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Radius Model for Some Cells in Human Body on Multiplicative Calculus

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Abstract

The cell is the basic structure and process unit that carries all the living characteristics of a living thing and has the ability to survive on its own under suitable conditions. The relationship of cell size with nutrient absorption and nutrient consumption in the cell membrane has been examined with the current model using the theory of differential equations in classical analysis. During these examinations, the cell considered was assumed to be spherical. In fact, the shapes of cells vary depending on their functional properties. Many have long appendages, cylindrical parts or branch-like structures. However, in this study, a simple global cell will be discussed, leaving all these complex situations aside. In the current model, the relationship between the change in the radius of the cell and the nutrient absorption and consumption in the cell membrane is detailed using classical differential equations. The answer to the question for which cell size is the consumption rate exactly balanced with the absorption rate was found in classical analysis. The current model consists of first-order differential equations. In this model, the dependent variables are the radius of the cell and the mass of the cell. The classical solutions of these models will be examined, the size of the cell and the cell membrane relationship will be examined, and details will be given with numerical examples. However, in order to consider this biological phenomenon from different perspectives and compare the results, the relevant event will be modeled using multiplicative analysis, one of the Non-Newtonian analyses. The new models will be solved using multiplicative analysis techniques, and the results will be compared with classical analysis. With this new model, it is planned to clarify the results obtained in the classical case, to reveal more clearly the relationship between the size of the cell and nutrient absorption and consumption in the cell membrane, and to obtain important results.

1. Introduction

A spherical cell absorbs nutrients at a rate proportional to its surface area S, but consumes nutrients at a rate proportional to its volume V (Figure 1.1). Some constants and their equivalents that will appear in the cell model to be established are as follows.

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Figure 1.1: Arbitrary Spherical Cell

A: Net absorption rate of nutrients per unit time,
C: Net rate of consumption of nutrients per unit time,
V: Volume of the cell,
S: Surface area of the cell,
r: Radius of the cell,
In this study, the change of four cell models radii with respect to time was analyzed in classical and multiplicative analyses and each case was numerically examined and comparisons were made. To establish model, following assumptions are considered [1–10].
1. The cell is roughly spherical.
2. The cell absorbs oxygen and nutrients from its surface. The larger the surface area S, the faster the overall absorption rate. The rate of absorption of nutrients (or oxygen) is assumed to be proportional to the surface area of the cell.
3. The rate at which nutrients are consumed (i.e. depleted) in metabolism is proportional to the volume V of the cell. The larger the volume, the more nutrients are needed to keep the cell alive.
Now let's restate the assumptions mathematically. According to second assumption, the absorption rate A is proportional to S. This means:

$$A = k_1 S,$$

where k_1 is proportional constant. Since absorbance and surface area are positive quantities, only positive values of the proportionality constant are significant, so k_1 must be positive (This is consistent with multiplicative analysis). The value of this constant depends on its properties, such as the permeability of the cell membrane or how many pores it contains to allow the passage of nutrients. By using a general constant called a parameter to represent this proportionality constant, the model is kept general enough to apply to many different cell types. According to third assumption, the rate of food consumption, C is proportional to V:

$$C = k_2 V,$$

where k_2 is positive proportional constant. k_2 depends on cell metabolism, that is, how fast it consumes nutrients while performing its activities. According to first assumption, the cell is spherical, so

$$S = 4\pi r^2, \quad V = \frac{4}{3}\pi r^3,$$

where S is surface area and V is its volume. Putting these rationales together gives the following relationships between nutrient absorption A, consumption C, and cell radius r:

$$\begin{aligned} A(r) &= (4\pi k_1) r^2, \\ C(r) &= \left(\frac{4}{3}\pi k_2\right) r^3. \end{aligned}$$

These equations will contribute to seeing how nutrient balance depends on cell size. Here, functions A and C are second and third degree polynomial functions, respectively, depending on the radius of the cell. Nutrient balance depends on the radius of a cell. First, the answer to the question of whether nutrient absorption or nutrient consumption is more effective for small, medium or large cells will be sought [11]. The problem expressed in the following classical case is the problem on which we base our study and make comparisons.

Motivation question: For what cell size is the consumption rate exactly balanced by the absorption rate? What ratio (consumption or absorption) dominates for small or large cells?
If the consumption rate for the cell is in equilibrium with the absorption rate, it yields

$$A(r) = C(r).$$

Then,

$$(4\pi k_1) r^2 = \left(\frac{4}{3}\pi k_2\right) r^3.$$

$r = 0$ is trivial solution of this equation. This is not biologically meaningful anyway. Non-trivial solutions are required for this study. From the above relation, we get

$$r = \frac{3k_1}{k_2},$$

where $r \neq 0$. This means that the rates of absorption and consumption are equal for cells of this size. For small r values, $C(r)$ dominates. Thus, absorption dominates for smaller cells, while consumption dominates for larger cells. From here, cells larger than the critical size $r = \frac{3k_1}{k_2}$ cannot meet the nutrient demand and the cell dies because consumption cannot meet nutrient absorption [1–8].

Using the simple geometric argument above, it can be concluded that cell size has strong effects on its ability to absorb nutrients or oxygen fast enough to feed itself. If a cell absorbs nutrients faster than the food consumed ($A > C$), some of the excess nutrients accumulate and this accumulation of nutrient mass can be converted into cell mass. This can cause growth (increase in cell mass).

Conversely, if the rate of consumption exceeds the rate of absorption of nutrients, $C > A$, the cell has a metabolic "fuel" shortage and must convert some of its own mass into energy reserves that can power its metabolism, resulting in a loss of cell mass. We can track such changes in cell mass using a simple "equilibrium equation" using differential equations in classical analysis. The equilibrium equation is the difference between the rate of change of cell mass (A) incoming nutrient (mass) ratio (C);

$$\frac{dm}{dt} = A - C, \tag{1.1}$$

Each term in this equation must have the same units of nutrient mass per unit time. A is a depletion rate that contributes positively to mass gain, while C is a depletion rate that negatively contributes to mass gain. This is already the basic logic in the creation of the model. If we consider the expressions

$$A = k_1 S, \quad C = k_2 V, \quad m = pV$$

in (1.1), we get

$$\frac{d(pV)}{dt} = k_1 S - k_2 V, \tag{1.2}$$

where S is surface area, V is volume and p is density of the cell. The above equation is quite general and does not depend on the cell shape. Let us now consider the special case of a cell being spherical. Eq. (1.1) will be converted into an equation showing the variation of the cell radius with time where

$$S = 4\pi r^2, \quad V = 4/3\pi r^3.$$

By (1.2) and after some adjustments and implementation of chain rule, we get

$$\frac{dr}{dt} = \frac{1}{p}\left(k_1 - \frac{k_2}{3} r\right). \tag{1.3}$$

With an explanation of how the cell mass changes, a result estimate of the rate of change of the cell radius is reached. This was done using classical analysis methods. The resulting equation is a differential equation that tells us about a growing cell. This model will be used as a tool to understand how it predicts the dynamics of cells with different initial sizes. The differential equation (1.3) is a linear differential equation. The general solution of this equation is given below as a preliminary conclusion for the study. Eq. (1.3) will be solved in the classical case by the method of variation of parameters. If this equation is adapted to the solution and some adjustments are made, we get

$$r(t) = \frac{3k_1}{k_2} + ke^{-\frac{k_2 t}{3p}}, \tag{1.4}$$

where k is an arbitrary constant. Using this solution, the variation of the radius of the cell with time can be obtained for different times [1]. Some considerations will be made on two problems involving stomach and blood cells.

Let's express some information about stomach cell, which we will examine on the first example.

Example 1.1 (Stomach Cell-Usual Case, [1–8])**.** *The stomach is a muscular, expandable digestive system organ. A healthy stomach cell radius value is approximately between* $10-30\mu m$ *and the cell density is* $p = 1.04gr/(cm)^3$ *. Based on this information, let's analyze the cell radius variation for classical model. Let's fix the cell radius at* $r(t) = 20\mu m$*. If we consider the initial condition* $r(0) = 20\mu m$ *in general solution (1.4), we get*

$$r(t) = \frac{3k_1}{k_2} + \left(20 - \frac{3k_1}{k_2}\right)e^{-\frac{k_2 t}{3(1,06)}},$$

and

$$\frac{k_1}{k_2} \cong 6.6.$$

This value is the equilibrium state. Now let's observe the change in the cell by changing this ratio. Since $r(0) = 20$ *for* $\frac{k_1}{k_2} \cong 6.6$*, it can be said that the balance situation continues. Now let's observe the state of the cell for different values of the ratio by changing the variable t for* $k_2 = 10$ *in Table 1.1.*

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
6,6	20	19.8081	19.8003	19.8000	19.800	...	19.800
6,5	20	19.5081	19.5003	19.5000	19.5000	...	19.5000
6,4	20	19.2081	19.2003	19.2000	19.2000	...	19.2000
6,3	20	18.9081	18.9003	18.9000	18.9000	...	18.9000
6,2	20	18.6081	18.6003	18.6000	18.6000	...	18.6000
6,1	20	18.3081	18.3003	18.3000	18.3000	...	18.3000
6,0	20	18.0081	18.0003	18.0000	18.0000	...	18.0000

Table 1.1: Change of stomach cell radius over time according to the change

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 19.800.$$

As can be seen in this example, for $\frac{k_1}{k_2} = 6.6$*, there is a 1% decrease, which is the most stable condition. For* $\frac{k_1}{k_2} = 6.3-6.4$*, there is a* $4-5.5\%$ *decrease, which is within normal limits. For* $\frac{k_1}{k_2} = 6.0-6.1$*, there is a* $8.5-10\%$ *decrease, which is the condition that should be monitored.*

Example 1.2 (Blood Cell-Usual Case, [1–8]). *Eristocytes, or red blood cells, are the main oxygen-carrying components of blood. Red blood cells are small* $(3.5\mu m)$*, round cells shaped in cross section as two concave discs. Let's examine the cell radius situation in general solution (1.4) according to these values. Dense of a blood cell is* $p = 1,10gr/cm^3$ *. The equilibrium state for this cell is*

$$\frac{k_1}{k_2} = 1.17,$$

for

$$r(t) = \frac{3k_1}{k_2} + \left(3.5 - \frac{3k_1}{k_2}\right)e^{-\frac{k_2 t}{3p}},$$

where $r(0) = 3.5$ *for* $k_2 = 10$*. Now let's observe the change in the blood cell by changing this ratio for* $p = 1,10gr/cm^3$ *by Table 1.2.*

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
1.17	3.5000	3.5096	3.5112	3.5116	3.5117	...	3.5117
1,16	3.5000	3.4796	3.4812	3.4816	3.4817	...	3.4817
1,15	3.5000	3.4496	3.4512	3.4516	3.4517	...	3.4517
1,14	3.5000	3.4196	3.4212	3.4216	3.4217	...	3.4217
1,13	3.5000	3.3896	3.3912	3.3916	3.3917	...	3.3917
1,12	3.5000	3.3596	3.3612	3.3616	3.3617	...	3.3617
1,11	3.5000	3.3296	3.3312	3.3316	3.3317	...	3.3317

Table 1.2: Change of blood cell radius over time according to the change

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 3.5117.$$

As can be seen in this example, for $\frac{k_1}{k_2} > 1.17$ *the cell expands and for* $\frac{k_1}{k_2} < 1.17$ *the cell shrinks. The changes are gradual and controlled and the final values are within physiological limits. The main changes occur in the first 3 hours. After the 4th hour, complete stability occurs.*

Example 1.3 (Brain Cell-Usual Case, [1–8]). *The brain cell is known as a neuron. The average radius of a healthy brain cell is* $r(t) = 10\mu m$ *and its density is* $p = 1.03gr/cm^3$ *. The equilibrium state for this cell is*

$$\frac{k_1}{k_2} = 3.3,$$

for

$$r(t) = \frac{3k_1}{k_2} + (10 - \frac{3k_1}{k_2})e^{-\frac{k_2 t}{3p}},$$

where $r(0) = 10$ *for* $k_2 = 10$*. Now let's observe the change in the blood cell by changing this ratio for* $p = 1,03gr/cm^3$ *by Table 1.3.*

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
3.3	10.0000	9.9039	9.9004	9.9001	9.9000	...	9.9000
3.2	10.0000	9.6039	9.6004	9.6001	9.6000	...	9.6000
3.1	10.0000	9.3039	9.3004	9.3001	9.3000	...	9.3000
3.0	10.0000	9.0039	9.004	9.0001	9.0000	...	9.0000
2.9	10.0000	8.7039	8.7004	8.7001	8.7000	...	8.7000
2.8	10.0000	8.4039	8.4004	8.4001	8.4000	...	8.4000
2.7	10.0000	8.1039	8.1004	8.1001	8.1000	...	8.1000

Table 1.3: Change of brain cell radius over time according to the change

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 9.9000.$$

As can be seen in this example, for $k_1/k_2 \geq 3.2$, there is less than 4% change in size and normal neuronal function is preserved. For $3.0 \leq k_1/k_2 < 3.1$, there is a $7-10\%$ change in size and functional changes may occur. For $k_1/k_2 \leq 2.9$, there is a 13% change in size and neuronal function is at risk.

Example 1.4 (Liver Cell-Usual Case, [1–8]). *Liver cells (hepatocytes) are the basic functional units of the liver. They are large cells with a polygonal shape, usually 25 micrometers in diameter. Their density is approximately $p = 1,09gr/cm^3$. These cells perform vital functions such as protein synthesis, detoxification of toxins, bile production, and glycogen storage. The equilibrium state for this cell is*

$$\frac{k_1}{k_2} = 8.3,$$

for

$$r(t) = \frac{3k_1}{k_2} + \left(25 - \frac{3k_1}{k_2}\right) e^{-\frac{k_2 t}{3p}}$$

where $r(0) = 25$ for $k_2 = 10$. Now let's observe the change in the blood cell by changing this ratio for $p = 1,09gr/cm^3$ by Table 1.4.

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
8.3	25.0000	24.3752	24.0282	23.8162	23.6799	...	23.4270
8.2	25.0000	24.0752	23.7282	23.5162	23.3799	...	23.1270
8.1	25.0000	23.7752	23.4282	23.2162	23.0799	...	22.8270
8.0	25.0000	23.4752	23.1282	22.9162	22.7799	...	22.5270
7.9	25.0000	23.1752	22.8282	22.6162	22.4799	...	22.2270
7.8	25.0000	22.8752	22.5282	22.3162	22.1799	...	21.9270
7.7	25.0000	22.5752	22.2282	21.0162	21.8799	...	21.6270

Table 1.4: Change of liver cell radius over time according to the change

In equilibrium, it can be easily seen that

$$r(5) = 23.5901, r(6) = 23.5294, r(7) = 23.4881, r(8) = 23.4599, r(9) = 23.4405.$$

As can be seen in this example, for $k_1/k_2 \geq 8.1$, there is less than 8% change in size and normal hepatocyte function is preserved. For $7.9 \leq k_1/k_2 \leq 8.0$, there is a 9-10% change in size and regular monitoring is required. For $k_1/k_2 \leq 7.8$, there is a risk because there is more than 12% change in size and close monitoring is required.

Remark 1.5. *There is no particular reason to examine only the stomach, blood, brain and liver cells here. The changes of four cells radii in both classical and multiplicative analysis will be examined.*
In the next section, we will examine the cell radius models on multiplicative analysis. For this reason, it would be useful to explain the multiplicative analysis in general terms in this section.
The classical analysis most commonly used today was founded by Gottfried Leibnitz and Isaac Newton in the second half of the 17th century. Since the basic operation in this analysis is addition, it is called additive (classical) analysis or Newtonian analysis. Many new types of analysis have emerged as a result of the ideas of establishing new analysis with different arithmetic operations based on classical analysis. An example of these analyzes is multiplicative analysis. This type of analysis is generally called non-Newtonian analysis in the literature. The first example of studies carried out with different arithmetic operations can be given as Volterra type analysis defined by Vito Volterra in 1887. Since this new approach is based on multiplication, it is called multiplicative analysis. The first study for Volterra type analysis was conducted by Volterra and Hostinsky in 1938 [9]. In the period from 1967 to 1970, Michael Grossman and Robert Katz gave definitions of a new type of derivative and integral, transferring the roles of subtraction and addition operations to division and multiplication operations, thus introducing a new calculus called multiplicative analysis [10, 11].
Multiplicative analysis is a field of study that can be easily used in solving many scientific problems and provides great advantages. As a result of the researches on the subject, it is seen that some problems encountered in applied sciences can be complicated to express with classical analysis. The multiplicative analysis facilitates the solution of these problems and offers a different perspective in the mathematical modeling of these problems. In this direction, multiplicative analysis emerged as an alternative to classical analysis. Many important studies have been carried out in different fields related to multiplicative analysis [12–16].

Definition 1.6. *Let $f : A \to \mathbb{R}^+$ be a positive function for all x on $A \subseteq \mathbb{R}$. The multiplicative derivative of f is defined by [17]*

$$f^*(x) = \lim_{h\to 0} \left(\frac{f(x+h)}{f(x)}\right)^{\frac{1}{h}}.$$

Theorem 1.7. *A positive function f is multiplicatively differentiable at x if and only if it is usual differentiable at that point [18]. There is a relationship between derivative in classical sense and derivative in multiplicative sense as [19],*

$$f^*(x) = e^{\frac{f'(x)}{f(x)}}.$$

Definition 1.8 ([19]). *$F:(a,b)\to\mathbb{R}$ is called multiplicative anti-derivative of $f:(a,b)\to\mathbb{R}$ where $F^*(x)=f(x)$ for each $x\in(a,b)$. Following presentation is used for this concept.*

$$\int f(x)^{dx}=F(x).$$

Remark 1.9 ([20]). *If f is positive and continuous on [a,b], it is integrable in multiplicative sense and*

$$\int_a^b f(t)^{dx}=e^{\int_a^b \ln(f(t))\,dt}.$$

Definition 1.10 ([20]). *An $n-th$ order multiplicative differential equation is defined by*

$$f(t,y,y^*,y^{(**)},\ldots,y^{(*(n-1))},y^{(*n)}(t))=1,\quad (t,y)\in\mathbb{R}\times\mathbb{R}^+$$

for a positive function f.

2. Radius Analysis for Some Cells in Multiplicative Calculus

In this section, it is thought that original results will be obtained regarding the change of cell size due to different definitions of derivative, integral and differential equations in multiplicative analysis. These solutions will then be evaluated with mathematical and numerical examples. The differential equation discussed in the article proposal will be established in multiplicative analysis and will be solved using multiplicative analysis techniques. Multiplicative analysis has a very strong literature and different application areas [9–20].
In the classical case, the equation (1.4) discussed in the first section can be written as follows in multiplicative analysis;

$$r^*(t)r^{\frac{k_2}{3p}}=e^{\frac{k_1}{p}}. \tag{2.1}$$

This multiplicative equation will be solved using the method of indefinite exponents in multiplicative analysis. According to this method, the homogeneous solution of the equation is

$$r_h=e^{c_1e^{\frac{-k_2t}{3p}}},$$

where

$$r+\frac{k_2}{3p}=0\quad\to\quad r=-\frac{k_2}{3p}.$$

Let $r_p(t)=e^A$ be particular solution. If multiplicative derivative is taken for $r_p(t)$ to find the constant A and substituted in Eq. (2.1), we get

$$r^*r^{\frac{k_2}{3p}}=e^{\frac{k_2}{p}}\quad\to\quad A=\frac{3k_1}{k_2}.$$

Then, the general solution of (2.1) is

$$r(t)=e^{c_1e^{\frac{-k_2t}{3p}}}e^{\frac{3k_1}{k_2}},$$

where $r=r_hr_p$.
Now, let's examine the change in the radii of the stomach, blood, brain and liver cells using this solution in multiplicative analysis.

Example 2.1 (Stomach Cell-Multiplicative Case). *Let $r(0)=20\mu m$ and $p=1,04gr/cm^3$ for a stomach cell. We will analyse the change of radius for a stomach cell in multiplicative analysis by using the following general solution;*

$$r(t)=e^{c_1e^{\frac{-k_2t}{3p}}}e^{\frac{3k_1}{k_2}}.$$

If this solution is used, the equilibrium ratio for stomach cell is obtained as;

$$\frac{k_1}{k_2}=0.99,$$

for $r(0)=20$. Here, calculations will be made for particular selections of $k_1=99$ and $k_2=100$ in Table 2.1.

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
0.99	20.000	19.9967	19.9967	19.9967	19.9967	...	19.9967
0.89	20.000	19.1967	19.1867	19.1867	19.1867	...	19.1867
0.79	20.000	18.3967	18.3767	18.3767	18.3767	...	18.3767
0.69	20.000	17.5967	17.5667	17.5667	17.5667	...	17.5667
0.59	20.000	16.7967	16.7567	16.7567	16.7567	...	16.7567
0.49	20.000	15.9967	15.9467	15.9467	15.9467	...	15.9467
0.39	20.000	15.1967	15.1367	15.1367	15.1367	...	15.1367

Table 2.1: Change of stomach cell radius over time according to the change in multiplicative case

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 19.9967$$

According to the results obtained in the multiplicative model, There is a decrease in the first 2 hours, and a constant value is reached after the 2nd hour. There is a different balance status for each k_1/k_2 *value. Health Status Assessment:* $k_1/k_2 > 0.89$ *is a safe status,* $0.69 \leq k_1/k_2 \leq 0.89$ *is a monitoring status, and* $k_1/k_2 < 0.69$ *is a risk status. The classical model is in normal physiological adaptation, while the multiplicative model may be an acute stress response. The classical model is in a safer range, while the multiplicative model shows riskier changes. These two models may represent the response of the stomach cell to different conditions (normal adaptation vs. severe stress).*

Example 2.2 (Blood Cell-Multiplicative Case)**.** *Let* $r(0) = 3.5\mu m$ *and* $p = 1.10 gr/cm^3$ *for a blood cell. Again, we will examine cell radius change for blood cell using the solution (2.1) in the multiplicative case. If this solution is used, the equilibrium ratio for the cell is obtained as;*

$$\frac{k_1}{k_2} = 0.41,$$

for $r(0) = 3.5$*. Here the calculations will be made for particular selections of* $k_1 = 41$ *and* $k_2 = 100$*. in Table 2.2.*

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
0.41	3.5000	3.4984	3.4984	3.4984	3.4984	...	3.4984
0.36	3.5000	3.3484	3.3484	3.3484	3.3484	...	3.3484
0.31	3.5000	3.1984	3.1984	3.1984	3.1984	...	3.1984
0.26	3.5000	3.0484	3.0484	3.0484	3.0484	...	3.0484
0.21	3.5000	2.8984	2.8984	2.8984	2.8984	...	2.8984
0.16	3.5000	2.7484	2.7484	2.7484	2.7484	...	2.7484
0.11	3.5000	2.5984	2.5984	2.5984	2.5984	...	2.5984

Table 2.2: Change of blood cell radius over time according to the change in multiplicative case

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 3.4984.$$

Similarly, in this example, for $k_1/k_2 \geq 0.36$*, there is less than 5% change in size and normal erythrocyte function is preserved. For* $0.26 \leq k_1/k_2 < 0.36$*, there is a 10-15% change in size and oxygen carrying capacity may be affected. For* $k_1/k_2 < 0.26$*, there is a change in size of more than 15%, in which case there is a risk of Hemolysis and cell function is seriously compromised.*
The classical model represents normal adaptation, while the multiplicative model represents the acute stress response. The classical model appears more physiological, while the multiplicative model represents pathological conditions. These two models may represent the response of erythrocytes to different conditions (normal vs. extreme stress).

Example 2.3 (Brain Cell-Multiplicative Case)**.** *Let* $r(0) = 10$ *and* $p = 1.03 gr/cm^3$ *for a brain cell. Again, we will examine the cell radius change for the brain cell using solution (2.1) in the multiplicative case. If this solution is used, the equilibrium ratio brain the cell is obtained as;*

$$\frac{k_1}{k_2} = 0.76,$$

for $r(0) = 10$*. Here the calculations will be made for particular selections of* $k_1 = 76$ *and* $k_2 = 100$ *in Table 2.3.*

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
0.76	10.0000	9.9984	9.9984	9.9984	9.9984	...	9.9984
0.66	10.0000	9.4984	9.4984	9.4984	9.4984	...	9.4984
0.56	10.0000	8.9984	8.9984	8.9984	8.9984	...	8.9984
0.46	10.0000	8.4984	8.4984	8.4984	8.4984	...	8.4984
0.36	10.0000	7.9984	7.9984	7.9984	7.9984	...	7.9984
0.26	10.0000	7.4984	7.4984	7.4984	7.4984	...	7.4984
0.16	10.0000	6.9984	6.9984	6.9984	6.9984	...	6.9984

Table 2.3: Change of brain cell radius over time according to the change in multiplicative case

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 9.9984.$$

In this example, for $k_1/k_2 > 0.66$ *there is minimal dimensional change and normal neuronal function is present. For* $0.46 \leq k_1/k_2 \leq 0.66$ *there is moderate change and neuroprotective treatment may be required. For* $k_1/k_2 < 0.46$ *there is significant dimensional change and intensive neuroprotective treatment may be required.*
There are following differences between brain cells model in classical analysis and multiplicative analysis. Multiplicative model shows more dramatic changes, while classical model appears more physiological. Both models reach stable end states. The multiplicative model responds faster.

Example 2.4 (Liver Cell-Multiplicative Case). *Let $r(0) = 25$ and $p = 1.09gr/cm^3$ for a liver cell. Again, we will examine the cell radius change for the liver cell using solution (2.1) in the multiplicative case. If this solution is used, the equilibrium ratio for the cell is obtained as;*

$$\frac{k_1}{k_2} = 1.07,$$

for $r(0) = 25$. Here the calculations will be made for particular selections of $k_1 = 107$ and $k_2 = 100$ in Table 2.4.

$\frac{k_1}{k_2}$	$r(0)$	$r(1)$	$r(2)$	$r(3)$	$r(4)$	...	$r(10)$
1.07	25.0000	24.9984	24.9984	24.9984	24.9984	...	24.9984
0.97	25.0000	23.9984	23.9984	23.9984	23.9984	...	23.9984
0.87	25.0000	22.9984	22.9984	22.9984	22.9984	...	22.9984
0.77	25.0000	21.9984	21.9984	21.9984	21.9984	...	21.9984
0.67	25.0000	20.9984	20.9984	20.9984	20.9984	...	20.9984
0.57	25.0000	19.9984	19.9984	19.9984	19.9984	...	19.9984
0.47	25.0000	18.9984	18.9984	18.9984	18.9984	...	18.9984

Table 2.4: Change of liver cell radius over time according to the change in multiplicative case

In equilibrium, it can be easily seen that

$$r(5) = r(6) = r(7) = r(8) = r(9) = 24.9984.$$

In this example, for $k_1/k_2 = 1.07$, there is a 0.006% change, which is the ideal situation. For $k_1/k_2 = 0.97 - 0.87$, a 4-8% reduction is within normal limits. For $k_1/k_2 = 0.77 - 0.67$, a 12-16% reduction is the situation that should be monitored. For $k_1/k_2 = 0.57 - 0.47$, there is a 20-24% reduction, which is a risk situation. Hepatoprotective treatment is required.
There are following differences between liver cells model in classical analysis and multiplicative analysis. In the classical case, the rate of change is slow and continuous, while in multiplicative analysis it is fast and one-time. While in the classical case there is a stability in the form of an asymptotic approach, in the multiplicative case there is an instantaneous stability.

3. Conclusion

In this study, four types of cells (Stomach, Blood, Brain, Liver) were considered and the change in the radii of the cells was examined by establishing the classical model in multiplicative analysis. In the classical and multiplicative case, the change in the radius of the cell membrane as time passes has been examined. In the multiplicative case, changes occur faster than in the classical case and the cells are in a more difficult situation than in the classical case.

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