $\Psi$ since $\pi GSO(\Psi)$ is closed under arbitrary unions.

$(\alpha_{vi}) \implies (\alpha_{iv})$: Let $\Psi$ be any subset of $\Psi$. Suppose that there exist an element $\mu \in \Delta(cl_{\pi g}s(\mathbb{N}))$ such that $\mu \notin \ker(\Delta(\mathbb{N}))$. Then there exists an open set $F$ of $\Phi$ such that $\Delta(\mathbb{N}) \subset F$ and $\mu \notin F$. Hence, there exists $\Theta = \Phi \setminus F \in C(\mu, \Phi)$ such that $\Delta(\mathbb{N}) \cap \Theta = \emptyset$ and $cl_{\pi g}s(\mathbb{N}) \subset \Delta^{-1}(\Theta) = \emptyset$. From here we obtain that $\Delta(cl_{\pi g}s(\mathbb{N})) \cap \Theta = \emptyset$ and $\mu \notin \Delta(cl_{\pi g}s(\mathbb{N}))$ which is a contradiction.

$(\alpha_{v}) \implies (\alpha_{iv})$: Let $\exists \in \Psi$. Then $\Delta^{-1}(\exists) \subset \Psi$. By $(\alpha_{v})$, $\Delta(cl_{\pi g}s(\Delta^{-1}(\exists))) \subset \ker(\Delta^{-1}(\exists)) \subset \ker(\exists)$.

$(\alpha_{v}) \implies (\alpha_i)$: Let $F$ be any open subset of $\Phi$. Then by $(\alpha_{iv})$ and by Lemma 2.2, $cl_{\pi g}s(\Delta^{-1}(F)) \subset \Delta^{-1}(\ker(F)) = \Delta^{-1}(F)$. So we have $cl_{\pi g}s(\Delta^{-1}(F)) = \Delta^{-1}(F)$. Since $\pi GSO(\Psi)$ is closed under arbitrary unions, $\pi GSC(\Psi)$ is closed under arbitrary intersections and hence $\Delta^{-1}(F) = cl_{\pi g}s(\Delta^{-1}(F))$ is $\pi g$s-closed.

Remark 3.1. Statements $(\alpha_i)$ and $(\alpha_{ii})$ in Theorem 3.1 are identical even if $\pi GSO(\Psi)$ is not closed under arbitrary unions (or likewise, $\pi GSC(\Psi)$ is not closed under arbitrary intersections).

Definition 3.2. $\Delta: (\Psi, \top) \to (\Phi, \bot)$ is categorized as:

$(\theta_1)$ perfectly continuous [36] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in \top \cap C(\Psi))$;

$(\theta_2)$ RC-continuous [9] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in RC(\Psi))$;

$(\theta_3)$ strongly continuous [37] $\iff (F \in \top \implies \Delta^{-1}(F) \in \top \cap C(\Psi))$ (identically $\mathbb{N} \subset \Psi \implies \Delta(cl(\mathbb{N})) \subset \Delta(\mathbb{N}))$;

$(\theta_4)$ contra-continuous [6] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in C(\Psi))$;

$(\theta_5)$ contra-super-continuous [38] $\iff (\forall \nu \in \Psi)(\forall \Theta \in C(\Delta(\nu), \Phi))(\exists F \in RO(\nu, \Psi))(\Delta(F) \subset \Theta)$;

$(\theta_6)$ contra-semicontinuous [9] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in SC(\Psi))$;

$(\theta_7)$ contra g-continuous [39] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in GC(\Psi))$;

$(\theta_8)$ contra gs-continuous [9] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in GSC(\Psi))$;

$(\theta_9)$ contra $\pi g$s-continuous [7] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in \pi GCS(\Psi))$;

$(\theta_{10})$ contra we$^*$-continuous [16] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in we^*C(\Psi))$;

$(\theta_{11})$ contra e$^*$-continuous [40] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in e^*C(\Psi))$;

$(\theta_{12})$ contra e$^*$-continuous [41] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in e^*C(\Psi))$;

$(\theta_{13})$ almost contra e$^*$-continuous [42] $\iff (F \in RO(\Phi) \implies \Delta^{-1}(F) \in e^*C(\Psi))$;

$(\theta_{14})$ almost contra e$^*$-continuous [42] $\iff (F \in RO(\Phi) \implies \Delta^{-1}(F) \in e^*C(\Psi))$;

$(\theta_{15})$ contra b$g$s-continuous [25] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in b\Delta(C(\Psi)))$;

$(\theta_{16})$ contra sb$g$s-continuous [43] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in sb\Delta(C(\Psi)))$;

$(\theta_{17})$ contra $\pi g$p-continuous function [8] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in \pi GPC(\Psi))$;

$(\theta_{18})$ contra $\pi g\gamma$-continuous function [20] $\iff (F \in \downarrow \implies \Delta^{-1}(F) \in \pi G\gamma C(\Psi))$. 


Remark 3.2.

\[
\begin{array}{c}
\iota_6 \leftarrow \iota_4 \leftarrow \iota_5 \leftarrow \iota_1 \leftarrow \iota_3 \\
\downarrow \downarrow \\
\iota_8 \leftarrow \iota_7 \\
\downarrow \downarrow \\
\text{contra } \pi gs\text{-continuous} \leftarrow \iota_9 \\
\downarrow \downarrow \\
\iota_{18} \leftarrow \iota_{17}
\end{array}
\]

Remark 3.3. As can be seen from the samples below, reversibility of the consequences in the above diagram need not to be true.

Example 3.1. \(T = \{\emptyset, \nu_2, \nu_1, \nu_4, \nu_2, \nu_3, \nu_1, \nu_2, \nu_3, \nu_4\}, \Psi\) is the topology on \(\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}\). Since mappings under \(\Delta : \Psi \rightarrow \Psi\) are listed as \(\Delta(\nu_1) = \nu_1, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \nu_3, \Delta(\nu_4) = \nu_4\) the contra \(\pi gs\)-continuity of \(\Delta\) is evident. However, it is neither contra \(\pi g\)-continuous nor contra gs-continuous since \(\Delta^{-1}(\{\nu_2\}) = \{\nu_2\} \notin \pi GC(\Psi)\) and \(\Delta^{-1}(\{\nu_2\}) = \{\nu_2\} \notin \pi GSC(\Psi)\).

Example 3.2. Let \(\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}, \ T = \{\emptyset, \nu_1, \nu_2, \nu_1, \nu_2, \nu_4\}\). Match-ups for \(\Delta : \Psi \rightarrow \Psi\) are

\[\Delta(\nu_1) = \Delta(\nu_2) = \Delta(\nu_3) = \nu_1, \Delta(\nu_4) = \nu_3.\]

\(\Delta\) is contra \(\pi gs\)-continuous, but it is not contra \(e^*\theta\)-continuous since \(\Delta^{-1}(\{\nu_1\}) = \Delta^{-1}(\{\nu_1, \nu_2\}) = \{\nu_1, \nu_2, \nu_3\}\) is not \(e^*\theta\)-closed w.r.t. \(T\).

Example 3.3. Given \(\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}, \ T = \{\emptyset, \nu_1, \nu_2, \nu_1, \nu_2, \nu_4\}\). Match-ups for \(\Delta : \Psi \rightarrow \Psi\) are

\[\Delta(\nu_1) = \nu_3, \Delta(\nu_2) = \nu_1, \Delta(\nu_3) = \Delta(\nu_4) = \nu_4.\]

Although \(\Delta\) is contra \(\pi gs\)-continuous, it is not almost contra \(e^*\theta\)-continuous, since \(\{\nu_1, \nu_3\}\) is regular open and \(\Delta^{-1}(\{\nu_1, \nu_3\}) = \{\nu_1, \nu_2\}\) is not an \(e^*\)-closed. By checking the connections between these class of functions in [42] we can easily state that \(\Delta\) cannot be almost contra \(e^*\theta\)-continuous, contra \(e^*\theta\)-continuous and contra \(e^*\theta\)-continuous.

Example 3.4. \(T = \{\emptyset, \nu_1, \nu_2, \nu_1, \nu_3, \nu_1, \nu_1, \nu_3, \nu_2, \nu_1, \nu_2, \nu_4\}\) is a topology on \(\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}\). Match-ups of \(\Delta : \Psi \rightarrow \Psi\) are

\[\Delta(\nu_1) = \nu_3, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \Delta(\nu_4) = \nu_4.\]

Since \(\Delta^{-1}(\{\nu_1, \nu_2\}) = \{\nu_1, \nu_2, \nu_3\}\) is not \(\pi GSC(\Psi)\), \(\Delta\) is not contra \(\pi gs\)-continuous. However, it is contra \(e^*\theta\)-continuous. So it is contra \(e^*\)-continuous, almost contra \(e^*\theta\)-continuous and almost contra \(e^*\theta\)-continuous.

As seen from the examples above contra \(\pi gs\)-continuity does not require almost contra \(e^*\theta\)-continuity, almost contra \(e^*\)-continuity, contra \(e^*\theta\)-continuity and contra \(e^*\)-continuity. It is also clear that almost contra \(e^*\theta\)-continuity, almost contra \(e^*\)-continuity, contra \(e^*\theta\)-continuity and contra \(e^*\)-continuity does not require contra \(\pi gs\)-continuity. As another result we can state that contra \(we^*\)-continuity does not require contra \(\pi gs\)-continuity.

Research Question Does contra \(\pi gs\)-continuity require contra \(we^*\)-continuity?

Example 3.5. \(T = \{\emptyset, \nu_1, \nu_2, \nu_1, \nu_3, \nu_1, \nu_4, \nu_2, \nu_1, \nu_4\}\) is a topology on \(\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}\). Match-ups of \(\Delta : \Psi \rightarrow \Psi\) are

\[\Delta(\nu_1) = \nu_3, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \nu_1, \Delta(\nu_4) = \nu_2.\]

\(\Delta\) is contra \(\pi gs\)-continuous, but it is not contra \(bg\)-continuous since \(\Delta^{-1}(\{\nu_1, \nu_3\}) = \{\nu_1, \nu_3\}\) is not \(bg\)-closed. So it cannot be contra \(bg\)-continuous.

Example 3.6. \(T = \{\emptyset, \nu_1, \nu_5, \nu_2, \nu_4, \nu_1, \nu_2, \nu_4, \nu_5\}\) is a topology on \(\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5\}\). Match-ups of \(\Delta : \Psi \rightarrow \Psi\) are

\[\Delta(\nu_1) = \nu_1, \Delta(\nu_2) = \nu_2, \Delta(\nu_3) = \Delta(\nu_4) = \nu_3, \Delta(\nu_5) = \nu_5.\]

\(\Delta\) is contra \(bg\)-continuous. However, since \(\Delta^{-1}(\{\nu_1, \nu_2, \nu_5\}) = \{\nu_1, \nu_2, \nu_5\} \notin \pi GSC(\Psi)\), it is not contra \(\pi gs\)-continuous.
As seen from the examples above there is no relation between contra $b\hat{g}$-continuity and contra $\pi gs$-continuity. As another result we see that a contra $\pi gs$-continuity does not require contra $sb\hat{g}$-continuity.

**Research Question** Does contra $sb\hat{g}$-continuity require contra $\pi gs$-continuity?

**Example 3.7.** [8] Let $\mathcal{T} = \{\emptyset, \{\nu_1\}, \{\nu_2\}, \{\nu_2, \nu_1\}, \{\nu_3, \nu_2\}, \{\nu_3, \nu_2, \nu_1\}, \Psi\}$ and $\perp = \{\emptyset, \{\nu_1\}, \Psi\}$ be two topologies on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. The identity function $\Delta : (\Psi, \mathcal{T}) \to (\Psi, \perp)$ is contra $\pi gs$-continuous, but it is not contra $\pi gp$-continuous.

**Example 3.8.** [8] Let $\mathcal{T} = \{\emptyset, \{\nu_2\}, \{\nu_2, \nu_1\}, \{\nu_1, \nu_4\}, \{\nu_2, \nu_4, \nu_4\}, \Psi\}$ and $\perp = \{\emptyset, \{\nu_4\}, \Psi\}$ be two topologies on $\Psi = \{\nu_1, \nu_2, \nu_3, \nu_4\}$. The identity function $\Delta : (\Psi, \mathcal{T}) \to (\Psi, \perp)$ is contra $\pi gp$-continuous and contra $\pi g\gamma$-continuous, but it is not contra $\pi gs$-continuous.

As seen from Example 3.7 and Example 3.8 there is no connection between contra $\pi gp$-continuity and contra $\pi gs$-continuity. Example 3.8 also shows that contra $\pi g\gamma$-continuity does not require contra $\pi gs$-continuity.

**Theorem 3.2.** [4] Let $\mathcal{K} \subset \Psi$, afterwards $\mathcal{K} \in RO(\Psi)$ if and only if $\mathcal{K} \in \pi O(\Psi) \cap \pi GSC(\Psi)$.

**Definition 3.3.** $\Delta : \Psi \to \Phi$ is called as:
1. $\pi$-continuous [3] $\iff (F \in \perp \Rightarrow \Delta^{-1}(F) \in \pi O(\Psi))$,
2. $\pi g$-continuous [3] $\iff (F \in \perp \Rightarrow \Delta^{-1}(F) \in \pi GO(\Psi))$,
3. $\pi gs$-continuous [4] $\iff (F \in C(\Phi) \Rightarrow \Delta^{-1}(F) \in \pi GSC(\Psi))$,
4. completely continuous [44] $\iff (F \in \perp \Rightarrow \Delta^{-1}(F) \in RO(\Psi))$.

**Theorem 3.3.** Whenever $\Delta : \Psi \to \Phi$, afterwards the statement below is satisfied; $\Delta$ is contra $\pi gs$-continuous and $\pi$-continuous if and only if $\Delta$ is completely continuous.

**Proof.** Obvious from Theorem 3.2.

**Theorem 3.4.** Under the circumstance $\pi GSO(\Psi)$ is closed under arbitrary unions, it can be stated that whenever $\Delta : \Psi \to \Phi$ is contra $\pi gs$-continuous and $\Phi$ is regular, afterwards $\Delta$ is $\pi gs$-continuous.

**Definition 3.4.** Whenever $\pi GSC(\Psi) \subset SC(\Psi)$ afterwards $\Psi$ is accepted as $\pi gs-T_2$ [4].

**Theorem 3.5.** Whenever $\Psi$ is considered as $\pi gs-T_2$ space afterwards, contra $\pi gs$-continuity, contra-semicontinuity and contra $gs$-continuity of $\Delta : \Psi \to \Phi$ are identical.

**Proof.** Assume that $\Psi$ as a $\pi gs-T_2$ space. Since $SC(\Psi) \subset \pi GSC(\Psi)$, we have $SC(\Psi) = \pi GSC(\Psi)$. Using the relation $SC(\Psi) \subset GSC(\Psi)$, we obtain $\pi GSC(\Psi) \subset GSC(\Psi)$. Since $GSC(\Psi) \subset \pi GSC(\Psi)$, we have $GSC(\Psi) = \pi GSC(\Psi)$. Therefore $\pi GSC(\Psi) = SC(\Psi) = GSC(\Psi)$.

**Theorem 3.6.** For each $i \in I$, $p_i$ stands for projection of $\prod \Phi_i$ onto $\Phi_i$. If $\Delta : \Psi \to \prod \Phi_i$ is contra $\pi gs$-continuous, then $p_i \circ \Delta : \Psi \to \Phi_i$ is contra $\pi gs$-continuous for each $i \in I$.

**Proof.** Since $p_i$ is continuous and $\Delta$ is contra $\pi gs$-continuous, we can state that $p_i^{-1}(U_i)$ is open in $\prod Y_i$ for any $U_i \in \perp_i$ and $(p_i \circ \Delta)^{-1}(U_i) = \Delta^{-1}(p_i^{-1}(U_i)) \in \pi GSC(\Psi)$. Hereby, $p_i \circ \Delta$ is contra $\pi gs$-continuous.

**Definition 3.5.** A topological space $\Psi$ is said to be locally $\pi gs$-indiscrete if $\pi GSO(\Psi) \subset C(\Psi)$.

**Theorem 3.7.** The fact that $\Psi$ is locally $\pi gs$-indiscrete for contra $\pi gs$-continuous $\Delta : \Psi \to \Phi$ requires that $\Delta$ is continuous.

**Proof.** Allow $F \in \perp$. Since $\Delta$ is contra $\pi gs$-continuous, $\Delta^{-1}(F) \in \pi GSC(\Psi)$. Since $\Psi$ is locally $\pi gs$-indiscrete, $\Delta^{-1}(F) \in \top$.

**Theorem 3.8.** Whenever $\Psi$ is a $\pi gs-T_2$ for any $\Delta : \Psi \to \Phi$, afterwards following are equivalent:
1. $\Delta$ is completely continuous;
2. $\Delta$ is $\pi$-continuous and contra $\pi gs$-continuous;
3. $\Delta$ is $\pi$-continuous and contra $gs$-continuous;
4. $\Delta$ is $\pi$-continuous and contra-semicontinuous.
Proof. Equivalence of (i_2), (i_3) and (i_4) is obvious from Theorem 3.5 and the equivalence of (i_1) and (i_2) can be easily seen from Theorem 3.2. □

Definition 3.6. The topological space $(\Psi, \tau)$ is called:

(i) submaximal [45] if $\forall N \subset \Psi (\text{cl}(N) = \Psi \Rightarrow N \in \tau)$,
(ii) extremally disconnected [45] if $\forall N \subset \Psi (N \in \tau \Rightarrow \text{cl}(N) \in \tau)$.

Definition 3.7. $\Delta : \Psi \to \Phi$ is called contra $\alpha$-continuous [46] (correspondingly contra precontinuous [46], contra $\beta$-continuous [47], contra $\gamma$-continuous [48]) if the preimage of every open subset of $\Phi$ is $\alpha$-closed (correspondingly preclosed, $\beta$-closed, $\gamma$-closed) in $\Psi$.

Lemma 3.1. For any $(\Psi, \tau)$, if $\pi GSC(\Psi)$ is closed under finite unions then, $\pi gs - \tau = \{ U \subset \Psi : \text{cl}_{\pi gs}(\Psi \setminus U) = \Psi \setminus U \}$.

Theorem 3.9. Whenever $\Psi$ is extremally disconnected, submaximal and $\pi gs$-T_2 for any $\Delta : \Psi \to \Phi$, afterwards the following are equivalent:

(i) $\Delta$ is contra $\pi gs$-continuous;
(ii) $\Delta$ is contra $gs$-continuous;
(iii) $\Delta$ is contra-semicontinuous;
(iv) $\Delta$ is contra-continuous;
(v) $\Delta$ is contra precontinuous;
(vi) $\Delta$ is contra $\beta$-continuous;
(vii) $\Delta$ is contra $\alpha$-continuous;
(viii) $\Delta$ is contra $\gamma$-continuous.

Proof. In an extremally disconnected submaximal space $(\Psi, \tau)$,

$$\tau = \alpha O(\Psi) = SO(\Psi) = PO(\Psi) = \gamma O(\Psi) = \beta O(\Psi).$$

From this fact we can say that (i_3), (i_4), (i_5), (i_6), (i_7), (i_8) are equivalent. The equivalence of (i_1), (i_2), (i_3) is obvious from Theorem 3.5. □

Theorem 3.10. Whenever $\Psi$ is said to be extremally disconnected, afterwards any $\Delta : \Psi \to \Phi$ is contra $\pi gs$-continuous and $\pi gs$-continuous.

Definition 3.8. $\Delta : \Psi \to \Phi$ is said to be $\pi gs$-irresolute [4] if $\Delta^{-1}(F) \in \pi GSO(\Psi)$ for each $F \in \pi GSO(\Phi)$.

Theorem 3.11. For $\Delta : \Psi \to \Phi$ and $\rho : \Phi \to \zeta$ following properties hold:

(i) If $\Delta$ is $\pi gs$-irresolute and $\rho$ is contra $\pi gs$-continuous, then $\rho \circ \Delta$ is contra $\pi gs$-continuous;
(ii) If $\Delta$ is contra $\pi gs$-continuous and $\rho$ is continuous, then $\rho \circ \Delta$ is contra $\pi gs$-continuous;
(iii) If $\Delta$ is $\pi gs$-continuous and $\rho$ is RC-continuous, then $\rho \circ \Delta$ is $\pi gs$-continuous;
(iv) If $\Delta$ is $\pi gs$-continuous and $\rho$ is contra continuous, then $\rho \circ \Delta$ is contra $\pi gs$-continuous;
(v) If $\Delta$ is $\pi gs$-irresolute and $\rho$ is RC-continuous (correspondingly contra $\pi$-continuous, contra-continuous, contra $g$-continuous, contra $\pi$-continuous, contra $g$-continuous, contra $\pi$-continuous, contra $g$-continuous), then $\rho \circ \Delta$ is contra $\pi gs$-continuous.

Definition 3.9. $\Delta : \Psi \to \Phi$ is characterized as $\pi gs$-open if $\Delta(8)$ is $\pi gs$-open in $\Phi$ for each $\pi gs$-open subset $8$ of $\Psi$.

Theorem 3.12. $\Delta : \Psi \to \Phi$ and $\rho : \Phi \to \zeta$ be two functions and suppose that $\pi GSC(\Phi)$ is closed under arbitrary intersections. Whenever $\Delta$ is surjective $\pi gs$-open function and $\rho \circ \Delta$ is contra $\pi gs$-continuous, afterwards $\rho$ is contra $\pi gs$-continuous.

Proof. Suppose $\mu \in \Phi$ and $\Theta \in C(\rho(\mu), \zeta)$. Since $\Delta$ is surjective, existence of $\nu \in \Psi$ satisfying $\Delta(\nu) = \mu$ is clear. Naturally, $\Theta \in C(\rho \circ \Delta(\nu), \zeta)$. Since $\rho \circ \Delta$ is contra $\pi gs$-continuous, $\zeta \in \pi GSO(\nu, \Phi)$ naturally appears satisfying $\rho \circ \Delta(\zeta) \subset \Theta$ relation. Since $\Delta$ is $\pi gs$-open, $\Delta(\zeta)$ is an element of $\pi GSO(\mu, \Phi)$. Hence, for each $\mu \in \Phi$ and for each $\Theta \in C(\rho(\mu), \zeta)$, existence of $\Delta(\zeta) = F \in \pi GSO(\mu, \Phi)$ is natural satisfying $\rho(F) \subset \Theta$. By Theorem 3.1 $\rho$ is contra $\pi gs$-continuous. □

Corollary 3.1. Whenever $\pi GSC(\Phi)$ is closed under arbitrary intersections and $\Delta : \Psi \to \Phi$ is surjective $\pi gs$-irresolute and $\pi gs$-open, afterwards for any $\rho : \Phi \to \zeta$, $\rho \circ \Delta$ is contra $\pi gs$-continuous if and only if $\rho$ is contra $\pi gs$-continuous.

Proof. Obvious from Theorems 3.11 and 3.12. □
Definition 3.10. \( \Delta : \Psi \to \Phi \) is characterized as weakly contra \( \pi_{gs} \)-continuous whenever \( \nu \in \Psi \) and \( \Theta \in C(\Delta(\nu), \Phi) \), afterwards a set \( F \in \pi \text{GSO}(\nu, \Psi) \) exists satisfying \( \text{int}(\Delta(F)) \subset \Theta \).

Definition 3.11. A function \( \Delta : \Psi \to \Phi \) is called as \( (\pi_{gs}s) \)-open whenever \( \Delta(F) \in \text{SO}(\Phi) \) for all \( F \in \pi \text{GSO}(\Psi) \).

Theorem 3.13. Whenever \( \Delta : \Psi \to \Phi \) is a weakly contra \( \pi_{gs} \)-continuous and \( (\pi_{gs}s) \)-open and \( \pi \text{GSO}(\Psi) \) is closed under arbitrary unions, afterwards \( \Delta \) is contra \( \pi_{gs} \)-continuous.

Proof. Whenever \( \nu \in \Psi \) and \( \Theta \in C(\Delta(\nu), \Phi) \), with the weakly contra \( \pi_{gs} \)-continuity of \( \Delta \), as a result the set \( F \in \pi \text{GSO}(\nu, \Psi) \) appears satisfying \( \text{int}(\Delta(F)) \subset \Theta \). Since \( \Delta \) is \( (\pi_{gs}s) \)-open, \( \Delta(F) \) is semi-open in \( \Phi \). Hence, \( \Delta(F) \subset \text{cl}(\text{int}(\Delta(F))) \subset \text{cl}(\Theta) = \Theta \). \( \square \)

Definition 3.12. \( \text{fr}_{\pi_{gs}}(\Psi) \) stands for \( \pi_{gs} \)-frontier of \( \Psi \) and characterized as \( \text{cl}_{\pi_{gs}}(\Psi) \cap \text{cl}_{\pi_{gs}}(\Psi \setminus \Psi) \).

Theorem 3.14. Let \( \Delta : \Psi \to \Phi \) be a function. Whenever \( \pi \text{GSC}(\Psi) \) is closed under arbitrary intersections then, the set of whole points \( \nu \in \Psi \) at which \( \Delta \) is not contra \( \pi_{gs} \)-continuous is equal to \( \bigcup \{ \text{fr}_{\pi_{gs}}(\Delta^{-1}(\Theta)) : \Theta \in C(\Delta(\nu), \Phi) \} \).

Proof. Let \( \nu \) be any element of \( \Psi \) at which \( \Delta \) is not contra \( \pi_{gs} \)-continuous. Then, there exists a closed subset \( \Theta \) of \( \Phi \) comprising \( \Delta(\nu) \) such that \( \Delta(F) \) is not contained in \( \Theta \) for every \( F \in \pi \text{GSO}(\nu, \Psi) \). So \( F \cap (\Psi \setminus \Delta^{-1}(\Theta)) \neq \emptyset \). Then, we have \( \nu \in \text{cl}_{\pi_{gs}}(\Psi \setminus \Delta^{-1}(\Theta)) \). Since \( \nu \in \Delta^{-1}(\Theta) \subset \text{cl}_{\pi_{gs}}(\Delta^{-1}(\Theta)), \nu \in \text{fr}_{\pi_{gs}}(\Delta^{-1}(\Theta)) \).

For the converse, assume that \( \Delta \) is contra \( \pi_{gs} \)-continuous at \( \nu \in \Psi \) and \( \Theta \in C(\Delta(\nu), \Phi) \). Naturally a set \( F \in \pi \text{GSO}(\nu, \Psi) \) appears satisfying \( F \subset \Delta^{-1}(\Theta) \). Therefore, \( \nu \in \text{int}_{\pi_{gs}}(\Delta^{-1}(\Theta)) \). Hence, \( \nu \notin \text{fr}_{\pi_{gs}}(\Delta^{-1}(\Theta)) \). \( \square \)

Corollary 3.2. For any \( \Delta : \Psi \to \Phi \), whenever \( \pi \text{GSC}(\Psi) \) is closed under arbitrary intersections, afterwards \( \Delta \) is not contra \( \pi_{gs} \)-continuous at \( \nu \) if and only if \( \Theta \in C(\Delta(\nu), \Phi) \) appears satisfying \( \nu \in \text{fr}_{\pi_{gs}}(\Delta^{-1}(\Theta)) \).

### 4. Preservation theorems

In this section, new separation axioms, connected spaces, compact spaces, covers and graphs related to \( \pi_{gs} \)-open sets are defined and various results are presented by examining the properties of these new concepts.

Definition 4.1. \( \Psi \) is said to be \( \pi_{gs} \)-T\(_1\) whenever \( \nu \) and \( \mu \) is in \( \Psi \) are distinct points, sets \( F \in \pi \text{GSO}(\nu, \Psi) \) and \( \tilde{U} \in \pi \text{GSO}(\mu, \Psi) \) naturally appears satisfying \( \mu \notin F \) and \( \nu \notin \tilde{U} \).

Definition 4.2. \( \Psi \) is said to be \( \pi_{gs} \)-T\(_2\) whenever \( \nu \) and \( \mu \) in \( \Psi \) are distinct points, sets \( F \in \pi \text{GSO}(\nu, \Psi) \) and \( \tilde{U} \in \pi \text{GSO}(\mu, \Psi) \) naturally appears satisfying \( F \cap \tilde{U} = \emptyset \).

Theorem 4.1. Under the assumption \( \tilde{U} \) is an Urysohn space, whenever \( \nu \) and \( \mu \) is distinct points in \( \Psi \) a function \( \Delta : \Psi \to \Phi \) naturally appears that is contra \( \pi_{gs} \)-continuous at \( \nu \) and \( \mu \) for which \( \Delta(\nu) \neq \Delta(\mu) \), afterwards \( \Psi \) is \( \pi_{gs} \)-T\(_2\).

Proof. Assume that \( \nu \) and \( \mu \) is distinct points in \( \Psi \). Also, let \( \Delta : \Psi \to \Phi \) be contra \( \pi_{gs} \)-continuous at \( \nu \) and \( \mu \) such that \( \Delta(\nu) \neq \Delta(\mu) \). Letting \( \nu' = \Delta(\nu) \) and \( \mu' = \Delta(\mu) \) with the knowledge of \( \Phi \) is Urysohn, existence of \( \tilde{Z} \in O(\nu', \Phi) \) and \( F \in O(\mu', \Phi) \) guaranteed such that \( \text{cl}(\tilde{Z}) \cap \text{cl}(F) = \emptyset \). Since \( \Delta \) is contra \( \pi_{gs} \)-continuous at \( \nu \) and \( \mu \), there exist \( \pi_{gs} \)-open subsets \( \Psi \) and \( \Omega \) of \( \Psi \) comprising \( \nu \) and \( \mu \), correspondingly, such that \( \Delta(\Psi) \subset \text{cl}(\tilde{Z}) \) and \( \Delta(\Omega) \subset \text{cl}(F) \). Hereby, \( \Delta(\Psi \cap \Omega) \subset \Delta(\Psi) \cap \Delta(\Omega) \subset \text{cl}(\tilde{Z}) \cap \text{cl}(F) = \emptyset \) which implies that \( \Psi \cap \Omega = \emptyset \). Hence, \( \Psi \) is \( \pi_{gs} \)-T\(_2\). \( \square \)

Corollary 4.1. Whenever \( \Delta : \Psi \to \Phi \) is contra \( \pi_{gs} \)-continuous injection and \( \Phi \) is an Urysohn space, afterwards \( \Psi \) is \( \pi_{gs} \)-T\(_2\).

Definition 4.3. The topological space \( \Psi \) is called as,
(\( i_1 \)) \( \pi_{gs} \)-connected space \( \Leftrightarrow \Psi \) is not the union of two disjoint non-empty \( \pi_{gs} \)-open sets,
(\( i_2 \)) \( gs \)-connected space [15] \( \Leftrightarrow \Psi \) is not the union of two disjoint non-empty \( gs \)-open sets.

Remark 4.1. Although \( \pi_{gs} \)-connected spaces are \( gs \)-connected, the contrary implication is not valid in general.

Example 4.1. Let \( \Psi = \{\nu, \mu\} \) and \( T = \{\emptyset, \{\nu\}, \{\mu\}\} \). \( \Psi \) is \( gs \)-connected, but it is not \( \pi_{gs} \)-connected since \( \{\nu\} \) and \( \{\mu\} \) are non-empty disjoint \( \pi_{gs} \)-open subsets of \( \Psi \).

Theorem 4.2. For a topological space \( \Psi \) the following are equivalent:
(\( i_1 \)) \( \Psi \) is \( \pi_{gs} \)-connected;
(\( i_2 \)) The only subsets of \( \Psi \) which are both \( \pi_{gs} \)-open and \( \pi_{gs} \)-closed are \( \emptyset \) and \( \Psi \);
(\( i_3 \)) Each \( \pi_{gs} \)-continuous function of \( \Psi \) into a discrete space \( \Phi \) with at least two points is a constant function.
Whenever the product space of two non-empty spaces is connected.

Theorem 4.6. The projection functions \( p_{\Phi} : \Psi \times \Phi \to \Psi \) and \( p_{\Psi} : \Psi \times \Phi \to \Phi \) are \( \pi gs \)-irresolute.

Proof. Let \( p_{\Psi} : \Psi \times \Phi \to \Psi \) be the projection function from \( \Psi \times \Phi \) onto \( \Psi \) and \( \mathcal{N} \) be any \( \pi gs \)-closed subset of \( \Psi \). Then, \( \mathcal{N} \times \Phi \) is a \( \pi gs \)-closed subset of \( \Psi \times \Phi \). Therefore, \( \mathcal{N} \times \Phi \) is connected in \( \Psi \times \Phi \). Hence, \( \mathcal{N} \times \Phi \) is connected in \( \Psi \times \Phi \).

Theorem 4.7. Whenever \( \Delta : \Psi \to \Phi \) is a \( \pi gs \)-irresolute surjection and \( \Psi \) is \( \pi gs \)-connected, afterwards \( \Phi \) has to be \( \pi gs \)-connected.

Proof. Assume that \( \Phi \) is not \( \pi gs \)-connected. Naturally, two non-empty disjoint \( \pi gs \)-open subsets \( F \) and \( \Omega \) of \( \Phi \) appear so that \( F \cup \Omega = \Phi \). Then \( \Delta^{-1}(F) \) and \( \Delta^{-1}(\Omega) \) are both \( \pi gs \)-open subsets of \( \Psi \), since \( \Delta \) is surjective and \( \pi gs \)-irresolute. Besides, \( \emptyset = \Delta^{-1}(F \cap \Omega) = \Delta^{-1}(F) \cap \Delta^{-1}(\Omega) \) and \( \Psi = \Delta^{-1}(F) \cup \Delta^{-1}(\Omega) \). Therefore, we reach the result that \( \Psi \) is not \( \pi gs \)-connected which is a contradiction. Hereby, \( \Phi \) is \( \pi gs \)-connected.

Theorem 4.8. Whenever the product space of two non-empty spaces is \( \pi gs \)-connected, each factor space has to be \( \pi gs \)-connected.
Proof. Accept $\Psi$ and $\Phi$ as non-empty topological spaces and the product space $\Psi \times \Phi$ as $\pi_{gs}$-connected. Since the projection functions are $\pi_{gs}$-irresolute and surjective, by Theorem 4.7, $\Psi$ and $\Phi$ are $\pi_{gs}$-connected. □

**Definition 4.4.** A topological space $\Psi$ is called as:

(i) $\pi_{gs}$-compact if every $\pi_{gs}$-open cover of $\Psi$ has a finite subcover,

(ii) countably $\pi_{gs}$-compact if every countable cover of $\Psi$ by $\pi_{gs}$-open sets has a finite subcover,

(iii) $\pi_{gs}$-Lindelöf if every $\pi_{gs}$-open cover of $\Psi$ has a countable subcover.

**Definition 4.5.** $\aleph \in \Psi$ is characterized to be $\pi_{gs}$-compact relative to $\Psi$ whenever every $\pi_{gs}$-open cover of $\aleph$ by $\pi_{gs}$-open sets of $\Psi$ has a finite subcover.

**Theorem 4.9.** Whenever $\Delta : \Psi \to \Phi$ is contra $\pi_{gs}$-continuous and $\aleph \subseteq \Psi$ is $\pi_{gs}$-compact relative to $\Psi$, afterwards $\Delta(\aleph)$ has to be strongly $S$-closed.

Proof. Let $\{\Theta_i : i \in I\}$ be a closed cover of $\Delta(\aleph)$ by closed subsets of the subspace $\Delta(\aleph)$. Then for each $i \in I$, there exists a closed set $\Theta_i$, in $\Phi$ such that $\Delta(\aleph) = \bigcap \{\Theta_i : i \in I\} = \bigcap \{\Theta_i \cap \Delta(\aleph) : i \in I\} = \bigcup \{\Theta_i : i \in I\} \cap \Delta(\aleph)$ and $\Theta_i = \Theta_i \cap \Delta(\aleph)$. Since for each $\nu \in \aleph$, we have $\Delta(\nu) \in \Delta(\aleph)$ and since $\Delta$ is contra $\pi_{gs}$-continuous, for each $\nu \in \aleph$ there exists $i(\nu) \in I$ and there exists $F(\nu) \in \pi_{GSO}(\nu, \Psi)$ such that $\Delta(\nu) \in \Theta_{i(\nu)}$ and $\Delta(F(\nu)) \subset \Theta_{i(\nu)}$. Then, $\{F(\nu) : \nu \in \aleph\}$ is a cover of $\aleph$ by $\pi_{gs}$-open sets of $\Psi$. Since $\aleph$ is $\pi_{gs}$-compact relative to $\Psi$, there exists a finite subset $\aleph_0$ of $\aleph$ such that $\aleph \subset \bigcup \{F(\nu) : \nu \in \aleph_0\}$. Then, we obtain $\Delta(\aleph) \subset \bigcup \{\Theta_{i(\nu)} : \nu \in \aleph_0\}$. Therefore, $\Delta(\aleph) = \Delta(\aleph) \cap \bigcup \{\Theta_{i(\nu)} : \nu \in \aleph_0\} = \bigcup \{\Delta(\Theta_i) : \nu \in \aleph_0\} = \bigcup \{\Theta_{i(\nu)} : \nu \in \aleph_0\}$ and this means that $\{\Theta_{i(\nu)} : \nu \in \aleph_0\}$ is a finite subcover of $\{\Theta_i : i \in I\}$. Hence, $\Delta(\aleph)$ is strongly $S$-closed. □

**Corollary 4.2.** Whenever $\Delta : \Psi \to \Phi$ is a contra $\pi_{gs}$-continuous surjection and $\Psi$ is $\pi_{gs}$-compact, afterwards $\Phi$ has to be strongly $S$-closed.

**Theorem 4.10.** Whenever the product space of two non-empty spaces is $\pi_{gs}$-compact, afterwards each factor space has to be $\pi_{gs}$-compact.

Proof. Let $\Psi \times \Phi$ be the product space of the non-empty topological spaces $\Psi$ and $\Phi$ and $\Psi \times \Phi$ be $\pi_{gs}$-compact. Let $\{\Delta_i : i \in I\}$ be any $\pi_{gs}$-open cover of $\Psi$. Then, $\Psi \times \Phi = \pi_{gs}^{-1}(\Psi) = \pi_{gs}^{-1}(\bigcup \{\Delta_i : i \in I\}) = \bigcup \{\pi_{gs}^{-1}(\Delta_i) : i \in I\}$. Since $\pi_{gs}$ is $\pi_{gs}$-irresolute, $\pi_{gs}^{-1}(\Delta_i) = \Delta_i \times \Phi$ is $\pi_{gs}$-open in $\Psi \times \Phi$ for each $i \in I$. Therefore, $\{\Delta_i \times \Phi : i \in I\}$ is a $\pi_{gs}$-open cover of $\Psi \times \Phi$. Since $\Psi \times \Phi$ is $\pi_{gs}$-compact, there exists a finite subset $I_0$ of $I$ such that $\bigcup \{\Delta_i \times \Phi : i \in I_0\} = \Psi \times \Phi$. Then, $\Psi = \pi_{gs}(\Psi \times \Phi) = \pi_{gs}(\bigcup \{\Delta_i \times \Phi : i \in I_0\}) = \pi_{gs}(\bigcup \{\Delta_i : i \in I_0\}) \times \Phi = \bigcup \{\Delta_i : i \in I_0\}$. Hence, $\Psi$ is $\pi_{gs}$-compact. The proof for the space $\Phi$ is similar. □

**Theorem 4.11.** Contra $\pi_{gs}$-continuous images of $\pi_{gs}$-Lindelöf (correspondingly countably $\pi_{gs}$-compact) spaces are strongly $S$-Lindelöf (correspondingly strongly countably $S$-closed).

Proof. Let $\Psi$ be a $\pi_{gs}$-Lindelöf space and $\Delta : \Psi \to \Phi$ be a surjective contra $\pi_{gs}$-continuous function. Let $\{\Theta_i : i \in I\}$ be a closed cover of $\Phi$. Since $\Delta$ is contra $\pi_{gs}$-continuous, $\{\Delta^{-1}(\Theta_i) : i \in I\}$ is a $\pi_{gs}$-open cover of $\Psi$. Since $\Psi$ is $\pi_{gs}$-Lindelöf, there exists a countable subset $I_0$ of $I$ such that $\bigcup \{\Delta^{-1}(\Theta_i) : i \in I_0\} = \Psi$. Since $\Delta$ is surjective, $\Phi = \Delta(\Psi) = \Delta(\bigcup \{\Delta^{-1}(\Theta_i) : i \in I_0\}) = \bigcup \{\Delta(\Delta^{-1}(\Theta_i)) : i \in I_0\} = \bigcup \{\Theta_i : i \in I_0\}$ and $\Phi = \bigcup \{\Theta_i : i \in I_0\}$. Hence, $\Phi$ is strongly $S$-Lindelöf. The proof for the contra $\pi_{gs}$-continuous images of countably $\pi_{gs}$-compact spaces is similar. □

**Definition 4.6.** The graph $G(\Delta)$ of $\Delta : \Psi \to \Phi$ is said to be a contra $\pi_{gs}$-graph if for each $(\nu, \mu)$ in $(\Psi \times \Phi) \setminus G(\Delta)$, there exist a set $\aleph$ in $\pi_{GSO}(\nu, \Psi)$ and a set $\Omega$ in $C(\mu, \Phi)$ such that $(\aleph \times \Omega) \cap G(\Delta) = \emptyset$.

**Theorem 4.12.** The following are equivalent for the graph $G(\Delta)$ of any $\Delta : \Psi \to \Phi$.

(i) $G(\Delta)$ is a contra $\pi_{gs}$-graph;

(ii) for all $(\nu, \mu) \in (\Psi \times \Phi) \setminus G(\Delta)$, there exist a $\pi_{gs}$-open set $\aleph \subset \Psi$ comprising $\nu$ and a closed set $\Omega \subset \Phi$ comprising $\mu$ such that $\Delta(\aleph) \cap \Omega = \emptyset$.

**Theorem 4.13.** Whenever $\Delta : \Psi \to \Phi$ is contra $\pi_{gs}$-continuous and $\Phi$ is an Uryshon space, afterwards $G(\Delta)$ has to be a contra $\pi_{gs}$-graph.

Proof. For all $(\nu, \mu) \in (\Psi \times \Phi) \setminus G(\Delta)$, it is clear that $\Delta(\nu) \neq \mu$. Since $\Phi$ is Uryshon space, there exist open sets $\Delta(\nu)$ and $\Delta(\mu)$ in $\Phi$ comprising $\Delta(\nu)$ and $\mu$, correspondingly, such that $\text{cl}(\Delta(\nu)) \cap \text{cl}(\Delta(\mu)) = \emptyset$. Since $\Delta$ is contra $\pi_{gs}$-continuous, a $\aleph \in \pi_{GSO}(\nu, \Psi)$ appears so that $\Delta(\aleph) \subset \text{cl}(\Delta(\nu))$. Then, $\Delta(\aleph) \cap \text{cl}(\Delta(\mu)) = \emptyset$. Hereby, $G(\Delta)$ is contra $\pi_{gs}$-graph. □
Theorem 4.14. Let $\Delta : \Psi \to \Phi$ be a function and $\rho : \Psi \to \Psi \times \Phi$ be the graph function of $\Delta$ defined as $\rho(\nu) = (\nu, \Delta(\nu))$ for every $\nu \in \Psi$. If $\rho$ is contra $\pi gs$-continuous, then $\Delta$ is contra $\pi gs$-continuous.

Proof. For all open set $F \subseteq \Phi$, it is clear that $\Psi \times F$ is open in $\Psi \times \Phi$. Since $\rho$ is a contra $\pi gs$-continuous function, $\Delta^{-1}(F) = \rho^{-1}(\Psi \times F)$ is $\pi gs$-closed in $\Psi$. Hence, $\Delta$ is contra $\pi gs$-continuous. □

Theorem 4.15. Let $\Delta : \Psi \to \Phi$ and $\rho : \Psi \to \Phi$ be two contra $\pi gs$-continuous functions. If $\Phi$ is an Uryshon space and $\pi GSO(\Psi)$ is closed under finite intersections then, the set $E = \{ \nu \in \Psi : \Delta(\nu) = \rho(\nu) \}$ is $\pi gs$-closed in $\Psi$.

Proof. If we show that $\nu \notin E \Rightarrow \nu \notin cl_{\pi gs}(E)$, then the theorem will be proved. Let $\nu \in \Psi \setminus E$. Then, $\Delta(\nu) \neq \rho(\nu)$. Since $\Phi$ is Uryshon, there exist open subsets $F$ and $G$ of $\Phi$ comprising $\Delta(\nu)$ and $\rho(\nu)$, correspondingly, such that $cl(F) \cap cl(G) = \emptyset$. Since $\Delta$ and $\rho$ are contra $\pi gs$-continuous, $\Delta^{-1}(cl(F))$ and $\rho^{-1}(cl(G))$ are $\pi gs$-open in $\Psi$. Let $\Delta^{-1}(cl(F)) = D_1$ and $\rho^{-1}(cl(G)) = D_2$. Then, $\nu \in D_1 \cap D_2$. Since $\pi GSO(\Psi)$ is closed under finite intersections, $\nu$ is a $\pi gs$-open set in $\Psi$ comprising $\nu$. So, $\Delta(\nu) \cap \rho(\nu) = \emptyset$. Hence, $\nu \cap E = \emptyset$. By Lemma 2.1, $\nu \notin cl_{\pi gs}(E)$. □

Definition 4.7. For a subset $\mathcal{N}$ of space $\Psi$, if $cl_{\pi gs}(\mathcal{N}) = \Psi$ then $\mathcal{N}$ is said to be $\pi gs$-dense in $\Psi$.

Theorem 4.16. Let $\Delta : \Psi \to \Phi$ and $\rho : \Psi \to \Phi$ be two functions. If
1. $\Phi$ is an Uryshon space and $\pi GSO(\Psi)$ is closed under finite intersections,
2. $\Delta$ and $\rho$ are contra $\pi gs$-continuous,
3. $\Delta = \rho$ on a $\pi gs$-dense subset $\mathcal{N}$ of $\Psi$, then $\Delta = \rho$ on $\mathcal{N}$.

Proof. By Theorem 4.15, the set $E = \{ \nu \in \Psi : \Delta(\nu) = \rho(\nu) \}$ is $\pi gs$-closed in $\Psi$. Since $\Delta = \rho$ on a $\pi gs$-dense subset $\mathcal{N}$, we have $\mathcal{N} \subseteq E$. Then, $\Psi = cl_{\pi gs}(\mathcal{N}) \subseteq cl_{\pi gs}(E) = E$. Hence, $\mathcal{N} = \Psi$. □

Definition 4.8. $\Psi$ is characterized to be weakly Hausdorff [49] if each element of $\Psi$ is an intersection of regular closed sets.

Theorem 4.17. Let $\Delta : \Psi \to \Phi$ be an injective contra $\pi gs$-continuous function. If $\Phi$ is weakly Hausdorff then, $\Psi$ is $\pi gs$-$T_1$.

Proof. Let $\nu$ and $\mu$ be any two elements in $\Psi$ such that $\nu \neq \mu$. Since $\Delta$ is injective, $\Delta(\nu) \neq \Delta(\mu)$. If $\Phi$ is weakly Hausdorff, regular closed subsets $\Theta_1$ and $\Theta_2$ of $\Phi$ comprising $\Delta(\nu)$ and $\Delta(\mu)$, correspondingly, appears such that $\Delta(\nu) \not\in \Theta_2$ and $\Delta(\mu) \not\in \Theta_1$. Since regular closed sets are closed and $\Delta$ is contra $\pi gs$-continuous, $\Delta^{-1}(\Theta_1)$ and $\Delta^{-1}(\Theta_2)$ are $\pi gs$-open subsets of $\Psi$ comprising $\nu$ and $\mu$, correspondingly, such that $\mu \not\in \Delta^{-1}(\Theta_1)$ and $\nu \not\in \Delta^{-1}(\Theta_2)$. Hence, $\Psi$ is $\pi gs$-$T_1$. □

Theorem 4.18. If $\Delta : \Psi \to \Phi$ is an injective function whose graph $G(\Delta)$ is contra $\pi gs$-graph then, $\Psi$ is $\pi gs$-$T_1$.

Proof. Let $\nu$ and $\mu$ be any two elements in $\Psi$ such that $\nu \neq \mu$. Since $\Delta$ is injective, $(\nu, \Delta(\mu)) \in (\Psi \times \Phi) \setminus G(\Delta)$. Since $G(\Delta)$ is contra $\pi gs$-graph, there exists a $\pi gs$-open subset $\Omega$ of $\Psi$ and a closed subset $\Theta$ of $\Phi$ comprising $\nu$ and $\mu$, correspondingly, such that $\Delta(\Omega) \cap \Theta = \emptyset$. Then $\Delta^{-1}(\Theta) \cap \Omega = \emptyset$ and $\mu \not\in \Omega$. Similarly, since $(\Delta(\nu), \mu) \in (\Psi \times \Phi) \setminus G(\Delta)$, there exists a $\pi gs$-open subset $\Omega$ of $\Psi$ comprising $\mu$ such that $\nu \notin \Omega$. Hence, $\Psi$ is $\pi gs$-$T_1$. □

Theorem 4.19. Let $\Delta : \Psi \to \Phi$ be an injective contra $\pi gs$-continuous function. Whenever $\Phi$ is an ultra Hausdorff space, $\Psi$ has to be $\pi gs$-$T_2$.

Proof. Let $\nu$ and $\mu$ be any two elements in $\Psi$ such that $\nu \neq \mu$. Since $\Delta$ is injective, $\Delta(\nu) \neq \Delta(\mu)$. If $\Phi$ is an ultra Hausdorff space, there exist disjoint clopen subsets $\Theta_1$ and $\Theta_2$ of $\Phi$ comprising $\Delta(\nu)$ and $\Delta(\mu)$, correspondingly. Then, $\Delta^{-1}(\Theta_1)$ and $\Delta^{-1}(\Theta_2)$ are disjoint subsets of $\Psi$ comprising $\nu$ and $\mu$, correspondingly, which are both $\pi gs$-open and $\pi gs$-closed in $\Psi$ since $\Delta$ is contra $\pi gs$-continuous. Hence, $\Psi$ is $\pi gs$-$T_2$. □

Definition 4.9. A space $\Psi$ is said to be $\pi gs$-normal if each pair of non-empty disjoint closed sets can be separated by disjoint $\pi gs$-open sets.

Theorem 4.20. Let $\Delta : \Psi \to \Phi$ be an injective closed contra $\pi gs$-continuous function. If $\Phi$ is ultra normal, then $\Psi$ is $\pi gs$-normal.

Proof. Let $\Theta_1$ and $\Theta_2$ be any two non-empty disjoint closed subsets of $\Psi$. Since $\Delta$ is injective and closed, $\Delta(\Theta_1)$ and $\Delta(\Theta_2)$ are non-empty disjoint closed subsets of $\Phi$. Since $\Phi$ is ultra normal, there exist disjoint clopen subsets $\Theta_1$ and $\Theta_2$ of $\Phi$ such that $\Delta(\Theta_1) \subseteq \Theta_1$ and $\Delta(\Theta_2) \subseteq \Theta_2$. Since $\Delta$ is contra $\pi gs$-continuous, $\Delta^{-1}(\Theta_1)$ and $\Delta^{-1}(\Theta_2)$ are disjoint $\pi gs$-open subsets of $\Psi$ such that $\Theta_1 \subseteq \Delta^{-1}(\Theta_1)$ and $\Theta_2 \subseteq \Delta^{-1}(\Theta_2)$. Hence, $\Psi$ is $\pi gs$-normal. □
5. Conclusion

It is understood from the studies of many researchers on contra continuity, which is one of the types of continuity that has been frequently studied recently as in the past, still arouses curiosity today. Researchers have not only examined various properties of the different types of contra continuous functions they have identified, but also examined the relationships between different contra continuities. In this study, we not only share the concept of contra \( \pi gs \)-continuity [8] related with \( \pi gs \)-open sets defined by Çaksu [4], but also investigated various properties of contra \( \pi gs \)-continuous functions and examined the relationships between different contra continuities. Remark 3.2 clearly shows that the concept of contra \( \pi gs \)-continuity is weaker than the concepts of contra \( \pi g \)-continuity [7], contra \( gs \)-continuity [9], contra \( g \)-continuity [39], contra semicontinuity [9], contra super continuity [38], contra continuity [6], strong contra continuity [37], perfect continuity [35] and RC continuity [9]. We also obtained important results by examining various properties related to separation axioms, connectedness, compactness, cover and graph concepts. We believe that our study will shed light on the studies researchers interested in contra continuous functions.

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**Affiliations**

**Nebiye Korkmaz**

**Address:** Muğla Sıtkı Koçman University, Education Faculty, Dept. of Mathematics and Science Education, 48000, Menteşe-Muğla/TURKEY

**E-mail:** nkorkmaz@mu.edu.tr

**ORCID ID:** 0000-0003-2248-4280