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An Investigation of Radial Vibration Modes of Embedded Double-Curved-Nanobeam-Systems

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Abstract: The vibration of two curved nanobeams with coupling radial springs is considered. A nonlocal Euler-Bernoulli curved nanobeam model has been assumed in order to investigate the radial vibration of the double-curved-nanobeam-system (DCNBS) embedded in an elastic medium. Natural frequencies for the DCNBS are obtained by using the Navier Method. Moreover, the effect of the angle of curvature on the natural frequencies is discussed. Comparison studies are also performed to verify the present formulation and solutions. It is shown that the results are in excellent agreement with the previous studies. Furthermore, it is shown that considering the effects of the curvature decreases the natural frequency of the DCNBS and that the natural frequency decreases by increasing the small scale coefficient. In addition, the variation of the frequency has been investigated based on the stiffness of the springs in a radial direction.

Keywords: Radial vibration; Double curved nanobeam; Coupling radial springs.

1. Introduction

Nano materials have been highly attractive for many researchers in recent years due to the improvement of their quality and properties. Both experimental and atomistic modelling studies show that when the dimensions of the structures become very small, the size effect gains importance. Therefore, the size effect plays an important role in the mechanical behaviour of micro- and nanostructures [1]. Among the various nanostructures, nanobeams have many potential important applications [2, 3]. Thai [4] proposed a theory of nonlocal

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beam for the buckling, bending and vibration of nanobeams. The solutions of deflection, buckling load, and natural frequency were obtained for simply supported nanobeams.

The influences of various thermal environments on buckling and vibration of nonlocal temperature-dependent functionally graded beams are analysed by Ebrahimi and Salari [5] using the Navier Analytical Solution. In another work, Ebrahimi and Salari [6] investigated thermo-mechanical vibration of FG nanobeams with arbitrary boundary conditions applying a differential transform method. Furthermore, Ebrahimi et al. [7] explored the effects of linear and non-linear temperature distributions on the vibration of FG nanobeams. Ebrahimi et al. [8] investigated the vibration behaviour of size-dependent nanoscale beams based on nonlocal Timoshenko Beam Theory. Therefore, the nonlocal elasticity theory has been used within the Euler-Bernoulli beam model with von Kármán type nonlinearity. Boundary characteristic orthogonal polynomials have been investigated by Chakraverty and Behera [9] by using the Rayleigh-Ritz Method to survey free vibration within the framework of non-uniform Euler-Bernoulli nanobeams based on nonlocal elasticity theory. Hence, the non-uniform cross section of nanobeams are considered by taking linear, as well as quadratic, variations of Young's modulus and density along the space coordinate. In recent years, the vibration of curved nanobeams and nanorings has been utilized in many empirical experiments and dynamic molecular simulations [10]. Thus, a number of researchers are interested in studying vibration curved nanobeams.

Yan and Jiang [11] have investigated a curved piezoelectric nanobeam's electro mechanical response. In their paper, an Euler-Bernoulli curved beam theory was used to obtain the clearly explained solutions for the electroelastic fields of a curved simply-clamped beam when subjected to mechanical and electrical loads. However, a new numerical technique, namely the differential quadrature method, was developed for the dynamic analysis of the nanobeams in the polar coordinate system by Kananipour et al [12]. Additionally, Khater et al [13] have investigated the effect of surface energy and thermal loading on the static stability of nanowires. In this research, nanowires have been considered as curved fixed-fixed Euler-Bernoulli beams. The Gurtin-Murdoch theory was used to represent the surface effects. The model takes into account both the von Kármán strain and the axial strain. Furthermore, Wang and Duan [10] have surveyed the free vibration problem of nanorings/arches. In their research, the problem was formulated on the framework of Eringen's nonlocal theory of elasticity according to allowing for the small length scale effect. In this article, defects and elastic boundary conditions were investigated. The small length scale effect lowered the vibration frequencies. In addition, an explicit solution has been shown by Assadi and Farshi [14] for size and geometry dependent free vibration of curved nanobeams with consideration of surface effects. In this paper, surface elasticity, surface residual stress and surface mass density were included in the study to popularize the existing classical theories. The deviations of actual dynamic characteristics from the classical theories for various geometries were found as new results in their research.

Double-nanobeam systems are extremely important in nano-optomechanical systems and sensor applications.

Vu et al [15] have investigated the vibration of a double-beam-system. The free transverse vibration of double beam systems has been presented by Oniszczuk [16]. In this research, simple support has been assumed to investigation of double beam system. The effects of compressive axial load on forced transverse vibrations of a double-beam system have been surveyed by Zhang et al [17]. Li et al [18] have presented an exact dynamic stiffness matrix for axially loaded double-beam systems. The nonlocal effects in the forced vibration of an elastically connected double-carbon nanotube system under a moving nanoparticle have been investigated by Şimşek [19]. Stojanovic and Kozic [20] have presented the forced transverse vibration of the Rayleigh and Timoshenko double-beam system with the effect of compressive axial load. The nonlocal vibration of a double-nanoplate system has been considered by Murmu and Adhikari [21]. The couple nanoplates are considered to be joined by an enclosing elastic medium. In the research, the expressions for free-bending vibration of double-nanoplate systems have been investigated employing Eringen’s theory. The natural frequencies have been derived by employing an analytical solution. In addition, they considered the axial instability of Double-nanobeam systems. In this investigation, the nonlocal elasticity theory has been used for modelling double-nanobeam systems. The nonlocal model accounts for the small-scale effects due to the nanoscale [22]. However, a vibration analysis of a double nanobeam under primary compressive stresses has been investigated by Murmu and Adhikari [23]. In this paper, a scrutinizing method has been employed to determine the frequencies of the double nanobeam. However, the surface effects on the transverse vibration and axial buckling of a double-nanobeam have been surveyed within the framework of the Euler-Bernoulli Beam Theory by Hui Wang and Feng Wang [24]. For three typical deformation modes of the double-nanobeam system, they derived the natural frequency and critical axial load accounting for both surface elasticity and residual surface tension respectively. Moreover, Ciekot and Kukla [25] have investigated and considered a problem of transverse free vibration of a double nanobeam system. The nanobeams of the system are coupled by an arbitrary number of translational springs. Therefore, the solution to the problem by using Green's function properties was obtained. Incidentally, Ghorbanpour et al [26] was concerned with the size-dependent wave propagation of double-piezoelectric nanobeam-systems based on the Euler-Bernoulli Beam Theory. Two piezoelectric nanobeams were coupled by an enclosing elastic medium which has been simulated by the Pasternak Foundation. In this research, nonlocal elasticity theory has been used to derive the general differential equation based on Hamilton's principal to include those scale effects. In addition, a theoretical study of the free longitudinal vibration of a nonlocal viscoelastic double nanorod system has been presented by Karličić et al [27]. It is assumed that a light viscoelastic layer continuously couples two parallel nonlocal viscoelastic nanorods. Moreover, Murmu and Adhikari [28] have investigated the transverse vibration of two nanobeams with coupling springs. The expression for free bending vibration of a double nanobeam has been established using the nonlocal elasticity model. In this research, an analytical method has been developed to obtain the frequencies. The two nanobeams are presumably joined by vertical springs. The mentioned springs can be representatives of the van der Waals forces due to optomechanical coupling between the two nanobeams, or an enclosed elastic medium's stiffness. It is assumed that the coupled nanobeams in the double nanobeam were identical and the four boundary conditions on the four ends were simple. The effects of nonlocal parameter stiffness of the coupling springs and the higher modes of vibration on the resonance natural frequency of the nonlocal-double-nanobeam system are also investigated.

To the best of the author's knowledge, there has been no record regarding double curved nanobeams. Therefore, there is a strong scientific need to understand the vibration behaviour of double nanobeams when considering the effect of curvature. Hence in this paper, free vibration of DCNBS is investigated within the context of nonlocal elasticity theory. The natural frequencies of the DCNBS are obtained by using the Navier Solution. The prominent point of this study is the investigation of the curvature angles affects the radial vibration of DCNBSs. Comparison studies are also performed to verify present formulation and solutions. It has been shown that the results are in excellent agreement with previous studies and that considering the curvature effects decreases the natural frequency of the DCNBS.

2. Formulation

2.1. Governing Equation

First, we consider a curved beam with radius R, density ρ and cross-sectional area A, as depicted in Figure 1.

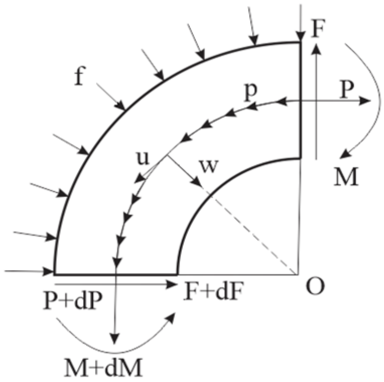

FIGURE 1. Element of a curved beam

Based on the Euler-Bernoulli beam theory, the equation of motion of the curved beam in the radial and tangential directions and the moment equation of motion in the middle plane of the ring can be expressed as

$$\frac{\partial F}{\partial \theta}+P+fR=\rho AR\frac{\partial^2 w}{\partial t^2} \quad (1)$$

$$\frac{\partial P}{\partial \theta}-F+pR=\rho AR\frac{\partial^2 u}{\partial t^2} \quad (2)$$

$$\frac{\partial M}{\partial \theta}+FR=0 \quad (3)$$

where $F(\theta,t)$ is the shearing force, $P(\theta,t)$ is the tensile force, $f(\theta,t)$ is the external distributed radial force, $p(\theta,t)$ is the distributed tangential force, ρ is the density, A is the cross-sectional area and R is the radius of the centre line of the ring [29]. Eliminating the normal stress resultant P from Eq. (1) and p=0 yields the single relation Eq. (4).

$$\rho AR\frac{\partial^3 w}{\partial t^2 \partial \theta}+\frac{1}{R}\frac{\partial^3 M}{\partial \theta^3}-R\frac{\partial f}{\partial \theta}+\frac{1}{R}\frac{\partial M}{\partial \theta}=\rho AR\frac{\partial^2 u}{\partial t^2} \quad (4)$$

By ignoring the products of small quantities

$$\frac{\partial u}{\partial \theta} = w \tag{5}$$

Substituting Eq. (5) into Eq. (4), and then simplifying the relation, obtains the equation of motion for an Euler-Bernoulli curved beam:

$$\rho AR\frac{\partial^4 w}{\partial t^2 \partial \theta^2} + \frac{1}{R}\frac{\partial^4 M}{\partial \theta^4} - R\frac{\partial^2 f}{\partial \theta^2} + \frac{1}{R}\frac{\partial^2 M}{\partial \theta^2} = \rho AR\frac{\partial^2 w}{\partial t^2} \tag{6}$$

By using the nonlocal theory, the equation of motion for a nonlocal Euler-Bernoulli curved beam is

$$\frac{EI}{R^3}w^{(6)} + \frac{2EI}{R^3}w^{(4)} + \frac{EI}{R^{(3)}}w^{(2)} - Rf^{(2)} + \frac{(e_0 a)^2}{R}f^{(4)} - \frac{\rho A(e_0 a)^2}{R}\ddot{w}^{(4)} + \rho A(R + \frac{(e_0 a)^2}{R})\ddot{w}^{(2)} - \rho AR\ddot{w} = 0 \tag{7}$$

2.2. Out-of-Phase Vibration of DCNBS; $\left(w_1 - w_2 \neq 0\right)$

We consider a DCNBS which connects two curved nanobeams with radial distributed springs, as shown in Figure 2. The material properties are considered to be that of single-walled carbon nanotube (SWCNT). The elastic modulus, E, is taken as 0.971 TPa and the mass density, ρ, is taken as 2300 kg/m^3. Employing Eq. (7), the governing nonlocal equations for the DCNBS, as shown in Figure 2, can be written as:

Curved nanobeam-1:

$$\begin{aligned}&\frac{E_1 I_1}{R^3{}_1}w_1^{(6)} + \frac{2E_1 I_1}{R^3{}_1}w_1^{(4)} + \frac{E_1 I_1}{R_1^3}w_1^{(2)} + R_1 k[w_1^{(2)} - w_2^{(2)}] - \frac{(e_0 a)^2}{R_1}k[w_1^{(4)} - w_2^{(4)}]\\ &- \frac{\rho_1 A_1 (e_0 a)^2}{R_1}\ddot{w}_1^{(4)} + \rho_1 A_1 (R_1 + \frac{(e_0 a)^2}{R_1})\ddot{w}_1^{(2)} - \rho_1 A_1 R_1 \ddot{w}_1 = 0\end{aligned} \tag{8}$$

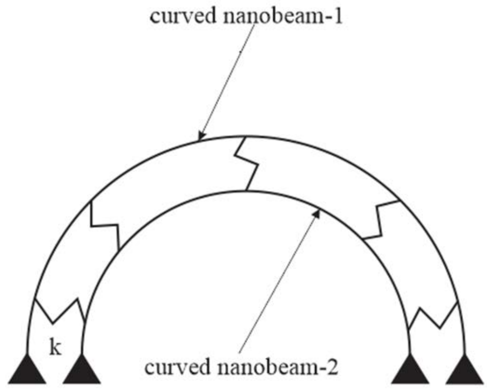

FIGURE 2. The configuration of a simply-supported DCNBS

Curved nanobeam-2:

$$\frac{E_2 I_2}{R^3{}_2} w_2{}^{(6)} + \frac{2E_2 I_2}{R^3{}_2} w_2^{(4)} + \frac{E_2 I_2}{R_2{}^3} w_2^{(2)} + R_2 k[w_2^{(2)} - w_1^{(2)}] - \frac{(e_0 a)^2}{R_2} k[w_2^{(4)} - w_1^{(4)}]$$
$$-\frac{\rho_2 A_2 (e_0 a)^2}{R_2} \ddot{w}_2^{(4)} + \rho_2 A_2 (R_2 + \frac{(e_0 a)^2}{R_2}) \ddot{w}_2^{(2)} - \rho_2 A_2 R_2 \ddot{w}_2 = 0 \quad (9)$$

We assume that

$$E_1 I_1 = E_2 I_2 = EI = C \quad (10)$$

$$\rho_1 A_1 = \rho_2 A_2 = \rho A = C \quad (11)$$

$$R_1 = R_2 = R = C \quad (12)$$

Considering Eqns. (8) and (9) and using the assumptions from Eqns. (10), (11) and (12), we get

Curved nanobeam-1:

$$\frac{EI}{R^3} w_1{}^{(6)} + \frac{2EI}{R^3} w_1^{(4)} + \frac{EI}{R^3} w_1^{(2)} + Rk[w_1^{(2)} - w_2^{(2)}] - \frac{(e_0 a)^2}{R} k[w_1^{(4)} - w_2^{(4)}]$$
$$-\frac{\rho A (e_0 a)^2}{R} \ddot{w}_1^{(4)} + \rho A (R + \frac{(e_0 a)^2}{R}) \ddot{w}_1^{(2)} - \rho A R \ddot{w}_1 = 0 \quad (13)$$

Curved nanobeam-2:

$$\frac{EI}{R^3}w_2^{(6)}+\frac{2EI}{R^3}w_2^{(4)}+\frac{EI}{R^3}w_2^{(2)}+Rk[w_2^{(2)}-w_1^{(2)}]-\frac{(e_0a)^2}{R}k[w_2^{(4)}-w_1^{(4)}]$$
$$-\frac{\rho A(e_0a)^2}{R}\ddot{w}_2^{(4)}+\rho A(R+\frac{(e_0a)^2}{R})\ddot{w}_2^{(2)}-\rho AR\ddot{w}_2=0 \tag{14}$$

In the case of the DCNBS, a change in variables is used by considering $w(\theta,t)$ as the relative motion of curved nanobeam-1 due to curved nanobeam-2:

$$w(\theta,t)=w_1(\theta,t)-w_2(\theta,t) \tag{15}$$

Subtracting Eq. (14) from Eq. (13) gives

$$\frac{EI}{R^3}[w_1^{(6)}-w_2^{(6)}]+\frac{2EI}{R^3}[w_1^{(4)}-w_2^{(4)}]+\frac{EI}{R^3}[w_1^{(2)}-w_2^{(2)}]+2Rk[w_1^{(2)}-w_2^{(2)}]-\frac{2(e_0a)^2}{R}k[w_1^{(4)}-w_2^{(4)}]$$
$$-\frac{\rho A(e_0a)^2}{R}[\ddot{w}_1^{(4)}-\ddot{w}_2^{(4)}]+\rho A(R+\frac{(e_0a)^2}{R})[\ddot{w}_1^{(2)}-\ddot{w}_2^{(2)}]-\rho AR[\ddot{w}_1-\ddot{w}_2]=0 \tag{16}$$

By employing Eq. (15) and using Eq. (16), we get

$$\frac{EI}{R^3}w^{(6)}+\frac{2EI}{R^3}w^{(4)}+\frac{EI}{R^3}w^{(2)}+2Rkw^{(2)}-\frac{2(e_0a)^2}{R}kw^{(4)}-\frac{\rho A(e_0a)^2}{R}\ddot{w}^{(4)}$$
$$+\rho A(R+\frac{(e_0a)^2}{R})\ddot{w}^{(2)}-\rho AR\ddot{w}=0 \tag{17}$$

It is essential to note that if the small scale coefficient is eliminated and a single curved beam is assumed, Eq. (17) turns into the equation of a classical curved beam.

3. Solution Method

3.1. Vibration of DCNBS

By considering that the relative motion $w(\theta,t)$ is a natural mode of vibration, the Navier solution for Eq. (17) can be expressed as

$$w=\sin\left(\frac{n\pi}{\alpha}\theta\right)e^{i\omega t} \tag{18}$$

where ω is the frequency, $\sin(n\pi\theta/\alpha)$ is the corresponding deformation shape of the DCNBS, and i is the conventional imaginary number $\sqrt{-1}$. Substituting Eq. (18) into Eq. (17) yields

$$\begin{aligned}&-\frac{EI}{R^3}\left(\frac{n\pi}{\alpha}\right)^6+\frac{2EI}{R^3}\left(\frac{n\pi}{\alpha}\right)^4-\frac{EI}{R^3}\left(\frac{n\pi}{\alpha}\right)^2-2Rk\left(\frac{n\pi}{\alpha}\right)^2-\frac{2\left(e_0a\right)^2}{R}k\left(\frac{n\pi}{\alpha}\right)^4\\&+\frac{\rho A(e_0a)^2}{R}\omega_n^2\left(\frac{n\pi}{\alpha}\right)^4+\rho A(R+\frac{(e_0a)^2}{R})\left(\frac{n\pi}{\alpha}\right)^2\omega_n^2+\rho AR\omega_n^2=0\end{aligned} \tag{19}$$

Here, we define the dimensionless natural frequency, stiffness and nonlocal parameter as:

$$\Omega_n^2=\omega_n^2\frac{\rho A}{EI}\left(R\alpha\right)^4 \tag{20.a}$$

$$\kappa=\frac{k\left(R\alpha\right)^4}{EI} \tag{20.b}$$

$$\mu=\frac{e_0a}{R\alpha} \tag{20.c}$$

By employing Eqns. (19) and (20), the dimensionless natural frequency of the DCNBS is as follows:

$$\Omega_n=\sqrt{\frac{\frac{\left(n\pi\right)^6}{\alpha^2}-2\left(n\pi\right)^4+\alpha^2\left(n\pi\right)^2+\frac{2\left(n\pi\right)^2}{\alpha^2}\kappa+\frac{2\mu^2\left(n\pi\right)^4}{\alpha^2}\kappa}{\frac{\mu^2\left(n\pi\right)^4}{\alpha^2}+\frac{\left(n\pi\right)^2}{\alpha^2}+\mu^2\left(n\pi\right)^2+1}} \tag{21}$$

3.2. Both Curved Nanobeams of DCNBS are Vibrating in Phase; ($w_1 - w_2 = 0$)

In this section, the in-phase vibration of the DCNBS will be surveyed. For the current DCNBS, the relative motion of the two curved nanobeams vanishes ($w_1 - w_2 = 0$). The dimensionless natural frequencies for this case can be expressed as

$$\Omega_n = \sqrt{\frac{\dfrac{(n\pi)^6}{\alpha^2} - 2(n\pi)^4 + \alpha^2 (n\pi)^2}{\dfrac{\mu^2 (n\pi)^4}{\alpha^2} + \dfrac{(n\pi)^2}{\alpha^2} + \mu^2 (n\pi)^2 + 1}} \tag{22}$$

The vibration of the DCNBS is not dependent of the stiffness of the radial spring for the in-phase sequence. Hence, the DCNBS vibrates as one single curved nanobeam.

3.3. One Curved Nanobeam of DCNBS is Stationary; ($w_2 = 0$)

Assume that the DCNBS that one of the two nanobeams is stationary ($w_2 = 0$). By employing the equations from Eringen's theory, the governing equation for the DCNBS [Eq. (17)] can be rewritten as

$$\begin{aligned} &\frac{EI}{R^3} w^{(6)} + \frac{2EI}{R^3} w^{(4)} + \frac{EI}{R^3} w^{(2)} + Rkw^{(2)} - \frac{(e_0 a)^2}{R} kw^{(4)} - \frac{\rho A (e_0 a)^2}{R} \ddot{w}^{(4)} \\ &+ \rho A (R + \frac{(e_0 a)^2}{R}) \ddot{w}^{(2)} - \rho A R \ddot{w} = 0 \end{aligned} \tag{23}$$

In this section, the DCNBS behaves as if the nanobeam is in a Winkler elastic foundation [30], and k can be defined as the stiffness of the Winkler elastic foundation. By employing the Navier solution, such as in the previous section, the natural frequency of DCNBS can be rewritten as

$$\Omega_r = \sqrt{\frac{\dfrac{(r\pi)^6}{\alpha^2} - 2(r\pi)^4 + \alpha^2 (r\pi)^2 + \dfrac{(r\pi)^2}{\alpha^2}\kappa + \dfrac{\mu^2 (r\pi)^4}{\alpha^2}\kappa}{\dfrac{\mu^2 (r\pi)^4}{\alpha^2} + \dfrac{(r\pi)^2}{\alpha^2} + \mu^2 (r\pi)^2 + 1}} \tag{24}$$

where κ is the stiffness of the radial springs and μ is the nonlocal parameter as expressed in Eq. (20). In fact, the DCNBS behaves as a nanobeam on an elastic foundation when one of the nanobeams in the DCNBS is stationary ($w_2 = 0$).

4. Numerical Results and Discussion

In this section, the dimensionless natural frequency of DCNBS (Ω) is compared with the dimensionless natural frequency of a straight double nanobeam subjected to a differently sized scale coefficient (μ), as in Figure 3. In our survey we made an extreme decrease in the curvature angle to simulate the straight double nanobeam, such as reference [28]. We achieved reasonable results in this survey such that they can represent the validity of our research, as shown in Figure 3.

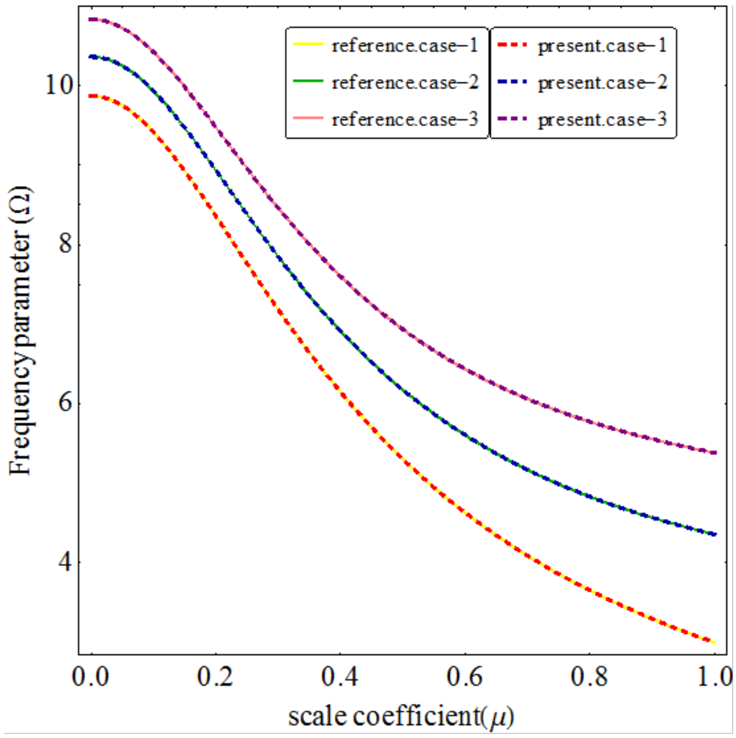

FIGURE 3. Comparison of present cases results and reference cases results [28]

4.1. Effect of Curvature Angle on Natural Frequency of DCNBS

As a first example, the first three dimensionless natural frequencies against the various angles of curvature for different cases of DCNBS are presented in Table 1. Case 1, Case 2 and Case 3 present the following conditions: (1) in-phase vibration of DCNBS $(w_1 - w_2 = 0)$, (2) when one of the curved nanotubes in DCNBS is not moving or not intended to be moved $(w_2 = 0)$, and (3) out-of-phase vibration of $(w_1 - w_2 \neq 0)$, respectively.

TABLE 1. The first three dimensionless natural frequencies against three angles of curvature for the three cases.

Case	α	Ω_1	Ω_2	Ω_3
Case 1	0	5.3002	11.9743	18.4389
	$\frac{\pi}{4}$	4.8206	11.6962	18.2476
	$\frac{\pi}{2}$	3.5555	10.8908	17.6828
	π	0	8.0326	15.5491
Case 2	0	6.1719	12.3849	18.7081
	$\frac{\pi}{4}$	6.4855	12.5098	18.7840
	$\frac{\pi}{2}$	5.3518	11.7232	18.2248
	π	3.1623	8.9735	16.1175
Case 3	0	6.9349	12.7822	18.9735
	$\frac{\pi}{4}$	7.8029	13.2735	19.3055
	$\frac{\pi}{2}$	6.6814	12.5003	18.7511
	π	4.4721	9.8246	16.6666

The results are in the dimensionless form of the natural frequency. The dimensionless stiffness of the springs is assumed to be constant ($\kappa = 10$), and the nonlocal parameter (μ) is assumed to be 0.5. The SWCNTs here are referred to as nanobeams. By increasing the angles of curvature in the three cases, the dimensionless natural frequencies will decrease as presented in Table 1. The values of the dimensionless natural frequency for these cases are different. As can be seen, the values of the frequencies in Case 1 are less than those of Case 2, and the values of

Case 2 are less than those of Case 3. The reason for these changes in values originates from Eqns. (21), (22) and (24). From these equations, it can be seen that the effect of stiffness in Case 2 is less than that of Case 3; therefore, the values of the natural frequencies in Case 3 are greater than the values of the natural frequency in Case 3. Moreover, the values of the natural frequencies in Case 1 are independent of stiffness of the coupling springs. Hence, the values of the natural frequencies for Case 1 are less than the other values. It may be noticed that the frequency parameters (Ω) are decreasing with an increase in α, as shown in Figure 4. There is also a decrease in the value of the dimensionless natural frequency with regard to the assimilation of angle of curvature in the DCNBS. Hence, the angle of curvature in the nonlocal elastic model is reflected in the vibration of the DCNBS. A comparison of the three cases of DCNBS shows that the dimensionless natural frequency for Case 3 is greater than the dimensionless natural frequency for Case 1 and Case 2. The relative higher frequency in Case 3 [Eq. 19] originates from the coupling effect of the springs.

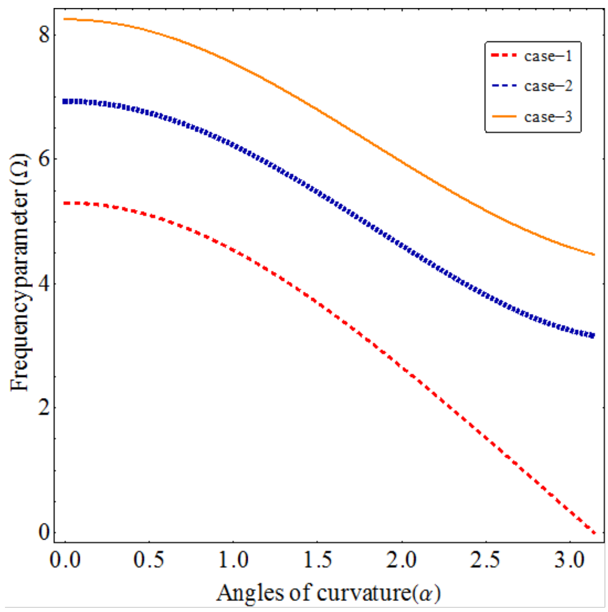

FIGURE 4. Variation in the dimensionless natural frequency (Ω) against the angle of curvature (α)

The presence of springs for Case 3 makes the DCNBS stiffer and increases the stiffness effect due to the system. For Case 2 the stiffness effect due to the auxiliary nanobeam (nanobeam 2) is absent. Hereupon there is effective lower stiffness parameter in Case 2.

4.2. Small-Scale Effect on Vibrating DCNBS

The first two dimensionless natural frequencies against the various scale coefficients for different cases of DCNBS are shown in Figures 5, 6 and 7. The variations of the frequency parameter of DCNBS with respect to nonlocal parameters are depicted for $\alpha = \pi/2$ in Figure 5. It is seen from the figure that the frequency parameter (Ω) decreases with the increase of the values of the dimensionless nonlocal parameter (μ). The decrease for the dimensionless natural frequency against the increasing nonlocal parameter occurs for all the three cases considered. The decrease is in the value of the dimensionless natural frequency with regard to the assimilation of small-scale effects in the DCNBS, hence the size effects in the nonlocal elastic model reflected in the vibration of the DCNBS.

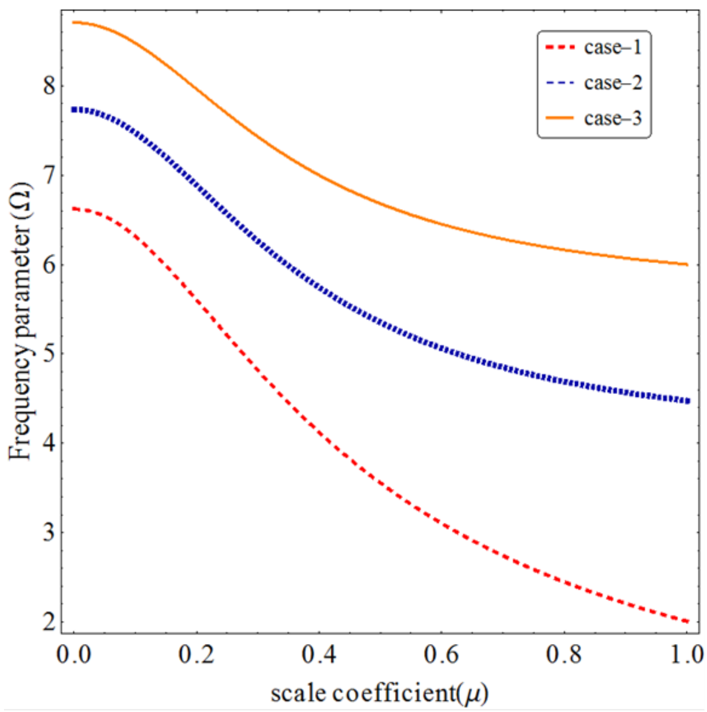

FIGURE 5. Variation in the dimensionless natural frequency (Ω) against the scale coefficient (μ) at ($\alpha = \pi/2$)

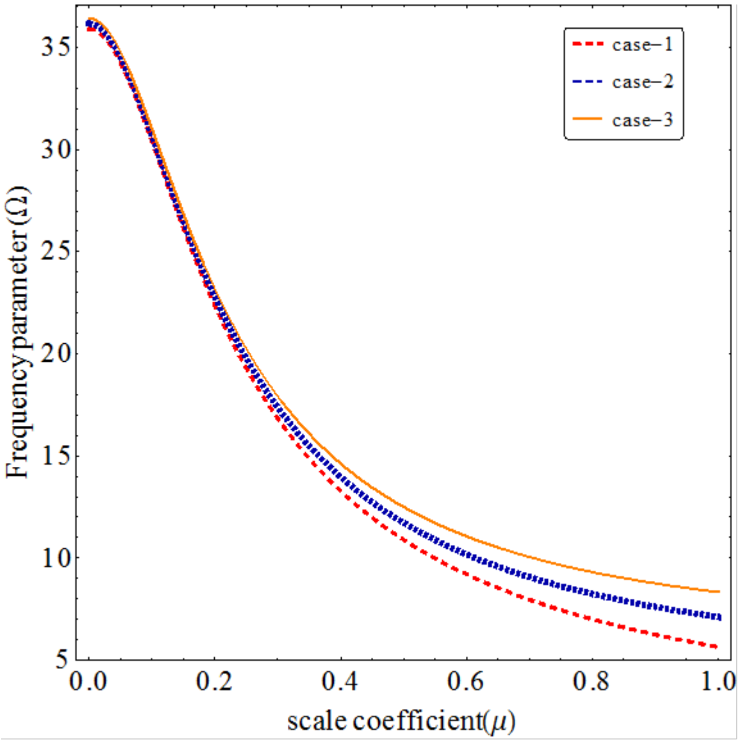

FIGURE 6. Variation in the dimensionless natural frequency (Ω) against the scale coefficient (μ) at (α = π/2).

To compare the three cases of DCNBS, the dimensionless natural frequency for Case 3 is greater than the dimensionless natural frequency for Case 1 and Case 2. The relative higher frequency in Case 3 [Eq. 19] originates from the coupling effect of the springs. The presence of springs for Case 3 makes the DCNBS stiffer and increases the stiffness effect due to the system. For Case 2, the stiffness effect due to the auxiliary nanobeam (nanobeam 2) is absent. Therefore, there is an effective lower stiffness parameter in Case 2.

In addition to that, it is observed that the value of the dimensionless natural frequency for Case 2 is greater than the value of the dimensionless natural frequency for Case 1. In Case 1, the frequency is relatively less because the DCNBS becomes independent of the stiffness of the springs. The DCNBS vibrates in an in-phase sequence. For Case 1, the DCNBS becomes analogous to the frequency of the single nanobeam without the coupling effect of springs. In other words, the entire DCNBS can behave as a vibrating single nanobeam. The variation in the natural frequency with respect to scale coefficient of the curved nanobeam for three cases at the second mode of vibration is considered as Figure 6. Hence, the figure is expressed in higher modes and the natural frequencies of the three cases are in better agreement.

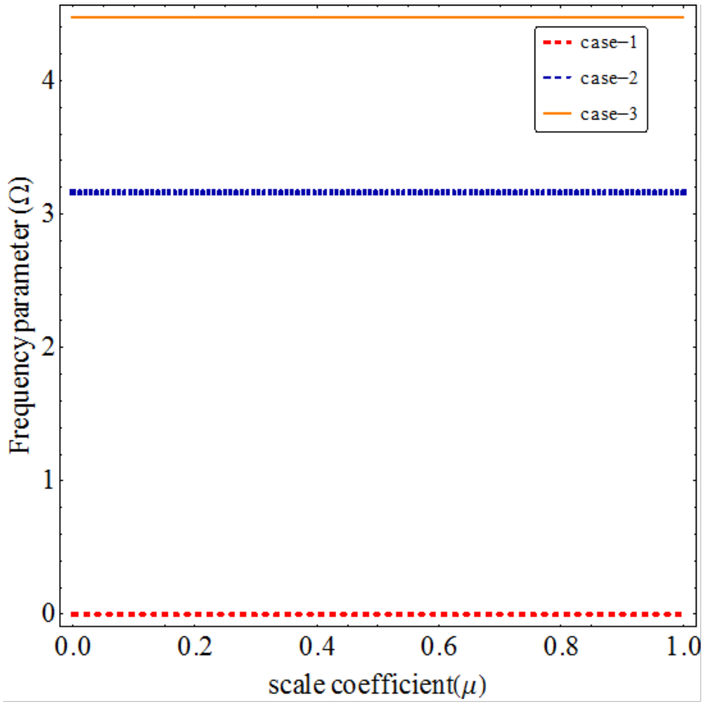

FIGURE 7. Variation in the dimensionless natural frequency (Ω) against the scale coefficient (μ) at α = π.

Thus, the vibration behaviours of the DCNBS in higher modes become more similar to each other.

The variation of the fundamental natural frequency with respect to the nonlocality parameter presented as it shown in figure 7. The figure reveals that the dimensionless natural frequency at curvature angle (α = π) is independent of the scale coefficient.

4.3. Analysis of Higher Modes of DCNBS

In this part, the higher natural frequencies of the DCNBS against the various angles of curvature for the different cases of DCNBS are tabulated in Table 2. In this case, the nonlocal parameter (μ) is assumed to be 0.5 and κ is considered to be 250. From Table 2, it can be seen that the dimensionless natural frequency increases with an increase in the number of modes (wave numbers).

TABLE 2. Variation in the dimensionless natural frequency (Ω) against the number of modes (n) for the three cases

Case	α	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
Case 1	0	5.3002	11.9744	18.4390	24.8203	31.1643	37.4887
	$\frac{\pi}{4}$	4.8206	11.6963	18.2477	24.6752	31.0476	37.3911
	$\frac{\pi}{2}$	3.5555	10.8908	17.6828	24.2439	30.6996	37.0997
	π	0	8.0326	15.5491	22.5743	29.3367	35.9514
Case 2	0	22.9803	25.3650	28.9827	33.4073	38.3564	43.6509
	$\frac{\pi}{4}$	22.2222	25.0821	28.8015	33.2704	38.2454	43.5572
	$\frac{\pi}{2}$	20.3136	24.2734	28.2696	32.8644	37.9145	43.2775
	π	15.8114	21.5528	26.3016	31.3080	36.6253	42.1781
Case 3	0	32.0639	33.8140	36.6059	40.2001	44.3984	49.0449
	$\frac{\pi}{4}$	31.0550	33.4876	36.4154	40.0622	44.2884	48.9527
	$\frac{\pi}{2}$	28.5069	32.5543	35.8560	39.6532	43.9609	48.6774
	π	22.3607	29.4028	33.7901	38.0891	42.6871	47.5970

The value of the higher natural frequency for Case 2 is greater than value of the higher natural frequency for Case 1. In Case 1, the higher frequency is relatively less because the DCNBS becomes independent of the stiffness of the springs. Similarly, this can be observed in Cases 2 and 3. By comparison in Eqns. (21) and (24), it displays the effects of coupling springs in Case 2 being less than Case 3.

4.4. Effect of the Stiffness of the Coupling Springs on DCNBS

In order to investigate the effect of higher and lower values of the stiffness of the coupling springs, diagrams have been drawn for the dimensionless natural frequency with respect to the dimensionless stiffness of the springs. The variation in values of the dimensionless natural frequency (Ω) with the dimensionless stiffness of the springs (κ) for the three angle of curvature of DCNBS ($\alpha = \frac{\pi}{4}, \frac{\pi}{2}, \pi$) are depicted in Figure 8. The dimensionless stiffness of the springs is taken to be κ = 0 to 100. The dimensionless nonlocal parameter μ is assumed to be 0.5 in this section. It is seen from the figure that the dimensionless natural frequency increases with the increase in the dimensionless stiffness of the springs. In addition, the dimensionless natural frequencies decrease with an increase in the angle of curvature of the DCNBS.

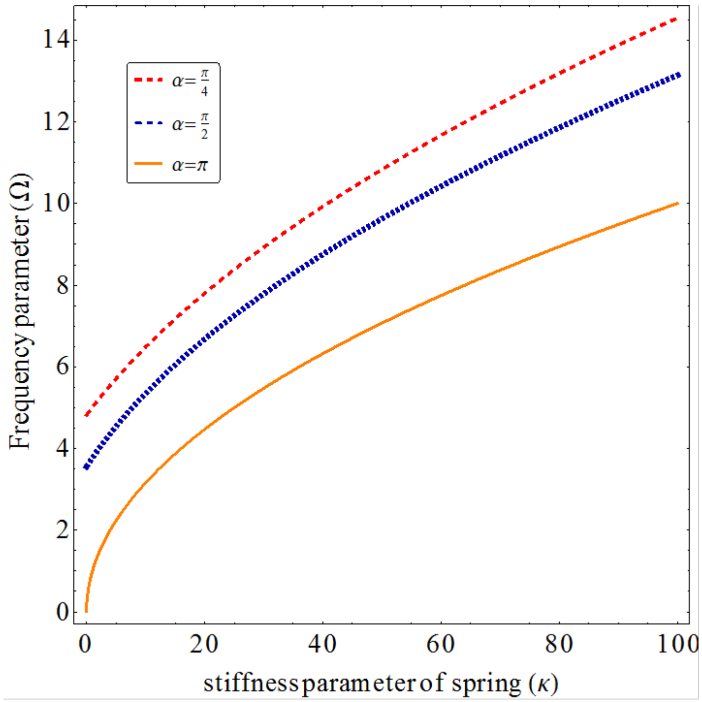

FIGURE 8. Variation in the dimensionless natural frequency (Ω) against the dimensionless stiffness of coupling springs (κ) for different angles of curvature at the fundamental mode of vibration

5. Conclusions

In this study, the radial vibration of double curved nanobeams has been investigated. Theoretical nonlocal elasticity and equilibrium equations were developed to determine the governing equation of a double-curved-nanobeam-system (DCNBS). To the best of the authors' knowledge, this is the first report regarding the vibration behaviour of double curved nanobeams. The numerical results have been determined by employing the Navier Method. The effect of various parameters, such as curvature angle, nonlocal parameters, stiffness of coupling springs and the mode number of natural frequencies of the DCNBS were investigated. This study has revealed that the curvature angle of the DCNBS plays an important role in their radial vibration wherein the natural frequencies decrease with increases in the curvature angle. In addition, nonlocal effects reduce the frequencies of the DCNBS. Moreover, in-phase and out-of-phase vibrations were surveyed with detail in this research. The frequencies of the in-phase vibration are independent of the stiffness of the springs, and the stiffness of the coupling springs are effective natural frequencies of the DCNBS. Hence, the natural frequencies of the DCNBS increase with an increase in the stiffness of the coupling springs.

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