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MHD Mixed Convective Flow Past a Vertical Plate Embedded in a Porous Medium with Radiation Effects and Convective Boundary Condition Considering Chemical Reaction

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Özet. Bu makale, gözenekli bir ortama gömülü, taşınım sınır koşullu bir dikey plaka üzerinde kararlı MHD birleşik taşınımına ısıl radyasyon etkisini incelemektedir. Kimyasal tepkime etkisi de ayrıca hesaba katılmış ve bu yüzden bir birinci kerte kimyasal tepkime dikkate alınmıştır. Öncelikle, akışkanın korunum denklemleri normalize formda yazılmış ve daha sonra adi diferansiyel denklemlerin çözümü için bir kapalı sonlu-fark şeması uygulanmıştır. Çeşitli ilgili fiziksel parametreleri yorumlamak için açıklayıcı sonuçlar elde edilmiştir. Sonuçlar, $Bi = 0$ durumu için yüzey sürtünmesinin, ışınım parametresinin çeşitli değerleri için hemen hemen sabit olduğunu ve Nusselt sayısında bir değişim olmadığını göstermiştir. Bununla birlikte, $Bi > 0$ için, ışınım parametresindeki artış, yüzey sürtünme değerlerinde artışa Nusselt sayısı değerlerinde azalmaya yol açmıştır.[†]

Anahtar Kelimeler. Işınım etkisi, MHD akışı, taşınım sınır koşulu, kimyasal tepkime, Biot sayısı.

Abstract. The present paper examines the effect of thermal radiation on steady MHD mixed convection over a vertical plate with a convective boundary condition embedded in a porous medium. The effect of chemical reaction is also taken into account and hence a first-order chemical reaction is considered. Firstly, the governing equations of fluid have been written into a normalized form and then an implicit finite-difference scheme is applied for solving the ordinary differential equations. Illustrative outcomes are obtained to interpret the different physical parameters of interest. The results show that, for the case $Bi = 0$, the skin-friction is almost constant for various values of radiation parameter and no change in the Nusselt number takes place. However, for $Bi > 0$, increasing the radiation parameter is leading to an increase in values of skin-friction and reduction in values of Nusselt number.

Keywords. Radiation effect, MHD flow, convection boundary condition, chemical reaction, Biot number.

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1. Introduction

The coupled heat and mass transfer convection flows under the influence of magnetic field are found in many applications and engineering processes such as cooling of nuclear reactors, the boundary layer control in aerodynamics, plasma studies and etc. Analysis of free and forced convection flow for electrically conducting fluids in the presence of chemical reaction is of substantial importance in many applications in Science and Technology such as packed-bed catalytic reactors, cooling of nuclear reactors, geothermal reservoirs, thermal insulation and etc. Magneto-hydro-dynamic (MHD) flows are also frequently arisen in a porous media in order to controlling transport phenomena such as petroleum reservoirs recovery, radioactive waste disposal and etc.

There has been an interest in analyzing MHD flow under the influence of thermal radiation effects due to the effect of radiation on the performance of many engineering systems applying the electrically conducting fluids. Moreover, by taking into account the radiation heat transfer the operating temperature rises and then the fluid is ionized. Many new engineering processes take place at high temperatures and thus the effect of thermal radiation cannot be ignored. Therefore, having the knowledge of radiation heat transfer is of considerable importance in modeling the engineering issues.

Combined heat and mass transfer by laminar natural convection from a vertical plate were studied by Lin and Wu [1]. Hossain and Takhar [2] found the radiation effect on mixed convection along a vertical plate with uniform surface temperature. The effect of free convection on MHD coupled heat and mass transfer of a moving permeable vertical surface was reported by Yih [3]. Chamkha and Khaled [4] investigated hydro-magnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. Chamkha et al [5] investigated the natural convection from an inclined plate embedded in a variable porosity medium due to solar radiation. Makinde [6] studied the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Ramachandra Prasad et al [7] considered the radiation and mass transfer effects in two-dimensional flow past an impulsively started iso-thermal vertical plate. The effect of chemical reaction on the mixed MHD flow over a semi-infinite plate in a porous medium was studied by Rajeswari et al [8].

Recently, Makinde and Aziz [9] considered a convective boundary condition in a MHD mixed convection flow over an infinite vertical plate. The effects of permeability, suction and chemical reaction have been investigated in their model. The objective of the present analysis is focused on analyzing the effects of thermal radiation on steady MHD mixed convection flow over an infinite vertical plate considering a convective boundary condition and suction which the plate is embedded in a porous medium. Moreover, a first-order chemical reaction is also considered.

2. Physical and Mathematical Model

An infinite vertical plate considered in the present study is shown in Figure 1. The plate is embedded in a saturated porous medium. The Cartesian coordinate system is selected for the problem. The x and y axes are along and perpendicular to the surface, respectively. The cold fluid on the right side of the surface is assumed as viscous, incompressible, Newtonian and electrically conducting. The temperature of the fluid is T_∞ where all thermo-physical properties are taken constant and independent of temperature (T) and concentration (C) except for the density (ρ). In the present work, the mixed convection flow is considered laminar, steady and hydro-magnetic. The forced magnetic field of strength B_0 is homogeneous and normal to the plate. Because of the small magnetic Reynolds number for most fluids in engineering problems, the induced magnetic field is considered insignificant. The temperature of the hot fluid with a convective heat transfer coefficient h_f on the left side of the plate is T_f.

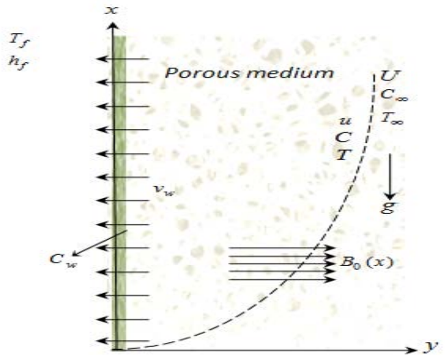

Figure 1. Schematic of coordinate system.

Under the foregoing conditions and Boussinesq approximation, the system of momentum, energy and concentration equations can be written as follows [9, 10]:

$$-v_w \frac{du}{dy} = \nu \frac{d^2u}{dy^2} + g\beta_t(T - T_\infty) + g\beta_m(C - C_\infty) + \frac{\nu}{K}(U - u) + \frac{\sigma B_0^2}{\rho}(U - u) \quad (1)$$

$$-v_w \frac{dT}{dy} = \frac{k}{\rho C_p}\frac{d^2T}{dy^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho C_p}(U - u)^2 - \frac{1}{\rho C_p}\frac{dq_r}{dy} \quad (2)$$

$$-v_w \frac{dC}{dy} = D\frac{d^2C}{dy^2} - \gamma(C - C_\infty) \quad (3)$$

The appropriate boundary conditions for this problem may be given by:

$$y = 0 : u = 0; \quad -k\frac{dT}{dy} = h_f(T_f - T); \quad C = C_w \quad (4)$$

$$y \to \infty : u \to U; \quad T \to T_\infty; \quad C \to C_\infty \quad (5)$$

where u is the velocity component parallel to the x-axis, ν, C_p and k are kinematic viscosity, specific heat at constant pressure and thermal conductivity of the fluid, respectively. β_t and β_m are thermal and concentration expansion coefficients. g, q_r, v_m, C_w and U are the gravitational acceleration, radiation heat flux, wall suction velocity, local concentration at plate surface and free stream velocity, respectively. σ is the electrical conductivity and K is the permeability parameter. Subscript ∞ denotes the free stream conditions of temperature and concentration. Furthermore, D is the mass diffusivity, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio, C_s is the concentration susceptibility and γ is the reaction rate coefficient.

By taking into account the Rosseland approximation for the radiation term [11] in the equation (2), the convective heat flux, q_r, can be modeled as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{dT^4}{dy} \quad (6)$$

where k^* is the Rosseland mean absorption coefficient and σ^* is the Stefan-Boltzmann constant. The Rosseland approximation is used for an optically thick medium, so the fluid is assumed to be optically thick medium. It seems reasonable to assume that the temperature differences within the flow are sufficiently small so that, T^4 can be stated in a simpler way by using a Taylor series about T_∞ and neglecting higher order terms, therefore:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

By replacing the above expression into equations (6) and (7), the energy equation (2) reduces to the following equation:

$$-v_w\frac{dT}{dy}=\frac{k}{\rho C_p}\frac{d^2T}{dy^2}+\frac{\nu}{C_p}\left(\frac{du}{dy}\right)^2+\frac{\sigma B_0^2}{\rho C_p}(U-u)^2+\frac{16\sigma^* T_\infty^3}{3\rho C_p k^*}\frac{d^2T}{dy^2} \tag{8}$$

To normalize the equations (1), (3) to (5) and (8), the following normalized quantities are introduced:

$$\begin{array}{lll}
\eta=\dfrac{v_w y}{\nu} & Pr=\dfrac{\rho\eta C_p}{k} & Ec=\dfrac{v_w^2}{C_p(T_f-T_\infty)} \\
f=\dfrac{u}{v_w} & Sc=\dfrac{\nu}{D} & \lambda=\dfrac{\gamma\nu^2}{Dv_w^2} \\
\theta=\dfrac{T-T_\infty}{T_f-T_\infty} & Gr=\dfrac{g\beta_t(T_f-T_\infty)\nu}{v_w^3} & R=\dfrac{4\sigma^* T_\infty^3}{kk^*} \\
\phi=\dfrac{C-C_\infty}{C_w-C_\infty} & Gc=\dfrac{g\beta_m(C_w-C_\infty)\nu}{v_w^3} & F=\dfrac{U}{v_m} \\
\kappa=\dfrac{v_w^2 K}{\nu^2} & M=\dfrac{\sigma B_0^2\nu}{\rho c} & Bi=\dfrac{h_f\nu}{kv_m}
\end{array} \tag{9}$$

where η, Pr, M, Ec, Gr, Gc are the normalized coordinate along the y-axis, the Prandtl number, magnetic field parameter, Eckert number, thermal Grashof number and mass transfer Grashof number, respectively. Also κ, Sc and λ are the permeability parameter, Schmidt number and reaction rate parameter, respectively. F is the normalized velocity of free stream and R is the radiation parameter. Furthermore, f, θ and ϕ are the normalized quantities of velocity, temperature and concentration, respectively.

In this work similar to [9], the calculations of y direction are considered while the variations are neglected in the x direction. Therefore, with help of normalized parameters the ordinary differential equations (1), (3) to (5) and (8) are converted to simplified ordinary differential equations as follows:

$$-\frac{df}{d\eta}=\frac{d^2f}{d\eta^2}+Gr\theta+Gc\theta+M(F-f)+\frac{F-f}{\kappa} \tag{10}$$

$$-\frac{d\theta}{d\eta}=\frac{1}{Pr}\frac{d^2\theta}{d\eta^2}+Ec\left(\frac{df}{d\eta}\right)^2+MEc(F-f)^2 \tag{11}$$

$$-\frac{d\phi}{d\eta}=\frac{1}{Sc}\frac{d^2\phi}{d\eta^2}-\frac{\lambda}{Sc}\phi \tag{12}$$

$$\eta = 0 : f = 0;\ \frac{d\theta}{d\eta} = Bi[\theta - 1];\ \phi = 1 \tag{13}$$

$$\eta \to \infty : f = F;\ \theta = \phi = 0 \tag{14}$$

From the engineering point of view, the skin-friction coefficient at the wall is given by:

$$C_f = \frac{2}{F^2}\, f'(0) \tag{15}$$

and also the Nusselt number is given by:

$$Nu = \frac{h\nu}{kv_w} = \frac{T_f - T_\infty}{T_w - T_\infty}\,\theta'(0) = -\frac{\theta'(0)}{\theta(0)} \tag{16}$$

where T_w is the plate surface temperature.

The Sherwood number is given by:

$$Sh = \frac{h_m\nu}{Dv_w} = -\phi'(0) \tag{17}$$

where h_m is the convection mass transfer coefficient.

3. Numerical Solution

The normalized equations (10) to (12) that are coupled, with the boundary conditions (13) and (14) are solved numerically using an implicit finite difference scheme of second order. The numbers of grids in numerical domain are chosen 10000 points and the convergence criterion is taken 10^{-8}. The integration is considered as a region limited to $\eta = 10$ which lies very well outside the momentum, energy and concentration boundary layers. More details of the integration scheme can be found in [10]. In order to access the accuracy of the present scheme, the values of the wall shear stress, Nusselt and Sherwood numbers have been compared with those obtained by Makinde and Aziz [9] for a special case, $R = 0$. It is seen from Table 1 that the comparison between the results is found in a close agreement. For this purpose, the values of F, Pr and Sc as those in [9] are 0.5, 0.72 and 0.24, respectively and also the value of κ, λ, M, Ec, Gr, Gc, is 0.1.

4. Results and Discussion

All the figures are obtained from $Pr = 0.72$ that corresponds to air and $Sc = 0.24$, 0.62, 0.78 which states the diffusion of hydrogen, water and ammonia in air, respectively. Moreover, $F = 0.5$ is chosen for all profiles.

Table 1. Comparison of values of $f'(0)$, $-\theta'(0)$ and $-\phi'(0)$ with $R = 0$.

Bi	Makinde and Aziz [9]			Present results		
	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$
0	1.892	0.000	0.459	1.888	0.000	0.459
1	1.908	0.398	0.459	1.904	0.398	0.459
10	1.917	0.639	0.459	1.914	0.639	0.459

Figure 2 illustrates the effects of the Biot number and magnetic field parameter on the normalized velocity profiles. It is observed that the velocity increases from zero at the boundary to its peak point and then falls to the velocity of free stream with $F = 0.5$. Figure 2 shows that the values of velocity increase with increasing the Biot number because of a rise in convective heat transfer to the fluid on the right side of the wall and decrease with increasing the magnetic parameter because of exerting a drag force on the fluid by the magnetic field.

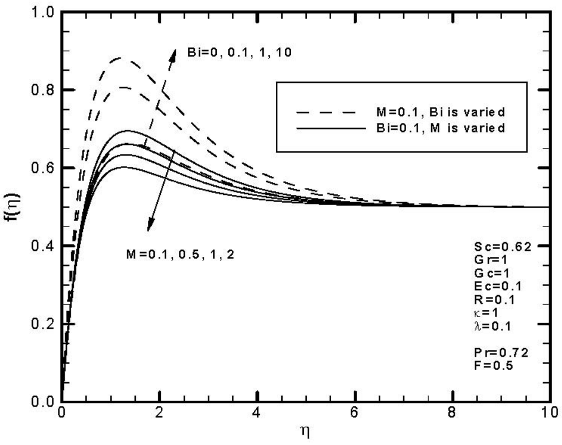

Figure 2. Velocity profiles for different values of Biot number and magnetic parameter.

The temperature profiles for different values of Biot number are demonstrated in Figure 3. Due to the high convective heat transfer to the cold fluid, it is clear that the temperature of the fluid increases with increasing the Biot number.

Figure 4 is made to demonstrate the effect of permeability parameter on the velocity and temperature profiles. It is observed that increasing the permeability parameter is concluded retarding effect of porous medium on the flow. For this reason, both the velocity and temperature increase with increasing the permeability parameter.

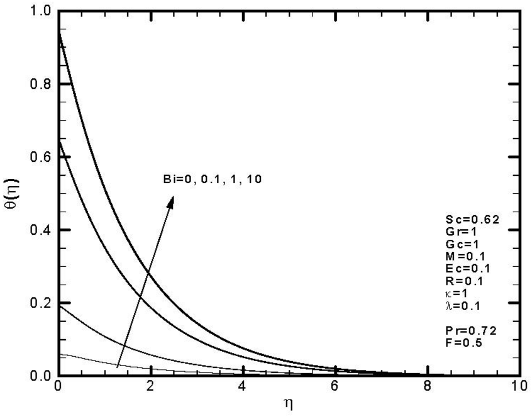

Figure 3. Temperature profiles for different values of Biot number.

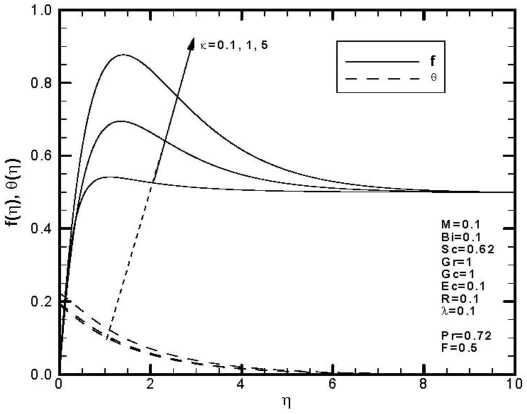

Figure 4. Effect of κ on the velocity and temperature profiles.

The Eckert number states the existence of viscous dissipation in the model. The viscous dissipation is known as a heat generation source inside the fluid and thus should increase the values of velocity and temperature. The effect of Eckert number is presented in Figure 5 where the temperature and velocity values are increased by increasing this number.

To illustrate the effects of the thermal and mass buoyancy forces, the velocity and temperature profiles for different values of thermal and mass transfer Grashof numbers are plotted in Figures 6 and 7. It is clear that increasing these numbers leads to increase in the thermal and mass buoyancy forces. Hence, the velocity and temperature increase with increasing the thermal and mass transfer Grashof numbers.

Figure 8 gives the effects of the Schmidt number and reaction rate parameter on concentration profiles. The Schmidt number is defined as the ratio of momentum

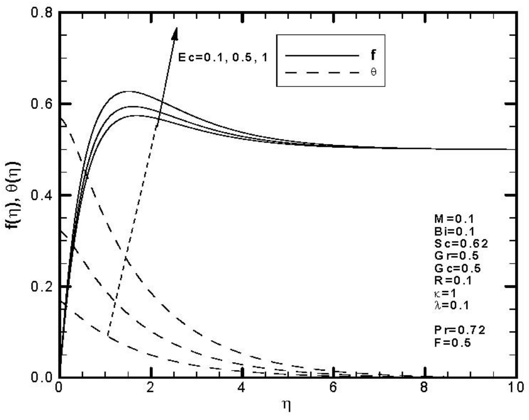

Figure 5. Effect of Ec on the velocity and temperature profiles.

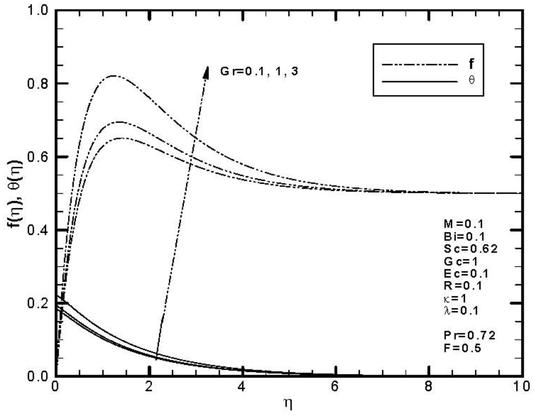

Figure 6. Effect of Gr on the velocity and temperature profiles.

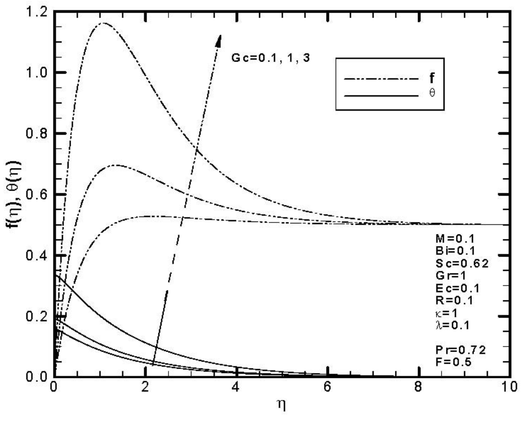

Figure 7. Effect of Gc on the velocity and temperature profiles.

diffusivity and mass diffusivity. Thus, increasing the Schmidt number is leading to a fall in the concentration values. Moreover, when the reaction rate parameter increases, the consumption of the species increases. This means that the concentration of the species tends to drop.

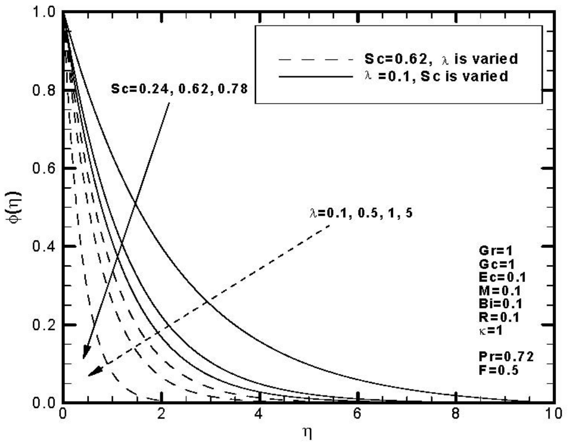

Figure 8. Effect of Sc and λ on the concentration profiles.

Figures 9 and 10 are plotted to explore the effect of radiation parameter on the velocity and temperature distributions, respectively. The positive sign of the radiation parameter ($R > 0$) shows that the fluid is heated by the thermal radiation. As shown in Figure 9 the values of the velocity are increased under the effect of the radiation parameter. Furthermore, Figure 10 shows clearly that the temperature of the fluid increases with increasing the radiation parameter.

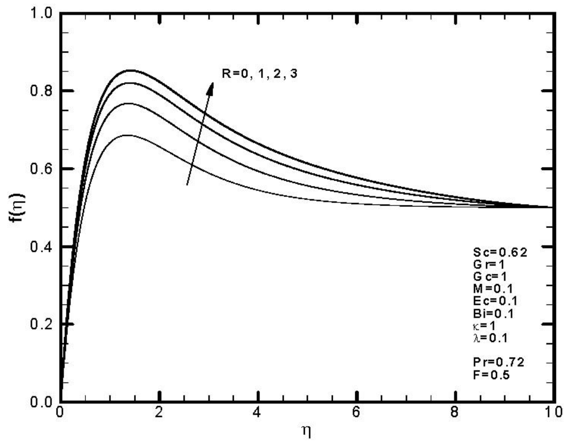

Figure 9. Effect of radiation on the velocity profiles.

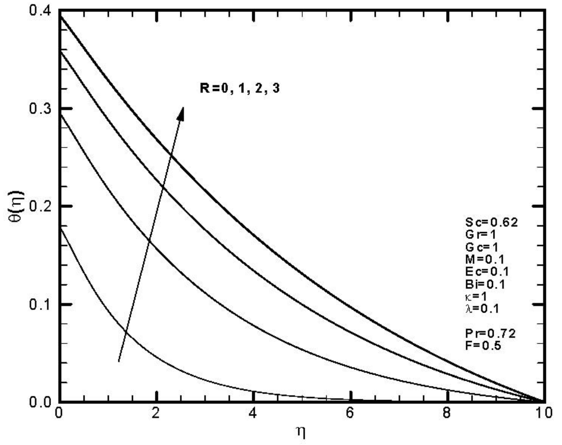

FIGURE 10. Effect of radiation on the temperature profiles.

To describe the effect of thermal radiation on the skin-friction and Nusselt number, Figures 11 and 12 are obtained for different values of Biot number. It is inferred from Figures 11 and 12 that skin-friction increases whereas the Nusselt number decreases with increase in radiation parameter. It can be seen from Figure 11 that the values of skin-friction increase with increasing the Biot number. Moreover, the skin-friction is almost constant, $C_f \approx 9.60$, for different values of radiation parameter for the case $Bi = 0$. Figure 12 illustrates that there is no significant effect on Nusselt number by increasing the Biot number, for $Bi > 0$. Also, this figure shows that no change in Nusselt number, $Nu = 0$, takes place for $Bi = 0$ due to the zero rate of heat transfer at the plate surface.

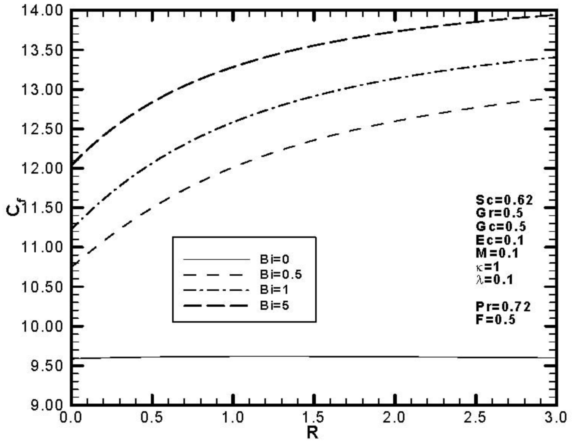

FIGURE 11. Variations in C_f with R for various values of Bi.

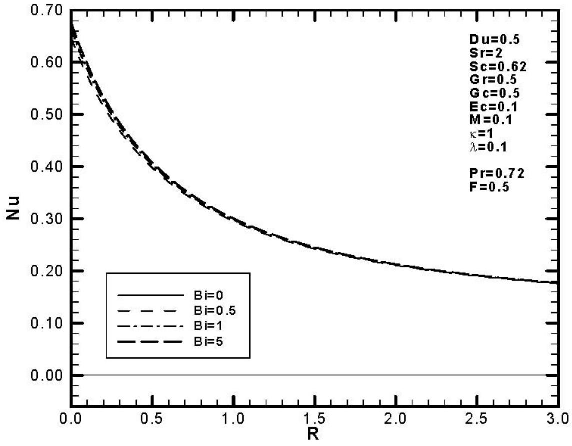

FIGURE 12. Variations in Nu with R for various values of Bi.

As expected, increasing the value of reaction rate parameter yields an increase in the Sherwood number and a decrease in the skin-friction in accordance with Figure 13.

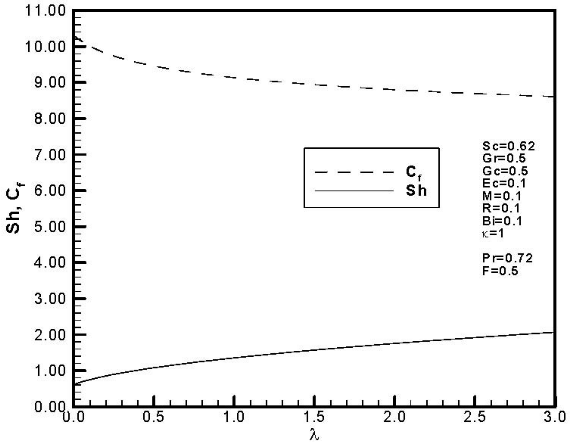

FIGURE 13. Variations in C_f and Sh with λ.

5. Conclusion

An analysis is carried out to study the effect of radiation on MHD mixed convection over a vertical plate with the convective boundary condition including the effects of chemical reaction, suction and viscous dissipation. An implicit finite difference scheme of the second order has been applied for solving the governing equations. The main outcomes of the paper can be itemized as follows:

- The results demonstrated that the values of velocity and temperature are enhanced with increasing the Biot number.
- The values of concentration are affected by chemical reaction and decrease with increasing the reaction rate parameter.
- Due to the importance of thermal radiation effect, the velocity and temperature increase by increasing the radiation parameter.
- Increasing the values of radiation parameter creates an increase in skin-friction and reduce in Nusselt number.
- Increasing the Biot number tends to increase the skin-friction.
- There is no significant effect on Nusselt number by increasing the Biot number.
- For the case Bi=0, the skin-friction is almost constant for various values of radiation parameter and no change in the Nusselt number takes place.
- Increasing the value of reaction rate parameter yields an increase in the Sherwood number and a decrease in the skin-friction.

6. Nomenclature

B_0	Homogeneous magnetic field
Bi	Biot number
C	Concentration of chemical species
C_p	Specific heat at constant pressure
D	Mass diffusivity
Ec	Eckert number
f	Dimensionless velocity
F	Dimensionless velocity of free stream
g	Gravitational acceleration
Gr	Thermal Grashof number
Gc	Mass transfer Grashof number
h	Convection heat transfer coefficient
k	Thermal conductivity of fluid
K	Permeability coefficient
k^*	Rosseland mean absorption coefficient

U	Free steam velocity
v	Velocity component along the y-axis
x	Coordinate in horizontal direction
y	Coordinate in vertical direction

Greek symbols

β	Expansion coefficient
γ	Reaction rate coefficient
ϕ	Dimensionless concentration
η	Dimensionless coordinate along the y-axis
κ	Permeability parameter
λ	Reaction rate parameter
ν	Kinematic viscosity
ρ	Density of fluid
σ	Electrical conductivity of fluid
σ^*	Stefan-Boltzmann constant

M	Magnetic field parameter
Pr	Prandtl number
q	Heat flux
R	Radiation parameter
Sc	Schmidt number
T	Temperature of fluid
u	Velocity component along the x-axis

Subscripts

∞	Free stream
f	Hot fluid
m	Related to concentration
r	Radiation
t	Related to temperature
w	wall

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