Journal of Computer and Education Research
October 2024 Volume 12 Issue 24 (ISSN:2148-2896)
http://dergipark.org.tr/jcer

Research Article

Investigation of Gifted Students' Errors Related to Height and Diagonal Concepts in Geometry Teaching

Gülşah SALTIK AYHANÖZ [1] Solmaz Damla GEDİK ALTUN [2,*]

[1] *Niğde Akşemsettin Science and Art Center, Niğde, Turkey* gulsah@windowslive.com

[2] *Nevşehir Hacı Bektaş Veli University, Nevşehir, Turkey* sdgedik@nevsehir.edu.tr

[*] *Corresponding Author:* sdgedik@nevsehir.edu.tr

Article Info

Received: *19 April 2024*
Accepted: *09 September 2024*

Keywords: Gifted students, geometry, error, height, diagonal

10.18009/jcer.1470685

Publication Language: English

Abstract

The aim of this study was to examine the errors of gifted students regarding the concepts of height and diagonal in Geometry. Eighteen gifted students studying at a Science and Art Center in central Anatolia participated in the study. A knowledge test consisting of 5 questions was prepared to collect data. The knowledge test was applied to gifted students. Descriptive analysis method was used to analyze the data in the study. The data obtained in line with the responses of the students to the knowledge test used as a data collection tool were classified under categories in line with the purpose of the study by coding the errors in the solutions of the questions. According to the data obtained, it was concluded that the concept of diagonal and height, which constitute the basis of many subjects in geometry education of gifted individuals, should be emphasized more.

To cite this article: Saltık-Ayhanöz, G. & Gedik-Altun, S.D. (2024). Investigation of gifted students' errors related to height and diagonal concepts in geometry teaching. *Journal of Computer and Education Research, 12* (24), 523-548. https://doi.org/10.18009/jcer.1470685

Introduction

Geometry is a branch of mathematics that examines the relations, measurements and properties of points, angles, surfaces and objects (Turkish Language Society [TLS], 2022). Geometry has an important place in the elementary mathematics curriculum. Geometry is not only a field of meaningless shapes and rules. In geometry teaching, it is aimed that students gain the skills of reasoning, proving, problem solving, critical thinking, identifying facts and using definitions (National Council of Teachers of Mathematics [NCTM], 2000). Therefore, it is very important to teach the definitions of geometric concepts and their relationships with each other. NCTM (2000) emphasized the importance of students knowing the definitions of geometric concepts, the properties of geometric objects and forming geometric relationships with them. Geometry is mostly seen by students as a

collection of rules, shapes and symbols. Meaningful relationships cannot be established between the concepts used in this field, and teachers have problems in making students comprehend the subjects in geometry lessons (Konyalıoğlu, 2013). This situation makes it difficult to achieve the desired goals in mathematics education in the 21st century. As a matter of fact, there are studies in the literature (Fazlı & Avcı, 2022) showing that students continue the wrong and incomplete learning they experienced in primary school years at the secondary education level.

In order for students to achieve the desired goals in geometry teaching, meaningful learning is required. Many national and international publications emphasize the importance of meaningful learning in mathematics teaching, that is, the structuring of knowledge in the mind (Ministry of National Education [MoNE], 2018). When new information is encountered in the process of meaningful learning, the individual needs to go through some mental processes such as recall, evaluation, comparison, and association in order to make sense of this information in his/her mind (Yanık, 2016). Some negativities that occur during students' construction of knowledge in their minds cause errors and concept deficiencies in individuals (Şahin et al., 2023).

Height and diagonal are frequently used concepts in geometry teaching, which is a sub-branch of mathematics. Height is the line segment connecting the point where the perpendicular drawn from any corner of a triangle to the opposite edge or extension cuts the extension or edge and this corner (Cunnigham & Roberts, 2010). The definition of height given in the literature as "the perpendicular drawn from the corner to the base of a triangle" is considered as an incomplete definition. A student who adopts the second definition is likely to have difficulties in finding the height of the acute-angled corner of an obtuse triangle. When it is called the perpendicular to the base, what the student is looking for in the triangle will be the base and incorrect learning will occur (Bütüner, 2017). Students' drawing the height incorrectly and not being able to locate it will cause difficulty for students in subjects where the concept of height should be known (Gürefe & Gültekin, 2016). The definition of diagonal is defined in the literature as the line segment connecting any two non-adjacent vertices in a polygon (Çoker & Karaçay, 1983). Knowing the definitions and properties of geometric concepts such as height and diagonal is very important in terms of creating geometric relationships between these concepts and learning geometry. A problem encountered in this fundamental subject will spread to all geometry subjects. The

foundations of the concept of "diagonal", which is the subject of geometry, are laid in the 3rd grade mathematics course of primary school. The concept of diagonal is included in the curriculum again in the 5th and 6th grade mathematics courses. In the 7th grade mathematics course curriculum, there is a learning outcome as "Identification of the concept of diagonal by students". The teaching and drawing of the concept of "height" is included in the measurement sub-learning area in the 6th grade in the curriculum (MONE, 2018). For this reason, the fact that students have learned these concepts incorrectly and incompletely in these classes may cause them to have difficulties in these subjects and in subsequent subjects that are based on these subjects in their later educational life.

Individuals with Special Talents

Gifted children, who have rich vocabulary, advanced verbal skills, extraordinary thinking skills, fluency, leadership capacity, creativity and high-level problem-solving skills compared to their peers, have been considered important for societies throughout history (Sak, 2011). Bringing the talents of all individuals to the best level is considered very necessary today. This understanding of education has made the education of gifted individuals more important. Ataman (2003) stated that the education programs prepared for students with normal development are not suitable for gifted students, they get bored at school because they learn fast and lose their motivation. The ability of gifted individuals to receive education appropriate to their abilities plays an important role in identifying and recognizing their giftedness and educating them according to their abilities (Siegle, 2001). Gifted individuals need special education, and students with special talents in the field of general ability need special education, especially in mathematics education. When the needs of children in need of special education are not met, inequality emerges, which is seen as a problem in the sociology of education. The education of gifted students constitutes an important step in ensuring equality of opportunity in education (MoNE, 1991:15). In this context, errors and misconceptions in mathematics education of gifted students, especially in the subjects that constitute the basis and are frequently used, are an important factor that should be taken into consideration in learning environments, so they need to be identified. It is imperative to intervene in students' misconceptions and misconceptions because it is not possible for students to overcome them with their own efforts (Zembat, 2015, p. 3).

Today, it is considered very necessary to optimize the talents of all individuals. This understanding of education has led to a greater emphasis on the education of gifted

individuals. Giftedness is generally defined for students in three groups. These groups are classified as those who have talent in areas such as sports, music, and art, those who have talent in an academic subject, and those who have versatile talents (Taber, 2017). Since the Geometry learning domain includes shape and space concepts, it contributes significantly to the development of students' intuitive thinking and visual perception, and to their reflective and critical view of the situations around them (Türnüklü et al., 2017). In addition, Geometry can be expressed as a learning area that allows students to recognize, analyze and evaluate their environment in a realistic way; consists of a set of intuition, knowledge and concepts; and has many meaningful relationships between these sets. These statements reveal that the skills that the Geometry course provides students with are supportive in developing the talent areas of gifted students. MONE (2018) stated that meaningful learning should be realized in order for students to use the relationships in Geometry in a meaningful way. In order to realize the meaningful learning mentioned in the mathematics curriculum, it is necessary to identify the errors and learning difficulties that students make about geometric concepts (Arabacı & Kanbolat, 2023; Öçal, 2017).

Importance of Research

In a study conducted with pre-service teachers, it was found that the participants had poor content knowledge of the concepts of height, diagonal and angle (Cunnigham & Roberts, 2010; Gutierrez & Jaime, 1999). Similar results were found in another study conducted with students (Monaghan, 2000). In another study, it was found that pre-service teachers made errors in drawing the heights of triangles (Gutierrez & Jaime, 1999). In another study, it was found that few students were able to correctly define the height and perpendicular center of a triangle (Hızarcı et al., 2006). In addition, Cunnigham and Roberts (2010) found that pre-service teachers often had the idea that you cannot draw a diagonal outside a polygon. When the literature was examined, no study was found in which the errors of gifted students about height and diagonal were identified. Therefore, it was necessary to conduct such a study.

Purpose of the Study

When Turkey's international achievement in geometry is analyzed, Turkish students scored below the international average in TIMSS 2019. Turkey ranked 22nd among 39 countries in the geometry learning domain (Mullis et al., 2019). Studies have shown that students have below-average achievement in geometry learning domain. In the light of the

research conducted and the results of TIMSS 2019, considering that the errors encountered by the students have a positive effect on the success of the students, it was aimed to conduct a study on the concepts of height and diagonal in the geometry learning domain to determine the errors of the students. In this way, by identifying students' errors, it will be possible to eliminate the deficiencies that cause these errors and provide students with a healthy education. The results of this study will enable mathematics teachers to have information about errors and to improve their teaching techniques accordingly. It is thought that the study to be conducted with gifted students will also be a guide for primary school students. In this context, the aim of the study is to determine the errors of gifted students in height and diagonal topics and the problem statement of the research is determined as "What are the errors of gifted students in height and diagonal topics?".

Method

Research Design

In this study, it was tried to determine the content knowledge of gifted students about the concepts of height and diagonal. In this study, qualitative research methods were used and case study design was utilized. In case studies, data are analyzed through in-depth examination of one or more special cases (Yıldırım & Şimşek, 2013). In addition, the errors of individuals regarding the concepts of height and diagonal were identified as a case. This design was defined by Yin (1984) as a design in which there is a single unit of analysis, a well-formulated theory is tested, and unique and contradictory situations are studied. In line with the above, it can be said that this study is a case study. With this method, answers to questions such as "what", "how" and "why" were sought and the data obtained were presented in the findings section.

Sample

The study was conducted with 18 gifted students at Akşemseddin Science and Art Center in Niğde province in the 2022-2023 academic year. Criterion sampling provides the opportunity to work with people, situations or events with the qualities determined in relation to the problem in a research (Yıldırım & Şimşek, 2016). Easily accessible criterion sampling method, one of the qualitative sampling methods, was included in the sampling method. It is important that students volunteer in order to maximize the efficiency of the product that emerges in practical studies. In this context, student selection for the study was

based on volunteerism. The students participating in the study were coded as K_1, K_2, ..., K_{18} and these codes were used in the presentation of the data.

Data Collection Tool

In order to collect the data in the study, the researchers prepared a 5-question knowledge test including the concepts of height and diagonal for gifted students. For these 5 questions, studies on these concepts were utilized (Altıntaş & İlgün, 2017; Bütüner, 2017; Cunnigham & Roberts, 2010; Gutierrez & Jaime, 1999). In the first question, students were asked to define the concepts of "height and diagonal". In the second question, students were asked to find the diagonals of the given geometric figures. In the third question, students were asked to find the heights of the given geometric figures. In the fourth and fifth questions, students were expected to answer questions such as whether the lines indicated by dashed lines indicate height and diagonal, and if not, why not.

Data Analysis

Descriptive analysis is the presentation of research data to the reader with direct quotations, adhering to its original form without any changes (Miles & Huberman, 1994). In this type of analysis, qualitative data are processed by adhering to a predetermined framework, findings are defined, and then the defined findings are interpreted (Yıldırım & Şimşek, 2016). In this study, in line with the responses of gifted students to the knowledge test used as a data collection tool, the data were coded according to the answers to the questions and classified according to the predetermined categories for the purpose of the study. In the evaluation of these definitions, the definitions made for this concept in the literature were taken into consideration. These definitions are given in Table 1.

Table 1. Definitions of diagonal and height concepts

Diagonal	A line drawn between two vertices that are not consecutive in a polygon or not on the same plane in a polyhedron (MoNE, 2023).
Height	The line segment joining the point where the perpendicular drawn from any corner of a triangle to the opposite edge or extension cuts the edge or extension and this corner is called the height of that edge. (MoNE, 2018, p. 80). The line segment joining a corner of a triangle with the point where the perpendicular drawn from any corner to the opposite side or extension intersects the side or extension. (Cunnigham & Roberts 2010)

Descriptive analysis method was used to analyze the data in the study. The data obtained in line with the answers given by the students to the achievement test used as a

data collection tool were classified under categories in line with the purpose of the study by coding the errors in the solutions of the questions. After all questions were evaluated, the results were categorized. The results were then evaluated by two experts in the field. A complete harmony was achieved during the analysis. After the data analysis process was completed, the data were presented in tables and interpreted. At the bottom of all categories, the answers given by the students were supported with direct quotations. Then, the data frequencies obtained from the students' responses were calculated and the reason statements were analyzed.

Reliability and Validity

The validity of this study was tried to be ensured by reporting the data obtained from the study in detail. One of the ways to ensure validity in qualitative studies is to report the data in detail (Yıldırım & Şimşek, 2016). In order to ensure the internal validity of the research, attention was paid to organizing the findings meaningfully and consistently, using data triangulation, clearly identifying the weaknesses and limitations of the research, and clarifying unclear facts. In addition, a conceptual framework was created by reviewing the relevant literature during the preparation of the data collection tools to be used to ensure internal validity in the research. In this way, while conducting descriptive analysis on the data obtained, it was tried to provide the scope to include the relevant concepts. In order to ensure the external validity of the study, the research process; determination of the participants, activity environments and selection, implementation of the practices, evaluation and data analysis were explained in detail. In addition, in order to ensure the external validity of the study, research problems and results were presented consistently and a detailed and rich narrative was used (Gül & Sözbilir, 2015). Students' personal information was kept confidential in the presentation of the findings. Qualitative data were coded and analyzed by two researchers to ensure the internal validity of the data. The consistency of the coding made by the researchers in the analysis of the data was checked.

In order to ensure internal reliability in the reliability dimension of the research, the data were given in descriptive ways and direct quotations. In addition, in order to ensure the reliability of the study, it was stated that the names of the students would not be used in any way. A classroom environment where students could feel comfortable during the application was created and no time limit was imposed for students to give their answers in detail. For the purpose of the study, the research was carried out by giving the necessary time to the

students in the appropriate time period. In this way, more accurate answers were tried to be obtained from the students.

In order to ensure external reliability, data collection tools were explained in detail and associated with the conceptual framework in the selection of data sources. The reliability of the research was calculated using Miles and Huberman's (1994) reliability formula by determining the number of agreement and disagreement in the comparisons. According to Miles and Huberman (1994), consensus among coders is expected to be at least 80%. Inter-coder reliability was calculated by dividing the number of codes agreed upon by the researchers by the total number of agreed and disagreed codes. The average reliability was calculated as 86% by determining the consensus and disagreement between the researchers.

Finding

This section presents the findings obtained from the analysis of gifted students' responses to the questions and their explanations about the reasons for these responses. In this direction, the data obtained from the students' answers were grouped according to the framework determined as "True, Partially True, Partially False, False and No Answer" and frequency values were calculated and the values for each question are presented in tables below. In addition, students' answers to the questions were supported with explanations and direct quotations. The first question of the data collection tool is given below as the definitions of the given geometric terms.

Question 1: Define the geometric terms given below.

Diagonal:
Height:

The answers given by the students regarding this question are categorized according to codes and given in Table 2 and Table 3.

Table 2. Answers related to the diagonal in question 1

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_8, K_{10}, K_{15}	3	%17
Partially Correct	K_1, K_2, K_3, K_4, K_5, K_9, K_{11}, K_{13}, K_{16}, K_{17}, K_{18}	11	%61
Partially False		0	
False	K_{12}, K_{14}	2	%11
No Answer	K_6, K_7	2	%11

Table 3. Answers related to the diagonal for question 1

Codes	**Pre-Servıce Teachers**	**F**	**%**
True		0	
Partially Correct	K_1, K_2, K_3, K_4, K_5, K_7, K_8, K_{11}, K_{15}, K_{17}, K_{18}	11	%61
Partially False		0	
False	K_9, K_{10}, K_{12}, K_{13}, K_{16}	5	%28
No Answer	K_6, K_{14}	2	%11

When the data in Table 2 are analyzed, it is seen that 3 students were able to correctly define the diagonal expression asked in the first question and 11 students were able to define it partially correctly. While 2 students defined it incorrectly, 2 students did not make any definition. In the first question, students were asked to define diagonal. Diagonal is defined as the line drawn between non-consecutive vertices of a polygon or between two vertices of a polyhedron that are not on the same plane. Taking this definition into account, the answers of the students were analyzed. The number of students who were able to define correctly in accordance with the given definition was 3. It was observed that the students who gave correct answers took into account that the number of sides of polygons must be 4 or more in order for them to have diagonals, and that they also took into account the lines drawn from non-neighboring vertices. The 11 students who defined it partially correctly were observed to consider the line segments connecting non-neighboring vertices. The 2 students who defined it incorrectly defined it as a line segment connecting opposite sides of a given shape. They did not take into account that the vertices should not be neighboring. One of the students also emphasized that the diagonals should be drawn on the inside of the shape. The definitions given by some students for this question are given below with direct quotations.

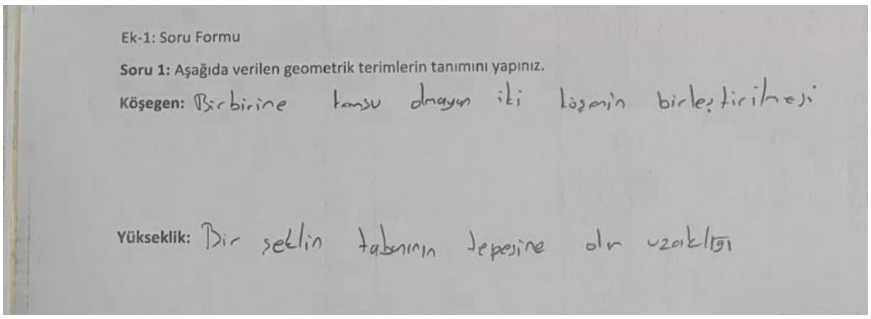


Figure 1. Student K3's answer to the first question

The response of the student coded K3 is as follows:
Diagonal: Two vertices that are not adjacent to each other.
Height: The distance from the base to the top of a shape.

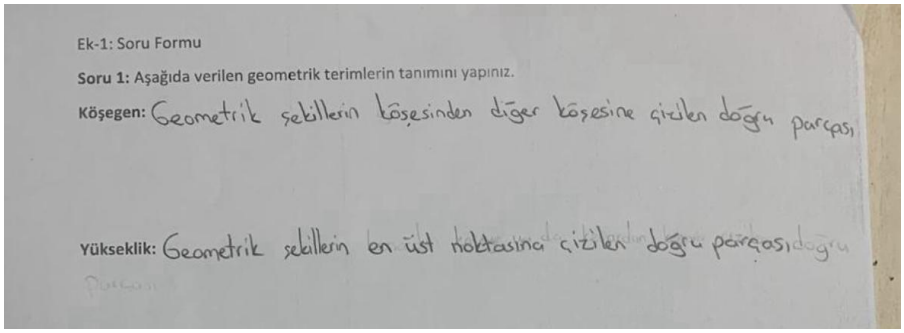


Figure 2. Student K12's answer to the first question

The response of the student coded K12 is as follows:
Diagonal: Line segment drawn from one corner of a geometric figure to the other corner.
Height: Line segment drawn to the top point of geometric shapes

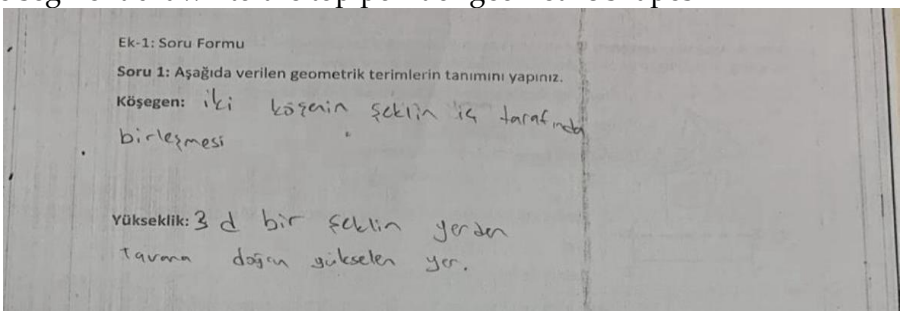


Figure 3. Student K16's answer to the first question

The response of the student coded K16 is as follows:
Diagonal: The union of two vertices on the inside of a figure.
Height: The place where a 3D shape rises from floor to ceiling

In the second question, students were asked to define height. Height is the farthest point of an object perpendicular to the base of the object from the base taken as reference. Taking this definition into account, the students' answers were coded. When the data in Table 3 are analyzed, it can be seen that while there were no students who could correctly define the definition of height in the second question, there were 11 students who could partially define it correctly. While 5 students defined it incorrectly, 2 students did not define

it at all. It was observed that the students who could define it partially correctly generally defined it as the length drawn from the corner to the base, from the base to the corner of a given shape, and as the perpendicular length. The students who answered incorrectly generally made definitions that were not related to height. Students did not specify polygons in their answers and used the expression "a shape". The definitions given by some students for this question are given below with direct quotations.

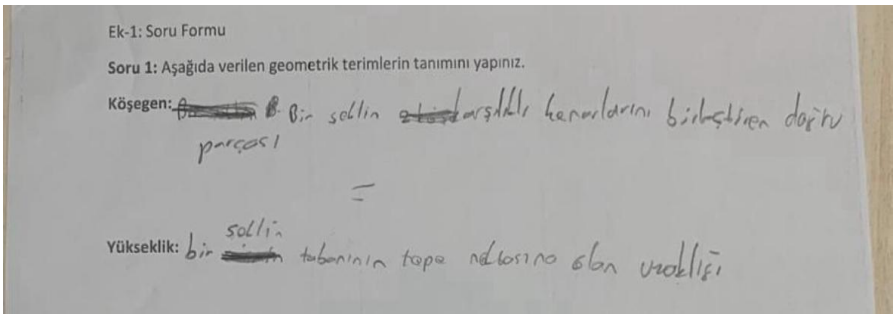


Figure 4. Student K5's answer to the second question

The response of the student coded K5 is as follows:
Diagonal: Line segment joining opposite sides of a shape
Height: The distance from the base of a shape to the vertex

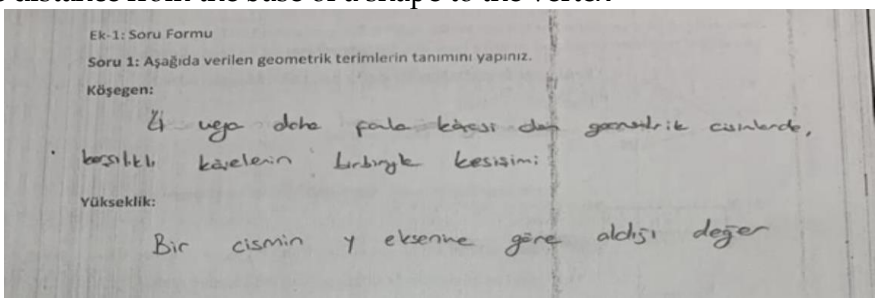


Figure 5. Student K9's answer to the second question

The response of the student coded K9 is as follows:
Diagonal: The intersection of opposite corners in geometric objects with four or more corners.
Height: The value of an object relative to the y-axis

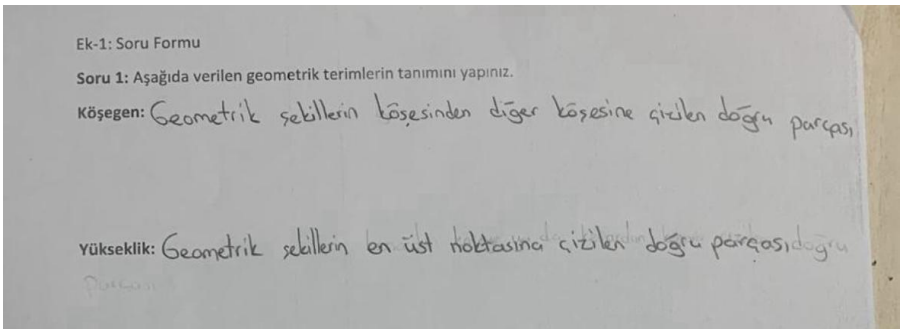


Figure 6. Student K13's answer to the second question

The response of the student coded K13 is as follows:
Diagonal: Line segment drawn from one corner of a geometric figure to the other corner.
Height: Line segment drawn to the top point of geometric shapes

Question 2. Draw the diagonals of the geometric shapes given below.

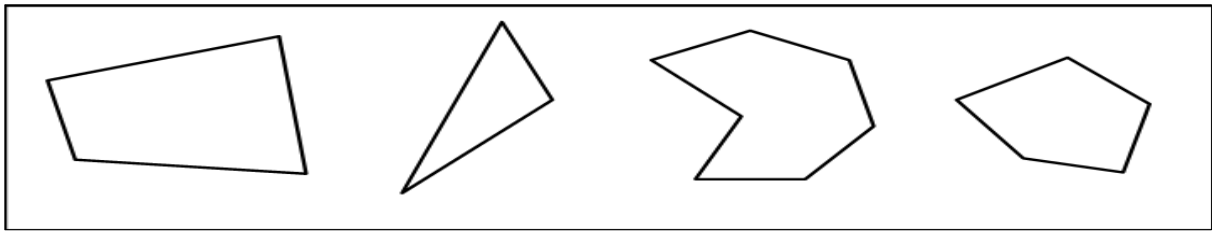


Table 4. Responses to question 2

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_7, K_9, K_{12}, K_{14}	4	%22
Partially Correct	K_3, K_5, K_6, K_8, K_{10}, K_{11}, K_{15}, K_{16}, K_{17}, K_{18}	10	%55
Partially False	K_1, K_2, K_4	3	%17
False	K_{13}	1	%6
No Answer		0	

When the data in Table 4 are analyzed, the students who drew all of the diagonals of the geometric figures given in the second question correctly were coded as correct. Students who drew 1 of them incorrectly and the diagonals of other geometric figures correctly were coded as partially correct. Students who drew 2 diagonals incorrectly were coded as partially incorrect, while students who drew only 1 diagonal correctly were coded as incorrect. When the table is analyzed, 4 students drew the diagonals of all geometric figures correctly, while 10 students drew the diagonals of 3 of the geometric figures correctly. While 3 students drew 2 diagonals incorrectly; 1 student drew all diagonals incorrectly. Excerpts from some students are given below.

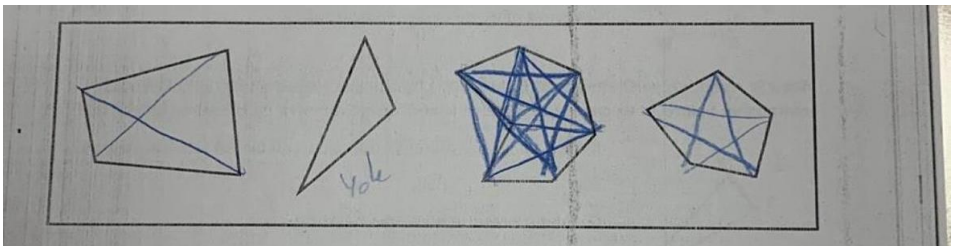


Figure 7. Student K_7's response to the second question

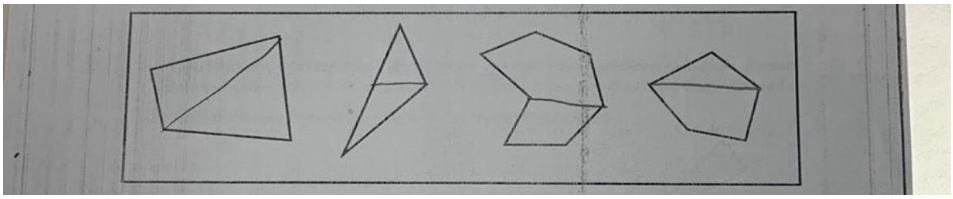


Figure 8. Student K_{13}'s response to the second question

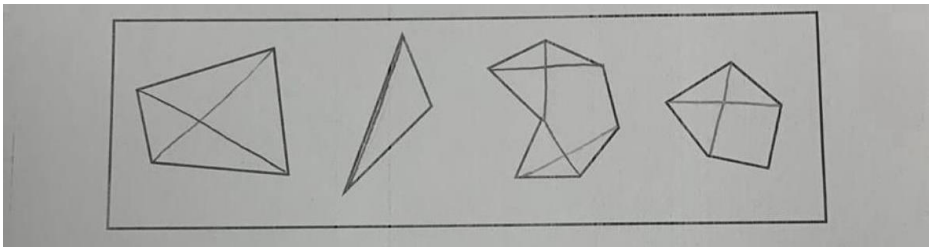


Figure 9. Student K_4's response to the second question

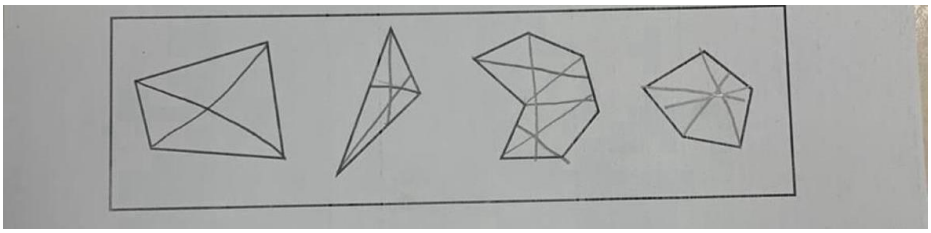


Figure 10. Student K_{15}'s response to the second question

Question 3. Find the heights of the triangles given below based on side A.

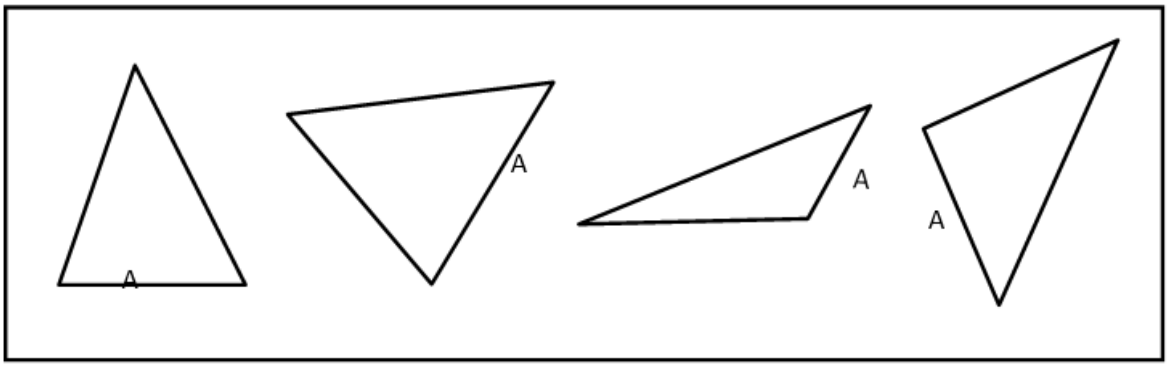


Table 5. Responses to question 3

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_9, K_{13}	2	%11
Partially Correct	K_1, K_2, K_7, K_8, K_{14}, K_{16}, K_{17}, K_{18}	8	%44
Partially False	K_3, K_4, K_5, K_6, K_{10}, K_{12}, K_{15}	7	%39
False	K_{11}	1	%6
No Answer		0	

When the data in Table 5 are examined, the students who correctly drew all of the heights of the base area of side A in the triangles given in the third question were coded as correct. Students who drew 1 of them incorrectly and the other heights correctly were coded as partially correct. Students who drew 2 and 3 heights incorrectly were coded as partially incorrect, while students who could not draw the heights in the triangles correctly were

coded as incorrect. When the table is analyzed, 2 students drew all the heights of the given triangles correctly by taking side A as the base, while 8 students drew the heights of 3 of the given triangles correctly. While 7 students drew 2 or 3 heights incorrectly; 1 student drew all heights incorrectly. Excerpts from some students are given below.

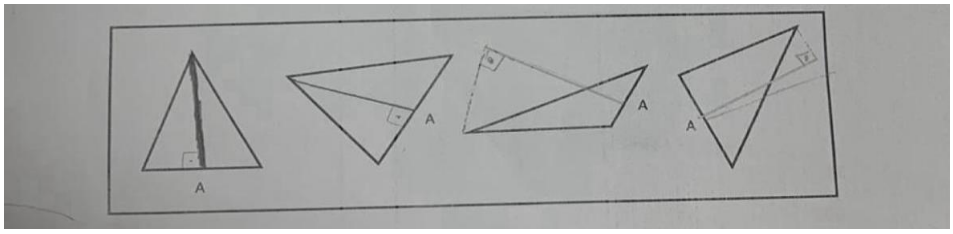


Figure 11. Student K18's response to the third question

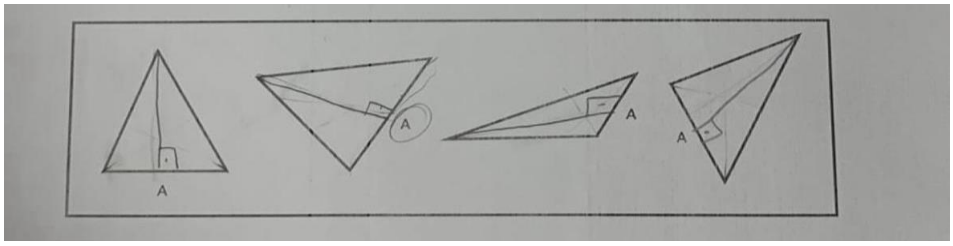


Figure 12. Student K6's response to the third question

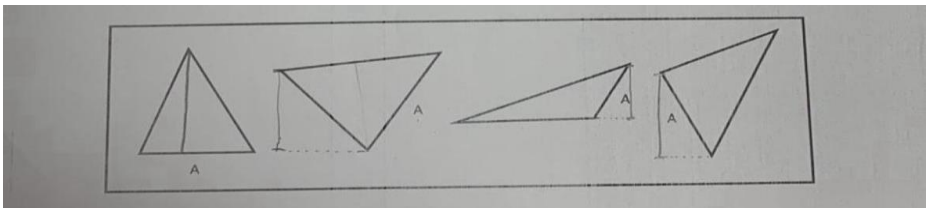


Figure 13. Student K15's response to the third question

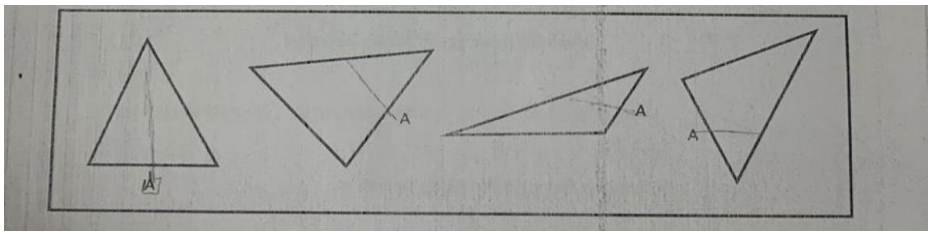
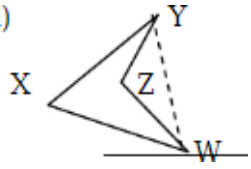


Figure 14. Student K11's response to the third question

Question 4. For each of the following figures, the dashed line segment has a diagonal.

Determine that it is not, and circle your answer. If the answer is no, please give a reason.

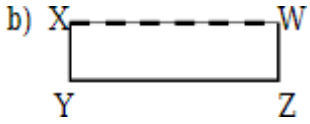
a)



Is the line segment YW denoted by the dashed line a diagonal?

Yes No

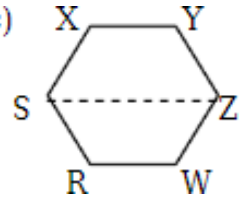
If the answer is no, please give a reason.

b) 

Is the line segment XW, denoted by the dashed line, a diagonal?

Yes No

If the answer is no, please give a reason.

c) 

Is the line segment SZ, denoted by a dashed line, a diagonal?

Yes No

If the answer is no, please give a reason.

Table 6. Answers related to question 4, option A

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_2, K_3, K_4, K_5, K_6, K_{11}, K_{12}, K_{14}	8	%44
Partially Correct		0	
Partially False	K_{15}, K_{17}	2	%12
False	K_1, K_7, K_8, K_9, K_{10}, K_{13}, K_{16}, K_{18}	8	%44
No Answer		0	

When Table 6 is examined, 8 of the students answered all of them correctly regarding whether the line segments with dashed lines in the figure given in the first option of the fourth question are diagonals. 2 students answered this question incorrectly. But they did not give any explanation. Therefore, they were coded as "Partially Incorrect". The remaining 8 students gave incorrect answers and gave incorrect explanations. Therefore, they were coded as incorrect. The students who gave wrong answers gave answers such as the diagonal should not be outside and it should be from corner to corner.

Table 7. Responses to Question 4, Option B

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_9, K_{10}, K_{14}, K_{15}, K_{16}, K_{17}, K_{18}	14	%78
Partially Correct	K_8, K_{12}	2	%11
Partially False		0	
False	K_{11}, K_{13}	2	%11
No Answer		0	

When Table 7 is examined, 16 of the students answered correctly regarding the reason why the line segment with dashed lines in the figure given in choice b of the fourth question is a diagonal. 2 students gave the wrong answer by answering "yes". No explanation was given for these students' answers. Two of the students who answered the question correctly gave correct answers but did not give any explanation. Therefore, they were coded as "Partially Correct". The explanations of those who gave correct answers were that the edge cannot be a diagonal and cannot be the union of two adjacent diagonals. The following are direct excerpts from some students' answers to the questions in the A and B options of Question 4.

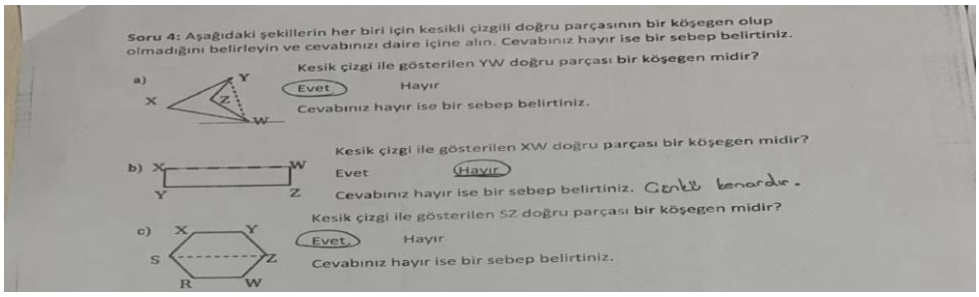


Figure15. Student K_2's response to the fourth question

The response of the student coded K_2 is as follows:

b) because it is the edge

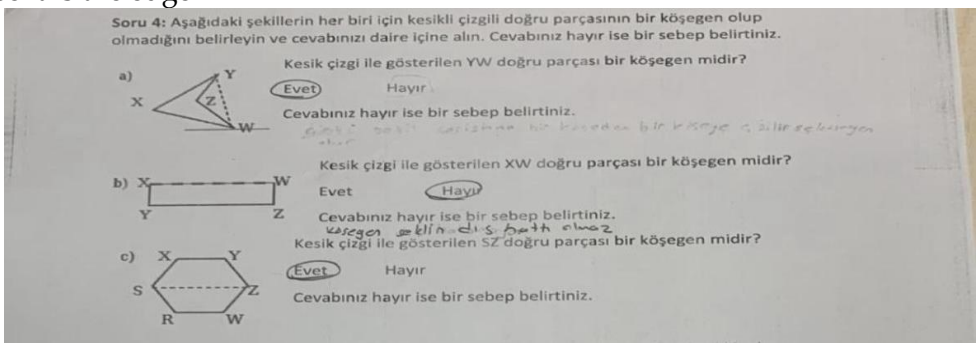


Figure16. Student K_{13}'s response to the fourth question

The response of the student coded K_{13} is as follows:

b) cannot be out of diagonal shape

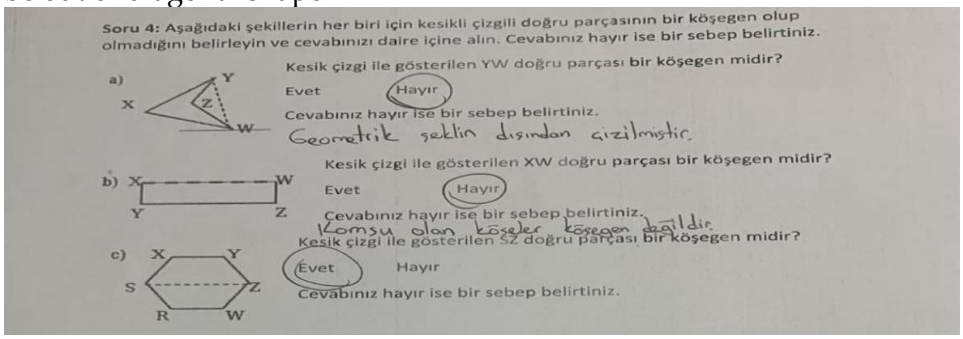


Figure17. Student K_{17}'s response to the fourth question

The response of the student coded K_{17} is as follows:

a) drawn from outside the geometric shape

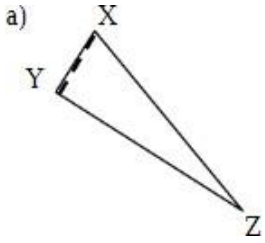
b) Adjacent vertices are not diagonal

All students answered choice c of the fourth question correctly by answering "Yes". They did not give any explanation for saying yes. Therefore, no table was created and no quotation was made.

Question 5: Is there a height of the dashed line segment for each of the following figures

Determine that it is not, and circle your answer. If the answer is no, please give a reason.

a)

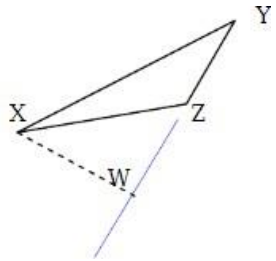


Does the line segment XY line segment indicated by the dashed line indicate a height?

Yes No

If the answer is no, please give a reason.

b)

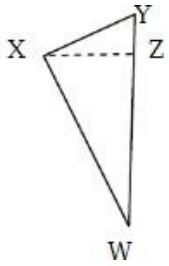


Does the line segment XW denoted by the dashed line indicate a height?

Yes No

If the answer is no, please give a reason.

c)



Does the line segment XZ, denoted by the dashed line, indicate a height?

Yes No

If the answer is no, please give a reason.

Table 8. Answers to Question 5, Option A

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_3, K_{14}	2	%11
Partially Correct	K_{16}, K_{17},	2	%11
Partially False	K_5, K_6, K_7, K_{10}, K_{11}, K_{18}	6	%33
False	K_1, K_2, K_4 K_8, K_9, K_{12}, K_{13}, K_{15}	8	%45
No Answer		0	

Table 8 shows that 2 of the students answered the question about whether the line segment with dashed lines in the figure given in the first option of the fifth question is a height correctly. These students explained that "if line segment XY is perpendicular to line segment YZ, then line segment XY is a height". 8 students gave partially incorrect answers to

this question. These students answered "No" to the question. Even though the answer was correct, their explanations were incorrect. Therefore, they were coded as "Partially Incorrect". Students generally stated that the edge cannot be a height. 2 students gave correct answers but did not give explanations. These were coded as "Partially Correct". The remaining 8 students answered incorrectly by saying "Yes". Therefore, they were coded as "Incorrect".

Table 9. Responses to Question 5, Option B

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_3, K_{13}	2	%11
Partially Correct	K_{15}, K_{17},	2	%11
Partially False	K_{18}	1	%6
False	K_1, K_2, K_4 K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{14}, K_{16},	13	%72
No Answer		0	

When Table 9 is analyzed, 2 of the students answered correctly whether the line segment with dashed lines in the figure given in choice B of the fifth question is a height. These students explained that "if line segment XW is perpendicular to line segment WY, then line segment XW is a height". 2 students answered "No" to this question and did not give any explanation. Therefore, it was coded as partially correct. 1 student answered "no" to this question but was coded as Partially Incorrect because he/she made the explanation incorrectly. This student stated that WZ would be a height. The remaining 13 students answered incorrectly by saying "Yes". Therefore, they were coded as "Incorrect".

Table 10. Responses to Question 5, Option C

Codes	**Pre-Service Teachers**	**F**	**%**
True	K_3	1	%6
Partially Correct	K_{17}	1	%6
Partially False		0	
False	K_1, K_2, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, K_{15}, K_{16}, K_{18}	16	%88
No Answer		0	

Looking at Table 9, only 1 of the students answered correctly as to whether the line segment with dashed lines is a height as given in option C of the fifth question. This student explained that "if the line segment XZ is perpendicular to the line segment WY, then the line segment XZ is a height". 1 student answered "No" to this question and did not give any explanation. Therefore, it is partially encoded correctly. The other 16 students answered

incorrectly by saying "Yes". That's why they are coded as "False". Direct quotations for the fifth question are given below.

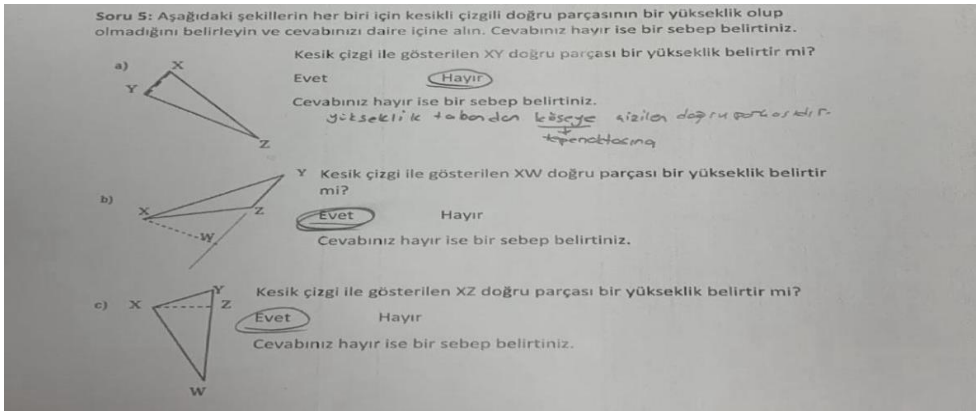


Figure 18. K3's answer to the fourth question

The response of the student coded K3 is as follows:

a) height is the segment drawn from the base to the corner

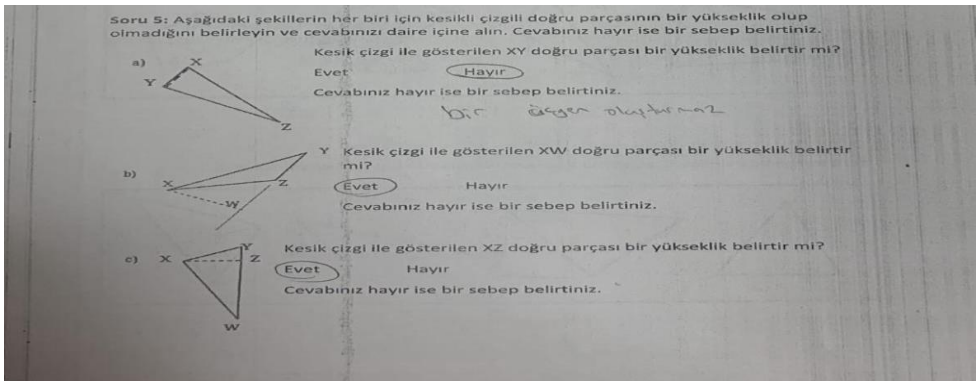


Figure 19. K5's answer to the fourth question

The response of the student coded K5 is as follows:

a) does not form a triangle

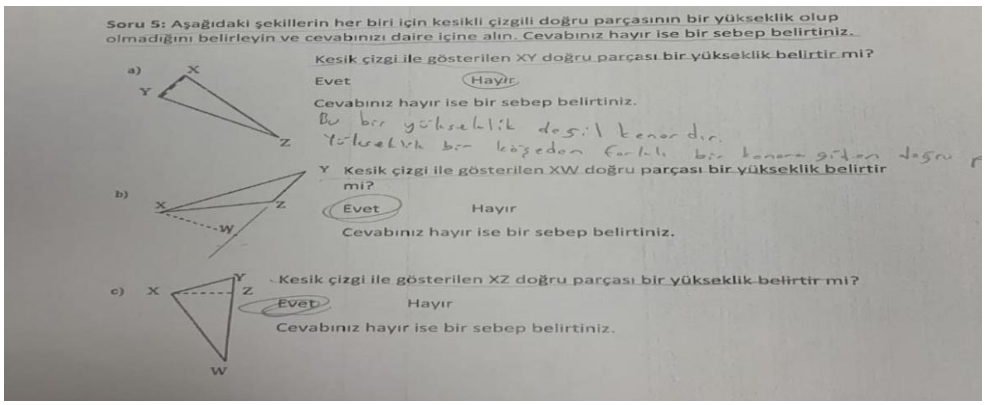


Figure 20. K9's answer to the fourth question

The response of the student coded K9 is as follows:

a) This is not a height but an edge. height is a line from a corner to a different edge

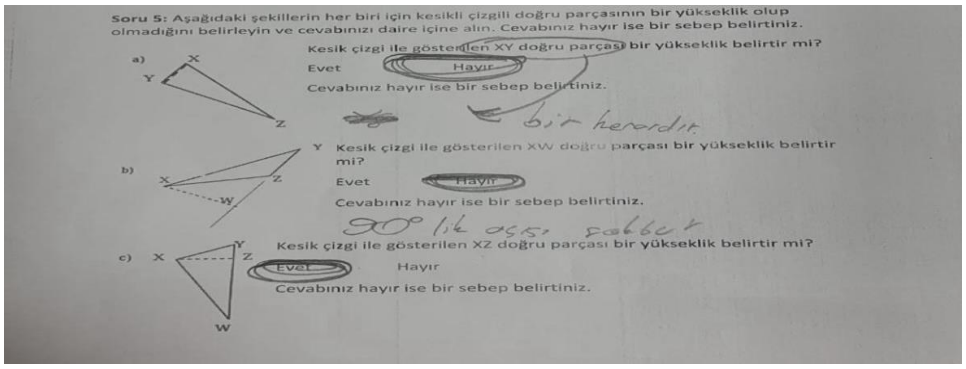


Figure 21. K_{11}'s answer to the fourth question

The response of the student coded K_{11} is as follows:

a) is an edge
b) There is no 90 degree angle

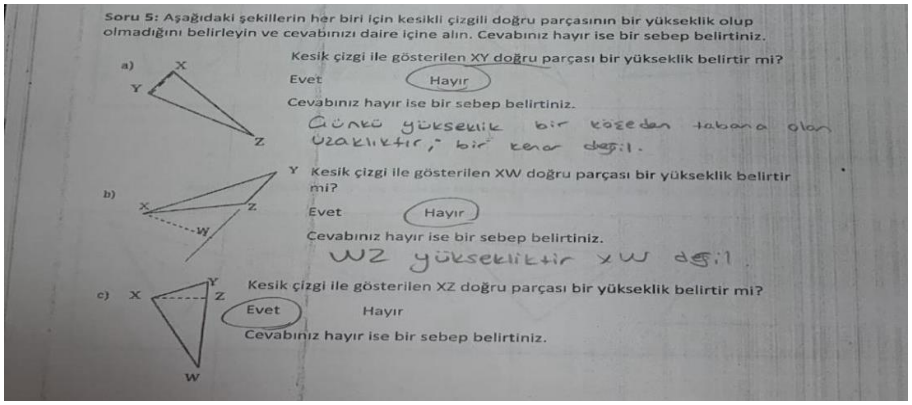


Figure 22. K_{18}'s answer to the fourth question

The response of the student coded K_{18} is as follows:

a) height is not an edge. Height is the distance from a corner to the base.
b) WZ is the height. XW isn't the height.

Discussion and Conclusion

In this study, the errors found in the five-question knowledge test prepared for the height and diagonal subjects of the gifted students were examined. It was tried to determine whether there were errors in the solutions of the questions given in the knowledge test. The responses given to the knowledge test by gifted students were evaluated.

When gifted students were asked about the definition of the concept of height and diagonal, 2 of the students did not answer for the definition of the concept of diagonal, 3 of them were correct, 11 were partially correct and 2 were wrong; For the concept of height, 2 did not respond, 11 students gave partially correct and 5 incorrect answers. Until the 8th grade, the conceptual definition of height was not mentioned in the books, and this concept was shown to the students by drawing on the figure. In the 8th grade mathematics textbook (MONE, 2023), the concept of height is defined as "the line segment drawn perpendicular to the opposite side or extension of the triangle from one corner of the triangle is called the

height of that side" and "the distance between the bases" is defined for geometric objects. The concept of diagonal; It is defined as "a line segment connecting two non-adjacent vertices in a polygon" and "a line segment connecting two vertices that are not on the same face in a polyhedron" (Çoker & Karaçay, 1983). This concept was first included in the geometry learning area in the 5th grade elementary mathematics curriculum at the primary education level (MONE, 2023). Vinner (1991) stated that if students' thoughts about the concept are incorrect, the definitions may also be incorrect. Some of the students may be due to the fact that they do not have enough conceptual knowledge or their thoughts about the concept are wrong. Because the inability to learn geometric shapes conceptually can cause students to make incorrect definitions (Linchevsky et. al., 1992). Similarly, Şengün and Yılmaz (2021) stated that in their research, students confused the concept of height with different concepts and had difficulty in determining height. In addition, fifteen of the forty-seven pre-service teachers in the Aksu et al. (2013) research confused the concept of diagonal with the concepts of vertex and edge, and in the study conducted by Ayvaz et al. (2017), it was concluded that the pre-service teachers made the definition of the diagonal incorrectly or incompletely.

In the second question, gifted students were asked to draw diagonals for some geometric shapes. 4 of the students answered correctly, 10 partially correct, 3 partially incorrect, and 1 incorrect. This shows that the vast majority of students can draw diagonals for different geometric shapes and answer correctly. It can be said that this situation has emerged thanks to the constructivist education given within the scope of the special education that the students receive. It is seen that the students who make mistakes make mistakes, especially regarding the drawing of a diagonal in a triangle, and the students confuse the concept of diagonal with different concepts in the triangle. Owens (2005) also stated in his research that participants had difficulties in forming diagonals. When the literature review is done, there are studies that conclude that students believe in the existence of a diagonal in the triangle and look for a diagonal in the triangle in the questions asked (Pickreign, 2007). For gifted students, it is seen that the situation stated by İnan et al. (2015) is the opposite. It is extremely important to use GeoGebra, Cabri and other mathematics software and web 2 tools in classroom environments in geometry education (Şahin et al., 2023). Because GeoGebra and other mathematics software have the potential to be used in many subjects in our curriculum (Kaba et al., 2010). It is thought that mathematics

software and web 2 tools included in classroom environments in special education given for gifted individuals are effective in the emergence of this situation.

Gifted students were asked to draw heights for some geometric shapes in the third question. 2 of the students answered correctly, 8 partially correct and 8 incorrect answers. It is seen that the number of students who gave correct answers and those who gave wrong answers are close to each other. In the research conducted by Şahin et al. (2023) with secondary school students, it was determined that students similarly had problems in drawing heights. Moreover, in the study conducted with secondary school mathematics teachers, it was stated that the participants made a mistake while determining the center of perpendicularity of the wide-angle triangle, they could not draw the heights in the wide-angle triangle, and they could not show that the right sides in the right triangle are heights (Yurtyapan & Karataş, 2020). In line with these statements, it is thought that both students and teachers have problems in drawing heights. This situation coincides with the conclusion that knowing a subject is necessary but not sufficient for learning (Konyalıoğlu et al.,2012).

In the fourth question, gifted students were asked whether the line segment with a dashed line for some geometric shapes was a diagonal. Although the fourth question consists of three parts, 8 students answered the A option of the question correctly, 8 students answered incorrectly and 2 students partially answered incorrectly. In option A of the fourth question, the majority of gifted students made a mistake and it is seen that the students have deficiencies in drawing diagonals in concave polygons. Similarly, Cunningham and Roberts' (2010) pre-service teachers could not draw all the diagonals of concave polygons with the idea that the diagonals would pass through the polygon. For option B of the fourth question, 14 students, 2 students partially answered correctly and 2 students answered incorrectly, and it is seen that the majority of the students answered this question correctly. This shows that students do not confuse the concepts of edge and diagonal. Option C of the fourth question was answered correctly by all students. These results contradict the conclusion of the study by Duatepe et al. (2013) and Kuzniak and Rauscher (2007) that students in questions involving diagonals had low levels of reasoning. It can be thought that this situation is due to the fact that specially talented individuals have high-level reasoning skills. Saltık-Ayhanöz (2022) stated that gifted students can produce original and logical solutions to many mathematical problems that their peers cannot solve, and use mathematical formulas effectively. The ability of gifted students to solve problems in different ways with unusual

speed and accuracy, and their ability to relate mathematics to real life has also been mentioned (Fıçıcı & Siegle, 2008). It is thought that these characteristics of gifted students support their use of the mathematical reasoning approach.

In the fifth question, gifted students were asked whether the dashed line segment was a height for some geometric shapes. It is seen that the vast majority of students answer the questions incorrectly. İnan et al. (2015) stated that although students were visually aware of the concept of triangle and its types, they had difficulty and made mistakes while defining or drawing the concept of height. The result of the research coincides with the results of the research conducted by Yurtyapan and Karataş (2020).

As a result of the research, it was seen that there were some errors regarding the concept of diagonal and height in specially talented individuals. If the concepts in geometry education are not adequately understood, it will be difficult to achieve the expected goals in education (Dağlı, 2010). A correct understanding of the concept of height is very important, especially for geometry topics such as volume and area (Van De Walle, 2014). In this direction, it has been concluded that the concept of diagonal and height, which is the basis of many subjects in the geometry education of specially talented individuals, should be emphasized more.

Recommendations

In geometry education, teachers teach through presentation in order to train the subjects in the curriculum. This situation prevents the structuring of geometry concepts by students, so concept deficiencies may occur. A study by Oberdorf and Cox (1999) found that students have errors due to insufficient experience. He stated that students need concrete thinking in order to identify and understand their mistakes (Koester, 2003). In order to make the information concrete, students should be offered rich environments based on the problems and interaction in life in constructivist learning environments. For this reason, it is very important to create learning environments based on errors, where learning activities will be carried out in abundance in Mathematics and Geometry lessons, and learning by doing and experiencing can take place. It is thought that adopting an activity and application-oriented teaching based on errors instead of a content-intensive teaching in mathematics lesson will be more beneficial in learning the concepts correctly.

Ethical Committee Permission Information

Name of the board that carries out ethical assessment: Niğde Ömer Halisdemir University Social and Humanities Scientific Research and Publication Ethics Board

The date and number of the ethical assessment decision: 31.01.2024 and 2024/01-16

Author Contribution Statement

Gülşah SALTIK AYHANÖZ: *Conceptualization, literature review, collection of data, review-writing and editing, editing the translation.*

Solmaz Damla GEDİK ALTUN: *Conceptualization, literature review, collection, processing, analysis of data, review-writing and editing, editing the translation.*

References

Altıntaş, E. & İlgün Ş. (2017). Ortaokul matematik öğretmenlerinin geometride "yükseklik" ve "diklik merkezi" kavramına ilişkin kavram yanılgıları [misconceptions of secondary school mathematics teachers regarding the concepts of "height" and "center of perpendicularity" in geometry]. *International Periodical for the Languages Literature and History of Turkish or Turkic, 12*(29), 73-86.

Arabacı, D., & Kanbolat, O. (2023). Pre-service elementary mathematics teachers' evaluations of meetings with expert and peer participation based on Van Hiele geometric thinking levels. *International e-Journal of Educational Studies, 7*(15), 752-768. https://doi.org/10.31458/iejes.1345567

Ataman, A. (2003). Üstün zekâlı ve üstün yetenekli çocuklar (Ed: A. Ataman). *Özel gereksinimli çocuklar ve özel eğitime giriş* [gifted and gifted children (Ed: A. Ataman) *Children with special needs and introduction to special education*]. (s. 173-195). Gündüz Eğitim ve Yayıncılık.

Ayvaz, Ü., Gündüz, N. & Bozkuş, F. (2017). Understanding of prospective mathematics teachers of the concept of diagonal. *Journal on Mathematics Education, 8*(2), 165-184.

Bütüner, S. Ö. (2017). Matematik öğretmen adaylarının geometri alan bilgilerinin belirlenmesi: Açı, köşegen, yükseklik, dörtgen [Determination of mathematics teacher candidates' geometry field knowledge: Angle, diagonal, height, quadrilateral]. *Amasya Üniversitesi Eğitim Fakültesi Dergisi, 6*(2), 501-530.

Cunningham, R. F., & Roberts, A. (2010). Reducing the mismatch of geometry concept definitions and concept images held by pre-service teachers. Erişim Tarihi: 7 Haziran 2023, www.k12prep. math.ttu.edu.tr

Çoker, D. & Karaçay, T. (1983). *Matematik terimleri sözlüğü [Dictionary of mathematical terms]. (1. Baskı)*. Türk Dil Kurumu Yayınları.

Dağlı, H. (2010). *İlköğretim beşinci sınıf öğrencilerinin çevre, alan ve hacim konularına ilişkin kavram yanılgıları* [*Misconceptions of primary school fifth grade students regarding perimeter, area and volume*]. Yüksek Lisans Tezi, Afyon Kocatepe Üniversitesi.

Fazlı, E., & Avcı, Ö. (2022). Matematik eğitiminde motivasyon ve öz-düzenleme: Tek bir durum çalışması [otivation and self-regulation in mathematics education: A single case study]. *Harran Maarif Dergisi, 7*(1), 1-45.

Fıçıcı A., & Siegle D. (2008). International teachers' judgment of gifted mathematics student characteristics. *Journal of Gifted Talented International, 23*(1), 22-37. https://doi.org/10.1080/15332276.2008.11673510

Gutierrez, A. & Jaime, A. (1999). Pre-service primary teachers' understanding of the concept of attitude of a triangle. *Journal of Mathematics Teacher of Education, 2*(3), 253-275.

Gül, Ş., & Sözbilir, M. (2015). Fen ve matematik eğitimi alanında gerçekleştirilen ölçek geliştirme araştırmalarına yönelik tematik içerik analizi [Thematic content analysis of scale development research in the field of science and mathematics education]. *Eğitim ve Bilim, 40*, 85-102.

Gürefe, N. & Gültekin, S.H. (2016). Yükseklik kavramına dair öğrenci bilgilerinin incelenmesi [Examination of student knowledge about the concept of height]. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi, 17*(2), 429-450.

Hızarcı, S., Ada, Ş. & Elmas, S. (2006). Geometride temel kavramların öğretilmesi ve öğrenilmesindeki hatalar [errors in teaching and learning basic concepts in geometry]. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi, 13*, 337-342.

Jones, S., & Tanner, H. (2000). Becoming a successful teacher of mathematics (pp. 86-104). Routledge Falmer.

Koester, B. A. (2003). Prisms and pyramids: Constructing three-dimensional models to build understanding, *Teaching Children Mathematics, 9(*8), 436- 442.

Konyalıoğlu, A.C., Özkaya, M. & Gedik, S. D. (2012). Matematik öğretmen adaylarının konu alan bilgilerinin hataya yaklaşımları açısından incelenmesi [Examining the subject knowledge of mathematics teacher candidates in terms of their approach to errors]. *Iğdır University Journal of the Institute of Science and Technology, 2*(2SpA), 27-32.

Linchevsky, L., Vinner, S., & Karsenty, R. (1992). To be or not to be minimal? Student teachers views about definitions in geometry. In W. Geeslin & K. Graham (Eds.), *Proceedings of the 16th international conference for the psychology of mathematics education* (pp. 48–55). Durham USA.

Ministry of National Education [MONE], (2018). *Matematik dersi öğretim program [Mathematics course curriculum].* MoNE Publications.

Ministry of National Education [MONE], (2023). *Ortaokul 5. sınıf matematik ders kitabı [secondary school 5th grade mathematics textbook].* MoNE Publications.

Ministry of National Education [MONE], (2023). *Ortaokul 8. sınıf matematik ders kitabı [secondary school 8th grade mathematics textbook].* MoNE Publications.

Ministry of National Education [MONE], 1991. *Üstün yetenekli çocuklar ve eğitimleri raporu [gifted children and their education report].* MoNE Publications.

Miles, M. B., & Huberman A. M. (1994). *Qualitative data analysis: An expanded source book*. (2nd Edition). Sage Publications.

Mullis, I.V.S., Martin, M. O., Foy, P., Kelly, D.L., & Fishbein B. (2019). *TIMSS 2019 Internatinal Results in Mathematis and Scıence*, Boston College, TIMSS & PIRLS International Study Center. Erişim Tarihi: 9 Haziran 2023

NCTM. (2000). *Principles and standarts for school mathematics*. NCTM.

Owens, K. (2005). Substantıve communication of space mathematics in upper primary school. Proceedings of the 29 *the Conference of the International Group for the Psychology of Mathematics Education,* 4, 33-40.

Öçal, M.F. (2017). Geometri sorularındaki kavramsal hataları belirlemede geogebra'nın rolü [the role of geogebra in identifying conceptual errors in geometry questions]. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi, 19(3), 204-224.*

Paksu, A. D., İymen, E. & Pakmak, G. S. (2013). Sınıf öğretmeni adaylarının dörtgenlerin köşegenleri konusundaki kavram görüntüleri [Concept images of classroom teacher candidates on the subject of diagonals of quadrilaterals]. *Eğitim ve Bilim, 38*(167), 162-178.

Saltık-Ayhanöz, G. (2022). The place of intelligence games in mathematics education for gifted students. In Zahal O. (Eds.), *Current research in edititon* (pp. 152-168), Gece Publishing.

Sak, U. (2011). *Üstün zekâlılar: Özellikleri, tanılanmaları, eğitimleri [Gifted people: Characteristics, diagnosis, education]*. Maya Akademi.

Siegle, D. (2001). Overcoming bias in gifted and talented referrals. *Gifted and Talented Communicator*, *32*(3), 22-25.

Şahin, M., Güven, M. B., & Cemalettin, I. (2023). Yükseklik kavramında ortaokul öğrencilerinin yaşadıkları zorluklar ve matematik öğretmenlerinin bu konudaki görüşleri [The difficulties experienced by secondary school students in the concept of height and the opinions of mathematics teachers on this subject]. *Nevşehir Hacı Bektaş Veli Üniversitesi SBE Dergisi*, *13*(3), 1479-1502.

Şengün, K. Ç., & Yılmaz, S. (2021). Ortaokul 8. sınıf öğrencilerinin üçgende açıortay ve kenarortay belirleme durumlarının incelenmesi [Examining the situations of 8th grade secondary school students in determining the angle bisector and median of a triangle]. *International Journal of Active Learning (IJAL), 6*(1), 81-97.

Taber, K. S. (2017). Neden üstün yetenekliler için fen eğitimi [why science education for the gifted]. Ed: *Gökdere, Üstün Yetenekliler İçin Fen Eğitimi*, 4-16.

Türnüklü, E., Ergin, A.S., & Aydoğdu, M.Z. (2017). Examination of 8th grade students' problem posing studies on triangles. *Bayburt Eğitim Fakültesi Dergisi, 12(24),* 467-486.

Van de Walle, J. A. (1998). *Elementary and middle school mathematics: Teaching developmentally*. Longman.

Yanık, B. (2016). Kavramsal ve işlemsel anlama [conceptual and operational understanding]. E. Bingölbali, S. Arslan. & İ. Ö. Zembat (Ed.), *Matematik eğitiminde teoriler* [*theories in mathematics education*] (pp. 101-114). Ankara: Pegem Publishing.

Yenilmez, K., & Yaşa, E. (2008). İlköğretim öğrencilerinin geometrideki kavram yanılgıları [Primary school students' misconceptions in geometry]. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 21* (2), 461-483.

Yıldırım, A. & Şimşek, H. (2016). *Sosyal bilimlerde nitel araştırma yöntemleri* [Qualitative research methods in the social sciences (10th ed.)]. Seçkin Publishing.

Yin, R. (1984). *Case study research: Design and methods (3th ed.).* Sage publications.

Yurtyapan, M. İ., & Karataş, İ. (2020). Examination of secondary school mathematics teachers' pedagogical content knowledge on the subject of triangles and quadrilaterals]. *Turkish Journal of Computer and Mathematics Education*, *11*(1), 53-90.

Zembat, İ. Ö. (2015). Kavram yanılgısı nedir? [What is a misconception?] M. F. Özmantar, E. Bingölbali ve H. Akkoç (Ed.), *Matematiksel kavram yanılgıları ve çözüm önerileri* (4. Baskı, s. 1-8). PegemA Yayıncılık.