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Generalized Jacobi Elliptic Function Method for Periodic Wave Solutions of SRLW Equation and (1+1)-Dimensional Dispersive Long Wave Equation

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Özet. Biz bu çalışmada SRLW ve (1+1)-boyutlu dispersive uzun dalga denkleminin periyodik dalga çözümlerini elde etmek için genelleştirilmiş Jacobi eliptik fonksiyon metodunu sunarız.

Anahtar Kelimeler. SRLW denklemi, (1+1)-boyutlu dispersive uzun dalga denklemi, genelleştirilmiş Jakobi eliptik fonksiyon metot, periyodik çözümler, hareket eden dalga çözümler.

Abstract. We implement the generalized Jacobi elliptic function method with symbolic computation to construct periodic solutions for the symmetric regularized long wave (SRLW) equation and (1+1)-dimensional dispersive long wave equation.

Keywords. SRLW equation, (1+1)-dimensional dispersive long wave equation, generalized Jacobi elliptic function method, periodic solutions, traveling wave solutions.

1. Introduction

The mathematical modeling of events in nature can be explained by differential equations. These equations are mathematical models of complex physical occurrences that arise in engineering, chemistry, biology, mechanics and physics. The theory of nonlinear dispersive wave motion has recently been the subject of much study. The solutions of nonlinear equations play a crucial role in applied mathematics and physics, because; solutions of nonlinear partial differential equations make a very significant contribution to people's knowledge about the nature of physical phenomenon. Furthermore, when an original nonlinear equation is directly calculated,

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the solution will preserve the actual physical characters of solutions. Therefore, interest in the solution of nonlinear partial differential equations has never decreased. Because of this interest, many techniques and methods have been developed [1-10].

Recently, there has been intensive study to obtain analytical solutions of nonlinear partial differential equations. Interest has focused on obtaining exact solutions of nonlinear partial differential equations through using symbolical computer programs, such as Maple, Matlab, and Mathematica that facilitate complex and tedious algebraic computations. Various methods are presented which are used by scientists to obtain exact and analytic solutions of nonlinear partial differential equations with help of symbolical computer programs. Most of these methods are based on finding the balance term with balancing of the highest order linear and nonlinear term. These methods can be only applied to nonlinear partial differential equations.

In this study, we analyze the generalized Jacobi elliptic function method [11] and we obtain periodic wave solutions of SRLW equations [12] and (1+1)-dimensional dispersive long wave equations [13] by using the generalized Jacobi elliptic function method. Then, we show three-dimensional and two-dimensional periodic wave graphics for the SRLW equation and (1+1)-dimensional dispersive long wave equation by using a solution of these equations.

A symmetric regularized long wave equation (SRLWE)

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \tag{1}$$

had been investigated in [14-16]. Eq. (1) is used to describe nonlinear ion-acoustic wave and space-charge waves. This equation is symmetrical with respect to x, and t. It arises in many nonlinear problems of mathematical physics and applied mathematics. Periodic wave solutions of SRLW have been given by using the Exp function method [17] and (G'/G)-expansion method [18].

The (1+1)-dimensional dispersive long wave equation

$$\begin{aligned} u_t + uu_x + v_x &= 0 \\ v_t + u_x v + uv_x + \frac{1}{3}u_{xxx} &= 0 \end{aligned} \tag{2}$$

has been studied in [4]. Eq. (2) is one of the basic equations of fluid dynamics. $v-1$ shows the height of water wave and u denotes the velocity water's surface along the x axis. This equation is used to model in the coastal edge waves.

2. An Analysis of the Method and Applications

Before starting to give a generalized Jacobi elliptic function method [11], we will give a simple description of the generalized Jacobi elliptic function method. For doing this, it is considered in a two variables general form of nonlinear PDE

$$Q(u, u_t, u_x, u_{xx}, \ldots) = 0, \tag{3}$$

with $u(x,t) = u(\xi)$, $\xi = kx + wt$. Eq. (3) converts to nonlinear ODE for $u(\xi)$ as follows, where k, w are constants.

$$Q'(u', u'', u''', \ldots) = 0. \tag{4}$$

The solution of the Eq. (4) is supposed in the form

$$u(\xi) = a_0 + \sum_{i=1}^{n} \left[a_i F^i(\xi) + b_i F^{-i}(\xi)\right], \tag{5}$$

where n is a positive integer that can be determined by balancing the highest order derivate and the highest nonlinear terms in equation, k, w, a_0, a_i, b_i and ξ can be determined. Substituting solution (5) into Eq. (4) yields a set of algebraic equations for $F^i\left(\sqrt{A+BF^2+CF^4}\right)^j$, $(j = 0, 1)$ and $(i = 0, 1, 2, \ldots)$ then, all coefficients of $F^i\left(\sqrt{A+BF^2+CF^4}\right)^j$ have to vanish. After this separated algebraic equation, we can find coefficients and k, w, a_0, a_i, b_i and ξ.

In this work, we study the solution of the SRLW equation and (1+1)-dimensional dispersive long wave equation by using the generalized Jacobi elliptic function method which is introduced by [11]. The fundamental idea of their method is to take full advantage of the elliptic equation and use its solutions F. The desired elliptic equation is given as

$$F'^2 = A + BF^2 + CF^4, \tag{6}$$

where $F' = \dfrac{dF}{d\xi}$ and A, B, C are constants. Some solutions of Eq. (6) are given in paper [11].

Example 1. Let's consider SRLW equation,

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \tag{7}$$

with $u(x,t) = u(\xi)$, $\xi = kx+wt$, and then Eq. (7) converts to an ordinary differential equation as follows

$$w^2u'' + k^2u'' + kwuu'' + kw(u')^2 + w^2k^2u^{(4)} = 0, \tag{8}$$

and integrating (8) yields, we find the following equation

$$w^2u' + k^2u' + kwuu' + k^2w^2u''' = 0, \tag{9}$$

where the integration constant is taken as zero. The balance term is obtained as $n = 2$ by balancing uu' with u''' in Eq. (9). Therefore, we may choose to give the solution of Eq. (9) as follows

$$u = a_0 + a_1F + a_2F^2 + \frac{b_1}{F} + \frac{b_2}{F^2}. \tag{10}$$

Substituting (10) into Eq. (9) yields a set of algebraic equations for $a_0, a_1, a_2, b_1, b_2, k, w$. The algebraic equation system is obtained as

$$\begin{aligned}
a_1k^2 + a_0a_1kw + a_2b_1kw + a_1w^2 + a_1Bk^2w^2 &= 0,\\
-2b_2^2kB - 24Ab_2k^2w^2 &= 0,\\
-3b_1b_2kw - 6Ab_1k^2w^2 - 2b_2k^2 - b_1^2kw - 2a_0b_2kw - 2b_2w^2 - 8Bb_2k^2w^2 &= 0,\\
-b_1k^2 - a_0b_1kw - a_1b_2kw - b_1w^2 - Bb_1k^2w^2 &= 0,\\
2a_2k^2 + a_1^2kw + 2a_0a_2kw + 2a_2w^2 + 8a_2Bk^2w^2 &= 0,\\
3a_1a_2kw + 6a_1Ck^2w^2 &= 0,\\
2a_2^2kw + 24a_2Ck^2w^2 &= 0.
\end{aligned} \tag{11}$$

If the system of algebraic equations is solved with the help of Mathematica, we have

$$\begin{aligned}
&a_0 = -\frac{k}{w} - \frac{w}{k} - 4Bkw,\ a_1 = 0,\ a_2 = -12Ckw,\\
&b_1 = 0,\ b_2 = -12Akw,\ k \neq 0,\ w \neq 0.
\end{aligned} \tag{12}$$

Substituting (12) into (10), we have obtained analytic solutions of equation (7) as follows

(i) $A = 1,\ B = -(1 + m^2),\ C = m^2,$

$$u_1 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\,\mathrm{sn}^2(kx + wt) - 12Akw\left(\frac{1}{\mathrm{sn}^2(kx + wt)}\right). \tag{13}$$

$$u_2 = -\frac{k}{w} - \frac{w}{k} - 4Bkw – 12Ckw\left(\frac{\mathrm{cn}(kx + wt)}{\mathrm{dn}(kx + wt)}\right)^2 – 12Akw\left(\frac{\mathrm{dn}(kx + wt)}{\mathrm{cn}(kx + wt)}\right)^2. \tag{14}$$

(ii) $A = 1 - m^2,\ B = 2m^2 - 1,\ C = -m^2,$

$$u_3 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\,\mathrm{cn}^2(kx + wt) - 12Akw\left(\frac{1}{\mathrm{cn}^2(kx + wt)}\right). \tag{15}$$

(iii) $A = m^2 - 1,\ B = 2 - m^2,\ C = -1,$

$$u_4 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\,\mathrm{dn}^2(kx+wt) - 12Akw\left(\frac{1}{\mathrm{dn}^2(kx+wt)}\right). \quad (16)$$

(iv) $A = -m^2(1-m^2),\ B = 2m^2 - 1,\ C = 1,$

$$u_5 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\mathrm{dn}(kx+wt)}{\mathrm{sn}(kx+wt)}\right)^2 - 12Akw\left(\frac{\mathrm{sn}(kx+wt)}{\mathrm{dn}(kx+wt)}\right)^2. \quad (17)$$

(v) $A = 1 - m^2,\ B = 2 - m^2,\ C = 1,$

$$u_6 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\mathrm{cn}(kx+wt)}{\mathrm{sn}(kx+wt)}\right)^2 - 12Akw\left(\frac{\mathrm{sn}(kx+wt)}{\mathrm{cn}(kx+wt)}\right)^2. \quad (18)$$

(vi) $A = \frac{1}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4},$

$$u_7 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\mathrm{sn}(kx+wt)}{1 \pm \mathrm{dn}(kx+wt)}\right)^2 - 12Akw\left(\frac{1 \pm \mathrm{dn}(kx+wt)}{\mathrm{sn}(kx+wt)}\right)^2 \quad (19)$$

(vii) $A = \frac{m^2}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4},$

$$u_8 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw(\mathrm{sn}(kx+wt) \pm i\,\mathrm{cn}(kx+wt))^2 - 12Akw\left(\frac{1}{\mathrm{sn}(kx+wt) \pm i\,\mathrm{cn}(kx+wt)}\right)^2. \quad (20)$$

$$u_9 = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\mathrm{dn}(kx+wt)}{i\sqrt{1-m^2}\,\mathrm{sn}(kx+wt) \pm \mathrm{cn}(kx+wt)}\right)^2 - 12Akw\left(\frac{i\sqrt{1-m^2}\,\mathrm{sn}(kx+wt) \pm \mathrm{cn}(kx+wt)}{\mathrm{dn}(kx+wt)}\right)^2. \quad (21)$$

(viii) $A = \frac{1}{4},\ B = \frac{1-2m^2}{2},\ C = \frac{1}{4},$

$$u_{10} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw(m\,\mathrm{sn}(kx+wt) \pm i\,\mathrm{dn}(kx+wt))^2 - 12Akw\left(\frac{1}{m\,\mathrm{sn}(kx+wt) \pm i\,\mathrm{dn}(kx+wt)}\right)^2. \quad (22)$$

$$u_{11} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{dn}(kx+wt)}{m\operatorname{cn}(kx+wt) \pm i\sqrt{1-m^2}}\right)^2 - 12Akw\left(\frac{m\operatorname{cn}(kx+wt) \pm i\sqrt{1-m^2}}{\operatorname{dn}(kx+wt)}\right)^2. \quad (23)$$

$$u_{12} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{sn}(kx+wt)}{1 \pm \operatorname{cn}(kx+wt)}\right)^2 - 12Akw\left(\frac{1 \pm \operatorname{cn}(kx+wt)}{\operatorname{sn}(kx+wt)}\right)^2. \quad (24)$$

$$u_{13} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{cn}(kx+wt)}{\sqrt{1-m^2}\operatorname{sn}(kx+wt) \pm \operatorname{dn}(kx+wt)}\right)^2 - 12Akw\left(\frac{\sqrt{1-m^2}\operatorname{sn}(kx+wt) \pm \operatorname{dn}(kx+wt)}{\operatorname{cn}(kx+wt)}\right)^2 \quad (25)$$

(ix) $A = \frac{m^2-1}{4}$, $B = \frac{m^2+1}{2}$, $C = \frac{m^2-1}{4}$,

$$u_{14} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{dn}(kx+wt)}{1 \pm m\operatorname{sn}(kx+wt)}\right)^2 - 12Akw\left(\frac{1 \pm m\operatorname{sn}(kx+wt)}{\operatorname{dn}(kx+wt)}\right)^2. \quad (26)$$

(x) $A = \frac{1-m^2}{4}$, $B = \frac{m^2+1}{2}$, $C = \frac{1-m^2}{4}$,

$$u_{15} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{cn}(kx+wt)}{1 \pm \operatorname{sn}(kx+wt)}\right)^2 - 12Akw\left(\frac{1 \pm \operatorname{sn}(kx+wt)}{\operatorname{cn}(kx+wt)}\right)^2. \quad (27)$$

(xi) $A = -\frac{(1-m^2)^2}{4}$, $B = \frac{m^2+1}{2}$, $C = -\frac{1}{4}$,

$$u_{16} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw(m\operatorname{cn}(kx+wt) \pm \operatorname{dn}(kx+wt))^2 - 12Akw\left(\frac{1}{m\operatorname{cn}(kx+wt) \pm \operatorname{dn}(kx+wt)}\right)^2. \quad (28)$$

(xii) $A = \frac{1}{4}$, $B = \frac{m^2+1}{2}$, $C = \frac{(1-m^2)^2}{4}$,

$$u_{17} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{sn}(kx+wt)}{\operatorname{dn}(kx+wt) \pm \operatorname{cn}(kx+wt)}\right)^2 - 12Akw\left(\frac{\operatorname{dn}(kx+wt) \pm \operatorname{cn}(kx+wt)}{\operatorname{sn}(kx+wt)}\right)^2. \quad (29)$$

(xiii) $A = \frac{1}{4}$, $B = \frac{m^2-2}{2}$, $C = \frac{m^2}{4}$,

$$u_{18} = -\frac{k}{w} - \frac{w}{k} - 4Bkw - 12Ckw\left(\frac{\operatorname{cn}(kx+wt)}{\sqrt{1-m^2} \pm \operatorname{dn}(kx+wt)}\right)^2 - 12Akw\left(\frac{\sqrt{1-m^2} \pm \operatorname{dn}(kx+wt)}{\operatorname{cn}(kx+wt)}\right)^2. \quad (30)$$

Remark. Here $\operatorname{sn}(\xi, m)$, $\operatorname{cn}(\xi, m)$, $\operatorname{dn}(\xi, m)$ are Jacobi elliptic functions and m shows the modulus of the Jacobi elliptic functions.

If $m \to 1$, then $\operatorname{sn}\xi \to \tanh\xi$, $\operatorname{cn}\xi \to \operatorname{sech}\xi$, $\operatorname{dn}\xi \to \operatorname{sech}\xi$. If $m \to 0$, then $\operatorname{sn}\xi \to \sin\xi$, $\operatorname{cn}\xi \to \cos\xi$, $\operatorname{dn}\xi \to 1$.

We can obtain the following periodic solutions of (7) by using solutions (13-30) for $m \to 0$,

$$u(x,t) = -\frac{k}{w} - \frac{w}{k} + 4kw - 12kw\frac{1}{\sin^2(kx+wt)}, \quad (31)$$

$$u(x,t) = -\frac{k}{w} - \frac{w}{k} + 4kw - 12kw\frac{1}{\cos^2(kx+wt)}, \quad (32)$$

$$u(x,t) = -\frac{k}{w} - \frac{w}{k} - 8kw - 12kw(\cot^2(kx+wt) + \tan^2(kx+wt)), \quad (33)$$

$$u(x,t) = -\frac{k}{w} - \frac{w}{k} - 2kw - 3kw\left(\frac{\sin^2(kx+wt)}{(1 \pm \cos(kx+wt))^2} + \frac{(1 \pm \cos(kx+wt))^2}{\sin^2(kx+wt)}\right), \quad (34)$$

$$u(x,t) = -\frac{k}{w} - \frac{w}{k} - 2kw - 3kw\left(\frac{\cos^2(kx+wt)}{(\sin(kx+wt) \pm 1)^2} + \frac{(\sin(kx+wt) \pm 1)^2}{\cos^2(kx+wt)}\right). \quad (35)$$

In Figure 1 and Figure 2 are shown graphics of periodic wave solutions of the SRLW equation in three and two dimensions, respectively. In Figure 2, the periodic waves move to the left with time. If $\xi = kx - wt$, the periodic waves move to the right.

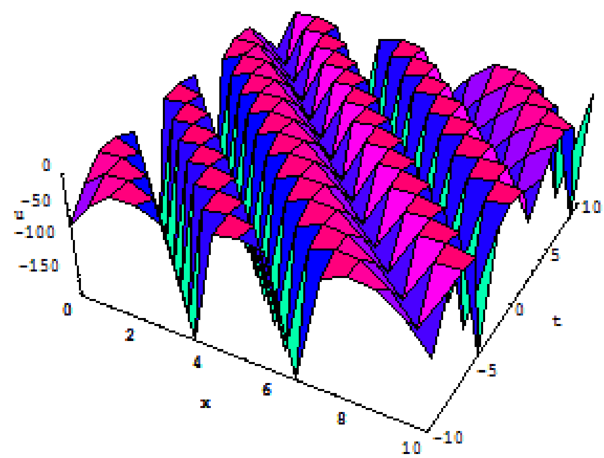


FIGURE 1. Periodic wave graphic of the SRLW equation for solution (32) in three dimensions, $k = 1$, $w = 0.5$.

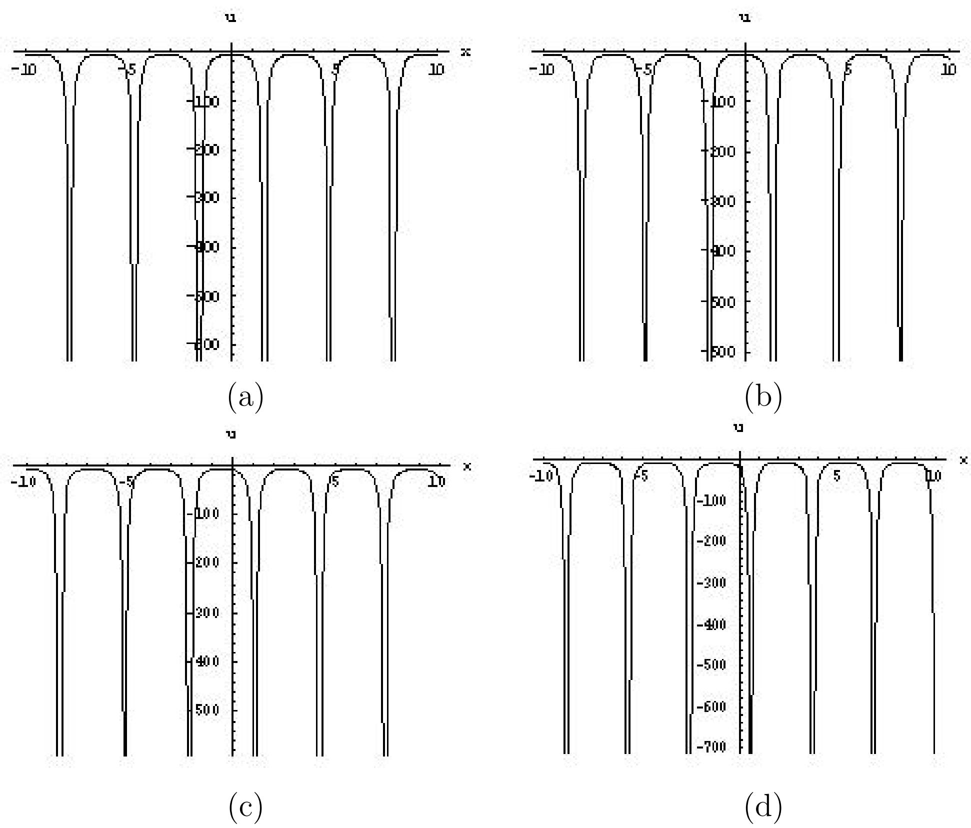


FIGURE 2. Periodic wave graphic of the SRLW equation for solution (32) in two dimensions (a) $t = 0$, (b) $t = 0.5$, (c) $t = 1$, (d) $t = 2$, $(k = 1,\ w = 0.5)$.

Example 2. Let's consider (1+1)-dimensional dispersive long wave equation

$$\begin{aligned} u_t + uu_x + v_x &= 0, \\ v_t + u_x v + uv_x + \frac{1}{3}u_{xxx} &= 0, \end{aligned} \tag{36}$$

with $u(x,t) = u(\xi)$, $\xi = kx + wt$, and then Eq. (36) converts to an ordinary differential equation as follows

$$\begin{aligned} wu' + kuu' + kv' &= 0, \\ wv' + k(uv)' + \frac{1}{3}k^3 u''' &= 0, \end{aligned} \tag{37}$$

and integrating (37) yields, we find the following equation

$$\begin{aligned} wu + \frac{k}{2}u^2 + kv &= 0, \\ wv + kuv + \frac{1}{3}k^3 u'' &= 0, \end{aligned} \tag{38}$$

where the integration constant is taken as zero. Balance terms are found as $n_1 = 1$, $n_2 = 2$ respectively, by balancing v with y^2 and uv with u'' in Eq. (38). Therefore, we may choose

$$\begin{cases} u = a_0 + a_1 F + \dfrac{b_1}{F}, \\ v = c_0 + c_1 F + \dfrac{d_1}{F} + c_2 F^2 + \dfrac{d_2}{F^2}. \end{cases} \tag{39}$$

Substituting (39) into Eq. (38) yields a set of algebraic equations for a_0, a_1, b_1, c_0, c_1, c_2, d_1, d_2, k, and w. The algebraic equation system is obtained as

$$\begin{aligned} b_1 d_2 k + \frac{2Ab_1k^3}{3} &= 0, & a_0c_0k + b_1c_1k + a_1d_1k + c_0w &= 0, \\ b_1d_1k + a_0d_2k + d_2w &= 0, & b_1c_0k + a_0d_1k + a_1d_2k + \frac{Bb_1k^3}{3} + d_1w &= 0, \\ a_1c_1k + a_0c_2k + c_2w &= 0, & a_1c_0k + a_0c_1k + b_1c_2k + \frac{1}{3}a_1Bk^3 + c_1w &= 0, \\ a_1c_2k + \frac{2}{3}a_1Ck^3 &= 0, & \frac{a_0^2k}{2} + a_1b_1k + c_0k + a_0w &= 0, \\ \frac{b_1^2k}{2} + d_2k &= 0, & a_0b_1k + d_1k + b_1w &= 0, \\ a_0a_1k + c_1k + a_1w &= 0, & \frac{1}{2}a_1^2k + c_2k &= 0, \end{aligned} \tag{40}$$

where $C \neq 0,\ k \neq 0$.

From the solutions of the above system, we can find

$$a_0 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}, \quad a_1 = -\frac{2\sqrt{C}k}{\sqrt{3}}, \quad b_1 = \frac{2\sqrt{A}k}{\sqrt{3}},$$
$$c_0 = \frac{1}{3}(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2), \qquad c_1 = 0, \qquad c_2 = -\frac{2Ck^2}{3}, \tag{41}$$
$$d_1 = 0, \qquad d_2 = -\frac{2Ak^2}{3}, \quad w = k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}.$$

Substituting (41) into (39), we have obtained analytic solutions of equation (36) as follows

(i) $A = 1$, $B = -(1+m^2)$, $C = m^2$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_1 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\operatorname{sn}(\xi) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1}{\operatorname{sn}(\xi)}\right) \\ v_1 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\operatorname{sn}^2(\xi) - \frac{2Ak^2}{3}\left(\frac{1}{\operatorname{sn}(\xi)}\right)^2 \end{cases} \tag{42}$$

$$\begin{cases} u_2 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\operatorname{cn}(\xi)}{\operatorname{dn}(\xi)}\right) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{\operatorname{dn}(\xi)}{\operatorname{cn}(\xi)}\right) \\ v_2 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\operatorname{cn}(\xi)}{\operatorname{dn}(\xi)}\right) - \frac{2Ak^2}{3}\left(\frac{\operatorname{dn}(\xi)}{\operatorname{cn}(\xi)}\right)^2 \end{cases} \tag{43}$$

(ii) $A = (1-m^2)$, $B = 2m^2 - 1$, $C = -m^2$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_3 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\operatorname{cn}(\xi) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1}{\operatorname{cn}(\xi)}\right) \\ v_3 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\operatorname{cn}^2(\xi) - \frac{2Ak^2}{3}\left(\frac{1}{\operatorname{cn}(\xi)}\right)^2 \end{cases} \tag{44}$$

(iii) $A = (m^2-1)$, $B = (2-m^2)$, $C = -1$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_4 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\operatorname{dn}(\xi) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1}{\operatorname{dn}(\xi)}\right) \\ v_4 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\operatorname{dn}^2(\xi) - \frac{2Ak^2}{3}\left(\frac{1}{\operatorname{dn}(\xi)}\right)^2 \end{cases} \tag{45}$$

(iv) $A = -m^2(1-m^2),\ B = 2m^2-1,\ C = 1,\ \xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t,$

$$\begin{cases} u_5 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\mathrm{dn}(\xi)}{\mathrm{sn}(\xi)}\right) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi)}\right) \\ v_5 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\mathrm{dn}(\xi)}{\mathrm{sn}(\xi)}\right)^2 - \frac{2Ak^2}{3}\left(\frac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi)}\right)^2 \end{cases} \tag{46}$$

(v) $A = (1-m^2),\ B = (2-m^2),\ C = 1,\ \xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t,$

$$\begin{cases} u_6 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}\right) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{\mathrm{sn}(\xi)}{\mathrm{cn}(\xi)}\right) \\ v_6 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}\right)^2 - \frac{2Ak^2}{3}\left(\frac{\mathrm{sn}(\xi)}{\mathrm{cn}(\xi)}\right)^2 \end{cases} \tag{47}$$

(vi) $A = \frac{1}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4},\ \xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t,$

$$\begin{cases} u_7 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\mathrm{sn}(\xi)}{1 \pm \mathrm{dn}(\xi)}\right) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1 \pm \mathrm{dn}(\xi)}{\mathrm{sn}(\xi)}\right) \\ v_7 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\mathrm{sn}(\xi)}{1 \pm \mathrm{dn}(\xi)}\right)^2 - \frac{2Ak^2}{3}\left(\frac{1 \pm \mathrm{dn}(\xi)}{\mathrm{sn}(\xi)}\right)^2 \end{cases} \tag{48}$$

(vii) $A = C = \frac{m^2}{4},\ B = \frac{m^2-2}{2},\ \xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t,$

$$\begin{cases} u_8 = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}(\mathrm{sn}(\xi) \pm i\,\mathrm{cn}(\xi)) \\ \qquad + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1}{(\mathrm{sn}(\xi) \pm i\,\mathrm{cn}(\xi))}\right) \\ v_8 = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}(\mathrm{sn}(\xi) \pm i\,\mathrm{cn}(\xi))^2 \\ \qquad - \frac{2Ak^2}{3}\left(\frac{1}{(\mathrm{sn}(\xi) \pm i\,\mathrm{cn}(\xi))}\right)^2 \end{cases} \tag{49}$$

$$\begin{cases} u_9 = -\sqrt{-\dfrac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \dfrac{2\sqrt{C}k}{\sqrt{3}}\left(\dfrac{\mathrm{dn}(\xi)}{i\sqrt{1-m^2}\,\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)}\right) \\ \qquad + \dfrac{2\sqrt{A}k}{\sqrt{3}}\left(\dfrac{i\sqrt{1-m^2}\,\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)}{\mathrm{dn}(\xi)}\right) \\ v_9 = \dfrac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \dfrac{2Ck^2}{3}\left(\dfrac{\mathrm{dn}(\xi)}{i\sqrt{1-m^2}\,\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)}\right)^2 \\ \qquad - \dfrac{2Ak^2}{3}\left(\dfrac{i\sqrt{1-m^2}\,\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)}{\mathrm{dn}(\xi)}\right)^2 \end{cases} \tag{50}$$

(viii) $A = C = \dfrac{1}{4}$, $B = \dfrac{1-2m^2}{2}$, $\xi = kx + k\sqrt{-\dfrac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_{10} = -\sqrt{-\dfrac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \dfrac{2\sqrt{C}k}{\sqrt{3}}(m\,\mathrm{sn}(\xi) \pm i\,\mathrm{dn}(\xi)) \\ \qquad + \dfrac{2\sqrt{A}k}{\sqrt{3}}\left(\dfrac{1}{(m\,\mathrm{sn}(\xi) \pm i\,\mathrm{dn}(\xi))}\right) \\ v_{10} = \dfrac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \dfrac{2Ck^2}{3}(m\,\mathrm{sn}(\xi) \pm i\,\mathrm{dn}(\xi))^2 \\ \qquad - \dfrac{2Ak^2}{3}\left(\dfrac{1}{(m\,\mathrm{sn}(\xi) \pm i\,\mathrm{dn}(\xi))}\right)^2 \end{cases} \tag{51}$$

$$\begin{cases} u_{11} = -\sqrt{-\dfrac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \dfrac{2\sqrt{C}k}{\sqrt{3}}\left(\dfrac{\mathrm{dn}(\xi)}{m\,\mathrm{cn}(\xi) \pm i\sqrt{1-m^2}}\right) \\ \qquad + \dfrac{2\sqrt{A}k}{\sqrt{3}}\left(\dfrac{m\,\mathrm{cn}(\xi) \pm i\sqrt{1-m^2}}{\mathrm{dn}(\xi)}\right) \\ v_{11} = \dfrac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \dfrac{2Ck^2}{3}\left(\dfrac{\mathrm{dn}(\xi)}{m\,\mathrm{cn}(\xi) \pm i\sqrt{1-m^2}}\right)^2 \\ \qquad - \dfrac{2Ak^2}{3}\left(\dfrac{m\,\mathrm{cn}(\xi) \pm i\sqrt{1-m^2}}{\mathrm{dn}(\xi)}\right)^2 \end{cases} \tag{52}$$

$$\begin{cases} u_{12} = -\sqrt{-\dfrac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \dfrac{2\sqrt{C}k}{\sqrt{3}}\left(\dfrac{\mathrm{sn}(\xi)}{1 \pm \mathrm{cn}(\xi)}\right) + \dfrac{2\sqrt{A}k}{\sqrt{3}}\left(\dfrac{1 \pm \mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}\right) \\ v_{12} = \dfrac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \dfrac{2Ck^2}{3}\left(\dfrac{\mathrm{sn}(\xi)}{1 \pm \mathrm{cn}(\xi)}\right)^2 - \dfrac{2Ak^2}{3}\left(\dfrac{1 \pm \mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}\right)^2 \end{cases} \tag{53}$$

$$\begin{cases} u_{13} = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\operatorname{cn}(\xi)}{\sqrt{1-m^2}\operatorname{sn}(\xi) \pm \operatorname{dn}(\xi)}\right) \\ \qquad\qquad + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{\sqrt{1-m^2}\operatorname{sn}(\xi) \pm \operatorname{dn}(\xi)}{\operatorname{cn}(\xi)}\right) \\ v_{13} = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\operatorname{cn}(\xi)}{\sqrt{1-m^2}\operatorname{sn}(\xi) \pm \operatorname{dn}(\xi)}\right)^2 \\ \qquad\qquad - \frac{2Ak^2}{3}\left(\frac{\sqrt{1-m^2}\operatorname{sn}(\xi) \pm \operatorname{dn}(\xi)}{\operatorname{cn}(\xi)}\right)^2 \end{cases} \tag{54}$$

(ix) $A = C = \frac{m^2-1}{4}$, $B = \frac{m^2+1}{2}$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_{14} = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\operatorname{dn}(\xi)}{1 \pm m\operatorname{sn}(\xi)}\right) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1 \pm m\operatorname{sn}(\xi)}{\operatorname{dn}(\xi)}\right) \\ v_{14} = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\operatorname{dn}(\xi)}{1 \pm m\operatorname{sn}(\xi)}\right)^2 - \frac{2Ak^2}{3}\left(\frac{1 \pm m\operatorname{sn}(\xi)}{\operatorname{dn}(\xi)}\right)^2 \end{cases} \tag{55}$$

(x) $A = C = \frac{1-m^2}{4}$, $B = \frac{1+m^2}{2}$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_{15} = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\operatorname{cn}(\xi)}{1 \pm \operatorname{sn}(\xi)}\right) + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1 \pm \operatorname{sn}(\xi)}{\operatorname{cn}(\xi)}\right) \\ v_{15} = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\operatorname{cn}(\xi)}{1 \pm \operatorname{sn}(\xi)}\right)^2 - \frac{2Ak^2}{3}\left(\frac{1 \pm \operatorname{sn}(\xi)}{\operatorname{cn}(\xi)}\right)^2 \end{cases} \tag{56}$$

(xi) $A = -\frac{(1-m^2)^2}{4}$, $B = \frac{1+m^2}{2}$, $C = -\frac{1}{4}$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_{16} = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}(m\operatorname{cn}(\xi) \pm \operatorname{dn}(\xi)) \\ \qquad\qquad + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{1}{(m\operatorname{cn}(\xi) \pm \operatorname{dn}(\xi))}\right) \\ v_{16} = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}(m\operatorname{cn}(\xi) \pm \operatorname{dn}(\xi))^2 \\ \qquad\qquad - \frac{2Ak^2}{3}\left(\frac{1}{(m\operatorname{cn}(\xi) \pm \operatorname{dn}(\xi))}\right)^2 \end{cases} \tag{57}$$

(xii) $A = \frac{1}{4}$, $B = \frac{1+m^2}{2}$, $C = \frac{(1-m^2)^2}{4}$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_{17} = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi) \pm \mathrm{cn}(\xi)}\right) \\ \qquad + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{\mathrm{dn}(\xi) \pm \mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}\right) \\ v_{17} = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi) \pm \mathrm{cn}(\xi)}\right)^2 \\ \qquad - \frac{2Ak^2}{3}\left(\frac{\mathrm{dn}(\xi) \pm \mathrm{cn}(\xi)}{\mathrm{sn}(\xi)}\right)^2 \end{cases} \tag{58}$$

(xiii) $A = \frac{1}{4}$, $B = \frac{m^2-2}{2}$, $C = \frac{m^2}{4}$, $\xi = kx + k\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2}t$,

$$\begin{cases} u_{18} = -\sqrt{-\frac{2Bk^2}{3} - 4\sqrt{A}\sqrt{C}k^2} - \frac{2\sqrt{C}k}{\sqrt{3}}\left(\frac{\mathrm{cn}(\xi)}{\sqrt{1-m^2} \pm \mathrm{dn}(\xi)}\right) \\ \qquad + \frac{2\sqrt{A}k}{\sqrt{3}}\left(\frac{\sqrt{1-m^2} \pm \mathrm{dn}(\xi)}{\mathrm{cn}(\xi)}\right) \\ v_{18} = \frac{1}{3}\left(-Bk^2 - 2\sqrt{A}\sqrt{C}k^2\right) - \frac{2Ck^2}{3}\left(\frac{\mathrm{cn}(\xi)}{\sqrt{1-m^2} \pm \mathrm{dn}(\xi)}\right)^2 \\ \qquad - \frac{2Ak^2}{3}\left(\frac{\sqrt{1-m^2} \pm \mathrm{dn}(\xi)}{\mathrm{cn}(\xi)}\right)^2 \end{cases} \tag{59}$$

We can obtain the following periodic solutions of Eq. (36) by using solutions (42-59) for $m \to 0$,

$$\begin{cases} u(x,t) = -\sqrt{\frac{2k^2}{3}} - \frac{2k}{\sqrt{3}} \frac{1}{\sin\left(kx + k\sqrt{\frac{2k^2}{3}}t\right)} \\ v(x,t) = \frac{k^2}{3} - \frac{2k^2}{3} \frac{1}{\sin\left(kx + k\sqrt{\frac{2k^2}{3}}t\right)^2} \end{cases} \tag{60}$$

$$\begin{cases} u(x,t) = -\sqrt{-\frac{16k^2}{3}} - \frac{2k}{\sqrt{3}}\cot\left(kx + k\sqrt{-\frac{16k^2}{3}}t\right) \\ \qquad\qquad + \frac{2k}{\sqrt{3}}\tan\left(kx + k\sqrt{-\frac{16k^2}{3}}t\right) \\ v(x,t) = \frac{1}{3}(-4k^2) - \frac{2k^2}{3}\cot^2\left(kx + k\sqrt{-\frac{16k^2}{3}}t\right) \\ \qquad\qquad - \frac{2k^2}{3}\tan^2\left(kx + k\sqrt{-\frac{16k^2}{3}}t\right) \end{cases} \tag{61}$$

$$\begin{cases} u(x,t) = -\sqrt{-\frac{4k^2}{3}} - \frac{k}{\sqrt{3}}\left(\frac{\sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{1 \pm \cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right) \\ \qquad\qquad + \frac{k}{\sqrt{3}}\left(\frac{1 \pm \cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{\sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right) \\ v(x,t) = -\frac{1}{3}(k^2) - \frac{k^2}{6}\left(\frac{\sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{1 \pm \cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right)^2 \\ \qquad\qquad - \frac{k^2}{6}\left(\frac{1 \pm \cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{\sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right)^2 \end{cases} \tag{62}$$

$$\begin{cases} u(x,t) = -\sqrt{-\frac{4k^2}{3}} - \frac{k}{\sqrt{3}}\left(\frac{\cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{1 \pm \sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right) \\ \qquad\qquad + \frac{k}{\sqrt{3}}\left(\frac{1 \pm \sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{\cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right) \\ v(x,t) = -\frac{1}{3}(k^2) - \frac{k^2}{6}\left(\frac{\cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{1 \pm \sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right)^2 \\ \qquad\qquad - \frac{k^2}{6}\left(\frac{1 \pm \sin\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}{\cos\left(kx + k\sqrt{-\frac{4k^2}{3}}t\right)}\right)^2 \end{cases} \tag{63}$$

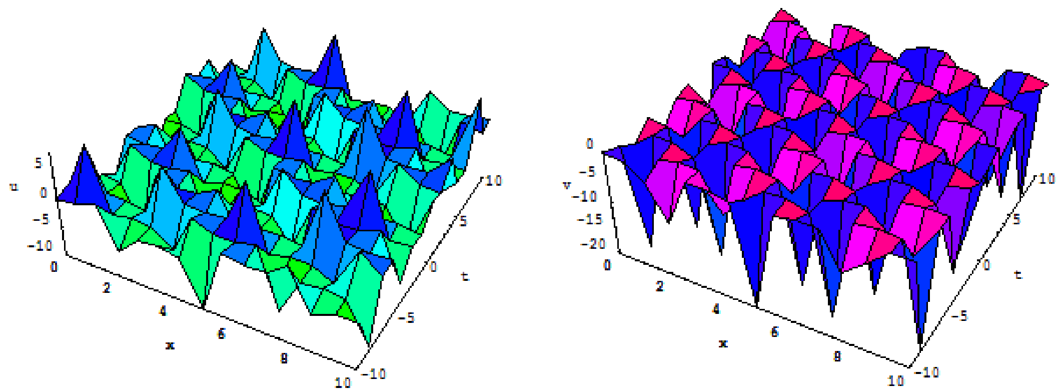


FIGURE 3. Periodic wave graphics of the (1+1)-dimensional dispersive long wave equation for $u(x, t)$ and $v(x, t)$ solutions of (60) in three dimensions, respectively $(k = 1)$.

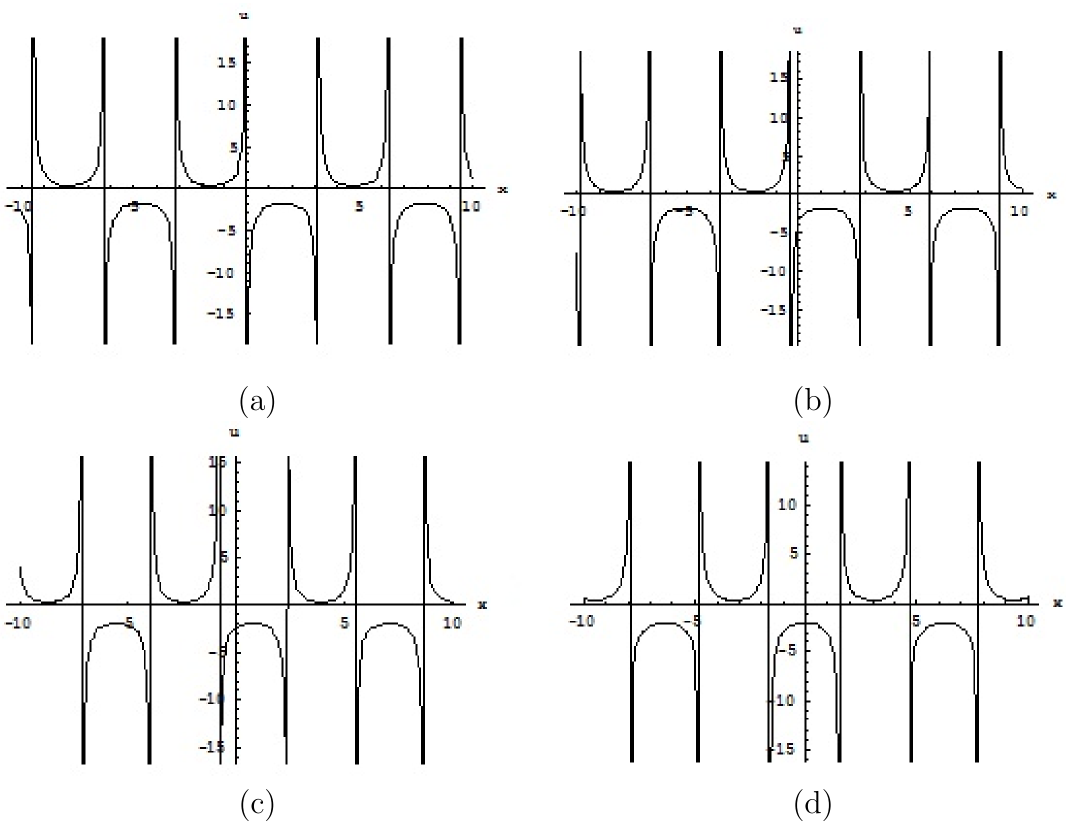
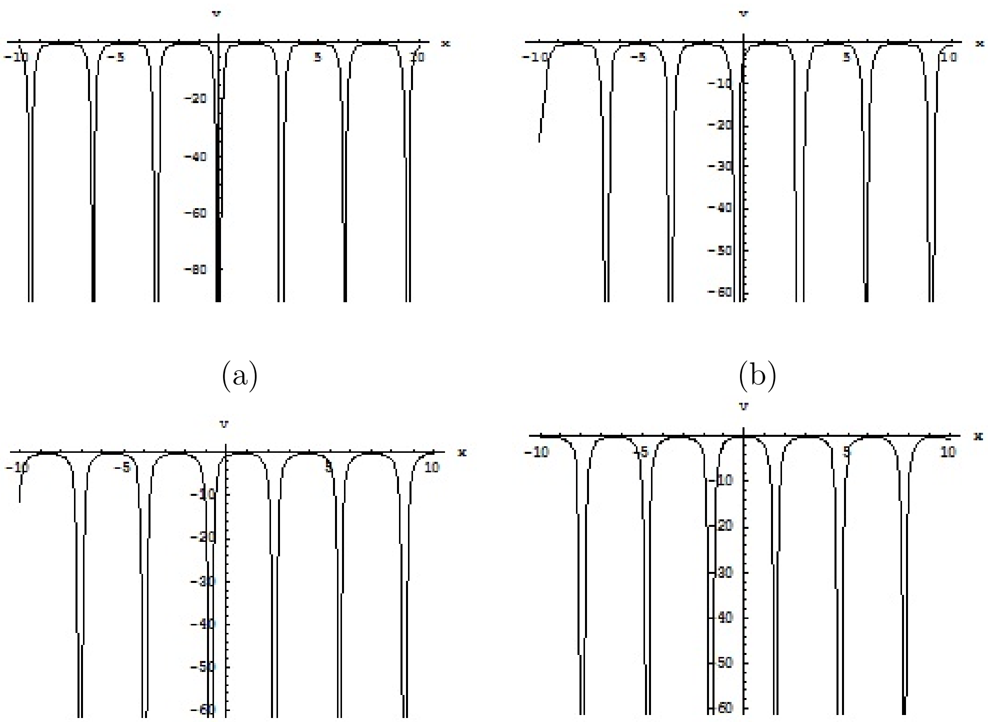


FIGURE 4. Periodic wave graphic of the (1+1)-dimensional dispersive long wave equation for $u(x, t)$ of solution (60) in two dimensions (a) $t = 0$, (b) $t = 0.5$, (c) $t = 1$, (d) $t = 2$, $(k = 1)$.



(a) (b)

(c) (d)

FIGURE 5. Periodic wave graphic of the (1+1)-dimensional dispersive long wave equation for $v(x,t)$ of solution (60) in two dimensions (a) $t=0$, (b) $t=0.5$, (c) $t=1$, (d) $t=2$, ($k=1$).

In Figure 3, Figure 4 and Figure 5, are shown graphics of periodic wave solutions of the (1+1)-dimensional dispersive long wave equation in three and two dimensions, respectively. In Figure 5, the periodic waves move to the left with time. If $\xi = kx - wt$, the periodic waves move to the right.

3. Conclusions

In this paper, we present the generalized Jacobi elliptic function method [11] by using ansatz (5) and, with aid of Mathematica, implement it in a computer algebraic system. An implementation of the method is given by applying it to the SRLW equation and (1+1)-dimensional dispersive long wave equation. We obtain some periodic solutions of these equations at the same time. The method can be used for many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculations on a computer.

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