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Weighted and Controlled Continuous g-Frames and their Multipliers in Hilbert Spaces

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Abstract: A generalization of weighted, multiplier, controlled from frame and Bessel sequences to continuous g-frames and continuous g-Bessel sequences in Hilbert spaces is presented in this study. Moreover, we find a dual of a continuous g-frame in the case that the multiplier operator is invertible. Finally, it is demonstrated that a controlled continuous g-frame is equivalent to a continuous g-frame.

Keywords: Continuous g-frame, continuous g-multiplier, weighted continuous g-frame, controlled continuous g-frame.

1. Introduction

Frames for Hilbert space were formally defined by Duffin and Schaeffer [11] in 1952 for studying some problems in non-harmonic Fourier series. Wenchang Sun [21] introduced a generalization of frames and showed that this includes more other cases of generalizations of the frame concept and proved that many basic properties can be derived within this more general context. Continuous frames were proposed by G. Kaiser [15] and independently by Ali, Antoine and Gazeau [2] to a family indexed by some locally compact space endowed with a Radon measure. Gabardo and Han in [13] denotes these frames as frames associated with measurable spaces.

Weighted and controlled frames have been introduced recently to improve the numerical efficiency of iterative algorithms for inverting the frame operator on abstract Hilbert spaces [4]. However, they have been used earlier in [6] for spherical wavelets. Gabor multipliers [10, 12], Gabor filters [16] and some other applications of frames led Peter Balazs to introduce Bessel and frame multipliers for abstract Hilbert spaces $\mathscr{H}_1$ and $\mathscr{H}_2$. These operators are defined by a fixed multiplication pattern (the symbol) which is inserted between the analysis and synthesis operators. A. Rahimi and A. Fereydooni [20] defined the concept of controlled g-frames and they showed that any controlled g-frame is equivalent a g-frame. P. Balazs, D. Bayer and A. Rahimi [5] defined continuous

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Bessel and continuous frame multipliers as generalizations of discrete Bessel and frame multipliers. They further developed their theory and proved a number of statements on the compactness of multipliers as well as on mapping properties with respect to Schatten classes.

A generalization of the concept of controlled from frame and Bessel sequences, which is proved in [4, 5, 18, 19, 20], to continuous g-frames and continuous g-Bessel sequences in Hilbert spaces is presented in this study. In this paper we extend the concepts of weighted and multiplier from continuous frames to continuous g-Bessel sequences and continuous g-frames. It is shown that the dual of a continuous g-frame in the case that the continuous g-multiplier operator is invertible, is $\{\overline{m(\omega)}\Theta_\omega \mathbf{M}^{-1}\}_{\omega\in\Omega}$ (see Theorem 1). Moreover, in Theorem 2, we show that a controlled continuous g-frame is equivalent to a continuous g-frame. Finally, we define the multiplier for C^2-controlled g-frames in Hilbert spaces.

2. Preliminaries

In the following, we briefly recall some definitions and basic properties of continuous g-frames in Hilbert spaces. We first give some notations which are needed later. Throughout this paper, (Ω,μ) is a measure space. $\mathscr{H}$ and $\mathscr{K}$ are two Hilbert spaces and $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$ is a sequence of closed Hilbert subspaces of $\mathscr{K}$. For each $\omega\in\Omega$, $\mathscr{B}(\mathscr{H},\mathscr{K}_\omega)$ is the collection of all bounded linear operators from $\mathscr{H}$ to $\mathscr{K}_\omega$. We also write

$$\bigoplus_{\omega\in\Omega}\mathscr{K}_\omega=\{g=\{g_\omega\}: g_\omega\in\mathscr{K}_\omega \quad \text{and} \quad \int_\Omega \|g_\omega\|^2 d\mu(\omega)<\infty\}.$$

A bounded operator $T:\mathscr{H}\longrightarrow\mathscr{K}$ is called positive (respectively non-negative), if $\langle Tf,f\rangle>0$ for all $f\neq 0$ (respectively $\langle Tf,f\rangle\geq 0$ for all $f\in\mathscr{H}$). Let $\mathscr{GL}(\mathscr{H})$ be the set of all bounded operators with a bounded inverse and $\mathscr{GL}^+(\mathscr{H})$ be the set of positive operators in $\mathscr{GL}(\mathscr{H})$.

Definition 1. We call a sequence $\{\Lambda_\omega\in\mathscr{B}(\mathscr{H},\mathscr{K}_\omega):\omega\in\Omega\}$ a continuous g-frame for $\mathscr{H}$ with respect to $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$, if

1. for each $f\in\mathscr{H}$, $\{\Lambda_\omega f\}_{\omega\in\Omega}$ is strongly measurable,
2. there are two constants $0<A\leq B<\infty$ such that,

$$A\|f\|^2\leq\int_\Omega\|\Lambda_\omega f\|^2 d\mu(\omega)\leq B\|f\|^2, \qquad (f\in\mathscr{H}). \tag{1}$$

We call A and B the lower and upper continuous g-frame bounds, respectively. If only the right-hand inequality of (1) is satisfied, we call $\{\Lambda_\omega\}_{\omega\in\Omega}$ the continuous g-Bessel sequence for $\mathscr{H}$ with respect to $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$ with continuous g-Bessel bound B. If $A=B=\lambda$, we call $\{\Lambda_\omega\}_{\omega\in\Omega}$ the λ-tight continuous g-frame. Moreover, if $\lambda=1$, $\{\Lambda_\omega\}_{\omega\in\Omega}$ is called the parseval continuous g-frame.

For any $\{f_\omega\}_{\omega\in\Omega}, \{g_\omega\}_{\omega\in\Omega} \in \bigoplus_{\omega\in\Omega}\mathscr{K}_\omega$, if the inner product is defined by

$$\langle f,g\rangle = \int_\Omega \langle f_\omega, g_\omega\rangle d\mu(\omega),$$

and the norm is defined by

$$\|f\| = |\langle f,f\rangle|^{\frac{1}{2}},$$

then $\bigoplus_{\omega\in\Omega}\mathscr{K}_\omega$ is a Hilbert space.

We define the synthesis operator for a continuous g-Bessel sequence $\{\Lambda_\omega\}_{\omega\in\Omega}$ as follows:

$$\langle T_\Lambda\{f_\omega\}_{\omega\in\Omega}, g\rangle = \int_\Omega \langle f_\omega, \Lambda_\omega g\rangle d\mu(\omega), \qquad \left(\{f_\omega\}_{\omega\in\Omega} \in \bigoplus_{\omega\in\Omega} K_\omega, g\in H\right).$$

Operator T_Λ is well-defined and bounded, therefore, operator T_Λ^* defined for map

$$T_\Lambda^* : H \longrightarrow \bigoplus_{\omega\in\Omega} K_\omega, \qquad T_\Lambda^*(f) = \{\Lambda_\omega f\}_{\omega\in\Omega},$$

is the adjoint of T_Λ and is called the analysis operator. The bounded linear operator S_Λ defined by

$$S_\Lambda : H \longrightarrow H, \qquad \langle S_\Lambda f, g\rangle = \int_\Omega \langle f, \Lambda_\omega^*\Lambda_\omega g\rangle d\mu(\omega),$$

is called the continuous g-frame operator of $\{\Lambda_\omega\}_{\omega\in\Omega}$.

Remark 2.1. A continuous frame is equivalent to a continuous g-frame, whenever $\mathscr{K}_\omega = \mathbb{C}$, for all $\omega\in\Omega$.

Definition 2. Let $\{\Lambda_\omega\}_{\omega\in\Omega}$ and $\{\Gamma_\omega\}_{\omega\in\Omega}$ be two continuous g-frames for $\mathscr{H}$ with respect to $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$ such that

$$\langle f,g\rangle = \int_\Omega \langle f, \Gamma_\omega^*\Lambda_\omega g\rangle d\mu(\omega). \tag{2}$$

Then, $\{\Gamma_\omega\}_{\omega\in\Omega}$ is called an alternate dual continuous g-frame of $\{\Lambda_\omega\}_{\omega\in\Omega}$.

Definition 3. Let $\{\Lambda_j \in \mathscr{B}(\mathscr{H},\mathscr{K}_j) : j\in J\}$ and $\{\Theta_j \in \mathscr{B}(\mathscr{H},\mathscr{K}_j) : j\in J\}$ be g-Bessel sequences. If for $m = \{m_j\}_{j\in J} \in l^\infty$, the operator

$$M_{m,\Lambda,\Theta} : \mathscr{H} \longrightarrow \mathscr{H}, \tag{3}$$

$$M_{m,\Lambda,\Theta}(f) = \sum_{j\in J} m_j \Lambda_j^*\Theta_j f, \tag{4}$$

is well-defined, then $M_{m,\Lambda,\Theta}$ is called the g-multiplier of Λ,Θ and m.

Definition 4. Let $C,C' \in \mathscr{GL}^+(\mathscr{H})$. The family $\{\Lambda_j\}_{j\in J}$ is called a (C,C')-controlled g-frame for $\mathscr{H}$ with respect to $\{\mathscr{K}_j\}_{j\in J}$, if $\{\Lambda_j\}_{j\in J}$ is a g-Bessel sequence and there exist constants $A>0$ and $B<\infty$ such that

$$A\|f\|^2 \le \sum_{j\in J} \langle \Lambda_\omega C f, \Lambda_\omega C' f\rangle \le B\|f\|^2, \qquad (f\in\mathscr{H}). \tag{5}$$

A and B will be called (C,C')-controlled g-frame bounds. If $C' = I$ (or, $C' = C$), we call $\{\Lambda_j\}_{j\in J}$ a C-controlled g-frame (respectively, C^2-controlled g-frame) for $\mathscr{H}$ with bounds A and B. If the second part of the above inequality holds, it will be called (C,C')-controlled g-Bessel sequence with bound B.

3. Main results

In this section we define the concept of weighted continuous g-frames and we extend the concept of multiplier from continuous frame to continuous g-Bessel sequences and continuous g-frames and show some of their properties.

Definition 5. Let (Ω,μ) be a measure space with positive measure μ and $m : \Omega \longrightarrow \mathbb{R}^+$. The g-sequence $\{\Delta_\omega\}_{\omega\in\Omega}$ for $\mathscr{H}$ with respect to $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$ is called a weighted continuous g-frame with respect to (Ω,μ) and m, if

1. for each $f \in \mathscr{H}$, $\{\Delta_\omega f\}_{\omega\in\Omega}$ is strongly measurable and $m \in L^\infty(\Omega,\mathbb{R}^+)$,
2. there are two constants $0 < A \leq B < \infty$ such that,

$$A\|f\|^2 \leq \int_\Omega m(\omega)\|\Delta_\omega f\|^2 d\mu(\omega) \leq B\|f\|^2, \qquad (f \in \mathscr{H}). \tag{6}$$

$\{\Delta_\omega\}_{\omega\in\Omega}$ is called weighted continuous g-Bessel if the second part of inequality in (6) holds.

Note that we call $\{\Delta_\omega\}_{\omega\in\Omega}$ a weighted continuous g-frame if the g-sequence $\{\sqrt{m(\omega)}\Delta_\omega\}_{\omega\in\Omega}$ is a continuous g-frame.

Definition 3 motivated us to define the continuous g-multiplier as follows.

Proposition 1. If $\{\Lambda_\omega\}_{\omega\in\Omega}$ and $\{\Theta_\omega\}_{\omega\in\Omega}$ are continuous g-Bessel sequences for $\mathscr{H}$ with respect to $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$ with bounds B_Λ and B_Θ respectively. Let $m \in L^\infty(\Omega,\mathbb{C})$, then the operator $\mathbf{M} := M_{m,\Lambda,\Theta} : \mathscr{H} \longrightarrow \mathscr{H}$ weakly defined by $\langle \mathbf{M}(f),g\rangle = \int_\Omega m(\omega)\langle f,\Theta_\omega^*\Lambda_\omega g\rangle d\mu(\omega)$, is well-defined and $(M_{m,\Lambda,\Theta})^* = M_{\overline{m},\Theta,\Lambda}$.

Proof. First we show that $\mathbf{M}$ is well-defined. For this, suppose that $f,g \in \mathscr{H}$, therefore,

$$\begin{aligned}
\left|\int_\Omega m(\omega)\langle f,\Theta_\omega^*\Lambda_\omega g\rangle d\mu(\omega)\right| &\leq \|m\|_\infty \int_\Omega |\langle\Theta_\omega f,\Lambda_\omega g\rangle| d\mu(\omega) \\
&\leq \|m\|_\infty \Big(\int_\Omega \|\Theta_\omega f\|^2 d\mu(\omega)\Big)^{\frac{1}{2}} \Big(\int_\Omega \|\Lambda_\omega f\|^2 d\mu(\omega)\Big)^{\frac{1}{2}} \\
&\leq \|m\|_\infty \sqrt{B_\Theta}\|f\|\sqrt{B_\Lambda}\|f\| \\
&= \|m\|_\infty \sqrt{B_\Theta B_\Lambda}\|f\|^2.
\end{aligned}$$

We next claim that $(M_{m,\Lambda,\Theta})^* = M_{\overline{m},\Theta,\Lambda}$, let $f,g \in \mathscr{H}$. Then

$$\begin{aligned}
\langle f, M^*_{m,\Lambda,\Theta} g\rangle = \langle M_{m,\Lambda,\Theta} f, g\rangle &= \int_\Omega m(\omega)\langle f, \Theta^*_\omega \Lambda_\omega g\rangle d\mu(\omega) \\
&= \int_\Omega m(\omega)\langle \Lambda^*_\omega \Theta_\omega f, g\rangle d\mu(\omega) \\
&= \overline{\int_\Omega \overline{m}(\omega)\langle g, \Lambda^*_\omega \Theta_\omega f\rangle d\mu(\omega)} \\
&= \overline{\langle M_{\overline{m},\Theta,\Lambda} g, f\rangle} \\
&= \langle f, M_{\overline{m},\Theta,\Lambda} g\rangle.
\end{aligned}$$

Hence, we have $M^*_{m,\Lambda,\Theta} = M_{\overline{m},\Theta,\Lambda}$. ■

Proposition 1 extends the concept g-multiplier and we called it ”the continuous g-multiplier”.

Lemma 1. If $\{\Theta_\omega\}_{\omega\in\Omega}$ is a continuous g-Bessel sequence with bounds B_Θ and $m \in L^\infty(\Omega, \mathbb{C})$, then $\{m(\omega)\Theta_\omega\}_{\omega\in\Omega}$ is a continuous g-Bessel sequence with bound $B_\Theta \|m\|^2_\infty$.

Proof. For any $f \in \mathscr{H}$, we have

$$\int_\Omega \|m(\omega)\Theta_\omega f\|^2 d\mu(\omega) \le \|m\|^2_\infty \int_\Omega \|\Theta_\omega f\|^2 d\mu(\omega) \le B_\Theta \|m\|^2_\infty \|f\|^2.$$

■

Note that, if $m \in L^\infty(\Omega, \mathbb{R}^+)$ by definition 5, $\{\sqrt{m(\omega)}\Theta_\omega\}_{\omega\in\Omega}$ can be called a weighted continuous g-Bessel sequence. Now we find a dual of a continuous g-frame in the case that the continuous g-multiplier operator is invertible.

Theorem 1. Let $\mathbf{M} = M_{m,\Lambda,\Theta}$ be invertible and $\{\Lambda_\omega\}_{\omega\in\Omega}$ be a continuous g-frame and $m \in L^\infty(\Omega, \mathbb{C})$. Then, $\{\overline{m(\omega)}\Theta_\omega \mathbf{M}^{-1}\}_{\omega\in\Omega}$ is a dual for the continuous g-frame $\{\Lambda_\omega\}_{\omega\in\Omega}$.

Proof. By replacing f with $\mathbf{M}^{-1} f$ in

$$\langle \mathbf{M}(f), g\rangle = \int_\Omega m(\omega)\langle f, \Theta^*_\omega \Lambda_\omega g\rangle d\mu(\omega),$$

we have

$$\begin{aligned}
\langle f, g\rangle &= \int_\Omega m(\omega)\langle \mathbf{M}^{-1} f, \Theta^*_\omega \Lambda_\omega g\rangle d\mu(\omega) \\
&= \int_\Omega \langle f, \overline{m(\omega)} (\mathbf{M}^{-1})^* \Theta^*_\omega \Lambda_\omega g\rangle d\mu(\omega) \\
&= \int_\Omega \langle f, \overline{m(\omega)} \big(\Theta_\omega \mathbf{M}^{-1}\big)^* \Lambda_\omega g\rangle d\mu(\omega).
\end{aligned}$$

■

Controlled frames have been introduced recently to improve the numerical efficiency of iterative algorithms for inverting the frame operator. In the following, the concepts of controlled frames and controlled Bessel sequences are extended to controlled continuous g-frames and controlled continuous g-Bessel frames and we show that controlled continuous g-frames are equivalent to continuous g-frames. Moreover, we define a controlled g-frame's multiplier for C^2-controlled g-frames in Hilbert spaces.

Definition 6. Let $C, C' \in \mathscr{GL}^+(\mathscr{H})$. The family $\{\Lambda_\omega\}_{\omega\in\Omega}$ will be called a (C,C')-controlled continuous g-frame for $\mathscr{H}$ with respect to $\{\mathscr{K}_\omega\}_{\omega\in\Omega}$, if $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a continuous g-Bessel sequence and there exist constants $A>0$ and $B<\infty$ such that

$$A\|f\|^2 \leq \int_\Omega \langle \Lambda_\omega C f, \Lambda_\omega C' f\rangle d\mu(\omega) \leq B\|f\|^2, \qquad (f\in\mathscr{H}). \tag{7}$$

A and B are called (C,C')-controlled continuous g-frame bounds. If $C' = I$ (or, $C' = C$), we call $\{\Lambda_\omega\}_{\omega\in\Omega}$ a C-controlled continuous g-frame (respectively, C^2-controlled continuous g-frame) for $\mathscr{H}$ with bounds A and B. If the second part of the above inequality holds, it will be called (C,C')-controlled continuous g-Bessel sequence with bound B.

Lemma 2. Let $C\in\mathscr{GL}^+(\mathscr{H})$. The continuous g-Bessel sequence $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a C^2-controlled continuous g-Bessel sequence (or C^2-controlled continuous g-frame) if and only if there exist constants $A>0$ and $B<\infty$ such that

$$\int_\Omega \|\Lambda_\omega C f\|^2 d\mu(\omega) \leq B\|f\|^2, \qquad (f\in\mathscr{H}).$$

$$\left(A\|f\|^2 \leq \int_\Omega \|\Lambda_\omega C f\|^2 d\mu(\omega) \leq B\|f\|^2, \qquad (f\in\mathscr{H})\right).$$

Proof. The proof is straightforward from the definition. ■

Theorem 2. Let $C\in\mathscr{GL}^+(\mathscr{H})$. The family $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a continuous g-frame if and only if $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a C^2-controlled continuous g-frame.

Proof. Suppose that $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a C^2-controlled continuous g-frame with bounds A,B. Then for all $f\in\mathscr{H}$,

$$A\|f\|^2 \leq \int_\Omega \|\Lambda_\omega C f\|^2 d\mu(\omega) \leq B\|f\|^2.$$

Therefore,

$$\begin{aligned} A\|f\|^2 = A\|CC^{-1}f\|^2 \leq A\|C\|^2\|C^{-1}f\|^2 &\leq \|C\|^2 \int_\Omega \|\Lambda_\omega CC^{-1}f\|^2 d\mu(\omega) \\ &= \|C\|^2 \int_\Omega \|\Lambda_\omega f\|^2 d\mu(\omega). \end{aligned}$$

Hence

$$A\|C\|^{-2}\|f\|^2 \leq \int_\Omega \|\Lambda_\omega f\|^2 d\mu(\omega).$$

On the other hand, for every $f \in \mathscr{H}$

$$\int_{\Omega} \|\Lambda_\omega f\|^2 d\mu(\omega) = \int_{\Omega} \|\Lambda_\omega CC^{-1} f\|^2 d\mu(\omega) \leq B\|C^{-1}\|^2 \|f\|^2.$$

These inequalities yield that $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a continuous g-frame with bounds $A\|C^{-1}\|^{-2}$ and $B\|C^{-1}\|^2$. For the converse, assume that $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a continuous g-frame with bounds A', B'. Then, for all $f \in \mathscr{H}$

$$A'\|f\|^2 \leq \int_{\Omega} \|\Lambda_\omega f\|^2 d\mu(\omega) \leq B'\|f\|^2.$$

Hence, for $f \in \mathscr{H}$

$$\int_{\Omega} \|\Lambda_\omega Cf\|^2 d\mu(\omega) \leq B'\|Cf\|^2 \leq B'\|C\|^2 \|f\|^2.$$

Similarly, we have

$$A'\|f\|^2 = A'\|CC^{-1}f\|^2 \leq A'\|C^{-1}\|^2 \|Cf\|^2 \leq \|C^{-1}\|^2 \int_{\Omega} \|\Lambda_\omega Cf\|^2 d\mu(\omega).$$

Therefore, $\{\Lambda_\omega\}_{\omega\in\Omega}$ is a C^2-controlled g-frame with bounds $A'\|C^{-1}\|^{-2}, B'\|C\|^2$. ■

We conclude this section with the following lemma, that shows that the concept of a ”g-multiplier” can be defined for two C^2-controlled and C'^2-controlled continuous g-Bessel sequences and we call it ”the (C,C')-controlled continuous g-multiplier operator”.

Lemma 3. Let $C, C' \in \mathscr{GL}^+(\mathscr{H})$ and $\{\Lambda_\omega\}_{\omega\in\Omega}$ and $\{\Theta_\omega\}_{\omega\in\Omega}$ be C^2-controlled and C'^2-controlled continuous g-Bessel sequences for $\mathscr{H}$, respectively, and $m \in L^\infty(\Omega,\mathbb{C})$. The operator

$$M_{m,C,\Lambda,\Theta,C'} : \mathscr{H} \longrightarrow \mathscr{H},$$

defined by

$$\langle M_{m,C,\Lambda,\Theta,C'}(f), g\rangle = \int_{\Omega} m(\omega)\langle f, C'\Theta_\omega^*\Lambda_\omega Cg\rangle d\mu(\omega),$$

is a well-defined bounded operator.

Proof. Assume that $\{\Lambda_\omega\}_{\omega\in\Omega}$ and $\{\Theta_\omega\}_{\omega\in\Omega}$ are C^2-controlled and C'^2-controlled continuous g-Bessel sequences for $\mathscr{H}$, with bounds B_Λ, B_Θ, respectively. Then, we have

$$\begin{aligned}
\|M_{m,C,\Lambda,\Theta,C'}(f)\| &= \sup_{\|g\|=1} \|\langle M_{m,C,\Lambda,\Theta,C'}(f), g\rangle\| \\
&= \sup_{\|g\|=1} \Big\| \int_{\Omega} m(\omega)\langle f, C'\Theta_\omega^*\Lambda_\omega Cg\rangle d\mu(\omega)\Big\| \\
&\leq \sup_{\|g\|=1} \int_{\Omega} |m(\omega)| |\langle f, C'\Theta_\omega^*\Lambda_\omega Cg\rangle| d\mu(\omega) \\
&\leq \sup_{\|g\|=1} \|m\|_\infty \Big(\int_{\Omega} \|\Lambda_\omega Cg\|^2 d\mu(\omega)\Big)^{\frac{1}{2}} \Big(\int_{\Omega} \|\Theta_\omega C'^* f\|^2 d\mu(\omega)\Big)^{\frac{1}{2}} \\
&\leq \|m\|_\infty \sqrt{B_\Lambda B_\Theta}\|f\|.
\end{aligned}$$

This shows that $M_{m,C,\Lambda,\Theta,C'}$ is well-defined and

$$\|M_{m,C,\Lambda,\Theta,C'}\| \leq \|m\|_\infty \sqrt{B_\Lambda B_\Theta}.$$

■

4. Conclusions

In this article, the concept of weighted continuous g-frames is defined and the concept of multipliers from continuous frames to continuous g-Bessel sequences and continuous g-frames is extended. Controlled frames and controlled Bessel sequences are extended to controlled continuous g-frames and controlled continuous g-Bessel sequences. At the end of this paper, the concept of a "g-multiplier" for C^2-controlled and C'^2-controlled continuous g-Bessel sequences is defined.

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