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Araştırma Makalesi / Research Article

A CONSTITUTIVE MATERIAL MODEL FOR RESTRAINED REINFORCED CONCRETE COLUMNS IN CASE OF FIRE PART I

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Abstract: In the eighties increased basic research was carried out in **SFB** (**S**onder**F**orschungs**B**ereich: a special fire research activity for structural elements in Braunschweig Technical University, 1971-1986) in order to clarify the discrepancies between calculation and measurement of restrained compression elements in case of fire. The work was particularly focused on the description of a universal material model for the concrete under elevated temperatures that could be applied successfully to estimate the relaxation response of restrained compression columns in fire. In the literature, various material models for structural concrete can be found. In many cases, however, the verification of such models lacks for the case of fire, especially when a mechanical restraining or caused by the entire structural interaction of the carrying system is present.

Keywords: Fire action, Structural fire safety, material model for the concrete, restrained compression elements

UN MODELE DE MATERIAU CONSTITUTIF POUR COLONNES EN BETON ARME RESTREINT EN CAS D'INCENDIE PART I

Résumé: Dans les années quatre-vingt a augmenté la recherche fondamentale a été réalisée en **SFB** (Une activité spéciale de recherche sur le feu pour les éléments structurels de l'Université Technique de Braunschweig, 1971 à 1986) afin de clarifier les écarts entre calcul et mesure des éléments de compression restreint en cas d'incendie. Le travail a été particulièrement axée sur la description d'un modèle de matériau universel pour le béton sous des températures élevées qui pourraient être appliquée avec succès pour estimer la réponse de relaxation de colonnes de compression restreint dans le feu. Dans la littérature, différents modèles de matériaux pour le béton structurel peuvent être trouvés. Dans de nombreux cas, cependant, la vérification de ces modèles ne dispose pas dans le cas d'incendie, surtout quand une restreint mécanique ou causée par l'ensemble de l'interaction structurelle du système est présent.

Mots clés: Action incendie, Structurel sécurité en cas d' incendie, modèle de matériau béton, relaxation de colonnes

INTRODUCTION

In the eighties increased basic research was carried out in SFB in order to clarify the discrepancies between calculation and measurement of restrained compression elements in case of fire. The work was particularly focused on the description of an universal material model for the concrete by a comprehensive mathematical formulation of the rheological phenomena's under elevated temperatures that could be applied successfully to estimate the relaxation response of restrained compression columns in fire.

It was well known that the use of such material model would require an immense effort in calculator. The computational studies have shown, however, that such effort is worthwhile, because only by a realistic material model fire behavior of restrained structural elements can be estimated satisfactorily for fire case **(Bazant, Z. P., 1982)**. It should be emphasized here that a material model can gain confidence only if it has been tested for various structural elements and fire conditions.

In particular, the model must apply for the assessment of the fire behavior of structural elements in practical sizes. In the literature, various material models for structural concrete elements can be found. In many cases, however, the verification of such models lacks for the case of fire, especially when a mechanical restraining or caused by the entire structural interaction is present.

Literature review (1973-2014)

Upmeyer, J., and Schaumann, P. show in their contribution studies to fire resistance of reinforced concrete columns, based on a structural analysis with an advanced calculation model. The analysis covers simply supported columns as well as cantilever columns with buckling length in case of fire up to 20.0 m. Finally safety considerations for reinforced concrete columns will be performed, which show, that existing concrete columns, which are designed in conformity with the minimum dimensions of German Standards (table 31 in DIN 4102-4), are stable in case of fire, if they are planned and built in compliance to acknowledged rules of technology.

Cyllok, M., and Achenbach, **M**. give information to their labor tests. The application of non-linear zone method is explained. The statistical evaluation leads to governing key data, which proof adequate safety according to DIN 4102-2. Research work shows that the nonlinear zone method (simplified calculation method) can be used for design of reinforced concrete columns exposed to fire. The decisive action effects of a fire exposure on the structural behavior are considered by this method and have been proofed by a comparison of ultimate loads and deformation curves computed by using nonlinear zone method as well as using advanced method. In addition, this article statistically evaluates how safe – according to **DIN 4102-2** – published full scale tests that can be modeled by using nonlinear zone method. Information to mentioned tests is shown.

Frank Fingerloos, F and Richter, E. explain the background of the structural fire design of reinforced concrete columns with a modified new fire design-table in the German standard **DIN 4102**. The limit condition of according to this table results show often with a maximum 6 m length for rectangular columns and 5 m length for circular columns very conservative reinforcements and dimensions. For these cases it is proposed an extension of the fire design- table. An example completes the paper.

Sven Huismann. S. And , Manfred Korzen, M, and Andreas Rogge, A. discuss in their paper following a critical analysis of the material parameters of normal strength (NSC) and high strength concrete (HSC) presented in **Eurocode 2** the thermo-mechanical material parameters of one representative HSC. Using these parameters and based on an appropriate material model the behavior of HSC columns was simulated. It was found that the strength as a characteristic parameter of the material model has to be identified on the basis of transient creep tests and not of stationary tests, respectively as realized usually for NSC

Quast. U. states that the consistent distinction between stress-dependent and thermal strains is essential for nonlinear calculation fire induced cross-sections. Even for non-linear cross section calculations with prestressed reinforcement, taking into account the effects of creep and shrinkage or other issues, the concept of stress-dependent strain is appropriate. It results in a total uniform approach for the formulation of the strain state.

Lange, D., and Sjöström, J. describe in their paper the effect of thermal exposure varying in both the horizontal and vertical axes to columns by means of including this thermal boundary in a solution of classical Euler beam theory. The resulting solution allows for a variation in the stiffness of the rotational restraint at both ends of the column and a non-uniform temperature exposure through the column's section and along its height.

Xu, Y., Wu, B. obtain in their experimental results that: When the axial load ratio is 0.55, the fire resistances of the columns with L-, T-, and +-shaped cross-sections subjected to fire on all sides were 60–73% that of the column with the square cross-section under design loads. A computer program RCSSCF was developed to calculate temperature, deformation, and fire resistance of the loaded columns with different shaped cross-sections. It is stated that the results of the numerical simulation were compared with those of the full-scale fire resistance tests.

Franssen, J.-M. presents the basic principles of the arc-length technique, first for the way it is applied traditionally at room temperature, then for the way it can be applied to extend a numerical simulation beyond the moment of local failures in case of fire. The technique is then applied to the case of restrained columns and it is shown how it is possible to obtain a safe estimate of the critical temperature of the column leading to the failure of the structure, even if the degree of restraint applied to the column is unknown.

Nguyen, T.-T., Tan, K. H. give a simplified analytical model to directly determine these so-called thermal-induced restraint forces. The model bases on the concepts of equivalent distributed temperature as well as eccentricity- and temperature-dependent reduction factor of axial stiffness. The model is validated by fire tests conducted at Nanyang Technological University on twelve restrained concrete column specimens subjected to uniaxial and biaxial bending. Relatively good agreement between the analytical and the experimental results of restraint force development is obtained.

Neves, N., Valente, J.C., Rodrigues, J. P. C., make a proposal which uses the results of a series of tests and calculations, with the aim of being applied as a simple method to correct the value of the critical temperature of steel columns free to elongate, in order to take into account the restraint effect of the structure to which they belong in a practical situation.

Consequences of the Material Modeling for Concrete

In this paper, an approach is shown to take into account the material behavior of normal concrete as realistically as possible in restraining condition of structural elements. Concrete is one of a group of materials showing time dependent deformation under acting load. The portion of the total deformation occurring, which remains after deduction of the elastic ε_{el} and thermal expansion ε_{th} as well as independent shrinkage ε_{s} deformations of the load is commonly referred to as a creep deformation ε_{cr}.

The creep of concrete under elevated temperatures has been studied in a variety of researchers (**Waubke, N. Y.,** 1973**., Schneider, U.,** 1973**. and**

Anderberg, Y., 1976). In the context of this article, however, focus is laid only on the work of the SFB and Swedish researchers who have carried out intensified work as special material research to clarify the high-temperature creep of concrete. In the Lund Institute of Technology material model presented by Anderberg describes the total deformation of the concrete as given Eq. 1:

$$\varepsilon_{tot} = \varepsilon(\sigma(t), T(t), \tilde{\sigma}) \qquad (1)$$

In Eq. 1 the stress history of the concrete $\tilde{\sigma}$ is represented under high temperature effect. The total deformation ε_{tot} consists of several components; the individual variables however are associated with a certain experimental procedure. The Eq. 1 is given in explicit form as follows:

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{\sigma}(\tilde{\sigma}, \sigma, T) + \varepsilon_{scr}(\sigma, T, t) + \varepsilon_{tcr}(\sigma, T) \qquad (2)$$

In this equation ε_{th} thermal expansion, ε_{σ} spontaneous stress dependent compression, ε_{scr} stationary creep and ε_{tcr} transient creep are the strains under a certain compression stress. The determination of the deformation components are discussed in detail in **(Anderberg, Y.,** 1976**).** Anderberg formulates the transition creep as a spontaneous reaction to the effect of temperature and converts it as a time dependent deformation which is assumed linearly related to the present stress. The Eq. 2 results in by this way into Eq. 3:

$$\varepsilon_{tot} - \varepsilon_{th}(T) - \varepsilon_{tcr}(\sigma, T) = \varepsilon_{\sigma} + \varepsilon_{scr} \qquad (3)$$

For the accurate determination of the right side of this equation, a special importance is attached, because the stress generating deformation components in the calculation must be determined successively with certain time intervals. Research in SFB of this nature has already been completed **(Waubke, N. V.,** 1973**., Schneider., U.,** 1973**).** As an illustration of the phenomenon that with a load of concrete, the deformations of the building material increase over time, a relationship implicit form is required as given in Eq. 4:

$$F = (\varepsilon, \sigma, \widetilde{\sigma}, t, T) = 0 \qquad (4)$$

In Eq. 4 it is assumed that the differential and integral operators of the functions ε, σ, and t are known. **(Schneider, U.,** 1973**)** therefore the total deformation in Eq. 4 attempted to describe in accordance with the usual method at room temperature. Then a creep relationship as easy as possible was developed by determination of *φ-values*. According to Eq. 5 it is possible to describe the total deformation of concrete at a constant compression stress as follows:

$$\varepsilon_{tot=} \varepsilon_{th} - \left(1 + \varphi(T, t)\right) \frac{\sigma}{E(T)} \qquad (5)$$

The Eq. 5 can also be expressed in another form to determine the stress condition (See Figure 6 for the function of modulus of elasticity):

$$\sigma = (\varepsilon_{tot} - \varepsilon_{th)}). E(T) - \varphi(T, t). \sigma \qquad (6)$$

By means of the determination of $\varphi(T,t)$ the present problem would be solved. The difference $(\varepsilon_{tot} - \varepsilon_{th})$ in Eq. 5 gives the desired deformation term, which can be split into an elastic and inelastic strain components. The latter component corresponds to the time dependent deformation parts which occur under compression stress and unsteady temperature effects. It is defined by **(Schneider, U.,** 1985**)** as it reads in Eq. 7:

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_{\sigma} = \sigma\left(\frac{\partial J(\sigma,t)}{\partial t}\right)_{\sigma} \qquad (7)$$

In Eq. 7 $J(\sigma,t)$ corresponds to the creep function. It was determined by means of creep tests at unsteady temperatures, for a given constant compression stress *(hot creep tests).* The total deformation can thus be determined according to Eq. 8 for a given compression stress and for a given initial deformationε_0:

$$\varepsilon_{tot=} \varepsilon_0 + \frac{\sigma}{E(T)} + \sigma. J(\sigma, T) \qquad (8)$$

From here it results in an equation for determining $J(\sigma,T)$:

$$J(\sigma, T) = \frac{1}{E(T)}. \varphi(\sigma, T) \qquad (9)$$

In case of fire, however, temperature and stress changes occur in a concrete element. It is therefore not possible to apply Eq. 8 for creep problems in this form, because the determination of the total deformation results in integral equations.

Problem Definition

Reinforced concrete columns are generally in interaction with their surroundings if they are monolithic constructed with the structural system **(Kordina, K.,** 1979**., Anderberg, Y.,** 1976**)**. Thereafter, against the free thermal expansion of a fire exposed reinforced concrete column in a building an elastic restraining takes action. In this case variable axial restraining forces can develop bound to the grade of elastic restraining during the fire.

The restraining tests carried out with reinforced concrete columns in the special furnace of SFB have proven that between the prediction and the measured axial restraining forces during the fire significant deviations occurred. Therefore to clarify this discrepancy an intensive research was accomplished. Research focused in particular on the realistic description of the material behavior of concrete in case of fire **(Kordina, etal.,** 41. Jahrg**)**. In addition, other new tests were carried out with different cross-sectional sizes and column slenderness. In fact, such a constitutive material law was developed through a close collaboration of scientists from different subprojects ***A and B3*** of the ***SFB.***

AN ANALYTICAL MATERIAL MODEL FOR HIGH-TEMPERATURE

Deformation Components of Concrete at High Temperatures

The total deformation of concrete under unsteady temperature exposure has at least 5 individual deformation components. The total deformation ε_{tot} can be described as given in Eq. 10:

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{s} + \varepsilon_{el} + \varepsilon_{pl} + \varepsilon_{cr} \tag{10}$$

It can be assumed that the shrinkage component was included in the thermal expansion from Eq. 10 results in Eq. 11:

$$\varepsilon_{tot} - \varepsilon_{th} = \varepsilon_{el} + \varepsilon_{pl} + \varepsilon_{cr} \tag{11}$$

Eq. 11 therefore displays the sum of the stress generating deformation components minus the thermal expansion from the total deformation.

Modeling of the Stress History

In this section it will be shown mathematically that the inclusion of a stress history for concrete is essential. Multiplication the both sides of Eq. 11 by *E(t)* gives Eq. 12 and with definition of new deformation components, the following equations can be written:

$$E(t).(\varepsilon_{tot} - \varepsilon_{th}) = E(t).(\varepsilon_{el} + \varepsilon_{pl} + \varepsilon_{cr}) \tag{12}$$

$$\varepsilon_1 = \varepsilon_{tot} - \varepsilon_{th} \tag{13}$$

$$\varepsilon_2 = \varepsilon_{el} + \varepsilon_{pl} + \varepsilon_{cr} \tag{14}$$

and the total differential Eq. 12 results in Eq. 15:

$$\frac{\partial}{\partial t}(E(t).\varepsilon_1).dt = \frac{\partial}{\partial t}(E(t).\varepsilon_2).dt \tag{15}$$

By introducing the functions *U* and *V*

$$V(t) = \varepsilon_1 \frac{\partial}{\partial t}E(t) + E(t)\frac{\partial}{\partial t}\varepsilon_1 \tag{16}$$

$$U(t) = \varepsilon_2 \frac{\partial}{\partial t}E(t) + E(t)\frac{\partial}{\delta t}\varepsilon_2 \tag{17}$$

The Eq. 17 contains the following strain rates:

For the first expression in Eq. 17

$$\varepsilon_2 \frac{\partial}{\partial t}E(t) = \varepsilon_{el}\frac{\partial}{\partial t}E(t) + (\varepsilon_{cr} + \varepsilon_{pl})\frac{\partial}{\partial t}E(t) \tag{18}$$

Following the usual procedure results in for the second term for room temperature:

$$E(t)\frac{\partial}{\partial t}\varepsilon_2 = E(t)\frac{\partial}{\partial t}\left[\frac{\sigma\varphi}{E(t)} + \varepsilon_{pl}\right] + E(t)\frac{\partial}{\partial t}\varepsilon_{pl} \tag{19}$$

After conversion of Eq. 19

$$E(t)\frac{\partial}{\partial t}\epsilon_2 = E(t)\frac{\partial}{\partial t}\varepsilon_{el} + E(t)\frac{\partial}{\partial t}\left[\sigma\frac{\varphi}{E(t)}\right] + E(t)\frac{\partial}{\partial t}\varepsilon_{pl} \tag{20}$$

The total stress variation results in consequently from the summation of Eqs. 18 and 20:

$$U(t) = E(t)\frac{\partial}{\partial t}\varepsilon_{el} + \varepsilon_{el}\frac{\partial}{\partial t}E(t) + E(t)\frac{\partial}{\partial t}\left[\sigma\frac{\varphi}{E(t)}\right] + \varepsilon_{cr}\frac{\partial}{\partial t}E(t) + \varepsilon_{pl}\frac{\partial}{\partial t}E(t) + E(t)\frac{\partial}{\partial t}\varepsilon_{pl} \tag{21}$$

In case of leaving out of consideration the time independent deformation components in Eq. 20 the function *U(t)* gets the form as given in Eq. 22:

$$U(t) = \dot{\sigma} + E(t)\frac{\partial}{\partial t}\left(\sigma\frac{\varphi}{E(t)}\right) + (\sigma.\varphi)\frac{\dot{E}(t)}{E(t)} + \varepsilon_{pl}.\dot{E}(t) \tag{22}$$

and in Eq. 22 a new function defined as Z(t):

$$Z(t) = E(t)\frac{\partial}{\partial t}\left(\sigma\frac{\varphi}{E(t)}\right) \tag{23}$$

$$Z(T) = E(t)\left[\frac{\varphi}{E(t)}.\frac{\partial\sigma}{\partial t} + \sigma\frac{\partial}{\partial t}\left(\frac{\varphi}{E(t)}\right)\right] \tag{24}$$

and *U(t)* takes the form:

$$U(t) = \dot{\sigma} + \sigma.E(t)\frac{\partial}{\partial t}\left(\frac{\varphi}{E}\right) + \varphi.\frac{\partial\sigma}{\partial t} + (\sigma.\varphi)\frac{\dot{E}(t)}{E(t)} + \varepsilon_{pl}.\dot{E}(t) \tag{25}$$

After further conversion finally Eq. 26 can be obtained:

$$U(t) = (1+\varphi).\dot{\sigma} + \sigma\left[E(t).\frac{\partial}{\partial t}\left(\frac{\varphi}{E}\right) + \varphi.\frac{\dot{E}(t)}{E(t)}\right] + \varepsilon_{pl}.\dot{E}(t) \tag{26}$$

Now new function *Q(t)* can be defined using Eq. 27:

$$Q(t) = \frac{(U(t) - \varepsilon_{pl}.\dot{E}(t)}{(1+\varphi)} \tag{27}$$

then the decisive equation for describing the stress history is obtained:

$$Q(t) = \dot{\sigma} + \sigma\frac{1}{1+\varphi}\left[E(t)\frac{\partial}{\partial t}\left(\frac{\varphi}{E}\right) + \varphi.\frac{\dot{E}(t)}{E(t)}\right] \tag{28}$$

The Eq. 28 can be transformed into a simpler form as Eq. 29:

$$Q(t) = \dot{\sigma} + R(t) \tag{29}$$

It is clear from Eq. 29 that the determination of the actual compressive stress on a concrete element, the knowledge about the stress history is essential. By using Eq. 16 and Eq. 29 local compressive stresses can be determined:

$$V(t) = U(t) \tag{30}$$

The calculation process is described in detail in the following section.

Successive Calculation of the Compressive Stresses

In order to determine the current compression stress in a concrete element, it is necessary to integrate the Eq. 29. First partial derivatives E and φ of Eq. 28 show the significance of the functional dependence of these parameters. These arithmetic operations are given below in Eq. 32 through Eq. 35, wherein the relation with Eq. 31 is introduced.

$$\chi = \frac{\varphi(\sigma,T)}{E(\sigma,T)} \text{ s. also Fig. 28} \tag{31}$$

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial \sigma}.\frac{\partial \sigma}{\partial t} + \frac{\partial \sigma}{\partial T}.\frac{\partial T}{\partial t} \text{ s. also Fig. 6} \tag{32}$$

$$\frac{\partial \varphi}{\partial t} = \frac{\partial \varphi}{\partial \sigma}.\frac{\partial \sigma}{\partial t} + \frac{\partial \varphi}{\delta T}.\frac{\partial T}{\partial t} \text{ s. also Fig. 7} \tag{33}$$

$$\frac{\partial \chi}{\partial \sigma} = \frac{1}{E}.\frac{\partial \varphi}{\partial \sigma} - \frac{\varphi}{E^2}.\frac{\partial E}{\partial \sigma} \tag{34}$$

$$\frac{\partial \chi}{\partial T} = \frac{1}{E}.\frac{\partial \varphi}{\partial T} - \frac{\varphi}{E^2}.\frac{\partial E}{\partial t} \tag{35}$$

It is thus clear that, the Eq. 29 to Eq. 35 the effects of temperature rate $\dot{T}$ and its following appearances must be considered in the development of material law.

The general solution of Eq. 29 is known **(Bronstein-Semendjajew,** 1973**)** and can be written with Eq. 36:

$$\sigma = e^{-\int_0^t R(t)dt}.\left[\int Q(t).e^{\int R(t)dt}\ dt + C\right] \tag{36}$$

The evaluation of Eq. 36, however, in this form is hardly possible because the σ in Eq. 36 is implied.

Considered boundary conditions as

$$t = t_0 \text{ and } \sigma = \sigma(t_0) \quad \text{bzw} \quad \sigma = \sigma_0 \tag{37}$$

The compressive stress of concrete at a given time is therefore determined incrementally. The stress variation during a time step Δt can therefore be determined by total differentials of Eq. 36. From Eq. 36 the stress determination follows with finite extents stepwise with the help of Eq. 38.

$$\sigma_t = \sigma_{t-1}.e^{-R(t).\Delta t} + s(t) \tag{38}$$

in which

$$s(t) = e^{-R(t).\Delta t}\{Q(t).e^{R(t).\Delta t}.\Delta t\} \tag{39}$$

and determination of the stress variation is carried out according to Eq. 40

$$\Delta\sigma_t = \sigma_{t-1}\left(e^{-R(t).\Delta t} - 1\right) + s(t) \tag{40}$$

The determination of the function value of $R(t)$ in the above expression takes place in accordance with the agreements made in Eq. 29. In this regard, the equations 32 to 35 must be calculated successively.

According to some conversion operations $R(t)$ results in Eq. 41 for a certain time duration

$$R(t) = \frac{1}{1+\varphi_t(T,\sigma)}\left\{\varepsilon(t).\left[\frac{\Delta\chi}{\Delta\sigma}.\dot{\sigma} + \frac{\Delta\chi}{\Delta T}.\dot{T}\right] + \left(\frac{\varphi}{E}.\dot{E}\right)\right\} \tag{41}$$

In order to determine the function value $Q(t)$ in the stress expression Eq. 39, functional equation $V(t) = Q(t)$ must be solved in Eq. 27. Infinite differences form $V(t)$ can be written as in Eq. 42

$$V(t) = \varepsilon_1.\frac{\Delta E}{\Delta\sigma}.\dot{\sigma} + \varepsilon_1.\frac{\Delta E}{\Delta T}.\dot{T} + E(t).\dot{\varepsilon}_1 \tag{42}$$

From the equations 41 and 42 it is clear that at any given time of the fire duration besides the temperature rate $\dot{T}$, the stress rate $\dot{\sigma}$ has to be determined in order to calculate the actual stress in an element of concrete. Whereas the $\dot{T}$ can be determined without difficulty, however determination of $\dot{\sigma}$ is only possible iteratively.

The determination of the plastic deformations **(Schneider,** 1986**)** succeeds according to Eq. 43 for certain fire duration.

$$\varepsilon_{pl} = \frac{1}{3}.\left(\frac{\sigma}{E}\right)_t.\left(\frac{\sigma}{f_c}\right)^5 \tag{43}$$

The concrete strength f_c, E-modulus and creep function φ will be taken into account in the following functional relations:

$$f_c = f_c\left(T,\dot{T}\right) \tag{44}$$

$$E = E\left(T,\dot{T},\sigma\right) \tag{45}$$

$$\bar{\varphi} = \bar{\varphi}\,(\sigma,T,w) \tag{46}$$

In Eq. 46 w indicates the humidity of concrete in weight-percent.

The conducted extensive computational studies have shown that a modified new creep function $\bar{\varphi}$ must be represented as a product of two functions. This was therefore necessary because, in order to calculate the development of deformations of a concrete structural element at high temperatures, the long-time creep effects on one hand, and the influence of the cross-sectional shape on the other hand should be taken into account. The calculations results in

$$\bar{\varphi} = \psi(t,r_s,r_z).\varphi(\sigma,T,w) \tag{47}$$

whereby ψ is described **(Schneider,** 1986**)** with an empirical Eq. 48

$$\psi = \left(\frac{r_s}{r_z}\right)^{0.5} + 5.10^{-3}.t \tag{48}$$

In this equation, the time *"t"* is in minute and *"r_s"* applies for the hydraulic radius of the large specimen, and *"r_z"* for the hydraulic radius of the small specimen, which have been used in the material science. Here *"r_z"* is set 8.0 cm **(Bazant,** 1982**)**. Experimental investigations have also shown that the thermal expansion of the concrete is also affected by the dimensions of the test specimen. In order to consider this concern in calculations, only 85% of the thermal expansion of the concrete for big structural elements at high temperatures is taken into account

Approach to the Tensile Strength of Concrete

The tensile strength of the concrete is taken into account in the calculations up to 150 °C **(Waubke, N. V.,** 1973**).** Maximum permissible tensile strength is set 1/10 the size of the current temperature depending on the compressive strength. In the determination of the tensile strength, time influences will not be considered. The cracked concrete elements subsequently are used for the compressive stresses in case of compression.

THE STRESS-STRAIN PARAMETERS INFLUENCING THE DEFORMATION DEVELOPMENT AT HIGH TEMPERATURES

Thermal Expansion

Influence of the heating rate

In Figure 1, the measured thermal expansion of the specimen is shown via the furnace temperatures. In order to determine the influence of heating rates, two distinctly different temperature increases were simulated in the furnace. In Figure 1 it can be seen that at low heating rate due to almost homogeneous temperature distribution in the cross-section significantly higher thermal expansion occurs along the longitudinal axis of the samples. From this it can be concluded that at lower heating modes higher restraint forces shall be activated in the structural concrete elements than the ISO834 fire is present.

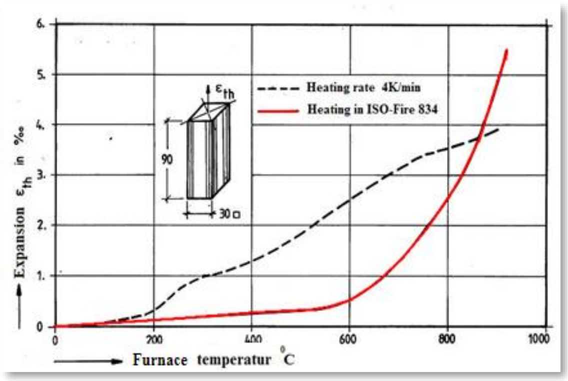

Figure 1: Influence of heating rates on the expansion of small concrete specimens obtained in SFB-Tests

Influence of the size of the specimens

In Figure 2, the thermal expansion of two different sized specimens is shown via the oven temperatures at a heating rate 4K/min. It is being clear that the specimens show at the same furnace temperature greatly differentiated thermal expansions. The higher thermal expansions occur at smaller specimens. The control calculations have shown, however, those other influences of the rheological side should be effective, such as the moisture transport and structural distortions in the element and the inner stress conditions.

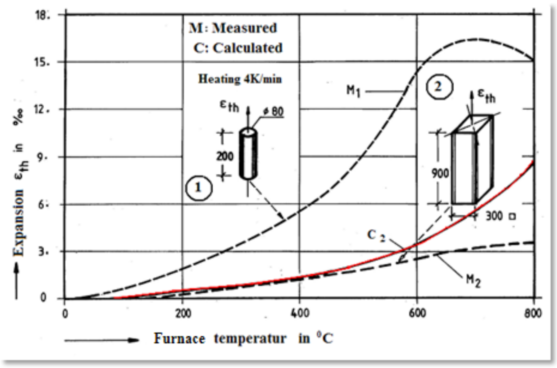

Figure 2: Measured and calculated thermal expansion of concrete specimens in different sizes

In Figure 3 however thermal expansions of the two different sized macro specimens are illustrated over time. The heating of the samples are carried out according to the ISO834-Fire Curve. The small sample body has a section along the column axis of the second macro specimen. The illustration of the test results shows also in this case, a significant difference in the development of thermal expansion of the specimens. It is clear that the small test specimens have, also in this case a higher thermal expansion

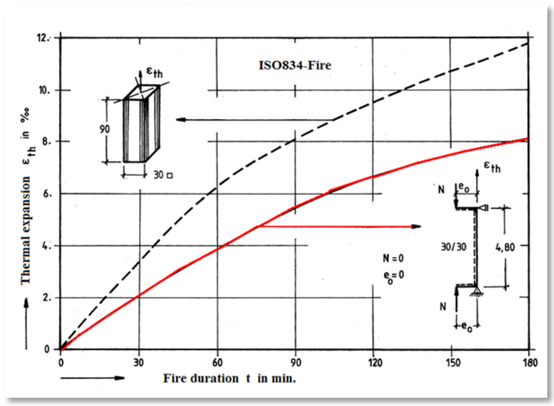

Figure 3: Thermal expansions of the two different sized macro specimens under the same heating conditions.

The results shown in figures 1 to 3 thus make the influence of specimen geometry on the development of pure thermal expansions of the specimens significant.

In order to take into account these influences and other imponderables in the numerical treatment of the material behavior, thermal expansion obtained from the small specimens was reduced by 15% (see conclusions in subsection *"Successive calculation of the compressive stresses"* and Figures 1, 2 and 3).

Influence of temperature history

The rheological tests carried out have shown that the thermal expansion of the concrete specimens indicates an irreversible behavior during the heating and cooling cycles **(Schneider, U., 1985)**. In particular, during the cooling phase of the concrete a permanent expansion remains.

In Figure 4, this behavior is illustrated via temperature. From the picture it is clear that the amount of the remaining strains depends on the maximum temperatures at a point of cross section in concrete attained. At higher temperatures also correspondingly high remaining expansion results in. They are up to a temperature of 400 °C negligible, so that in this temperature range, a quasi-reversible behavior of the thermal expansion of the concrete can be accepted. It is thus clear that the numerical treatment of concrete structures in fire, particularly in natural fires the consideration of the temperature history is inevitable.

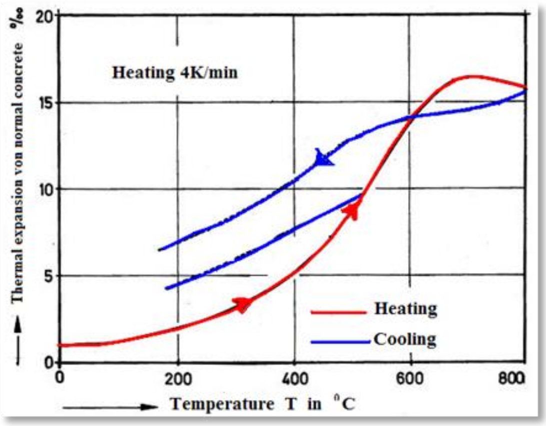

Figure 4: Effect of warming and cooling processes on the thermal expansion of the normal concrete

Influence of Heating Rate on the Concrete Strength

Loss of strength of concrete in the range of 70 to 200 °C is caused mainly by the evaporation of the physically bound water in concrete (**Ehm, C., 1985**). This process of evaporation of physically bound water is complete at about 200 °C. Until reaching this temperature limit a large pressure slope is formed in the concrete, but that is also dependent on the porosity. When a steam pressure is built up in the concrete, the high internal pore pressure and the external mechanical restrains may cause to premature failure of the specimen.

In Figure 5, this phenomenon is illustrated depending heating rate of the concrete. The figure shows the effect of temperature on the related loss of strength of concrete. The hatched area indicates the heating-rate dependent strength valley zone. In case of rapid heating rates in the range of 70 to 200 °C in normal concrete strength a strength valley is to observe, because in this area the phase change of the water is faster than the steam movement. Only by slow heating rates dehydration processes of the physically bound water have been completed. After that, the temperature-dependent strength of concrete has a steady development.

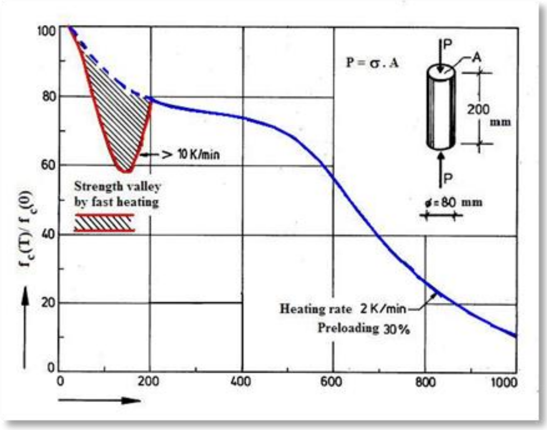

Figure 5: Related strength of concrete by elevated temperatures

In the numerical analyses the above discussed strength valley is taken into account in dependence of the heating rate. This area is regulated linearly in a temperature range of 50 to 200 °C. The consideration of a strength-valley in the calculations has initiated a significant improvement for the realistic analysis of relaxation problems at high temperatures.

Effect of Heating Rate on the Elastic Modulus of the Concrete

In accordance with the conclusions discussed in the previous subsection for the E-modulus of the concrete a similar temperature-dependent occurrence has been taken into account. Although no test results exist on an E-modulus-valley for the material concrete in a certain temperature range, it is analog assumed that the heating rate would influence the course of the modulus of elasticity respectively. This assumption is reasonable, since in many cases to estimate the modulus of elasticity the concrete strength is used.

The modulus of elasticity of the concrete is influenced not only by the temperature level attained, but also by the loading level. The related E-modulus of concrete is illustrated via temperature in Figure 6. The load level is chosen as a parameter. It is being clear that a higher load factor also causes consequent increase in modulus of elasticity. Depending on the load factor the development of the E-modulus for all temperature levels is steadily, if the heating of the structural element happens very slow.

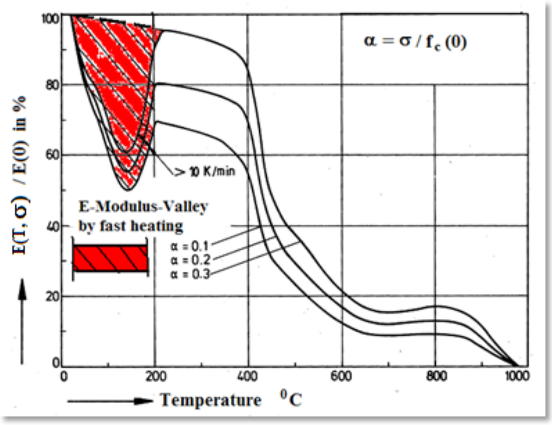

Figure 6: The related E-modulus of concrete by elevated temperatures depending on the load level.

Description of the Creep Function

The in this section presented $\bar{\varphi}$-functions were determined by extensive numerical and experimental analyses on normal concrete by the subproject B3 of SFB (**Subproject B3, 1974-1983**). The carried out theoretical investigations on large specimens have shown, however, that the thus obtained $\bar{\Phi}$-functions for normal concrete above 550 °C temperature range had to be modified. It has revealed that exceeding this temperature limit a rapid increase in the $\bar{\varphi}$-function for normal concrete should be applied. An analytical expression for the principal $\bar{\Phi}$-function can be found in **(Schneider, U., 1979).**

Influence of the load level

The experimental results show in **(Schneider, U., 1979)** that the temperature-dependent $\bar{\varphi}$-values (See Eq. 46) lie in a narrow distribution. It is however known that the modulus of elasticity of the concrete depends on both the temperature effect and also on the existing compression stress. When the results are evaluated from this point of view, then it is determined that the creep values lie no longer in a narrow zone.

The evaluated results and their development via temperature for a certain load level of $\bar{\varphi}$-values are illustrated in Figure 7. $\bar{\varphi}$-Function is shown in the figure in a set of curves, whereas parameter the moisture content in weight-percent is specified. It is determined that with increasing load factor the creep values also increase and the creep functions almost linearize (s. Figure 8). However, this property is valid up to the temperature limit of 550 °C.

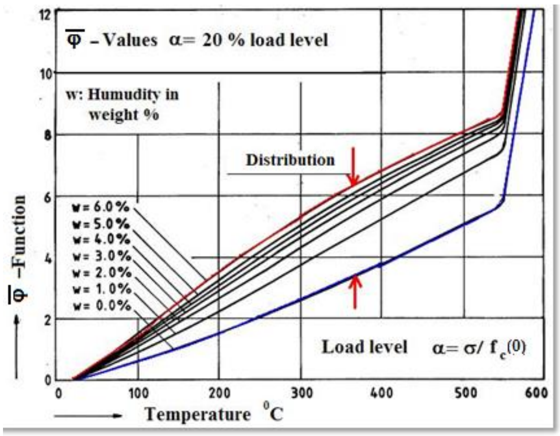

Figure 7: Creep function $\bar{\varphi}$ in case of 20 % load level

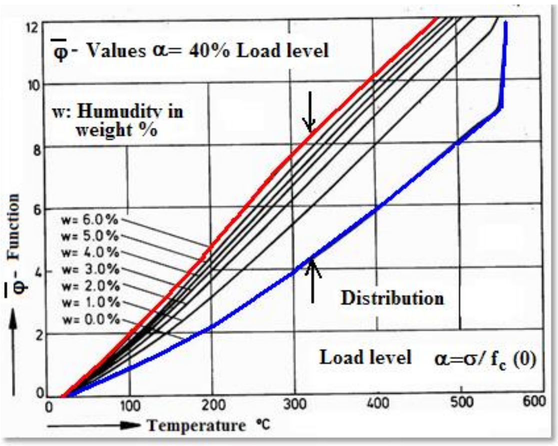

Figure 8: Creep function $\bar{\varphi}$ in case of 40 % load level

Influence of the concrete humidity on creep of concrete

Figure 9 shows the creep development for specific moisture content in the specimen. The load level has been chosen as a parameter. It will be clear that the load level apparently has a small influence in the temperature range of 20 to 500 °C on the $\bar{\Phi}$-values. In this area, the creep function is almost linear; after exceeding this limit, the linearity disappears and the load level shows its influence considerably.

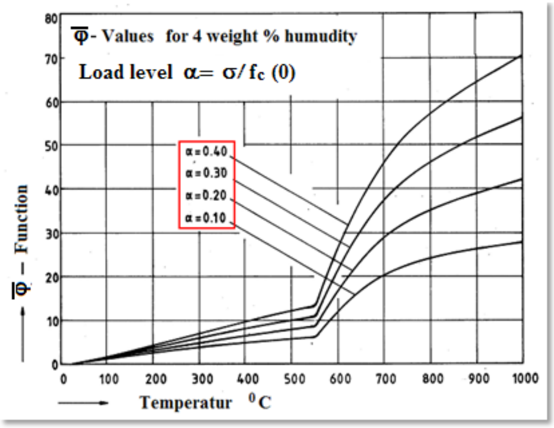

Figure 9: Creep function $\bar{\varphi}$ for 4 weight-percent concrete humidity.

Influence of stand-time

In the subsection *"Successive calculation of the compressive stresses"* it was shown that the creep function depends not only on the temperature and the load level, but must also be modified by a time parameter in order to take into account the long-term creep effects. Eq. 47 describes this characteristic of the creep function. The extensive calculations performed have shown that the creep function can be best illustrated in the way that the stress and temperature dependent effects are multiplied by an appropriate time function. These functional relationships are in Eq. 48 displayed. It also contains the geometric influence of the specimen.

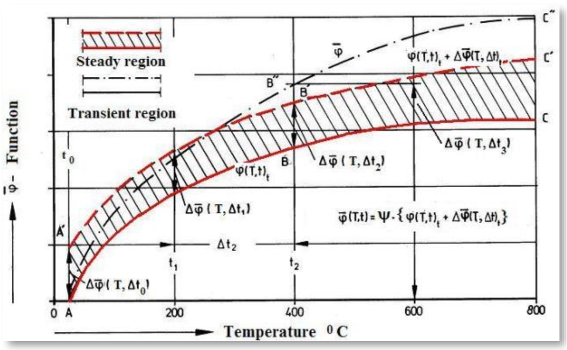

Figure 10: Influence of temperature stand-time on creep-function.

The influence of stand-time on the development of creep function is illustrated in the Figure 10 via the temperature in principle. In the figure the influences of steady and unsteady temperature effects have been made distinct. The curve A-B-C gives the basic creep-factor of a test specimen, which is heated with load up to a certain temperature, under a certain heating rate. When the temperature attained is then maintained stabile, by this additional creep deformations occur. This process is represented by a hatched area in Figure 10, which reflects the increase in the creep factors in accordance with the progressive stand-time.

The new creep function is shown in Figure 10 for a certain time Δt_1-step with the curve A'-B'-C '. In case of fire, however, unsteady temperatures are acting, so that the final creep factor is calculated successively for a specific temperature level (Eq. 49):

$$\bar{\varphi}\,(T,t) = \psi . \{\varphi(T,t)_{t-1} + \overline{\Delta\varphi}(T,\Delta t)_t\} \qquad (49)$$

The resulting new $\bar{\varphi}$-function is also shown in Figure 10 principally with the curve $A'' - B'' - C''$.

Irreversible Application for Reinforcing Steel

Extensive studies have indicated that taking into account the time effects for realistic material behavior of steel does not provide a special contribution (**Hoffend, F., 1981-1983**). Therefore, the stresses in the calculation during an acting load are non-linear-elastic determined. On the other hand a linear-elastic behavior to the building material is assigned when an unloading exists. That means a Hysteresis-Curve is applied for the stress-strain relationship for the constant steel temperature. Thus, during the cooling phase of the component temperatures, for example, remaining expansions are determined in the calculation.

SUMMARIES

It has been presented in SFB and in other research works a general calculation model for concrete materials for high temperatures. The application of these mathematical models presented has shown, however, that the material description was not complete and not enough realistic. Therefore, some models could be used successfully for certain boundary conditions. This reality is due to the fact that either experimental data have been obtained from steady state, or from unsteady temperature conditions. In some cases even partly mixed approach was applied. Numerical studies have shown that the material models that have been developed through the analysis of unsteady data are the best method to describe the material behavior in case of fire.

The results of the research works in SFB 148 showed that a material model for concrete has to include the transition creep or at least proper deformation effects. Such conditions shall in particular represent a benchmark for the fire test when a component is under restraining effects while the deformations and restraints shall be predicted and compared with test results (**Haksever, A., 1980**). The here proposed material model bases on the research works of **SFB 148**. The material model includes important premises that takes into account the transition creep and therefore for the fire protection design of reinforced concrete elements finds a reliable application also in restraining conditions.

The decisive solution of the relaxation problem was achieved by taking into account the most important physical factors *($\dot{T}$, w, σ)* in the determination of the concrete strength and the modulus of elasticity *E(T, σ)* (**Weiß, R., 1977**). Also, the modification of the creep function in consideration of the geometric shape of the test specimen and long-term time effects has been an important improvement. The validity of the model presented here will be shown in another contribution (Part II) of the author.

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NOTATIONS

f	Strength	[N/ mm^2]
f_c	Concrete strength	[N/ mm^2]
$f_c(T)$	Temperature dependent concrete strength	[N/ mm^2]
$f_c(0)$	Initial concrete strength	[N/ mm^2]
l	Length	[mm]
T	Temperature	[K]
$\dot{T}$	Heating rate	[K/min]
t	Time	[sec]
$E(t)$	Modulus of Elasticity for a certain time duration	[N/mm^2]
$E(0)$	Initial modulus of Elasticity	[N/mm^2]
$E(T, \sigma)$	Temperature and stress dependent modulus of Elasticity	[N/mm^2]
$\dot{E}$	Modulus of Elasticity differential by time	[N/mm^2/s]
r_s	Hydraulic radius of large specimen	[mm]
r_z	Hydraulic radius of small specimen	[mm]
w	Humidity	[%]

Additional Symbols

α	Load level	
ε	Strain	
ε_0	Initial strain	
ε_s	Shrinkage	
ε_{el}	Elastic expansion	
ε_{pl}	Plastic strain	
ε_{th}	Thermal expansion	
ε_c	Creep deformation	
ε_σ	Spontaneous stress dependent compression	
ε_{tot}	Total expansion	
ε_{scr}	Stationary creep	
ε_{tcr}	Transient creep	
φ(T, t)	Creep function	
$\bar{\Phi}$(σ, T, w)	Modified creep function	
$\psi(t, r_s, r_z)$	Modification factor for creep function	
J(σ, t)	Function for creep deformations	
σ	Stress	[N/mm^2]
σ_t	Actual stress	[N/mm^2]
σ_{t-1}	Previous stress	[N/mm^2]
$\dot{\sigma}$	Stress differential by time	[N/mm^2/s]
$\tilde{\sigma}$	Stress history	[N/mm^2]
Δt	Time increment	[s]
$\Delta\bar{\varphi}(\Delta t)$	Stationary creep increment	

The other notations are defined where they appear in the text.

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