A BINARY INTEGER PROGRAMMING MODEL FOR EXAM SCHEDULING PROBLEM WITH SEVERAL DEPARTMENTS

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Abstract

The problem of exam scheduling is a kind of scheduling problem that is studied academically, in which the exams of a certain number of courses is assigned to specific time intervals taking certain constraints into consideration. In most faculties of universities, exam schedules are made manually, which is both time-consuming and error-prone. The objective of this study is to develop a mathematical model that can solve exam scheduling problem in a shorter time than manual schedules and without error. Minimizing total number of classrooms assigned is determined as the objective function of the model. When the total number of classrooms assigned is minimized, the number of tasks per exam invigilator (exam superviser or assistant, proctor) is also minimized. A binary integer programming model is developed for this purpose.

Keywords: Exam scheduling, timetabling, mathematical model

ÇOK DEPARTMANLI SINAV ÇİZELGLEME PROBLEMİ İÇİN 0-1 TAMSAYILI PROGRAMLAMA MODELİ

Özet

Sınav çizelgeleme problemi belli sayıda dersin sınavlarının belli kısıtları dikkate alarak zaman aralıklarına atandığı, akademik olarak çalışılan bir çizelgeleme problemidir. Üniversitelerin pek çok fakültelerinde sınav çizelgeleri elle yapılmaktadır, bu da hem zaman alıcı hem de hataya açıktır. Bu çalışmanın amacı sınav çizelgeleme problemini elle yapmaktan daha kısa sürede ve hatasız olarak çözebilen bir matematiksel model geliştirmektir. Sınav dönemi boyunca açılan toplam derslik sayısının minimize edilmesi amaç fonksiyonu olarak belirlenmiştir. Açılan toplam derslik sayısı minimize edildiğinde, sınav gözetmeni başına düşen görev sayısı da minimize olacaktır. Bu amaç için bir 0-1 tamsayılı programlama modeli geliştirilmiştir.

Anahtar Kelimeler: Sınav çizelgeleme, çizelgeleme, matematiksel model

Introduction

Exam scheduling is one of the popular problems among timetabling problems like course scheduling, train and bus timetabling, nurse rostering (Acar *et al.* 2013). As in course scheduling, exam scheduling problem is encountered almost every college and university throughout the world (Carter *et al.*, 1997). Exam scheduling deals with assigning exams to timeslots, in addition exam-classroom assignment assigns exams to classrooms (Dammak *et al.* 2006). Here in this study exam scheduling refers to both assigning to time slots (sessions) and classrooms. This paper is based upon a previous proceeding (Aslan *et al.*, 2016). However the model of this paper is developed that exams of more than one department may be scheduled, and there are some additional constraints which guarrantees the number of invigilators not to be exceeded in a session.

The rest of the paper is organized as follows: In section 2 the litearture about exam scheduling is reviewed. In section 3 problem is formulated; sets, parameters, decision variables and the

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model is presented. In section 4 a small, medium and large size problems are generated and model is tested on the problems. In the 5th and the last section the paper is concluded with a summary and evaluation.

I. Literature

Exam scheduling problem has many variations; some papers are about assigning exams to time slots, some papers assign exams to both time slots and classrooms, some papers also deals with assigning invigilators. As the problem type differs from paper to paper it is hard to classify the literature according to problem type. Some studies in the literature is mentioned according to the methods used. For a comprehensive review; Qu *et al.* (2009) made a comprehensive litearture review and searched new trends about exam scheduling. According to problem type this study makes assignments to both time slots and classrooms. And according to method used, this study uses mathematical model for solving the problem.

Some papers about scheduling used mathematical modelling: Al-Yakoob *et al.* (2010) developed two seperate mixed integer-programming models for exam scheduling and proctor (invigilator) assignment. İlkuçar (2011) developed a mathematical model and a software for exam scheduling problem. Kağnıcıoğlu *et al.* (2006) developed a binary integer fuzzy goal programming model for multiobjective exam scheduling problem with fuzziness. Sagir et.al (2010) developed a nonlinear optimization model and used Analtical Network Process to prioritize the objectives of invigilator-exam scheduling problem.

Some studies developed a heuristic in addition to mathematical model: Dammak *et al.* (2006) developed a binary linear integer program for exam scheduling problem with classroom assignment. They also applied some heuristic procedures for the problem. Acar *et al.* (2013) developed a mathematical model for exam scheduling problem and a heuristic procedure for large problems.

And some papers developed (meta)heuristic methods for the problem: Yaldır *et al.* (2012) developed a software for exam scheduling automation based on evolutionary algorithms. Zhaohui *et al.* (2000) developed heuristic procedures for campus-wide exam scheduling problem. Malkawi *et al.* (2008) used graph coloring algorithm for exam scheduling. Burke *et al.* (2008) developed “late acceptance strategy” based on local search, Cheong et al (2007) developed a multi-objective evolutionary algorithm, Corne et al (1993) used genetic algorithm and Kordalewski *et al.* (2009) compared genetic algorithm, random search and simulated annealing for exam scheduling problem.

2. Problem Formulation

The model in this study assigns the exams of more than one department to the time slots (sessions) and classrooms. It doesn’t assign invigilators but considers the total number of invigilators for each session. The objective function minimizes total classrooms assigned during the whole exam period.

<u>Sets and parameters</u>

z : departments $\{1,\ldots,Z\}$
j : grades $\{1,\ldots,J\}$
k : exams $\{1,\ldots,K\}$
g : days $\{1,\ldots,G\}$
s : sessions $\{1,\ldots,S\}$
l : classrooms $\{1,\ldots,L\}$
M : set for exams of grades of departments

D_l : capacity of classroom *l*
C_{zjk} : number of students taking the exam *k* of grade *j* of department *z*
Q_l : invigilator needed for classroom *l*
R : total number of invigilators

<u>Decision Variables</u>
AB_{zjkgs} : 1 if the exam *k* of grade *j* of department *z* is assigned to session *s* on day *g*; 0 else
A_{zjkgsl} : 1 if the exam *k* of grade *j* of department *z* is assigned to classroom *l* in session *s* on day *g*; 0 else

$$Min \sum_{z\in M} \sum_{j\in M} \sum_{k\in M} \sum_{g}^{G} \sum_{s}^{S} \sum_{l}^{L} A_{zjkgsl} \tag{1}$$

subject to

$$\sum_{s}^{S} \sum_{k\in M} AB_{zjkgs} \leq 1 \quad \forall\, z,j \in M;\ \forall g \tag{2}$$

$$\sum_{j\in M} \sum_{k\in M} AB_{zjkgs} \leq 1 \quad \forall z,g,s;\ \ j,k \in M \tag{3}$$

$$\sum_{l}^{L} A_{zjkgsl} \times D_l \geq C_{zjk} \times AB_{zjkgs} \quad \forall z,j,k \in M;\ \forall g,s \tag{4}$$

$$\sum_{z\in M} \sum_{j\in M} \sum_{k\in M} A_{zjkgsl} \leq 1 \quad \forall g,s,l \tag{5}$$

$$\sum_{g}^{G} \sum_{s}^{S} AB_{zjkgs} = 1 \quad \forall z,j,k \in M \tag{6}$$

$$\sum_{z\in M} \sum_{j\in M} \sum_{k\in M} \sum_{l}^{L} \left(A_{zjkgsl} \times Q_l\right) \leq R \quad \forall g,s \tag{7}$$

The objective function (1) minimizes the sum of the classroom assignments of the all exams of all grades of all departments at all sessions of all days.

Constratint #2 provides that all grades of any department has only one exam a day. So if a student does not have any repeat exam, will have only one exam a day. Constraint #3 ensures the exams of different grades of the same department not to be on the same session. By this constraint the exams of a student who repeats at least one of them do not overlap. Constraint #4 guarrantees total capacities of the classrooms assigned to an exam of a grade of a department to be greater than or equal to the number of students taking that exam. By the help of constraint #5 exams of different departments on the same session are assigned to different classrooms. Constraint #6 assures that an exam of a grade of a department is guarranteed to be assigned to any session of any day. Number of classrooms assigned to an exam on a session of a day do not exceed number of invigilators via Constraint #7.

3. Data and Method

Benchmark problems for exam scheduling in the literature is not used because of the inconsistency with the model in this study. To test the model a small, medium and large size sample problems were generated. Table 1 summarizes the features of the test problems.

Table 1: Information about Test Problems

	# of departments	# of grades	# of exams per grade	# of total exams	Days	Sessions	Classrooms
Small	2	4	2	16	2	4	4
Medium	3	4	4	48	4	4	5
Large	4	4	6	96	6	4	7

Small test problem has 2 departments, each department has 4 grades, 16 exams (8 exams for each department), 4 classrooms, 8 sessions (2 days x 4 sessions). To simplify the instance, all classrooms are adjusted to have the same capacity (20 students) and each classrooms needs 1 invigilator. There are 4 invigilators. Number of students taking exams are shown in Table 2.

Table 2: Number of Students Taking Exams in Small Test Problem

Department	Grade	Exams	# of students
1	1	1, 2	30
1	2	3, 4	25
1	3	5, 6	20
1	4	7, 8	20
2	1	9, 10	30
2	2	11, 12	30
2	3	13, 14	15
2	4	15, 16	25

Medium size test problem has 3 departments, each department has 4 grades, 48 exams (16 exams for each department), 5 classrooms, 16 sessions (4 days x 4 sessions). To simplify the instance all classrooms are adjusted to have the same capacity (20 students) and each classroom needs 1 invigilator. There are 5 invigilators. Number of students taking exams are shown in Table 3.

Table 3: Number of Students Taking Exams in Medium Test Problem

Department	Grade	Exams	# of students
1	1	1, 2, 3, 4	30
1	2	5, 6, 7, 8	25
1	3	9, 10, 11, 12	20
1	4	13, 14, 15, 16	20
2	1	17, 18, 19, 20	30
2	2	21, 22, 23, 24	30
2	3	25, 26, 27, 28	15
2	4	29, 30, 31, 32	25
3	1	33, 34, 35, 36	30
3	2	37, 38, 39, 40	30
3	3	41, 42, 43, 44	20
3	4	45, 46, 47, 48	25

Large size test problem has 4 departments, each department has 4 grades, 96 exams (24 exams for each department), 7 classrooms, 24 sessions (6 days x 4 sessions). To simplify the instance all classrooms are adjusted to have the same capacity (20 students) and each classroom needs 1 invigilator. There are 7 invigilators. Number of students taking exams are shown in Table 4.

Table 4: Number of Students Taking Exams in Large Test Problem

Department	Grade	Exams	# of students
1	1	1, 2, 3, 4, 5, 6	30
1	2	7, 8, 9, 10, 11, 12	25
1	3	13, 14, 15, 16, 17, 18	20
1	4	19, 20, 21, 22, 23, 24	20
2	1	25, 26, 27, 28, 29, 30	30
2	2	31, 32, 33, 34, 35, 36	30
2	3	37, 38, 39, 40, 41, 42	15
2	4	43, 44, 45, 46, 47, 48	25
3	1	49, 50, 51, 52, 53, 54	30
3	2	55, 56, 57, 58, 59, 60	25
3	3	61, 62, 63, 64, 65, 66	20
3	4	67, 68, 69, 70, 71, 72	20
4	1	73, 74, 75, 76, 77, 78	30
4	2	79, 80, 81, 82, 83, 84	30
4	3	85, 86, 87, 88, 89, 90	15
4	4	91, 92, 93, 94, 95, 96	25

The model is written in the GAMS language (V.23.0.2) and is run at a computer with an Intel Core i5 3.30GHz CPU, 8GB RAM and Windows 10 64bit OS. Optimum solution is found for all test problems and elapsed time for optimal solutions is given at Table 5.

Table 5: Optimal Solutions and Elapsed time for Test Problems

	Elapsed time (sec.)	Optimal Solution
Small	0.13	26
Medium	4.54	80
Large	39.34	156

Solution for small test problem is displayed for validity at Figure 1. Exams shown with yellow shading from number 1 to 8 belongs to department 1, green shading from number 9 to 15 belongs to department 2. The model gives feasible solution. The objective function value is 26, meaning that 26 classrooms assigned to 16 exams belonging 2 departments, on 2 days with 4 sessions. Figure 1 shows the model works properly, as expected within the constraints.

Figure 1: The optimal timetable for the small test problem

	DAY→	DAY 1				DAY 2			
	SESSION→	1	2	3	4	1	2	3	4
CLASS↓	CLASSROOM↓								
1	1			9			10	2	
	2	1		9					
	3	1							
	4						10	2	
2	1	11							4
	2								12
	3			3					12
	4	11		3					4
3	1				6	5			
	2		14						
	3								
	4					13			
4	1								
	2						8	15	
	3				16			15	
	4		7		16				

The optimal solution for medium size test problem is shown at Figure 2. Exams shown with yellow shading from number 1 to 16 belongs to department 1, green shading from number 17 to 32 belongs to department 2 and blue shading from number 33 to 48 belongs to department 3. The objective function value is 80 in the optimal solution.

Figure 2: The optimal timetable for medium size test problem

DAY→		DAY 1				DAY 2				DAY 3				DAY 4			
CLASS↓	SESSION→ / CLASSROOM↓	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	1	2				34					33		17		35		
	2		36						3		33						
	3	20							18		1				35		
	4	2				34			3		1			19	4		
	5	20	36						18				17	19	4		
2	1				5			23		24							
	2			21			8			24			37			38	22
	3			21	39		8	40									7
	4				39			40		6			37			38	7
	5				5			23		6							22
3	1						27		41								43
	2	42			28									9	26		
	3					11				44							
	4			12								10					
	5										25						
4	1		32	46								30		47		29	
	2					31		14				48					
	3		32									48	16	47		29	
	4		15				45										
	5			46		31	45					30				13	

The optimal solution for large size test problem is shown at Figure 3, which displays the grades of the departments together (not seperately) and it can be seen that different exams do not overlap. Exams shown with yellow shading from number 1 to 24 belongs to department 1, green shading from number 25 to 48 belongs to department 2, blue shading from number 49 to 72 belongs to department 3 and red shading from number 73 to 96 belngs to department 4. The objective function value is 156 in the optimal solution.

Figure 3: The optimal timetable for the large test problem

DAY→	DAY 1				DAY 2				DAY 3				DAY 4				DAY 5				DAY 6			
SESSION→ / CLASSROOM↓	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	45	71	1	82	11	28	91	84	43	32	92	73	74	80	93	87	46	94	33	12	31	54	25	57
2	77	96	90	82	76	69	23	39	79	3	92	58	48	80	36		78	42	59	64	95	54	19	44
3	60	17	34	9	35		47	84		49	65	73	48	37	93	52	78			27	95	83	66	14
4	60	40	1	26	35	5	56	53	43	3	29	58	74	2		30	51	6	33	81	70	4	75	57
5	77	96	50	9	62	89	91	53	79	85	7	38	61	55	36	52	21	6	59	27	8	4	25	88
6	45		34	26	11	28	47	16	22	32	7		10	55	72	30	51	67	86	12	8	41	75	
7	24		50	63	76	5	56		68	49	29	13	10	2	18	20	46	94	15	81	31	83		44

Conclusions

The model developed in this study assigns exams of more than one department to sessions and classrooms optimally. However the model is not tested on a real life problem and it is expected that while the number of department and herewith number of exams increasing, finding optimal solution in a reasonable time will be hard. Exam scheduling problem may be included in design problems, which is a kind of problem that optimal solution is preferred to quick solution. Therefore if need for frequent rescheduling because of requests does not exist, long time of the solution will not be an issue. If rescheduling is common because of requests from instructors or students; heuristic or metaheuristic methods would be more appropriate. They can reach to acceptable solutions in a shorter time for big size real life problems.

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