Çankaya University Journal of Science and Engineering
Volume 7 (2010), No. 1, 51–57

(G'/G)-Expansion Method for Traveling Wave Solutions of the Sixth-Order Ramani Equation

İbrahim Enam İnan

Faculty of Education, Fırat University, Elazığ 23119, Turkey
ieinan@yahoo.com

Özet. Bu çalışmada, Ramani denkleminin hareket eden dalga çözümleri için (G'/G)-açılım metodu sunuldu. Bu metod yardımı ile yukarıda bahsedilen denklemin bazı hareket eden dalga çözümleri bulundu.

Anahtar Kelimeler. Altıncı mertebeden Ramani denklemi, (G'/G)-açılım metodu, hareket eden dalga çözümleri, lineer olmayan denklem.

Abstract. In this study, we implemented the (G'/G)-expansion method the traveling wave solutions of the sixth-order Ramani equation. By using this scheme, we found some traveling wave solutions of the above-mentioned equation.

Keywords. Sixth-order Ramani equation, (G'/G)-expansion method, traveling wave solutions, nonlinear equation.

1. Introduction

It has recently become more interesting to obtain exact solutions of nonlinear partial differential equations through using symbolical computer programs such as Maple, Matlab, and Mathematica, that facilitate complex and tedious algebraic computations. Calculating the exact and numerical solutions, in particular traveling wave solutions, of nonlinear equations in mathematical physics plays an important role in soliton theory [1,2]. Mathematical modeling of physical systems generally is explained with nonlinear equations. Hence, it is very important to find exact solutions of nonlinear partial differential equations. The exact solution of a differential equation gives information about its construction. These equations are mathematical models of complex physical phenomena that arise in engineering, chemistry, biology, mechanics and physics. Various effective methods have been developed to understand the mechanisms of these physical models, and these help physicists and

Received March 3, 2010; accepted July 8, 2010.

ISSN 1309 - 6788

engineers to ensure they have the knowledge to solve physical problems and to find the applications of the models.

Recently, interest has increased in traveling wave solutions of differential equations. There are many methods to solve these equations in literature. Some of them are: Hirota's dependent variable transformation [3], Bäcklund transformation [4], Cole-Hopf transformation [5], generalized Miura transformation [6], inverse scattering transform [7], Darboux transformation [8], Painlevé method [9], homogeneous balance method [10], similarity reduction method [11], sine-cosine method [12], Jacobi elliptic function method [13], and exp-function method [14].

The solutions of many nonlinear differential equations can be stated with tanh function terms [15,16]. The tanh function terms firstly were used on an *ad hoc* basis in 1990 and 1991 [17,18]. Then, Malfliet [19] formalized the tanh method in 1992 and illustrated it with several examples, Parkes and Duffy presented the automatic tanh method [20] in 1996, and after that, Fan defined the extended tanh method [21] in 2000, and Elwakil presented the modified extended tanh method [22] in 2002. The generalized extended tanh method [23] was defined by Zheng in 2003, the improved extended tanh method [24] by Yomba in 2004, and the tanh function method [25] by Chen and Zhang in 2004.

In this work, we will consider the traveling wave solutions of the sixth-order Ramani equation by using the (G'/G)-expansion method which is introduced by Mingliang Wang, Xiangzheng Li and Jinliang Zhang [26].

2. An Analysis of the Method and Applications

Before starting to apply the (G'/G)-expansion method, we will give a simple description of the method. For doing this, one can consider in a two-variables general form of nonlinear PDE

$$Q(u, u_t, u_x, u_{xx}, ...) = 0, \tag{1}$$

and transform (1) with $u(x,t) = u(\xi)$, $\xi = x - kt$ where k is a constant. After transformation, we get a nonlinear ODE for $u(\xi)$

$$Q'(u', u'', u''', ...) = 0. \tag{2}$$

The solution of (2) we are looking for is expressed as

$$u(\xi) = a_m \left(\frac{G'}{G}\right)^m + \cdots, \tag{3}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0, \tag{4}$$

where $a_m, \ldots, \lambda$ and μ are constants to be determined later, $a_m \neq 0$, and the positive integer m can be determined by balancing the highest order derivative with the highest nonlinear terms into (2). Substituting (3) into (2) and using (4) yields a set of algebraic equations in the same order as (G'/G), then all coefficients in the same order as (G'/G) have to vanish. After this separated algebraic equation, we can find the $a_m, \ldots, k, \lambda$ and μ constants. The general solutions of (4) are well known us, then substituting $a_m, \ldots, k$ and the general solutions of (4) into (3) we have more traveling wave solutions of (1).

Example. Consider sixth-order Ramani equation

$$\begin{aligned} u_{xxxxxx} + 15u_x u_{xxxx} + 15u_{xx}u_{xxx} + 45u_x^2 u_{xx} \\ - 5u_{xxxt} - 15u_x u_{xt} - 15u_t u_{xx} - 5u_{tt} = 0. \end{aligned} \tag{5}$$

For this example, we can use transformation with (1) then (5) becomes

$$\begin{aligned} u^{(6)} + 15u'u^{(4)} + 15u''u''' + 45(u')^2u'' + 5ku^{(4)} \\ + 15ku'u'' + 15ku'u'' - 5k^2u'' = 0. \end{aligned} \tag{6}$$

Balancing $u''u'''$, $u'u^{(4)}$ with $u^{(6)}$ then gives $m = 1$. Therefore, we may choose

$$u(\xi) = a_1 \left(\frac{G'}{G}\right) + a_0. \tag{7}$$

Substituting (7) into (6) yields a set of algebraic equations for a_0, a_1. These systems are

$$\begin{aligned} -5a_1k^2\lambda\mu + 5a_1k\lambda^3\mu + a_1\lambda^5\mu + 40a_1k\lambda\mu^2 - 30a_1^2k\lambda\mu^2 + 52a_1\lambda^3\mu^2 \\ - 30a_1^2\lambda^3\mu^2 + 136a_1\lambda\mu^3 - 150a_1^2\lambda\mu^3 + 45a_1^3\lambda\mu^3 = 0, \end{aligned}$$

$$
\begin{aligned}
&-5a_1k^2\lambda^2+5a_1k\lambda^4+a_1\lambda^6-10a_1k^2\mu+110a_1k\lambda^2\mu-60a_1^2k\lambda^2\mu\\
&\quad+114a_1\lambda^4\mu-60a_1^2\lambda^4\mu+80a_1k\mu^2-60a_1^2k\mu^2+720a_1\lambda^2\mu^2-630a_1^2\lambda^2\mu^2\\
&\quad+135a_1^3\lambda^2\mu^2+272a_1\mu^3-300a_1^2\mu^3+90a_1^3\mu^3=0,
\end{aligned}
$$

$$
\begin{aligned}
&-15a_1k^2\lambda+75a_1k\lambda^3-30a_1^2k\lambda^3+63a_1\lambda^5-30a_1^2\lambda^5+300a_1k\lambda\mu\\
&\quad-180a_1^2k\lambda\mu+1176a_1\lambda^3\mu-870a_1^2\lambda^3\mu+135a_1^3\lambda^3\mu\\
&\quad+1848a_1\lambda\mu^2-1710a_1^2\lambda\mu^2+405a_1^3\lambda\mu^2=0,
\end{aligned}
$$

$$
\begin{aligned}
&-10a_1k^2+250a_1k\lambda^2-120a_1^2k\lambda^2+602a_1\lambda^4-390a_1^2k\lambda^4+45a_1^3\lambda^4\\
&\quad+200a_1k\mu-120a_1^2k\mu-3584a_1\lambda^2\mu-2880a_1^2\lambda^2\mu\\
&\quad+540a_1^3\lambda^2\mu+1232a_1\mu^2-1140a_1^2\mu^2+270a_1^3\mu^2=0,
\end{aligned}
$$

$$
\begin{aligned}
&300a_1k\lambda-150a_1^2k\lambda+2100a_1\lambda^3-1500a_1^2\lambda^3\\
&\quad+225a_1^3\lambda^3+4200a_1\lambda\mu-3450a_1^2\lambda\mu+675a_1^3\lambda\mu=0,
\end{aligned}
$$

$$120a_1k-60a_1^2k+3360a_1\lambda^2-2490a_1^2\lambda^2+405a_1^3\lambda^2+1680a_1\mu-1380a_1^2\mu+270a_1^3\mu=0,$$

$$2520a_1\lambda-1890a_1^2\lambda+315a_1^3\lambda=0,$$

$$720a_1-540a_1^2+90a_1^3=0.$$

From the solutions system, we obtain the following with the aid of Mathematica.

Case 1.

$$a_0=a_0,\ a_1=2,\ k=\frac{5\lambda^2-20\mu+3\sqrt{5}\sqrt{\lambda^4-8\lambda^2\mu+16\mu^2}}{10}.$$

Case 2.

$$a_0=a_0,\ a_1=2,\ \lambda=0,\quad k=\pm\frac{2}{5}(\mp5\mu+3\sqrt{5}\mu). \tag{8}$$

Substituting (8) into (7) we have three types of traveling wave solutions of (5)

For Case 1.

(i) When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function traveling wave solutions

$$u_1(\xi) = 2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)\right) + a_0 - \lambda,$$

where

$$\xi = x - \left(\frac{5\lambda^2 - 20\mu + 3\sqrt{5}\sqrt{\lambda^4 - 8\lambda^2\mu + 16\mu^2}}{10}\right)t,$$

C_1 and C_2 are arbitrary constants.

(ii) When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function traveling wave solutions

$$u_2(\xi) = 2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right)\right) + a_0 - \lambda,$$

where

$$\xi = x - \left(\frac{5\lambda^2 - 20\mu + 3\sqrt{5}\sqrt{\lambda^4 - 8\lambda^2\mu + 16\mu^2}}{10}\right)t,$$

C_1 and C_2 are arbitrary constants.

(iii) When $\lambda^2 - 4\mu = 0$, we obtain the rational function solutions

$$u_3(\xi) = 2\left(\frac{C_2}{C_1 + C_2 x}\right) + a_0 - \lambda,$$

where C_1 and C_2 are arbitrary constants.

For Case 2.

(i) When $\mu < 0$, we obtain the hyperbolic function traveling wave solutions

$$u_1(\xi) = 2\left(\frac{\sqrt{-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{-4\mu}}{2}\xi\right)}\right)\right) + a_0,$$

where

$$\xi = x - \left[\pm\frac{2}{5}(\mp 5\mu + 3\sqrt{5}\mu)\right]t,$$

C_1 and C_2 are arbitrary constants.

(ii) When $\mu > 0$, we obtain the trigonometric function traveling wave solutions

$$u_2(\xi) = 2\left(\frac{\sqrt{4\mu}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu}}{2}\xi\right) + C_2\cos\left(\frac{\sqrt{4\mu}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu}}{2}\xi\right) + C_2\sin\left(\frac{\sqrt{4\mu}}{2}\xi\right)}\right)\right) + a_0,$$

where

$$\xi = x - \left[\pm\frac{2}{5}(\mp 5\mu + 3\sqrt{5}\mu)\right]t,$$

C_1 and C_2 are arbitrary constants.

(iii) When $\mu = 0$, we obtain the rational function solutions,

$$u_3(\xi) = 2\left(\frac{C_2}{C_1 + C_2 x}\right) + a_0,$$

where C_1 and C_2 are arbitrary constants.

3. Conclusion

In this study, we considered the sixth-order Ramani equation. We implemented the (G'/G)-expansion method for the exact solution of this nonlinear equation. Of course this method can be implemented in more complicated nonlinear equations by using symbolic computations.

References

[1] L. Debtnath, *Nonlinear Partial Differential Equations for Scientist and Engineers*, Birkhauser, Boston, MA, 1997.

[2] A. M. Wazwaz, *Partial Differential Equations: Methods and Applications*, Balkema, Rotterdam, 2002.

[3] X. B. Hu and W. X. Ma, Application of Hirota's bilinear formalism to the Toeplitz lattice-some special soliton-like solutions, *Phys. Lett. A* **293** (2002), 161–165.

[4] M. L. Wang and Y. M. Wang, A new Bäcklund transformation and multi-soliton solutions to the KdV equation with general variable coefficients, *Phys. Lett. A* **287** (2001), 211–216.

[5] A. M. Abourabia and M. M. El Horbaty, On solitary wave solutions for the two-dimensional nonlinear modified Kortweg-de Vries-Burger equation, *Chaos, Solitons and Fractals* **29** (2006), 354–364.

[6] T. L. Bock and M. D. Kruskal, A two-parameter Miura transformation of the Benjamin-Ono equation, *Phys. Lett. A* **74** (1979), 173–176.

[7] P. G. Drazin and R. S. Jhonson, *Solitons: An Introduction,* Cambridge University Press, Cambridge, 1989.

[8] V. B. Matveev and M. A. Salle, *Darboux transformations and solitons*, Springer, Berlin, 1991.

[9] F. Cariello and M. Tabor, Painlevé expansions for nonintegrable evolution equations, *Physica D* **39** (1989), 77–94.

[10] E. Fan, Two new applications of the homogeneous balance method, *Phys. Lett. A* **265** (2000), 353–357.

[11] P. A. Clarkson, New Similarity Solutions for the Modified Boussinesq Equation, *J. Phys. A: Math. Gen.* **22** (1989), 2355–2367.

[12] Y. Chuntao, A simple transformation for nonlinear waves, *Phys. Lett. A* **224** (1996), 77–84.

[13] F. Zuntao, L. Shikuo, L. Shida and Z. Qiang, New Jacobi elliptic function expansion and new periodic solutions of nonlinear wave equations, *Phys. Lett. A* **290** (2001), 72–76.

[14] J. H. He and X. H. Wu, Exp-function method for nonlinear wave equations, *Chaos, Solitons & Fractals* **30** (2006), 700–708.

[15] W. Hereman, A. Korpel and P. P. Banerjee, *Wave Motion* **7** (1985), 283–289.

[16] W. Hereman and M. Takaoka, Solitary wave solutions of nonlinear evolution and wave equations using a direct method and MACSYMA, *J. Phys. A: Math. Gen.* **23** (1990), 4805–4822.

[17] H. Lan and K. Wang, Exact solutions for two nonlinear equations, *J. Phys. A: Math. Gen.* **23** (1990), 3923–3928.

[18] S. Lou, G. Huang and H. Ruan, Exact solutions for two nonlinear equations, *J. Phys. A: Math. Gen.* **24** (1991), L587–L590.

[19] W. Malfliet, Solitary wave solutions of nonlinear wave equations, *Am. J. Phys.* **60** (1992), 650–654.

[20] E. J. Parkes and B. R. Duffy, An automated tanh-function method for finding solitary wave solutions to non-linear evolution equations, *Comput. Phys. Commun.* **98** (1996), 288–300.

[21] E. Fan, Extended tanh-function method and its applications to nonlinear equations, *Phys. Lett. A* **277** (2000), 212–218.

[22] S. A. Elwakil, S. K. El-labany, M. A. Zahran, and R. Sabry, Modified extended tanh-function method for solving nonlinear partial differential equations, *Phys. Lett. A* **299** (2002), 179–188.

[23] X. Zheng, Y. Chen, and H. Zhang, Generalized extended tanh-function method and its application to (1+1)-dimensional dispersive long wave equation, *Phys. Lett. A* **311** (2003), 145–157.

[24] E. Yomba, Construction of new soliton-like solutions of the (2+1) dimensional dispersive long wave equation, *Chaos, Solitons & Fractals* **20** (2004), 1135–1139.

[25] H. Chen and H. Zhang, New multiple soliton solutions to the general Burgers-Fisher equation and the Kuramoto-Sivashinsky equation, *Chaos, Solitons & Fractals* **19** (2004), 71–76.

[26] M. Wang, J. Zhang and X. Li Application of the (G'/G)-expansion to travelling wave solutions of the Broer-Kaup and the approximate long water wave equations, *Appl. Math. and Comput.* **206** (2008), 321–326.