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RESEARCH ARTICLE

SOFT INTERSECTION ALMOST QUASI-INTERIOR IDEALS OF SEMIGROUPS

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Abstract

Similar to how the quasi-interior ideal generalizes the ideal and interior ideal of a semigroup, the concept of soft intersection quasi-interior ideal generalizes the idea of soft intersection ideal and soft intersection interior ideal of a semigroup. In this study, we provide the notion of soft intersection almost quasi-interior ideal as well as the soft intersection weakly almost quasi-interior ideal in a semigroup. We show that any nonnull soft intersection quasi-interior ideal is a soft intersection almost quasi-interior ideal; and soft intersection almost quasi-interior ideal is a soft intersection weakly almost quasi-interior ideal, but the converses are not true. We further demonstrate that any idempotent soft intersection almost quasi-interior ideal is a soft intersection almost subsemigroup. With the established theorem that states that if a nonempty set A is almost quasi-interior ideal, then its soft characteristic function is a soft intersection almost quasi-interior ideal, and vice versa, we are also able to derive several intriguing relationships concerning minimality, primeness, semiprimeness, and strongly primeness between almost quasi-interior ideals, and soft intersection almost quasi-interior ideals.

Keywords

Soft set,
Semigroup,
(Almost) quasi-interior ideals,
Soft intersection (almost) quasi-interior ideals

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1. INTRODUCTION

As semigroups provide the abstract algebraic basis for "memoryless" systems, which restart on each iteration, semigroups are essential in many disciplines of mathematics. The formal study of semigroups began in the early 1900s. In practical mathematics, semigroups are essential models for linear time-invariant systems. Since finite semigroups are inherently connected to finite automata, studying them is essential to theoretical computer science. Furthermore, in probability theory, semigroups, and Markov processes are related.

Ideals are necessary to understand algebraic structures and their applications. The earliest ideals to help with the study of algebraic numbers were offered by Dedekind. Noether generalized them further by adding associative rings. Bi-ideals for semigroups were first introduced by Good and Hughes in 1952 [1]. Steinfeld [2] first established the idea of quasi-ideals for semigroups and later expanded it to rings. For many mathematicians, generalizing ideals in algebraic structures has been a key field of research.

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The concept of almost left, right, and two-sided ideals of semigroups was first developed by Grosek and Satko [3] in 1980. Later, in 1981, Bogdanovic [4] extended the concept of bi-ideals to almost bi-ideals in semigroups. By combining the concepts of almost ideals and quasi-ideals of semigroups, Wattanatripop et al. [5] proposed almost quasi-ideals in 2018. In 2020, Kaopusek et al. [6] introduced almost interior ideals and weakly almost interior ideals of semigroups, extending and analyzing the notions of almost ideals and interior ideals of semigroups. Iampan [7] in 2022, Gaketem [9] in 2022, Chinram and Nakkhasen [8] in 2022, and Gaketem and Chinram [10] in 2023 introduced almost subsemigroups, almost bi-quasi-interior ideals, almost bi-interior ideals, and almost bi-quasi-ideals of semigroups, respectively. Furthermore, in [5, 7–12], several almost fuzzy semigroup ideal types were studied.

The concept of soft sets as a means of modeling uncertainty was initially proposed by Molodtsov [13] in 1999, and it has since attracted interest from several of disciplines. The basic operations of soft sets were studied in [14–29]. Çağman and Enginoğlu [30] modified the concept of soft sets and soft set operations and in [31] Çağman et al. introduced soft intersection groups by which research on other soft algebraic systems was inspired. Soft sets were also conveyed to semigroup theory through the notions of soft intersection semigroup, left, right, and two-sided, quasi-ideals, interior ideals, and (generalized) bi-ideals, which were extensively explored in [32–33]. Sezgin and Orbay [34] studied soft intersection substructures to classify various semigroups. A variety of soft algebraic structures became the subject of additional investigation in [35–44].

Bi-interior ideals, bi-quasi-interior ideals, bi-quasi-ideals, quasi-interior ideals, and weak-interior ideals are some of the new semigroup types that Rao [45–48] has proposed. These ideals are expansions of existing ideals. Moreover, Baupradist et al. [49] proposed the notion of essential ideals in semigroups.

While the quasi-interior ideal of semigroups was introduced by Rao [47] as a generalization of ideal and interior ideal of a semigroup; soft intersection quasi-interior ideal of a semigroup was proposed in [50] as a generalization of the soft intersection ideal and interior ideal of a semigroup. In this study, as a further generalization of the nonnull soft intersection quasi-interior ideal, the concept of soft intersection almost quasi-interior and its generalization soft intersection weakly almost quasi-interior ideals are introduced. Moreover, we demonstrate that an idempotent soft intersection almost quasi-interior ideal is a soft intersection almost subsemigroup. We observe that under the binary operation of soft union, a semigroup may be constructed by soft intersection almost quasi-interior ideals; however, this is not the case under the soft intersection operation. We also establish the relationship between the soft intersection almost quasi-interior ideal and almost quasi-interior ideal of a semigroup in terms of minimality, primeness, semiprimeness, and strongly primeness. This is achieved by deriving that if a nonempty set A is an almost quasi-interior ideal, then its soft characteristic function is also a soft intersection almost quasi-interior ideal, and vice versa. This paper is organized in the following manner. Section 2 reviews the fundamental principles of soft set theory, including semigroup ideals. Section 3 looks at the definition and thorough study of soft intersection almost quasi-interior ideals. In the conclusion section, we highlight the significance of the study's findings and their possible impact on the field.

2. PRELIMINARIES

In this section, we review several fundamental notions related to semigroups and soft sets.

Definition 2.1. Let U be the universal set, E be the parameter set, and $P(U)$ be the power set of U and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K\colon E \to P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = \{(x, f_K(x))\colon x \in E, f_K(x) \in P(U)\}$$

[13, 30]. Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.2. Let $f_A \in S_E(U)$. If $f_A(\text{x}) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then f_A is called an absolute soft set and denoted by U_E [30].

Definition 2.3. Let f_A, $f_B \in S_E(U)$. If for all $x \in E$, $f_A(x) \subseteq f_B(x)$, then f_A is a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft equal to f_B and denoted by $f_A = f_B$ [30].

Definition 2.4. Let $f_A, f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$ [30].

Definition 2.5. For a soft set f_A, the support of f_A is defined by

$$supp(f_A) = \{x \in A: f_A(x) \neq \emptyset\}[18]$$

Thus, a null soft set in is indeed a soft set with an empty support, and we say that a soft set f_A is nonnull if $supp(f_A) \neq \emptyset$ [18].

Note 2.6. If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$ [51].

A semigroup S is a nonempty set with an associative binary operation and throughout this paper, S stands for a semigroup and all the soft sets are the elements of $S_S(U)$ unless otherwise specified. A nonempty subset A of S is called a left quasi-interior ideal of S if $SASA \subseteq A$; and is called a right quasi-interior ideal of S if $ASAS \subseteq A$; and is called a quasi-interior ideal of S if A is both a left quasi-interior ideal and a right quasi-interior ideal of S [47].

Definition 2.7. A nonempty subset A of S is called an almost left quasi-interior ideal of S if for all $x, y \in S$; $xAyA \cap A \neq \emptyset$, and is called an almost right quasi-interior ideal of S if for all $x, y \in S$, $AxAy \cap A \neq \emptyset$; and is called an almost quasi-interior ideal of S when A is both an almost left quasi-interior ideal of S and an almost right quasi-interior ideal of S.

Example 2.8. Let $S = \mathbb{Z}$ and $\emptyset \neq 2\mathbb{Z} \subseteq \mathbb{Z}$. Since $x(2\mathbb{Z})y(2\mathbb{Z}) \cap 2\mathbb{Z} \neq \emptyset$ and $(2\mathbb{Z})x(2\mathbb{Z})y \cap 2\mathbb{Z} \neq \emptyset$ for all $x, y \in \mathbb{Z}$, $2\mathbb{Z}$ is an almost quasi-interior ideal of S.

An almost (left/right) quasi-interior ideal A of S is called a minimal almost (left/right) quasi-interior ideal of S if for any almost (left/right) quasi-interior ideal B of S if whenever $B \subseteq A$, then $A = B$.

An almost (left/right) quasi-interior ideal P of S is called a prime almost (left/right) quasi-interior ideal if for any almost (left/right) quasi-interior ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost (left/right) quasi-interior ideal P of S is called a semiprime almost (left/right) quasi-interior ideal if for any almost (left/right) quasi-interior ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost (left/right) quasi-interior ideal P of S is called a strongly prime almost (left/right) quasi-interior ideal if for any almost (left/right) quasi-interior ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 2.9. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by [32]

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup_{x=yz} \{f_S(y) \cap g_S(z)\}, & if\ \exists y, z \in S\ such\ that\ x = yz \\ \emptyset, & otherwise \end{cases}$$

Theorem 2.10. Let $f_S, g_S, h_S \in S_S(U)$. Then,

i) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.

ii) $f_S \circ g_S \neq g_S \circ f_S$, *generally.*

iii) $f_S \circ (g_S \tilde{\cup} h_S) = (f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)$ and $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.

iv) $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ and $(f_S \tilde{\cap} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cap} (g_S \circ h_S)$.

v) If $f_S \tilde{\subseteq} g_S$, then $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ and $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$.

vi) If $t_S, k_S \in S_S(U)$ such that $t_S \tilde{\subseteq} f_S$ and $k_S \tilde{\subseteq} g_S$, then $t_S \circ k_S \tilde{\subseteq} f_S \circ g_S$ [32].

Lemma 2.11. Let f_S and g_S be soft sets over U. Then, $f_S \circ g_S = \emptyset_S$ if and only if $f_S = \emptyset_S$ or $g_S = \emptyset_S$.

Definition 2.12. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & if\ x \in A \\ \emptyset, & if\ x \in S\backslash A \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$ [32].

Corollary 2.13. $supp(S_A) = A$ [51].

Theorem 2.14. Let X and Y be nonempty subsets of S. Then, the following properties hold [32,51]:

i) $X \subseteq Y$ if and only if $S_X \tilde{\subseteq} S_Y$

ii) $S_X \tilde{\cap} S_Y = S_{X\cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X\cup Y}$

iii) $S_X \circ S_Y = S_{XY}$

Proof: In [32], (i) is given as if $X \subseteq Y$, then if $S_X \tilde{\subseteq} S_Y$. In [51], it was shown that if $S_X \tilde{\subseteq} S_Y$, then $X \subseteq Y$. Let $S_X \tilde{\subseteq} S_Y$ and $x \in X$. Then, $S_X(x) = U$, and this implies that $S_Y(x) = U$ since $S_X \tilde{\subseteq} S_Y$, Hence, $x \in Y$, and so $X \subseteq Y$. Now let $x \notin Y$. Then, $S_Y(x) = \emptyset$, and this implies that $S_X(x) = \emptyset$ since $S_X \tilde{\subseteq} S_Y$. Hence, $x \notin X$, and so $Y' \subseteq X'$, implying that $X \subseteq Y$.

Definition 2.15. Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & if\ \ y = x \\ \emptyset, & if\ \ y \neq x \end{cases}$$

The soft characteristic function of x is a soft set over U, that is, $S_x: S \longrightarrow P(U)$ [52].

Corollary 2.16. Let $x, y \in S$, f_S and S_x be soft sets over U. Then, $S_x \circ f_S \circ S_y \circ f_S = \emptyset_S$ if and only if $f_S = \emptyset_S$ and $f_S \circ S_x \circ f_S \circ S_y = \emptyset_S$ if and only if $f_S = \emptyset_S$.

Proof: By Lemma 2.11, $S_x \circ f_S \circ S_y \circ f_S = \emptyset_S$ if and only if $f_S = \emptyset_S$ or $S_x = \emptyset_S$ or $S_y = \emptyset_S$. Since $S_x \neq \emptyset_S$ and $S_y \neq \emptyset_S$ for all $x, y \in S$ by Definition 2.15, hence $f_S = \emptyset_S$. The rest of the proof is obvious by Lemma 2.11.

Definition 2.17. A soft set f_S over U is called a soft intersection left (resp. right) quasi-interior ideal of S over U if $f_S(xyzt) \supseteq f_S(y) \cap f_S(t)$ ($f_S(xyzt) \supseteq f_S(x) \cap f_S(z)$) for all $x, y, z, t \in S$. A soft set f_S over U is called a soft intersection quasi-interior ideal of S if it is both a soft intersection left quasi-interior ideal and a soft intersection right quasi-interior ideal of S over U [50].

It is easy to see that if $f_S(x) = U$ for all $x \in S$, then f_S is a soft intersection (left/right) quasi-interior ideal of S. We denote such a kind of (left/right) quasi-interior ideal by $\widetilde{\mathbb{S}}$. It is obvious that $\widetilde{\mathbb{S}} = S_S$, that is, $\widetilde{\mathbb{S}}(x) = U$ for all $x \in S$ [50].

Theorem 2.18. Let f_S be a soft set over U. Then, f_S is a soft intersection left (resp. right) quasi-interior ideal of S if and only if $\widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \circ f_S \,\widetilde{\subseteq}\, f_S$ $\left(f_S \circ \widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \,\widetilde{\subseteq}\, f_S\right)$. f_S is a soft intersection quasi-interior ideal of S if and only if $\widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \circ f_S \,\widetilde{\subseteq}\, f_S$ and $f_S \circ \widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \,\widetilde{\subseteq}\, f_S$ [50].

From now on, soft intersection left (right) quasi-interior ideal of S is denoted by SI-left (right) QI-ideal.

Definition 2.19. Let f_S be a soft set over U. Then, f_S is called a soft intersection almost subsemigroup of S if $(f_S \circ f_S) \,\widetilde{\cap}\, f_S \neq \emptyset_S$ [51].

We refer to [53] for the implications of network analysis and graph applications with respect to soft sets, which are determined by the divisibility of determinants and to [54-56] for more about soft set operations.

3. SOFT INTERSECTION ALMOST QUASI-INTERIOR IDEALS OF SEMIGROUPS

Definition 3.1. Let f_S be a soft set over U.

1) f_S is called a soft intersection almost left (resp. right) quasi-interior ideal of S if for all $x, y \in S$,

$$\left(S_x \circ f_S \circ S_y \circ f_S\right) \widetilde{\cap}\, f_S \neq \emptyset_S \;\left(\left(f_S \circ S_x \circ f_S \circ S_y\right) \widetilde{\cap}\, f_S \neq \emptyset_S\right).$$

f_S is called a soft intersection almost quasi-interior ideal of S if f_S is both a soft intersection almost left quasi-interior ideal of S and a soft intersection almost right quasi-interior ideal of S.

2) f_S is called a soft intersection weakly almost left (resp. right) quasi-interior ideal of S if for all $x \in S$,

$$(S_x \circ f_S \circ S_x \circ f_S)\, \widetilde{\cap}\, f_S \neq \emptyset_S \;\left((f_S \circ S_x \circ f_S \circ S_x)\, \widetilde{\cap}\, f_S \neq \emptyset_S\right).$$

f_S is called a soft intersection weakly almost quasi-interior ideal of S if f_S is both a soft intersection weakly almost left quasi-interior ideal of S and a soft intersection weakly almost right quasi-interior ideal of S.

Hereafter, for brevity, soft intersection is abbreviated as SI, left (right) quasi-interior is as left (right) QI; so soft intersection (weakly) almost left (right) quasi-interior ideal of S is denoted by SI-(weakly) almost left (right) QI-ideal.

Example 3.2. Let $S = \{z, k\}$ be the semigroup with the following Cayley Table.

	z	k
z	z	z
k	z	k

Let f_S, h_S, and g_S be soft sets over $U = \mathbb{Z}^-$ as follows:

$$f_S = \{(z, \{-3, -2\}), (k, \{-5\})\}$$

$$h_S = \{(z, \{-9, -8\}), (k, \{-1\})\}$$

$$g_S = \{(z, \emptyset), (k, \{-7, -4\})\}$$

Here, f_S and h_S are SI-almost QI-ideals. Let's first show that f_S is an SI-weakly almost QI-ideal:

$[(S_k \circ f_S \circ S_k \circ f_S) \tilde{\cap} f_S](z) = [(S_k \circ f_S) \circ (S_k \circ f_S)](z) \cap f_S(z) = \Big[\big((S_k \circ f_S)(z) \cap (S_k \circ f_S)(z)\big) \cup \big((S_k \circ f_S)(z) \cap (S_k \circ f_S)(k)\big) \cup \big((S_k \circ f_S)(k) \cap (S_k \circ f_S)(z)\big)\Big] \cap f_S(z) = \Big[\Big[\Big(\big(S_k(z) \cap f_S(z)\big) \cup \big(S_k(z) \cap f_S(k)\big) \cup \big(S_k(k) \cap f_S(z)\big)\Big) \cap \Big(\big(S_k(z) \cap f_S(z)\big) \cup \big(S_k(z) \cap f_S(k)\big) \cup \big(S_k(k) \cap f_S(z)\big)\Big)\Big] \cup \Big[\Big(\big(S_k(z) \cap f_S(z)\big) \cup \big(S_k(z) \cap f_S(k)\big) \cup \big(S_k(k) \cap f_S(z)\big)\Big) \cap \Big(\big(S_k(k) \cap f_S(k)\big)\Big)\Big] \cup \Big[\Big(\big(S_k(k) \cap f_S(k)\big)\Big) \cap \Big(\big(S_k(z) \cap f_S(z)\big) \cup \big(S_k(z) \cap f_S(k)\big) \cup \big(S_k(k) \cap f_S(z)\big)\Big)\Big]\Big] \cap f_S(z) = \Big[\big(f_S(z) \cap f_S(z)\big) \cup \big(f_S(z) \cap f_S(k)\big) \cup \big(f_S(k) \cap f_S(z)\big)\Big] \cap f_S(z) = f_S(z) = \{-3, -2\}$

$[(S_k \circ f_S \circ S_k \circ f_S) \tilde{\cap} f_S](k) = [(S_k \circ f_S) \circ (S_k \circ f_S)](k) \cap f_S(k) = [(S_k \circ f_S)(k) \cap (S_k \circ f_S)(k)] \cap f_S(k) = \big[\big(S_k(k) \cap f_S(k)\big) \cap \big(S_k(k) \cap f_S(k)\big)\big] \cap f_S(k) = f_S(k) \cap f_S(k) \cap f_S(k) = f_S(k) = \{-5\}$

Consequently,

$$(S_k \circ f_S \circ S_k \circ f_S) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \{-5\})\} \neq \emptyset_S$$

Similarly,

$$(S_z \circ f_S \circ S_z \circ f_S) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \emptyset)\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(S_x \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-weakly almost left QI-ideal. And also,

$$(f_S \circ S_k \circ f_S \circ S_k) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \{-5\})\} \neq \emptyset_S$$

$$(f_S \circ S_z \circ f_S \circ S_z) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \emptyset)\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(f_S \circ S_x \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-weakly almost right QI-ideal. Thus, f_S is an SI-weakly almost QI-ideal.

With similar calculations, we have

$$(S_k \circ f_S \circ S_z \circ f_S) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \emptyset)\} \neq \emptyset_S$$

$$(S_z \circ f_S \circ S_k \circ f_S) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \emptyset)\} \neq \emptyset_S$$

Therefore, we have shown that for all $x, y \in S$, $\big(S_x \circ f_S \circ S_y \circ f_S\big) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-almost left QI-ideal. Now let's show that f_S is an SI-almost right QI-ideal. As usual calculations, we have

$$(f_S \circ S_k \circ f_S \circ S_z) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \emptyset)\} \neq \emptyset_S$$

$$(f_S \circ S_z \circ f_S \circ S_k) \tilde{\cap} f_S = \{(z, \{-3, -2\}), (k, \emptyset)\} \neq \emptyset_S$$

Therefore, we have shown that for all $x, y \in S$, $\big(f_S \circ S_x \circ f_S \circ S_y\big) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-almost right QI-ideal. Thus f_S is an SI-almost QI-ideal. Similarly, h_S is an SI-weakly almost QI-ideal since

$$(S_k \circ h_S \circ S_k \circ h_S) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \{-1\})\} \neq \emptyset_S$$

$$(S_z \circ h_S \circ S_z \circ h_S) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \emptyset)\} \neq \emptyset_S$$

$$(h_S \circ S_k \circ h_S \circ S_k) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \{-1\})\} \neq \emptyset_S$$

$$(h_S \circ S_z \circ h_S \circ S_z) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \emptyset)\} \neq \emptyset_S$$

and also since

$$(S_k \circ h_S \circ S_z \circ h_S) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \emptyset)\} \neq \emptyset_S$$
$$(S_z \circ h_S \circ S_k \circ h_S) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \emptyset)\} \neq \emptyset_S$$
$$(h_S \circ S_k \circ h_S \circ S_z) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \emptyset)\} \neq \emptyset_S$$
$$(h_S \circ S_z \circ h_S \circ S_k) \tilde{\cap} h_S = \{(z, \{-9, -8\}), (k, \emptyset)\} \neq \emptyset_S$$

h_S is an SI-almost QI-ideal. One can also show that g_S is not an SI-almost QI-ideal. In fact;

$[(S_z \circ g_S \circ S_z \circ g_S) \tilde{\cap} g_S](z) = [(S_z \circ g_S) \circ (S_z \circ g_S)](z) \cap g_S(z) = \Big[\big((S_z \circ g_S)(z) \cap (S_z \circ g_S)(z)\big) \cup \big((S_z \circ g_S)(z) \cap (S_z \circ g_S)(k)\big) \cup \big((S_z \circ g_S)(k) \cap (S_z \circ g_S)(z)\big)\Big] \cap g_S(z) = \Bigg[\Big[\Big(\big(S_z(z) \cap g_S(z)\big) \cup \big(S_z(z) \cap g_S(k)\big) \cup \big(S_z(k) \cap g_S(z)\big)\Big) \cap \Big(\big(S_z(z) \cap g_S(z)\big) \cup \big(S_z(z) \cap g_S(k)\big) \cup \big(S_z(k) \cap g_S(z)\big)\Big)\Big] \cup \Big[\Big(\big(S_z(z) \cap g_S(z)\big) \cup \big(S_z(z) \cap g_S(k)\big) \cup \big(S_z(k) \cap g_S(z)\big)\Big) \cap \Big(\big(S_z(k) \cap g_S(k)\big)\Big)\Big] \cup \Big[\Big(\big(S_z(k) \cap g_S(k)\big)\Big) \cap \Big(\big(S_z(z) \cap g_S(z)\big) \cup \big(S_z(z) \cap g_S(k)\big) \cup \big(S_z(k) \cap g_S(z)\big)\Big)\Big]\Bigg] \cap g_S(z) = \Big[\big[\big(g_S(z) \cup g_S(k)\big) \cap \big(g_S(z) \cup g_S(k)\big)\big] \cup \big[\big(g_S(z) \cup g_S(k)\big) \cap \emptyset\big] \cup \big[\emptyset \cap \big(g_S(z) \cup g_S(k)\big)\big]\Big] \cap g_S(z) = \big[\big(g_S(z) \cup g_S(k)\big) \cup \emptyset \cup \emptyset\big] \cap g_S(z) = g_S(z) = \emptyset$

$[(S_z \circ g_S \circ S_z \circ g_S) \tilde{\cap} g_S](k) = [(S_z \circ g_S) \circ (S_z \circ g_S)](k) \cap g_S(k) = [(S_z \circ g_S)(k) \cap (S_z \circ g_S)(k)] \cap g_S(k) = \big[\big(S_z(k) \cap g_S(k)\big) \cap \big(S_z(k) \cap g_S(k)\big)\big] \cap g_S(k) = \emptyset \cap \emptyset \cap g_S(k) = \emptyset$

Thus, for $z \in S$;

$$(S_z \circ g_S \circ S_z \circ g_S) \tilde{\cap} g_S = \{(z, \emptyset), (k, \emptyset)\} = \emptyset_S$$

Hence, g_S is not an SI-weakly almost QI-ideal, thus it is not an SI-almost QI-ideal.

Proposition 3.3. Every SI-almost QI-ideal is an SI-weakly almost QI-ideal.

Proof: Let f_S be an SI-almost QI-ideal of S. Then, for all $x, y \in S$, $\big(S_x \circ f_S \circ S_y \circ f_S\big) \tilde{\cap} f_S \neq \emptyset_S$ and $\big(f_S \circ S_x \circ f_S \circ S_y\big) \tilde{\cap} f_S \neq \emptyset_S$. Hence, for all $x \in S$, $(S_x \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ and $(f_S \circ S_x \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$. So, f_S is an SI-weakly almost QI-ideal.

Since SI-weakly almost QI-ideal is a generalization of SI-almost QI-ideal, from now on, all the theorems and proofs are given for SI-almost QI-ideals instead of SI-weakly almost QI-ideals.

The converse of Proposition 3.3 is not true in general as shown in the following example:

Example 3.4. Let $S = \{a, r\}$ be the semigroup with the following Cayley Table.

	a	r
a	r	a
r	a	r

f_s be soft sets over $U = \mathbb{Z}$ as follows:

$$f_S = \{(a, \emptyset), (r, \{-6,3,6\})\}$$

Let's first show that f_S is an SI-weakly almost QI-ideal, that is for all $x \in S$,

$$(S_x \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

Let's start with S_a, S_a:

$[(S_a \circ f_S \circ S_a \circ f_S) \tilde{\cap} f_S](a) = [(S_a \circ f_S) \circ (S_a \circ f_S)](a) \cap f_S(a) = \Big[\big((S_a \circ f_S)(a) \cap (S_a \circ f_S)(r)\big) \cup \big((S_a \circ f_S)(r) \cap (S_a \circ f_S)(a)\big)\Big] \cap f_S(a) = \bigg[\Big[\Big(\big(S_a(a) \cap f_S(r)\big) \cup \big(S_a(r) \cap f_S(a)\big)\Big) \cap \Big(\big(S_a(a) \cap f_S(a)\big) \cup \big(S_a(r) \cap f_S(r)\big)\Big)\Big] \cup \Big[\Big(\big(S_a(a) \cap f_S(a)\big) \cup \big(S_a(r) \cap f_S(r)\big)\Big) \cap \Big(\big(S_a(a) \cap f_S(r)\big) \cup \big(S_a(r) \cap f_S(a)\big)\Big)\Big]\bigg] \cap f_S(a) = \big[\big(f_S(r) \cap f_S(a)\big) \cup \big(f_S(a) \cap f_S(r)\big)\big] \cap f_S(a) = \big(f_S(a) \cap f_S(r)\big) \cap f_S(a) = f_S(a) \cap f_S(r) = \emptyset$

$[(S_a \circ f_S \circ S_a \circ f_S) \tilde{\cap} f_S](r) = [(S_a \circ f_S) \circ (S_a \circ f_S)](r) \cap f_S(r) = \Big[\big((S_a \circ f_S)(a) \cap (S_a \circ f_S)(a)\big) \cup \big((S_a \circ f_S)(r) \cap (S_a \circ f_S)(r)\big)\Big] \cap f_S(r) = \bigg[\Big[\Big(\big(S_a(a) \cap f_S(r)\big) \cup \big(S_a(r) \cap f_S(a)\big)\Big) \cap \Big(\big(S_a(a) \cap f_S(r)\big) \cup \big(S_a(r) \cap f_S(a)\big)\Big)\Big] \cup \Big[\Big(\big(S_a(a) \cap f_S(a)\big) \cup \big(S_a(r) \cap f_S(r)\big)\Big) \cap \Big(\big(S_a(a) \cap f_S(a)\big) \cup \big(S_a(r) \cap f_S(r)\big)\Big)\Big]\bigg] \cap f_S(r) = \big[\big(f_S(r) \cap f_S(r)\big) \cup \big(f_S(a) \cap f_S(a)\big)\big] \cap f_S(r) = \big(f_S(r) \cup f_S(a)\big) \cap f_S(r) = f_S(r) = \{-6,3,6\}$

Hence,

$$(S_a \circ f_S \circ S_a \circ f_S) \tilde{\cap} f_S = \{(a, \emptyset), (r, \{-6,3,6\})\} \neq \emptyset_S$$

And also,

$$(S_r \circ f_S \circ S_r \circ f_S) \tilde{\cap} f_S = \{(a, \emptyset), (r, \{-6,3,6\})\} \neq \emptyset_S$$

Therefore, for all $x \in S$, $(S_x \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-weakly almost left QI-ideal. Similarly,

$$(f_S \circ S_a \circ f_S \circ S_a) \tilde{\cap} f_S = \{(a, \emptyset), (r, \{-6,3,6\})\} \neq \emptyset_S$$

$$(f_S \circ S_r \circ f_S \circ S_r) \tilde{\cap} f_S = \{(a, \emptyset), (r, \{-6,3,6\})\} \neq \emptyset_S$$

Hence, for all $x \in S$, $(f_S \circ S_x \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-weakly almost right QI-ideal. Thus f_S is an SI-weakly almost QI-ideal.

However, here not that f_S is not an SI-almost QI-ideal. In deed;

$[(S_a \circ f_S \circ S_r \circ f_S) \tilde{\cap} f_S](a) = [(S_a \circ f_S) \circ (S_r \circ f_S)](a) \cap f_S(a) = \Big[\big((S_a \circ f_S)(a) \cap (S_r \circ f_S)(r)\big) \cup \big((S_a \circ f_S)(r) \cap (S_r \circ f_S)(a)\big)\Big] \cap f_S(a) = \bigg[\Big[\Big(\big(S_a(a) \cap f_S(r)\big) \cup \big(S_a(r) \cap f_S(a)\big)\Big) \cap \Big(\big(S_r(a) \cap f_S(a)\big) \cup \big(S_r(r) \cap f_S(r)\big)\Big)\Big] \cup \Big[\Big(\big(S_a(a) \cap f_S(a)\big) \cup \big(S_a(r) \cap f_S(r)\big)\Big) \cap \Big(\big(S_r(a) \cap f_S(r)\big) \cup \big(S_r(r) \cap f_S(a)\big)\Big)\Big]\bigg] \cap f_S(a) = \big[\big(f_S(r) \cap f_S(r)\big) \cup \big(f_S(a) \cap f_S(a)\big)\big] \cap f_S(a) = \big(f_S(r) \cup f_S(a)\big) \cap f_S(a) = f_S(a) = \emptyset$

$[(S_a \circ f_S \circ S_r \circ f_S) \widetilde{\cap} f_S](r) = [(S_a \circ f_S)\circ(S_r \circ f_S)](r) \cap f_S(r) = \big[\big((S_a \circ f_S)(a) \cap (S_r \circ f_S)(a)\big) \cup \big((S_a \circ f_S)(r) \cap (S_r \circ f_S)(r)\big)\big] \cap f_S(r) = \Big[\Big[\Big(\big(S_a(a) \cap f_S(r)\big) \cup \big(S_a(r) \cap f_S(a)\big)\Big) \cap \Big(\big(S_r(a) \cap f_S(r)\big) \cup \big(S_r(r) \cap f_S(a)\big)\Big)\Big] \cup \Big[\Big(\big(S_a(a) \cap f_S(a)\big) \cup \big(S_a(r) \cap f_S(r)\big)\Big) \cap \Big(\big(S_r(a) \cap f_S(a)\big) \cup \big(S_r(r) \cap f_S(r)\big)\Big)\Big]\Big] \cap f_S(r) = \big[\big(f_S(r) \cap f_S(a)\big) \cup \big(f_S(a) \cap f_S(r)\big)\big] \cap f_S(r) = \big(f_S(a) \cap f_S(r)\big) \cap f_S(r) = f_S(a) \cap f_S(r) = \emptyset$

Consequently,

$$(S_a \circ f_S \circ S_r \circ f_S) \widetilde{\cap} f_S = \{(a,\emptyset),(r,\emptyset)\} = \emptyset_S$$

Thus, f_S is not an SI-almost QI-ideal.

Proposition 3.5. If f_S is an SI-left (resp. right) QI-ideal such that $f_S \neq \emptyset_S$, then f_S is an SI-almost left (resp. right) QI-ideal.

Proof: Let $f_S \neq \emptyset_S$ be an SI-left QI-ideal, then $\widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \circ f_S \widetilde{\subseteq} f_S$. Since $f_S \neq \emptyset_S$, by Corollary 2.16, it follows that $S_x \circ f_S \circ S_y \circ f_S \neq \emptyset_S$. We need to show that for all $x, y \in S$,

$$\big(S_x \circ f_S \circ S_y \circ f_S\big) \widetilde{\cap} f_S \neq \emptyset_S.$$

Since $S_x \circ f_S \circ S_y \circ f_S \widetilde{\subseteq} \widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \circ f_S \widetilde{\subseteq} f_S$, it follows that $S_x \circ f_S \circ S_y \circ f_S \widetilde{\subseteq} f_S$. Thus,

$$\big(S_x \circ f_S \circ S_y \circ f_S\big) \widetilde{\cap} f_S = S_x \circ f_S \circ S_y \circ f_S \neq \emptyset_S$$

implying that f_S is an SI-almost left QI-ideal.

Here it is obvious that $\emptyset_S$ is an SI-left QI-ideal, as $\widetilde{\mathbb{S}} \circ \emptyset_S \circ \widetilde{\mathbb{S}} \circ \emptyset_S \widetilde{\subseteq} \emptyset_S$; but it is not an SI-almost QI-ideal, since $\big(S_x \circ \emptyset_S \circ S_y \circ \emptyset_S\big) \widetilde{\cap} \emptyset_S = \emptyset_S \widetilde{\cap} \emptyset_S = \emptyset_S$ for all $x, y \in S$.

Here note that if f_S is an SI-almost left (resp. right) QI-ideal, then f_S needs not to be an SI-left (resp. right) QI-ideal as shown in the following example:

Example 3.6. In Example 3.2, it is shown that f_S and h_S are SI-almost QI-ideals; however f_S and h_S are not SI-QI-ideals. In fact;

$(\widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}} \circ f_S)(z) = \big[\big(\widetilde{\mathbb{S}} \circ f_S\big)(z) \cap \big(\widetilde{\mathbb{S}} \circ f_S\big)(z)\big] \cup \big[\big(\widetilde{\mathbb{S}} \circ f_S\big)(z) \cap \big(\widetilde{\mathbb{S}} \circ f_S\big)(k)\big] \cup \big[\big(\widetilde{\mathbb{S}} \circ f_S\big)(k) \cap \big(\widetilde{\mathbb{S}} \circ f_S\big)(z)\big] = \Big[\Big[\Big(\widetilde{\mathbb{S}}(z) \cap f_S(z)\Big) \cup \Big(\widetilde{\mathbb{S}}(z) \cap f_S(k)\Big) \cup \Big(\widetilde{\mathbb{S}}(k) \cap f_S(z)\Big)\Big] \cap \Big[\Big(\widetilde{\mathbb{S}}(z) \cap f_S(z)\Big) \cup \Big(\widetilde{\mathbb{S}}(z) \cap f_S(k)\Big) \cup \Big(\widetilde{\mathbb{S}}(k) \cap f_S(z)\Big)\Big]\Big] \cup \Big[\Big[\Big(\widetilde{\mathbb{S}}(z) \cap f_S(z)\Big) \cup \Big(\widetilde{\mathbb{S}}(z) \cap f_S(k)\Big) \cup \Big(\widetilde{\mathbb{S}}(k) \cap f_S(z)\Big)\Big] \cap \Big[\Big(\widetilde{\mathbb{S}}(k) \cap f_S(k)\Big)\Big]\Big] \cup \Big[\Big[\Big(\widetilde{\mathbb{S}}(k) \cap f_S(k)\Big)\Big] \cap \Big[\Big(\widetilde{\mathbb{S}}(z) \cap f_S(z)\Big) \cup \Big(\widetilde{\mathbb{S}}(z) \cap f_S(k)\Big) \cup \Big(\widetilde{\mathbb{S}}(k) \cap f_S(z)\Big)\Big]\Big] = \big[\big(f_S(z) \cup f_S(k) \cup f_S(z)\big) \cap \big(f_S(z) \cup f_S(k) \cup f_S(z)\big)\big] \cup \big[\big(f_S(z) \cup f_S(k) \cup f_S(z)\big) \cap f_S(k)\big] \cup \big[\big(f_S(k) \cap \big(f_S(z) \cup f_S(k) \cup f_S(z)\big)\big] = \big(f_S(z) \cup f_S(k)\big) \cup f_s(k) \cup f_s(k) = \big(f_S(z) \cup f_S(k)\big) = \{-5,-3,-2\} \not\subseteq f_S(z) = \{-3,-2\}$

Or similarly,

$(f_S \circ \widetilde{\mathbb{S}} \circ f_S \circ \widetilde{\mathbb{S}})(z) = \big[(f_S \circ \widetilde{\mathbb{S}})(z) \cap (f_S \circ \widetilde{\mathbb{S}})(z)\big] \cup \big[(f_S \circ \widetilde{\mathbb{S}})(z) \cap (f_S \circ \widetilde{\mathbb{S}})(k)\big] \cup \big[(f_S \circ \widetilde{\mathbb{S}})(k) \cap (f_S \circ \widetilde{\mathbb{S}})(z)\big] = \Big[\Big[\Big(f_S(z) \cap \widetilde{\mathbb{S}}(z)\Big) \cup \Big(f_S(z) \cap \widetilde{\mathbb{S}}(k)\Big) \cup \Big(f_S(k) \cap \widetilde{\mathbb{S}}(z)\Big)\Big] \cap \Big[\Big(f_S(z) \cap \widetilde{\mathbb{S}}(z)\Big) \cup \Big(f_S(z) \cap \widetilde{\mathbb{S}}(k)\Big) \cup \Big(f_S(k) \cap \widetilde{\mathbb{S}}(z)\Big)\Big]\Big] \cup \Big[\Big[\Big(f_S(z) \cap \widetilde{\mathbb{S}}(z)\Big) \cup \Big(f_S(z) \cap \widetilde{\mathbb{S}}(k)\Big) \cup \Big(f_S(k) \cap \widetilde{\mathbb{S}}(z)\Big)\Big] \cap \Big[\Big(f_S(k) \cap$

$\widetilde{\mathbb{S}}(k))]] \cup [[(f_S(k) \cap \widetilde{\mathbb{S}}(k))] \cap [(f_S(z) \cap \widetilde{\mathbb{S}}(z)) \cup (f_S(z) \cap \widetilde{\mathbb{S}}(k)) \cup (f_S(k) \cap \widetilde{\mathbb{S}}(z))]] = [(f_S(z) \cup f_S(z) \cup f_S(k)) \cap (f_S(z) \cup f_S(z) \cup f_S(k))] \cup [(f_S(z) \cup f_S(z) \cup f_S(k)) \cap f_S(k)] \cup [(f_S(k) \cap (f_S(z) \cup f_S(z) \cup f_S(k))] = (f_S(z) \cup f_S(k)) \cup f_S(k) \cup f_S(k) = (f_S(z) \cup f_S(k)) = \{-5,-3,-2\} \nsubseteq f_S(z) = \{-3,-2\}$

Thus, f_S is not an SI-QI-ideal. Similarly,

$$(\widetilde{\mathbb{S}} \circ h_S \circ \widetilde{\mathbb{S}} \circ h_S)(z) = (h_S(z) \cup h_S(k)) = \{-9,-8,-1\} \nsubseteq h_S(z) = \{-9,-8\} \text{ or}$$

$$(h_S \circ \widetilde{\mathbb{S}} \circ h_S \circ \widetilde{\mathbb{S}})(z) = (h_S(z) \cup h_S(k)) = \{-9,-8,-1\} \nsubseteq h_S(z) = \{-9,-8\}$$

Hence, h_S is not an SI-QI-ideal.

Proposition 3.7. Let f_S be an idempotent SI-almost left (right) QI-ideal. Then, f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an idempotent SI-almost left QI-ideal. Then, $f_S \circ f_S = f_S$ and for all $x, y \in S$, $(f_S \circ S_x \circ f_S \circ S_y) \widetilde{\cap} f_S \neq \emptyset_S$. We need to show that for all $x \in S$,

$$(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$$

Since,

$$\emptyset_S \neq (f_S \circ S_x \circ f_S \circ S_y) \widetilde{\cap} f_S = [(f_S \circ S_x \circ f_S \circ S_y) \widetilde{\cap} f_S] \widetilde{\cap} f_S = [(f_S \circ S_x \circ f_S \circ S_y) \widetilde{\cap} f_S] \widetilde{\cap} (f_S \circ f_S) \widetilde{\subseteq} (f_S \circ f_S) \widetilde{\cap} f_S$$

hence f_S is an SI-almost subsemigroup.

Theorem 3.7. Let $f_S \widetilde{\subseteq} h_S$ such that f_S is an SI-almost left (resp. right) QI-ideal, then h_S is an SI-almost left (resp. right) QI-ideal.

Proof: Assume that f_S is an SI-almost left QI-ideal. Hence, for all $x, y \in S$, $(S_x \circ f_S \circ S_y \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$. We need to show that $(S_x \circ h_S \circ S_y \circ h_S) \widetilde{\cap} h_S \neq \emptyset_S$. In fact,

$$(S_x \circ f_S \circ S_y \circ f_S) \widetilde{\cap} f_S \widetilde{\subseteq} (S_x \circ h_S \circ S_y \circ h_S) \widetilde{\cap} h_S$$

Since $(S_x \circ f_S \circ S_y \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(S_x \circ h_S \circ S_y \circ h_S) \widetilde{\cap} h_S \neq \emptyset_S$. This completes the proof.

Theorem 3.8. Let f_S and h_S be SI-almost left (resp. right) QI-ideals. Then, $f_S \widetilde{\cup} h_S$ is an SI-almost left (resp. right) QI-ideal.

Proof: Since f_S is an SI-almost left QI-ideal by assumption and $f_S \widetilde{\subseteq} f_S \widetilde{\cup} h_S$, $f_S \widetilde{\cup} h_S$ is an SI-almost left QI-ideal by Theorem 3.7.

Here note that if f_S and h_S are SI-almost left (resp. right) QI-ideals, then $f_S \widetilde{\cap} h_S$ needs not to be an SI-almost left (resp. right) QI-ideal.

Example 3.9. Consider the SI-almost QI-ideals f_S and h_S in Example 3.2. Since,

$$f_S \widetilde{\cap} h_S = \{(z, \emptyset), (k, \emptyset)\} = \emptyset_S$$

$f_S \widetilde{\cap} h_S$ is not an SI-almost QI-ideals.

Now, we give the relationship between almost QI-ideal and SI-almost QI-ideal. But first of all, we remind the following lemma to use it in Theorem 3.11.

Lemma 3.10. Let $x \in S$ and Y be nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$. If X is a nonempty subset of S and $y \in S$, then it is obvious that $S_X \circ S_y = S_{Xy}$ [52].

Theorem 3.11. Let A be a nonempty subset of S. Then, A is an almost left (resp. right) QI-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost left (resp. right) QI-ideal.

Proof: Assume that $\emptyset \neq A$ is an almost left QI-ideal. Then, $xAyA \cap A \neq \emptyset$ for all $x, y \in S$, and so there exist $t \in S$ such that $t \in xAyA \cap A$. Since,

$$\left(\left(S_x \circ S_A \circ S_y \circ S_A\right) \tilde{\cap} S_A\right)(t) = \left(S_{xAyA} \tilde{\cap} S_A\right)(t) = \left(S_{xAyA \cap A}\right)(t) = U \neq \emptyset$$

it follows that $\left(S_x \circ S_A \circ S_y \circ S_A\right) \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost left QI-ideal.

Conversely assume that S_A is an SI-almost left QI-ideal. Hence, we have $\left(S_x \circ S_A \circ S_y \circ S_A\right) \tilde{\cap} S_A \neq \emptyset_S$ for all $x, y \in S$. To show that A is an almost left QI-ideal of S, we should show that $A \neq \emptyset$ and $xAyA \cap A \neq \emptyset$ for all $x, y \in S$. $A \neq \emptyset$ is obvious from assumption. Now,

$$\begin{aligned}
\emptyset_S \neq \left(S_x \circ S_A \circ S_y \circ S_A\right) \tilde{\cap} S_A &\Rightarrow \exists n \in S \,;\, \left(\left(S_x \circ S_A \circ S_y \circ S_A\right) \tilde{\cap} S_A\right)(n) \neq \emptyset \\
&\Rightarrow \exists n \in S \,;\, \left(S_{xAyA} \tilde{\cap} S_A\right)(n) \neq \emptyset \\
&\Rightarrow \exists n \in S \,;\, \left(S_{xAyA \cap A}\right)(n) \neq \emptyset \\
&\Rightarrow \exists n \in S \,;\, \left(S_{xAyA \cap A}\right)(n) = U \\
&\Rightarrow n \in xAyA \cap A
\end{aligned}$$

Hence, $xAyA \cap A \neq \emptyset$ for all $x, y \in S$. Consequently, A is an almost left QI-ideal.

Lemma 3.12. Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$ [51].

Theorem 3.13. If f_S is an SI-almost left (resp. right) QI-ideal, then $supp(f_S)$ is an almost left (resp. right) QI-ideal.

Proof: Assume that f_S is an SI-almost left QI-ideal. Thus, $f_S \neq \emptyset_S$, $supp(f_S) \neq \emptyset$ and $\left(S_x \circ f_S \circ S_y \circ f_S\right) \tilde{\cap} f_S \neq \emptyset_S$ for all $x, y \in S$. To show that $supp(f_S)$ is an almost left QI-ideal, by Theorem 3.11, it is enough to show that $S_{supp(f_S)}$ is an SI-almost left QI-ideal. By Lemma 3.12,

$$\left(S_x \circ f_S \circ S_y \circ f_S\right) \tilde{\cap} f_S \tilde{\subseteq} \left(S_x \circ S_{supp(f_S)} \circ S_y \circ S_{supp(f_S)}\right) \tilde{\cap} S_{supp(f_S)}$$

and since $\left(S_x \circ f_S \circ S_y \circ f_S\right) \tilde{\cap} f_S \neq \emptyset_S$, it implies that $\left(S_x \circ S_{supp(f_S)} \circ S_y \circ S_{supp(f_S)}\right) \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost left QI-ideal of S and by Theorem 3.11, $supp(f_S)$ is an almost left QI-ideal.

Here note that the converse of Theorem 3.13 is not true in general as shown in the following example.

Example 3.14. Let $S = \{h, i, k\}$ be the semigroup with the following Cayley Table.

	h	i	k
h	k	h	h
i	h	k	k
k	h	k	k

f_S be soft sets over $U = \mathbb{Z}$ as follows:

$$f_S = \{(h,\{1,9\}),(i,\{0,5\}),(k,\emptyset)\}$$

Let's first show that f_S is not an SI-almost QI-ideal:

$[(f_S \circ S_i \circ f_S \circ S_i) \,\widetilde{\cap}\, f_S](h) = [(f_S \circ S_i) \circ (f_S \circ S_i)](h) \cap f_S(h) = \big[\big((f_S \circ S_i)(h) \cap (f_S \circ S_i)(i)\big) \cup \big((f_S \circ S_i)(h) \cap (f_S \circ S_i)(k)\big) \cup \big((f_S \circ S_i)(i) \cap (f_S \circ S_i)(h)\big) \cup \big((f_S \circ S_i)(k) \cap (f_S \circ S_i)(h)\big)\big] \cap f_S(h) = \Big[\Big[\Big(\big(f_S(h) \cap S_i(i)\big) \cup \big(f_S(h) \cap S_i(k)\big) \cup \big(f_S(i) \cap S_i(h)\big) \cup \big(f_S(k) \cap S_i(h)\big)\Big) \cap \emptyset\Big] \cup \Big[\Big(\big(f_S(h) \cap S_i(i)\big) \cup \big(f_S(h) \cap S_i(k)\big) \cup \big(f_S(i) \cap S_i(h)\big) \cup \big(f_S(k) \cap S_i(h)\big)\Big) \cap \Big(\big(f_S(h) \cap S_i(h)\big) \cup \big(f_S(i) \cap S_i(i)\big) \cup \big(f_S(i) \cap S_i(k)\big) \cup \big(f_S(k) \cap S_i(i)\big) \cup \big(f_S(k) \cap S_i(k)\big)\Big)\Big] \cup \Big[\emptyset \cap \Big(\big(f_S(h) \cap S_i(i)\big) \cup \big(f_S(h) \cap S_i(k)\big) \cup \big(f_S(i) \cap S_i(h)\big) \cup \big(f_S(k) \cap S_i(h)\big)\Big)\Big] \cup \Big[\Big(\big(f_S(h) \cap S_i(h)\big) \cup \big(f_S(i) \cap S_i(i)\big) \cup \big(f_S(i) \cap S_i(k)\big) \cup \big(f_S(k) \cap S_i(i)\big) \cup \big(f_S(k) \cap S_i(k)\big)\Big) \cap \Big(\big(f_S(h) \cap S_i(i)\big) \cup \big(f_S(h) \cap S_i(k)\big) \cup \big(f_S(i) \cap S_i(h)\big) \cup \big(f_S(k) \cap S_i(h)\big)\Big)\Big]\Big] \cap f_S(h) = \Big[\emptyset \cup \big[f_S(h) \cap \big(f_S(i) \cup f_S(k)\big)\big] \cup \emptyset \cup \big[\big(f_S(i) \cup f_S(k)\big) \cap f_S(h)\big]\Big] \cap f_S(h) = \big[\big(f_S(i) \cup f_S(k)\big) \cap f_S(h)\big] \cap f_S(h) = \emptyset.$

Similarly, $[(f_S \circ S_i \circ f_S \circ S_i) \,\widetilde{\cap}\, f_S](i) = \emptyset$ and $[(f_S \circ S_i \circ f_S \circ S_i) \,\widetilde{\cap}\, f_S](k) = \emptyset$. Thus,

$$(f_S \circ S_i \circ f_S \circ S_i) \,\widetilde{\cap}\, f_S = \{(h,\emptyset),(i,\emptyset),(k,\emptyset)\} = \emptyset_S$$

Thus f_s is not an SI-almost QI-ideal. Let's show that $supp(f_S) = \{h,i\}$ is an almost QI-ideal. In deed

$$[\{h\}supp(f_S)\{h\}supp(f_S)] \cap supp(f_S) = [\{h\}\{h,i\}\{h\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{h\}supp(f_S)\{i\}supp(f_S)] \cap supp(f_S) = [\{h\}\{h,i\}\{i\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{h\}supp(f_S)\{k\}supp(f_S)] \cap supp(f_S) = [\{h\}\{h,i\}\{k\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{i\}supp(f_S)\{h\}supp(f_S)] \cap supp(f_S) = [\{i\}\{h,i\}\{h\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{i\}supp(f_S)\{i\}supp(f_S)] \cap supp(f_S) = [\{i\}\{h,i\}\{i\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{i\}supp(f_S)\{k\}supp(f_S)] \cap supp(f_S) = [\{i\}\{h,i\}\{k\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{k\}supp(f_S)\{h\}supp(f_S)] \cap supp(f_S) = [\{k\}\{h,i\}\{h\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{k\}supp(f_S)\{i\}supp(f_S)] \cap supp(f_S) = [\{k\}\{h,i\}\{i\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[\{k\}supp(f_S)\{k\}supp(f_S)] \cap supp(f_S) = [\{k\}\{h,i\}\{k\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

it is seen that $[\{x\}supp(f_S)\{y\}supp(f_S)] \cap supp(f_S) \neq \emptyset$ for all $x, y \in S$. That is to say, $supp(f_S)$ is an almost left QI-ideal of S. Similarly,

$$[supp(f_S)\{h\}supp(f_S)\{h\}] \cap supp(f_S) = [\{h,i\}\{h\}\{h,i\}\{h\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{h\}supp(f_S)\{i\}] \cap supp(f_S) = [\{h,i\}\{h\}\{h,i\}\{i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{h\}supp(f_S)\{k\}] \cap supp(f_S) = [\{h,i\}\{h\}\{h,i\}\{k\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{i\}supp(f_S)\{h\}] \cap supp(f_S) = [\{h,i\}\{i\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{i\}supp(f_S)\{i\}] \cap supp(f_S) = [\{h,i\}\{i\}\{h,i\}\{i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{i\}supp(f_S)\{k\}] \cap supp(f_S) = [\{h,i\}\{i\}\{h,i\}\{k\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{k\}supp(f_S)\{h\}] \cap supp(f_S) = [\{h,i\}\{k\}\{h,i\}\{h\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{k\}supp(f_S)\{i\}] \cap supp(f_S) = [\{h,i\}\{k\}\{h,i\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

$$[supp(f_S)\{k\}supp(f_S)\{k\}] \cap supp(f_S) = [\{h,i\}\{k\}\{h,i\}\{k\}] \cap \{h,i\} = \{h\} \neq \emptyset$$

It is seen that $[supp(f_S)\{x\}supp(f_S)\{y\}] \cap supp(f_S) \neq \emptyset$ for all $x, y \in S$. That is to say, $supp(f_S)$ is an almost right QI-ideal of S. Consequently, $supp(f_S)$ is an almost QI-ideal of S; however f_S is not an SI-almost QI-ideal.

Definition 3.15. An SI-almost left (resp. right) QI-ideal f_S is called minimal if any SI-almost left (resp. right) QI-ideal h_S if whenever $h_S \tilde{\subseteq} f_S$, then $supp(h_S) = supp(f_S)$.

Theorem 3.16. A is a minimal almost left (resp. right) QI-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost left (resp. right) QI-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that A is a minimal almost left QI-ideal. Thus, A is an almost left QI-ideal of S, and so S_A is an SI-almost left QI-ideal by Theorem 3.11. Let f_S be an SI-almost left QI-ideal such that $f_S \tilde{\subseteq} S_A$. By Theorem 3.13, $supp(f_S)$ is an almost left QI-ideal and by Note 2.6 and Corollary 2.13,

$$supp(f_S) \subseteq supp(S_A) = A.$$

Since A is a minimal almost left QI-ideal, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost left QI-ideal by Definition 3.15.

Conversely, let S_A be a minimal SI-almost left QI-ideal. Thus, S_A is an SI-almost left QI-ideal of S and A is an almost left QI-ideal by Theorem 3.13. Let B be an almost left QI-ideal such that $B \subseteq A$. By Theorem 3.11, S_B is an SI-almost left QI-ideal, and by Theorem 2.14 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost left QI-ideal,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.13. Thus, A is a minimal almost left QI-ideal.

Definition 3.17. Let f_S, g_S and h_S be any SI-almost left (resp. right) QI-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost left (resp. right) QI-ideal.

Definition 3.18. Let f_S and h_S be any SI-almost left (resp. right) QI-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost left (resp. right) QI-ideal.

Definition 3.19. Let f_S, g_S and h_S be any SI-almost left (resp. right) QI-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost left (resp. right) QI-ideal.

It is obvious that every SI-strongly prime almost left (resp. right) QI-ideal of S is an SI-prime almost left (resp. right) QI-ideal and every SI-prime almost (left/right) QI-ideal of S is an SI-semiprime almost left (resp. right) QI-ideal.

Theorem 3.20. If S_P, the soft characteristic function of P, is an SI-prime almost left (resp. right) QI-ideal, then P is a prime almost left (resp. right) QI-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost left QI-ideal. Thus, S_P is an SI-almost left QI-ideal of S and thus, P is an almost left QI-ideal by Theorem 3.11. Let A and B be almost left QI-ideals such that $AB \subseteq P$. Thus, by Theorem 3.11, S_A and S_B are SI-almost left QI-ideals and by Theorem 2.14 (i) and (iii),

$$S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P.$$

Since S_P is an SI-prime almost left QI-ideal and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by Theorem 2.14 (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost left QI-ideal.

Theorem 3.21. If S_P, the soft characteristic function of P, is an SI-semiprime left (resp. right) almost QI-ideal of S, then P is a semiprime almost left (resp. right) QI-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost left QI-ideal. Thus, S_P is an SI-almost left QI-ideal and thus, P is an almost left QI-ideal of S by Theorem 3.11. Let A be an almost left QI-ideal such that $AA \subseteq P$. Thus, by Theorem 3.11, S_A is an SI-almost left QI-ideal and by Theorem 2.14 (i) and (iii),

$$S_A \circ S_A = S_{AA} \tilde{\subseteq} S_P$$

Since S_P is an SI-semiprime almost left QI-ideal of S, and $S_A \circ S_A \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$. Therefore, by Theorem 2.14 (i), $A \subseteq P$. Consequently, P is a semiprime almost left QI-ideal.

Theorem 3.22. If S_P, the soft characteristic function of P, is an SI-strongly prime almost left (resp. right) QI-ideal, then P is a strongly prime almost left (resp. right) QI-ideal of S, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-strongly prime almost left QI-ideal. Thus, S_P is an SI-almost left QI-ideal of S and thus, P is an almost left QI-ideal by Theorem 3.11. Let A and B be almost left QI-ideals such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.11, S_A and S_B are SI-almost left QI-ideals and by Theorem 2.14,

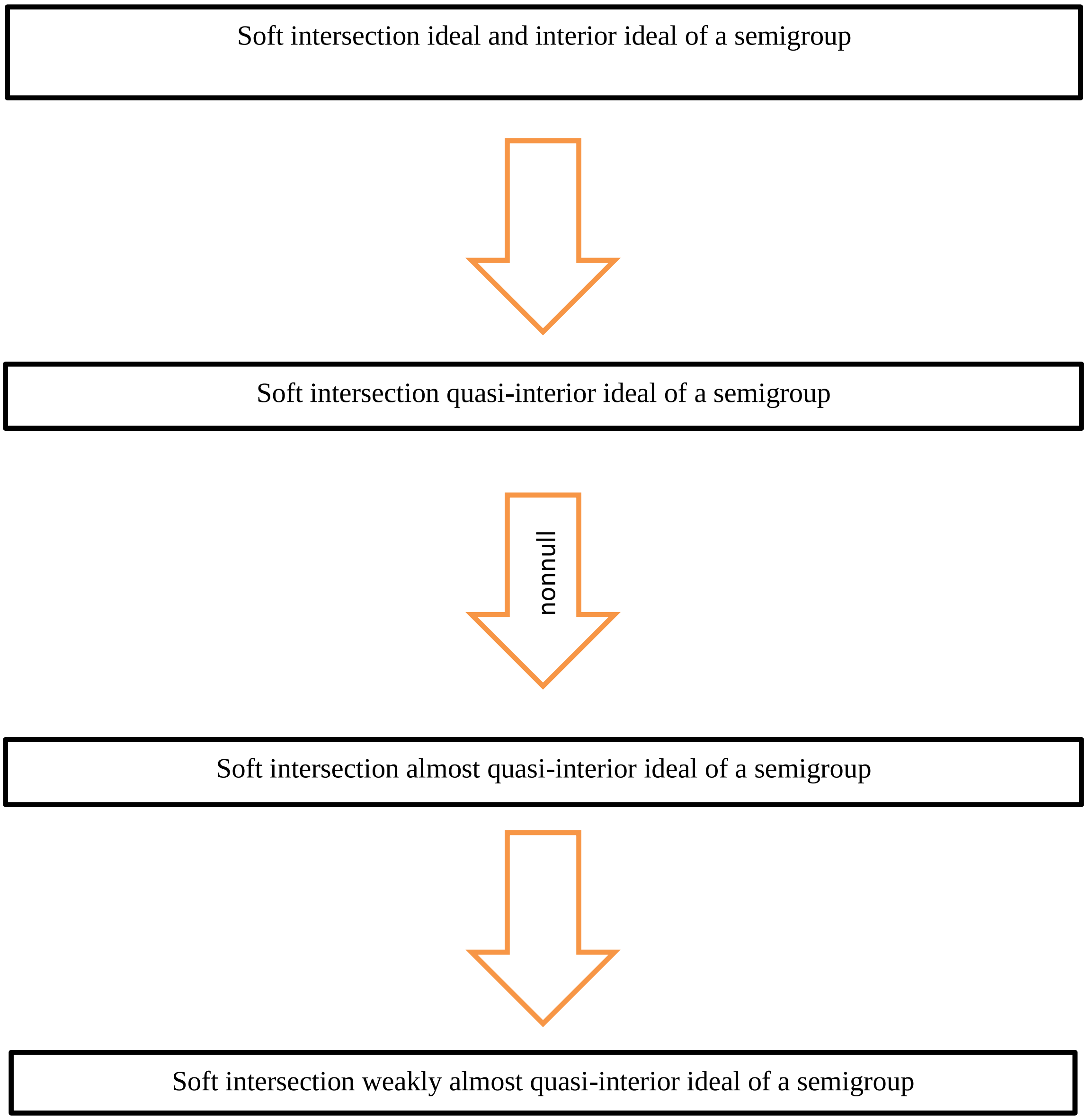
$$(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) = S_{AB} \tilde{\cap} S_{BA} = S_{AB \cap BA} \tilde{\subseteq} S_P$$

Since S_P is an SI-strongly prime almost left QI-ideal and $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Thus, by Theorem 2.14 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime almost left QI-ideal

4. CONCLUSION

In this study, we defined two notions of semigroups: "soft intersection almost quasi-interior ideal" and "soft intersection weakly almost quasi-interior ideal". We showed that while every nonnull soft intersection quasi-interior ideal is a soft intersection almost quasi-interior, and every soft intersection almost quasi-interior is a soft intersection weakly almost quasi-interior ideal; the converses are not true for counterexamples. Furthermore, it was demonstrated that an idempotent soft intersection almost quasi-interior ideal is a soft intersection almost subsemigroup. With the obtained theorem that if a nonempty set A is almost quasi-interior ideal, then its soft characteristic function is soft intersection almost quasi-interior ideal and vice versa, we obtained the relation among soft intersection almost quasi-interior ideal of a semigroup and almost quasi-interior ideal of a semigroup according with minimality, primeness, semiprimeness, and strongly primeness. Furthermore, we derived that the binary operation of soft union, in contrast to soft intersection operation, constructs a semigroup with the collection of soft intersection almost quasi-interior ideals. In future studies, many types of soft intersection almost ideals, including quasi-ideal, interior ideal, bi-ideal, bi-interior ideal, bi-quasi ideal, and bi-quasi-interior ideal of semigroups and their interrelations can be examined.

The relation between several soft intersection ideals and their generalized ideals is depicted in the following figure, where A➡B denotes that A is B but B may not always be A.



CONFLICT OF INTEREST

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REFERENCES

[1] Good RA, Hughes DR. Associated groups for a semigroup. Bull Amer Math Soc 1952; 58: 624–625.

[2] Steinfeld O. Uher die quasi ideals. Von halbgruppend Publ Math Debrecen 1956; 4: 262–275.

[3] Grosek O, Satko L. A new notion in the theory of semigroup. Semigroup Forum 1980; 20: 233–240.

[4] Bogdanovic S. Semigroups in which some bi-ideal is a group. Univ u Novom Sadu Zb Rad Prirod Mat Fak Ser Mat 1981; 11: 261–266.

[5] Wattanatripop K, Chinram R, Changphas T. Quasi-A-ideals and fuzzy A-ideals in semigroups. J Discrete Math Sci Cryptogr 2018; 21: 1131–1138.

[6] Kaopusek N, Kaewnoi T, Chinram R. On almost interior ideals and weakly almost interior ideals of semigroups. J Discrete Math Sci Cryptogr 2020; 23: 773–778.

[7] Iampan A, Chinram R, Petchkaew P. A note on almost subsemigroups of semigroups. Int J Math Comput Sci 2021; 16(4): 1623–1629.

[8] Chinram R, Nakkhasen W. Almost bi-quasi-interior ideals and fuzzy almost bi-quasi-interior ideals of semigroups. J Math Comput Sci 2022; 26: 128–136.

[9] Gaketem T. Almost bi-interior ideal in semigroups and their fuzzifications. Eur J Pure Appl Math 2022; 15(1): 281–289.

[10] Gaketem T, Chinram R. Almost bi-quasi ideals and their fuzzifcations in semigroups. Ann Univ Craiova Math Comput Sci Ser 2023; 50(2): 42–352.

[11] Wattanatripop K, Chinram R, Changphas T. Fuzzy almost bi-ideals in semigroups. Int J Math Comput Sci 2018; 13: 51–58.

[12] Krailoet W, Simuen A, Chinram R, Petchkaew P. A note on fuzzy almost interior ideals in semigroups. Int J Math Comput Sci 2021; 16: 803–808.

[13] Molodtsov D. Soft set theory-first results. Comput Math Appl 1999; 37(1): 19–31.

[14] Maji PK, Biswas R, Roy AR. Soft set theory. Comput Math Appl 2003; 45(1): 555–562.

[15] Pei D, Miao D. From soft sets to information systems. In: Proceedings of Granular Computing. IEEE 2005; 2: 617–621.

[16] Ali MI, Feng F, Liu X, Min WK, Shabir M. On some new operations in soft set theory. Comput Math Appl 2009; 57(9): 1547–1553.

[17] Sezgin A, Atagün AO. On operations of soft sets. Comput Math Appl 2011; 61(5):1457–1467.

[18] Feng F, Jun YB, Zhao X. Soft semirings. Comput Math Appl 2008; 56(10): 2621–2628.

[19] Ali MI, Shabir M, Naz M. Algebraic structures of soft sets associated with new operations. Comput Math Appl 2011; 61: 2647–2654.

[20] Sezgin A, Shahzad A, Mehmood A. New operation on soft sets: Extended difference of soft sets. J New Theory 2019; 27: 33–42.

[21] Stojanovic NS. A new operation on soft sets: Extended symmetric difference of soft sets. Military Technical Courier 2021; 69(4): 779–791.

[22] Sezgin A, Atagün AO. New soft set operation: Complementary soft binary piecewise plus operation. Matrix Science Mathematic 2023; 7(2): 125–142.

[23] Sezgin A, Aybek FN. New soft set operation: Complementary soft binary piecewise gamma operation. Matrix Science Mathematic 2023; (7)1: 27–45.

[24] Sezgin A, Aybek FN, Atagün AO. New soft set operation: Complementary soft binary piecewise intersection operation. BSJ Eng Sci 2023; 6(4): 330–346.

[25] Sezgin A, Aybek FN, Güngör NB. New soft set operation: Complementary soft binary piecewise union operation. Acta Informatica Malaysia 2023; 7(1): 38–53.

[26] Sezgin A, Demirci AM. New soft set operation: Complementary soft binary piecewise star operation. Ikonion Journal of Mathematics 2023; 5(2): 24–52.

[27] Sezgin A, Yavuz E. New soft set operation: Complementary Soft Binary Piecewise Lambda Operation. Sinop University Journal of Natural Sciences 2023; 8(2): 101–133.

[28] Sezgin A, Yavuz E. A new soft set operation: Soft binary piecewise symmetric difference operation. Necmettin Erbakan University Journal of Science and Engineering 2023; 5(2): 189–208.

[29] Sezgin A, Çağman N. New soft set operation: Complementary soft binary piecewise difference operation. Osmaniye Korkut Ata Üniv Fen Biliml Derg 2024; 7(1): 58–94.

[30] Çağman N, Enginoğlu S. Soft set theory and uni-int decision making. Eur J Oper Res 2010; 20, 7(2): 848–855.

[31] Çağman N, Çitak F, Aktaş H. Soft int-group and its applications to group theory. Neural Comput Appl 2012; 2: 151–158.

[32] Sezer AS, Çağman N, Atagün AO, Ali MI, Türkmen E. Soft intersection semigroups, ideals and bi-ideals; a new application on semigroup theory I. Filomat 2015; 29(5): 917–946.

[33] Sezer AS, Çağman N, Atagün AO. Soft intersection interior ideals, quasi-ideals and generalized bi-ideals; a new approach to semigroup theory II. J Mult-Valued Log Soft Comput 2014; 23(1-2): 161–207.

[34] Sezgin A, Orbay M. Analysis of semigroups with soft intersection ideals. Acta Univ Sapientiae Math 2022; 14(1): 166–210.

[35] Mahmood T, Rehman ZU, Sezgin A. Lattice ordered soft near rings. Korean J Math 2018; 26(3): 503–517.

[36] Jana C, Pal M, Karaaslan F, Sezgin A. (α, β)-soft intersectional rings and ideals with their applications. New Math Nat Comput 2019; 15(2): 333–350.

[37] Muştuoğlu E, Sezgin A, Türk ZK. Some characterizations on soft uni-groups and normal soft uni-groups. Int J Comput Appl 2016; 155(10): 1–8.

[38] Sezer AS, Çağman N, Atagün AO. Uni-soft substructures of groups. Ann Fuzzy Math Inform 2015; 9(2): 235–246.

[39] Sezer AS. Certain Characterizations of LA-semigroups by soft sets. J Intel Fuzzy Syst 2014; 27(2): 1035–1046.

[40] Özlü Ş, Sezgin A. Soft covered ideals in semigroups. Acta Univ Sapientiae Math 2020; 12(2): 317–346.

[41] Atagün AO, Sezgin A. Soft subnear-rings, soft ideals and soft n-subgroups of near-rings. Math Sci Letters 2018; 7(1): 37–42.

[42] Sezgin A. A new view on AG-groupoid theory via soft sets for uncertainty modeling. Filomat 2018; 32(8): 2995–3030.

[43] Sezgin A, Çağman N, Atagün AO. A completely new view to soft intersection rings via soft uni-int product. Appl Soft Comput 2017; 54: 366–392.

[44] Sezgin A, Atagün AO, Çağman N, Demir H. On near-rings with soft union ideals and applications. New Math Nat Comput 2022; 18(2): 495–511.

[45] Rao MMK. Bi-interior ideals of semigroups. Discuss Math Gen Algebra Appl 2018; 38: 69–78.

[46] Rao MMK. A study of a generalization of bi-ideal, quasi ideal and interior ideal of semigroup. Mathematica Moravica 2018; 22: 103–115.

[47] Rao MMK. Left bi-quasi ideals of semigroups. Southeast Asian Bull Math 2020; 44: 369–376.

[48] Rao MMK. Quasi-interior ideals and weak-interior ideals. Asia Pac Journal Mat 2020; 7(21): 1–20.

[49] Baupradist S, Chemat B, Palanivel K, Chinram R. Essential ideals and essential fuzzy ideals in semigroups. J Discrete Math Sci Cryptogr 2021; 24(1): 223–233.

[50] Sezgin A, Kocakaya FZ. Soft intersection quasi-interior ideals of semigroups. JuTISI 2024; (in press).

[51] Sezgin A, İlgin A. Soft intersection almost subsemigroups of semigroups. Int J Math Phys 2024; 15(1): 13-20.

[52] Sezgin A, İlgin A. Soft intersection almost ideals of semigroups. J Innovative Eng Nat Sci 2024; 4(2): 466-481.

[53] Pant S, Dagtoros K, Kholil MI, Vivas A. Matrices: Peculiar determinant property. OPS Journal 2024, 1: 1–7.

[54] Sezgin A, Çalışıcı H. A comprehensive study on soft binary piecewise difference operation. Eskişehir Teknik Univ Bilim Teknol Derg Teor Bilim 2024; 12(1): 32-54.

[55] Sezgin A, Dagtoros K. Complementary soft binary piecewise symmetric difference operation: A novel soft set operation. Scientific Journal of Mehmet Akif Ersoy University 2023; 6(2): 31-45.

[56] Sezgin A, Sarıalioğlu M. New soft set operation: Complementary soft binary piecewise theta operation. Journal of Kadirli Faculty of Applied Sciences 2023; 4(2): 1-33.