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Research Article

A NEW SOFT SET OPERATION: COMPLEMENTARY EXTENDED STAR OPERATION

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ABSTRACT

Soft set theory has established itself as a valuable mathematical framework for tackling issues marked by uncertainty, demonstrating its applicability across a range of theoretical and practical fields since its inception. Central of this theory is the operations of soft sets. To enhance the theory and to make a theoretical contribution to the theory, a new type of soft set operation, called "complementary extended star operation" for soft set, is proposed. An exhaustive examination of the properties of this operation has been undertaken, including its distributions over other soft set operations, with the goal of clarifying the relationship between the complementary extended star operation and other soft set operations. This paper also attempts to make a contribution to the literature of soft sets in the sense that studying the algebraic structure of soft sets from the standpoint of soft set operations offers a comprehensive understanding of their application as well as an appreciation of how soft set can be applied to classical and nonclassical logic.

Keywords: Soft Set, Soft Set Operations, Complementary Extended Soft Set Operations

1. INTRODUCTION

In our everyday experiences, we frequently encounter subjective notions like beauty, warmth, and boredom, which lack the objective certainty of scientific knowledge and vary from individual to individual. To navigate the intricacies of uncertainty, people have sought diverse solutions over time. However, existing methodologies have demonstrated inconsistencies in addressing novel, complex challenges stemming from changing circumstances. Among the array of theories proposed to manage uncertain scenarios, Zadeh's theory of fuzzy sets emerges as particularly prominent. Fuzzy sets are delineated by their membership functions. As the theory of fuzzy sets underwent rapid evolution, certain structural dilemmas surfaced. In response, Molodtsov [1] introduced soft set theory as a remedy for these structural intricacies.

Since its introduction, the soft set has been used in a wide range of theoretical and practical contexts, and several new research have been released in the literature. By defining the equality of two soft sets, the subset and superset of a soft set, the complement of a soft set, and soft binary operations like and/or, union, and intersection operations for soft sets, Maji et al. [2] cleared the way for further research in the field of soft set theory. Based on ideas from set theory, Pei and Miao [3] redefined the terms "soft subset" and "intersection of two soft sets." Next, several additional soft set operations were proposed by Ali et al. [4], and they were thoroughly examined by Sezgin and Atagün [5] and Ali et al. [6]. Extended difference and extended symmetric difference of soft sets were described by Sezgin et al. [7] and Stojanovic [8], respectively, and their characteristics were thoroughly examined in connection to other operations on soft sets.

Analysis of the research done to date reveals that soft set operations often fall into two categories: restricted soft set operations and extended soft set operations. The soft binary piecewise difference operation for soft sets was defined and analyzed by Eren and Çalışıcı [9], while Sezgin and Çalışıcı [10] conducted a detailed study of this soft set operation's properties. In 2021, Çağman [11] introduced a new notion in set theory: the inclusive and exclusive complement of sets. He then used this concept in group theory. Five new ideas related to binary complement operations presented by Çağman [11] were introduced by Sezgin et al. [12]. Aybek [13] presented several additional restricted and extended soft set operations and examined their properties. Furthermore, by taking the complement of the image set in the first row, the soft binary piecewise operation form-of which Eren and Çağman [9] were the pioneers-was somewhat changed. As a result, the complementary soft binary piecewise operations have been thoroughly examined by a number of scholars [14-22]. On the other hand, by taking the complement of the image set in the first and second rows and defining the complementary extended difference, lambda and union, plus and theta, respectively, and giving their algebraic properties and relations with other soft set operations, Akbulut [23] and Demirci [24] changed the form of the existing extended soft set operations in the literature. We refer to the following for more uses of soft sets in relation to algebraic structures: [25-32].

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Studying the properties of specified operations on sets, alongside the sets themselves, constitutes an essential aspect of algebraic structures, aiming to classify mathematical structures. Within the framework of algebra, this examination plays a crucial role. When considering soft sets as algebraic structures, attention is drawn to two primary types of soft set collections: one characterized by a fixed set of parameters, and the other by variable parameter sets. Just as classical set theory operations are foundational for soft sets, concepts pertaining to soft set operations are equally indispensable.

In this paper, in an effort to propel the theory of soft sets forward, this paper introduces a new soft set operation called "complementary extended star operation" and conducts a comprehensive examination of its properties. Furthermore, an analysis is undertaken to explore the interaction between the complementary extended star operation and other types of soft set operations, with the objective of elucidating their relationships. This subject is important within the framework as an understanding of the applications of soft sets requires a comprehension of their algebraic structures in connection to innovative operations.

PRELIMINARIES

Definition 2.1. Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let D ⊆ E. A pair (F, D) is called a soft set on U. Here, F is a function given by $F: D \rightarrow P(U)$ [1].

The notation of the soft set (F,D) is also shown as F_D, however, we prefer to use the notation of (F,D) as is used by Molodtsov [1] and Maji et al. [2].

The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$. In other words, in the collection $S_K(U)$, only soft sets with the parameter set K are included, while in the collection $S_E(U)$, soft sets over U with any parameter set can be included. Clearly, the set $S_K(U)$ is a subset of the set $S_E(U)$.

Definition 2.2. Let (F,D) be a soft set over U. If F(ℵ)=∅ or all ℵ∈D, then the soft set (F,D) is called a null soft set with respect to D, denoted by $\emptyset_D$. Similarly, let (F,E) be a soft set over U. If F(e)=∅ for all ℵ∈E, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ [4].

A soft set can be defined as F: ∅ ⟶ P(U), where U is a universal set. Such a soft set is called an empty soft set and is denoted by $\emptyset_\emptyset$. Thus, $\emptyset_\emptyset$ is the only soft set with an empty parameter set [6].

Definition 2.3. Let (F,D) be a soft set over U. If F(ℵ)=U for all ℵ∈D, then the soft set (F,K) is called a relative whole soft set with respect to D, denoted by U_D. Similarly, let (F,E) be a soft set over U. If F(e)=U for all ℵ∈E, then the soft set (F,E) is called a whole soft set with respect to E, denoted by U_E [4].

Definition 2.4. Let (F,D) and (G,Y) be soft sets over U. If D⊆Y and F(ℵ) ⊆G(ℵ) for all ℵ∈D, then (F,D) is said to be a soft subset of (G,Y), denoted by (F,D)$\widetilde{\subseteq}$(G,Y). If (G,Y) is a soft subset of (F,D), then (F,D) is said to be a soft superset of (G,Y), denoted by (F,D)$\widetilde{\supseteq}$(G,Y). If (F,D)$\widetilde{\subseteq}$(G,Y) and (G,Y)$\widetilde{\subseteq}$(F,D), then (F,D) and (G,Y) are called soft equal sets [3].

Definition 2.5. Let (F,D) be a soft set over U. The soft complement of (F,D), denoted by $(F,D)^r=(F^r,D)$, is defined as follows: for all ℵ∈D, F^r(ℵ)=U-F(ℵ) [4]

Çağman [11] introduced two new complements as a novel concept in set theory, termed as the inclusive complement and exclusive complement. For ease of representation, we denote these binary operations as + and θ, respectively. For two sets D and Y, these binary operations are defined as D+Y=D'∪Y, DθY=D'∩Y'. Sezgin et al. [12] examined the relations between these two operations and also defined three new binary operations and analyzed their relations with each other. Let D and Y be two sets D*Y=D'∪Y', DγY= D'∩Y, DλY=D∪Y'.

As a summary for soft set operations, we can categorize all types of soft set operations as follows: Let "⍟" be used to represent the set operations (i.e., here ⍟ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), then all type of soft set operations are defined as follows:

Definition 2.6. Let $(F,D),(G,Y) \in S_E(U)$. The restricted ⍟ operation of (F, D) and (G, Y) is the soft set (H,P), denoted by $(F,D)⍟_{\mathfrak{R}}(G,Y)=(H,P)$, where P=D ∩ Y ≠ ∅ and H(ℵ)=F(ℵ)⍟G(ℵ) for all ℵ∈P. Here, if P=D∩Y=∅, then $(F,D)⍟_R(G,Y)=\emptyset_\emptyset$ [4,5,6,13].

Definition 2.7. Let $(F,D),(G,Y) \in S_E(U)$. The extended ⍟ operation (F, D) and (G, Y) is the soft set (H, P), denoted by (F, D) $⍟_\varepsilon$(G, Y) = (H, P), where P = D ∪ Y and for all ℵ ∈ P,

$$H(\aleph)=\begin{cases}F(\aleph), & \aleph\in D-Y\\ G(\aleph), & \aleph\in Y-D\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[2,4,6,7,8,13].

Definition 2.8. Let $(F,D),(G,Y)\in S_E(U)$. The complementary extended $\circledast$ operation (F,D) and (G,Y) is the soft set (H,P), denoted by $(F,D)\overset{*}{\circledast}_{\varepsilon}(G,Y)=(H,P)$, where $P=D\cup Y$ and for all $\aleph\in P$,

$$H(\aleph)=\begin{cases}F'(\aleph), & \aleph\in D-Y\\ G'(\aleph), & \aleph\in Y-D\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[23,24].

Definition 2.9. Let $(F,D),(G,Y)\in S_E(U)$. The soft binary piecewise $\circledast$ of (F,D) and (G,Y) is the soft set (H,D), denoted by $(F,D)\underset{\circledast}{\sim}(G,Y)=(H,D)$, where for all $\aleph\in D$

$$H(\aleph)=\begin{cases}F(\aleph), & \aleph\in D-Y\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[9,10,33,34].

Definition 2.10. Let $(F,D),(G,Y)\in S_E(U)$. The complementary soft binary piecewise $\circledast$ of (F,D) and (G,Y) is the soft set (H,D), denoted by $(F,D)\overset{*}{\underset{\circledast}{\sim}}(G,Y)=(H,D)$, where for all $\aleph\in D$

$$H(\aleph)=\begin{cases}F'(\aleph), & \aleph\in D-Y\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[14-22].

For the considerations of graph applications and network analysis as regards soft sets, we refere to [35], which is motivated by the divisibility of determinants.

3. COMPLEMENTARY EXTENDED STAR OPERATION

In this section, a new soft set operation called complementary extended star operation of soft sets is introduced with its example and its full algebraic properties are analyzed.

Definition 3.1. Let (F,Z), (G,C) be soft sets over U. The complementary extended star operation $(*)$ operation of (F,Z) and (G,C) is the soft set (H,K), denoted by $(F,Z)\overset{*}{*}_{\varepsilon}(G,C)=(H,K)$, where for all $\aleph\in K=Z\cup C$,

$$H(\aleph)=\begin{cases}F'(\aleph), & \aleph\in Z-C\\ G'(\aleph), & \aleph\in C-Z\\ F(\aleph)*G(\aleph), & \aleph\in Z\cap C\end{cases}$$

where $F(\aleph)*G(\aleph)=F'(\aleph)\cup G'(\aleph)$ for all $\aleph\in Z\cap C$.

Example 3.2 Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set and $Z=\{e_1,e_3\}$ and $C=\{e_2,e_3,e_4\}$ be two subsets of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Assume that $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,C)=\{(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$ be two soft sets over U. Let $(F,Z)\overset{*}{*}_{\varepsilon}(G,C)=(L,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$L(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Here, since $Z\cup C=\{e_1,e_2,e_3,e_4\}$, $Z\text{-}C=\{e_1\}$, $C\text{-}Z=\{e_2,e_4\}$, $Z\cap C=\{e_3\}$, $L(e_1)=F'(e_1)=\{h_1,h_3,h_4\}$, $L(e_2)=G'(e_2)=\{h_2,h_3\}$, $L(e_4)=G'(e_4)=\{h_1,h_2,h_4\}$, $H(e_3)=F'(e_3)\cup G'(e_3)=\{h_3,h_4\}\cup\{h_1,h_5\}=\{h_1,h_3,h_4,h_5\}$. Hence,

$$(F,Z)\overset{*}{*_{\varepsilon}}(G,C)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_2,h_3\}),(e_3,\{h_1,h_3,h_4,h_5\}),(e_4,\{h_1,h_2,h_4\})\}$$

Theorem 3.3. (Algebraic Properties of Operation)

1) The set $S_E(U)$ is closed under $\overset{*}{*_{\varepsilon}}$.

***Proof*:** It is clear that $\overset{*}{*_{\varepsilon}}$ is a binary operation on $S_E(U)$. Indeed,

$$\overset{*}{*_{\varepsilon}}: S_E(U)\times S_E(U)\to S_E(U)$$

$$((F,Z),(G,C))\to (F,Z)\overset{*}{*_{\varepsilon}}(G,C)=(L,Z\cup C)$$

Similarly,

$$\overset{*}{*_{\varepsilon}}: S_Z(U)\times S_Z(U)\to S_Z(U)$$

$$((F,Z),(G,Z))\to (F,Z)\overset{*}{*_{\varepsilon}}(G,Z)=(T,Z\cup Z)=(T,Z)$$

That is, when Z is a fixed subset of the set E and (F,Z) and (G,Z) be elements of $S_Z(U)$, then so is $(F,Z)\overset{*}{*_{\varepsilon}}(G,Z)$. Namely, $S_Z(U)$ is closed under $\overset{*}{*_{\varepsilon}}$ either.

2) $[(F,Z)\overset{*}{*_{\varepsilon}}(G,C)]\overset{*}{*_{\varepsilon}}(H,R)\neq(F,Z)\overset{*}{*_{\varepsilon}}[(G,C)\overset{*}{*_{\varepsilon}}(H,R)]$.

Proof : First, let's handle the left hand side (LHS). Let $(F,Z)\overset{*}{*_{\varepsilon}}(G,C)=(T,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(T,Z\cup C)\overset{*}{*_{\varepsilon}}(H,R)=(M,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in(Z\cup C)\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}(Z\cup C) \\ T'(\aleph)\cup H'(\aleph), & \aleph\in(Z\cup C)\cap R \end{cases}$$

Thus,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}C)\text{-}R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C\text{-}Z)\text{-}R=Z'\cap C\cap R' \\ F(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)\text{-}R=Z\cap C\cap R' \\ H'(\aleph) & \aleph\in R\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}C)\cap R=Z\cap C'\cap R \\ G(\aleph)\cup H'(\aleph), & \aleph\in(C\text{-}Z)\cap R=Z'\cap C\cap R \\ \big(F(\aleph)\cap G(\aleph)\big)\cup H'(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now let's handle the right hand side (RHS) of the equation. Let (G,C) $\underset{*_\varepsilon}{*}$(H,R)=(K,C∪R). Here, for all ℵ∈C∪R,

$$K(ℵ)=\begin{cases} G'(ℵ), & ℵ\in C-R \\ H'(ℵ), & ℵ\in R-C \\ G'(ℵ)\cup H'(ℵ), & ℵ\in C\cap R \end{cases}$$

Assume that (F,Z) $\underset{*_\varepsilon}{*}$ (K,C∪R) = (S,Z∪C∪R), where for all ℵ∈Z∪C∪R,

$$S(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-(C\cup R) \\ K'(ℵ), & ℵ\in (C\cup R)-Z \\ F'(ℵ)\cup K'(ℵ), & ℵ\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$S(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-(C\cup R)=Z\cap C'\cap R' \\ G(ℵ), & ℵ\in (C-R)-Z=Z'\cap C\cap R' \\ H(ℵ) & ℵ\in (R-C)-Z=Z'\cap C'\cap R \\ G(ℵ)\cap H(ℵ), & ℵ\in (C\cap R)-Z=Z'\cap C\cap R \\ F'(ℵ)\cup G(ℵ), & ℵ\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(ℵ)\cup H(ℵ), & ℵ\in Z\cap(R-C)=Z\cap C'\cap R \\ F'(ℵ)\cup(G(ℵ)\cap H(ℵ)), & ℵ\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that M≠ S. That is, in the set $S_E(U)$, $\underset{*_\varepsilon}{*}$ is not associative.

3) [(F,Z) $\underset{*_\varepsilon}{*}$(G,Z)] $\underset{*_\varepsilon}{*}$(H,Z) ≠ (F,Z) $\underset{*_\varepsilon}{*}$[(G,Z) $\underset{*_\varepsilon}{*}$(H,Z)].

Proof : Firstly, let's look at the LHS. Let (F,Z) $\underset{*_\varepsilon}{*}$ (G,Z)=(T,Z∪Z), where for all ℵ ∈ Z∪Z=Z,

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-Z=\emptyset \\ G'(ℵ), & ℵ\in Z-Z=\emptyset \\ F'(ℵ)\cup G'(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Let (T,Z) $\underset{*_\varepsilon}{*}$ (H,Z) = (M, Z∪Z), where for all ℵ∈Z,

$$M(ℵ)=\begin{cases} T'(ℵ), & ℵ\in Z-Z=\emptyset \\ H'(ℵ), & ℵ\in Z-Z=\emptyset \\ T'(ℵ)\cup H'(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Thus,

$$M(ℵ)=\begin{cases} T'(ℵ), & ℵ\in Z-Z=\emptyset \\ H'(ℵ), & ℵ\in Z-Z=\emptyset \\ (F(ℵ)\cap G(ℵ))\cup H'(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Now, let's handle RHS. Let (G,Z) $\underset{*_\varepsilon}{*}$ (H,Z)=(L, Z∪Z), where for all ℵ∈Z,

$$L(ℵ)=\begin{cases} G'(ℵ), & ℵ\in Z-Z=\emptyset \\ H'(ℵ), & ℵ\in Z-Z=\emptyset \\ G'(ℵ)\cup H'(ℵ), & ℵ\in Z\cap Z=Z \end{cases}$$

Let (F,Z) $\overset{*}{*_\varepsilon}$ (L,Z) = (N,Z∪Z), where for all ℵ∈Z,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ∈Z\text{-}Z=\emptyset \\ L'(ℵ), & ℵ∈Z\text{-}Z=\emptyset \\ F'(ℵ)∪L'(ℵ), & ℵ∈Z∩Z=Z \end{cases}$$

Hence,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ∈Z\text{-}Z=\emptyset \\ L'(ℵ), & ℵ∈Z\text{-}Z=\emptyset \\ F'(ℵ)∪(G(ℵ)∩H(ℵ)), & ℵ∈Z∩Z=Z \end{cases}$$

It is seen that M≠N. That is, in the set $S_Z(U)$, $\overset{*}{*_\varepsilon}$ does not have associative property, where Z is a fixed subset of E.

4) (F,Z) $\overset{*}{*_\varepsilon}$(G,C)=(G,C) $\overset{*}{*_\varepsilon}$ (F,Z).

Proof: Firstly, we observe that the parameter set of the soft set on both sides of the equation is Z∪C, and thus the first condition of the soft equality is satisfied. Now let us look at the LHS. Let (F,Z) $\overset{*}{*_\varepsilon}$ (G,C)=(H,Z∪C), where for all ℵ∈Z∪C,

$$H(ℵ)=\begin{cases} F'(ℵ), & ℵ∈Z\text{-}C \\ G'(ℵ), & ℵ∈C\text{-}Z \\ F'(ℵ)∪G'(ℵ), & ℵ∈Z∩C \end{cases}$$

Now let's handle the RHS. Assume that (G,C) $\overset{*}{*_\varepsilon}$(F,Z)=(T,C∪Z), where for all ℵ∈C∪Z,

$$T(ℵ)=\begin{cases} G'(ℵ), & ℵ∈C\text{-}Z \\ F'(ℵ), & ℵ∈Z\text{-}C \\ G'(ℵ)∪F'(ℵ), & ℵ∈C∩Z \end{cases}$$

Thus, it is seen that H=T. Similarly, it is easily seen that (F,Z) $\overset{*}{*_\varepsilon}$(G,Z)= (G,Z) $\overset{*}{*_\varepsilon}$(F,Z). That is, $\overset{*}{*_\varepsilon}$ operation is not commutative in both $S_E(U)$ and $S_Z(U)$.

5) (F,Z) $\overset{*}{*_\varepsilon}$(F,Z)=$(F,Z)^r$.

***Proof*:** Let (F,Z) $\overset{*}{*_\varepsilon}$ (F,Z)=(H,Z∪Z), where for all ℵ∈Z,

$$H(ℵ)=\begin{cases} F'(ℵ), & ℵ∈Z\text{-}Z=\emptyset \\ F'(ℵ), & ℵ∈Z\text{-}Z=\emptyset \\ F'(ℵ)∪F'(ℵ), & ℵ∈Z∩Z=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)=F'(ℵ)∪F'(ℵ)=F'(ℵ), and so (H,Z)= $(F,Z)^r$ That is, $\overset{*}{*_\varepsilon}$ is not idempotent in $S_E(U)$.

6) (F,Z) $\overset{*}{*_\varepsilon}$ $\emptyset_Z$=$\emptyset_Z$ $\overset{*}{*_\varepsilon}$(F,Z)= U_Z.

Proof: Let $\emptyset_Z$=(T,Z). Thus, for all ℵ∈Z, T(ℵ)=∅. Let (F, Z) $\overset{*}{*_\varepsilon}$(T, Z) =(H,Z∪Z), where for all ℵ∈Z,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ T'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cup T'(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)= F'(ℵ)∪T'(ℵ)= F'(ℵ)∪ U= U and so (H,Z)= U_Z.

7) (F,Z) $\overset{*}{*_\varepsilon}$ $\emptyset_E=\emptyset_E$ $\overset{*}{*_\varepsilon}$(F,Z) =$U_E$.

***Proof*:** Let $\emptyset_E$ =(S,E). Thus, for all ℵ∈E, S(ℵ)= ∅**.** Let (F,Z) $\overset{*}{*_\varepsilon}$ (S,E)=(H,Z∪E), where for all ℵ∈E,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}E=\emptyset \\ S'(\aleph), & \aleph\in E\text{-}Z=Z' \\ F'(\aleph)\cup S'(\aleph), & \aleph\in Z\cap E=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)= F'(ℵ)∪S'(ℵ)=F'(ℵ)∪U=U. Thus,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}E=\emptyset \\ U, & \aleph\in E\text{-}Z=Z' \\ U, & \aleph\in Z\cap E=Z \end{cases}$$

Thus, (H,Z)= U_E.

8) (F,Z) $\overset{*}{*_\varepsilon}$ $\emptyset_\emptyset=\emptyset_\emptyset$ $\overset{*}{*_\varepsilon}$(F,Z) = $(F,Z)^r$

***Proof*:** Let $\emptyset_\emptyset$= (K, ∅) and (F,Z) $\overset{*}{*_\varepsilon}$ (K, ∅) = (Q,Z ∪ ∅) = (Q,Z), where for all ℵ∈Z,

$$Q(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}\emptyset=Z \\ K'(\aleph), & \aleph\in \emptyset\text{-}Z=\emptyset \\ F'(\aleph)\cup K'(\aleph), & \aleph\in Z\cap\emptyset=\emptyset \end{cases}$$

Hence, for all ℵ∈Z, Q(ℵ)= F'(ℵ) and thus (Q,Z) = $(F,Z)^r$. Similarly, let $\emptyset_\emptyset$ $\overset{*}{*_\varepsilon}$(F,Z)= (W, ∅ ∪ Z) = (W,Z), where for all ℵ∈Z,

$$W(\aleph)=\begin{cases} K'(\aleph), & \aleph\in \emptyset\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}\emptyset=Z \\ K'(\aleph)\cup F'(\aleph), & \aleph\in \emptyset\cap Z=\emptyset \end{cases}$$

Hence, for all ℵ∈Z, W(ℵ)= F'(ℵ), and thus (W,Z) = $(F,Z)^r$.

9) (F,Z) $\overset{*}{*_\varepsilon}$ $U_Z=U_Z$ $\overset{*}{*_\varepsilon}$(F,Z)= $(F,Z)^r$.

***Proof*:** Let U_Z =(H,Z), where for all ℵ∈Z, H(ℵ)=U. Let (F,Z) $\overset{*}{*_\varepsilon}$(H,Z)=(T,Z∪Z) where for all ℵ∈Z,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all ℵ∈Z, T(ℵ)= F'(ℵ)∪H'(ℵ)= F'(ℵ) ∪ ∅=F'(ℵ) , and thus (T,Z)= $(F,Z)^r$. Similarly, let U_Z $\overset{*}{*_\varepsilon}$(F,Z)= (W, Z ∪ Z) = (W,Z), where for all ℵ∈Z,

$$W(\aleph)=\begin{cases} H'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph), & \aleph\in Z-Z=\emptyset \\ H'(\aleph)\cup F'(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $W(\aleph)= H'(\aleph)\cup F'(\aleph)=\emptyset) \cup F'(\aleph) = F'(\aleph)$, and thus $(W,Z)= (F,Z)^r$.

10) $(F,Z) \overset{*}{*_\varepsilon} (F,Z)^r = (F,Z)^r \overset{*}{*_\varepsilon} (F,Z)= U_Z$.

***Proof*:** Let $(F,Z)^r = (H,Z)$, where for all $\aleph\in Z$, $H(\aleph)=F'(\aleph)$. Let $(F,Z) \overset{*}{*_\varepsilon} (H,Z)=(T,Z\cup Z)$ where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ H'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= F'(\aleph)\cup H'(\aleph)= F'(\aleph) \cup F(\aleph)=U$, and thus $(T,Z)= U_Z$.

11) $[(F,Z) \overset{*}{*_\varepsilon} (G,C)]^r = (F,Z) \cap_\varepsilon (G,C)$.

***Proof*:** Let $(F,Z) \overset{*}{*_\varepsilon} (G,C)=(H,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(H,Z\cup C)^r = (T,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in Z-C \\ G(\aleph), & \aleph\in C-Z \\ F(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Hence, $(T,Z\cup C) = (F,Z) \cap_\varepsilon (G,C)$.

12) $(F,Z) \overset{*}{*_\varepsilon} (G,C)= \emptyset_{Z\cup C} \Leftrightarrow (F,Z) = U_Z$ and $(G,C) = U_C$.

Proof: Let $(F,Z) \overset{*}{*_\varepsilon} (G,C) = (T,Z\cup C)$. Here, for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Since $(T,Z\cup C)= \emptyset_{Z\cup C}$, $T(\aleph)=\emptyset$ for all $\aleph\in Z\cup C$. Hence, for all $\aleph\in Z-C$, $F'(\aleph)=\emptyset$; for all $\aleph\in C-Z$, $G'(\aleph)=\emptyset$ and for all $\aleph\in Z\cap C$, $F'(\aleph)\cup G'(\aleph)= \emptyset$. Thus, for all $\aleph \in Z - C$, $F(\aleph)=U$, for all $\aleph \in C - Z$, $G(\aleph)=U$, for all $\aleph \in Z \cap C$, $F'(\aleph) = \emptyset$, $G'(\aleph) = \emptyset$. Thus, for all $\aleph \in Z \cap C$, $F(\aleph) = U$, $G(\aleph) = U$. Thus, for all $\aleph\in Z$, $F(\aleph)=U$ and for all $\aleph\in C$, $G(\aleph)=U$. Thus, $(F,Z) = U_Z$ and $(G,C) = U_C$.

13) $\emptyset_Z \tilde{\subseteq} (F,Z) \overset{*}{*_\varepsilon} (G,C)$, $\emptyset_C \tilde{\subseteq} (F,Z) \overset{*}{*_\varepsilon} (G,C)$, $\emptyset_{Z\cup C} \tilde{\subseteq} (F,Z) \overset{*}{*_\varepsilon} (G,C)$, $(F,Z) \overset{*}{*_\varepsilon} (G,C) \tilde{\subseteq} U_{Z\cup C}$.

14) $(F,Z) \,{}^r\tilde{\subseteq}\, (F,Z) \overset{*}{*_\varepsilon} (G,C)$ and $(G,C) \,{}^r\tilde{\subseteq}\, (F,Z) \overset{*}{*_\varepsilon} (G,C)$.

***Proof*:** Let $(F,Z) \overset{*}{*_\varepsilon} (G,C)=(H, Z \cup C)$ where $Z\subseteq Z \cup C$ and for all $\aleph\in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Here, for all $\aleph\in Z-C$, $F'(\aleph) \subseteq F'(\aleph)=H(\aleph)$ and for all $\aleph\in Z\cap C$, $F'(\aleph) \subseteq F'(\aleph) \cup G'(\aleph)=H(\aleph)$. Thus for all $\aleph\in Z$, $F'(\aleph)\subseteq H(\aleph)$. So $(F,Z)^r \tilde{\subseteq} (F,Z) \overset{*}{*_{\varepsilon}} (G,C)$. Similarly, for all $\aleph\in C-Z$, $G'(\aleph)\subseteq G'(\aleph)=H(\aleph)$ and for all $\aleph\in C\cap Z$, $G'(\aleph)\subseteq F'(\aleph) \cup G'(\aleph)=H(\aleph)$. Thus for all $\aleph\in C$, $G'(\aleph)\subseteq H(\aleph)$. So $(G,C)^r \tilde{\subseteq} (F,Z) \overset{*}{*_{\varepsilon}} (G,C)$.

15) If $(F,Z) \tilde{\subseteq} (G,Z)$ then $(F,Z) \overset{*}{*_{\varepsilon}} (G,Z) =(F,Z)^r$.

***Proof*:** Let $(F,Z)\tilde{\subseteq} (G,Z)$. Then, for all $\aleph\in Z$, $F(\aleph) \subseteq G(\aleph)$. Let $(F,Z) \overset{*}{*_{\varepsilon}} (G,Z)=(H,Z\cup Z)$, where for all $\aleph\in Z$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ G'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence, $F(\aleph)\subseteq G(\aleph)$ for all $\aleph\in Z$ and so $G'(\aleph)\subseteq F'(\aleph)$. Thus, for all $\aleph\in Z$, $H(\aleph)=F'(\aleph) \cup G'(\aleph)=F'(\aleph)$. Therefore, $(F,Z) \overset{*}{*_{\varepsilon}} (G,Z)=(F,Z)^r$.

16) If $(F,Z) \tilde{\subseteq}(G,Z)$, then $(G,Z) \overset{*}{*_{\varepsilon}} (H,C) \tilde{\subseteq} (F,Z) \overset{*}{*_{\varepsilon}} (H,C)$.

***Proof*:** Let $(F,Z) \tilde{\subseteq} (G,Z)$. Then, $F(\aleph) \subseteq G(\aleph)$ for all $\aleph\in Z$, and so $G'(\aleph)\subseteq F'(\aleph)$. Let $(G,Z) \overset{*}{*_{\varepsilon}} (H,C)=(W,Z\cup C)$ where for all $\aleph\in Z\cup C$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in Z-C \\ H'(\aleph), & \aleph\in C-Z \\ G'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z) \overset{*}{*_{\varepsilon}} (H,C) = (L,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$L(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ H'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus, $W(\aleph)=G'(\aleph) \subseteq G'(\aleph)=L(\aleph)$ for all $\aleph\in Z-C$; $W(\aleph)=H'(\aleph) \subseteq H'(\aleph)=L(\aleph)$ for all $\aleph\in C-Z$; $W(\aleph)= G'(\aleph)\cup H'(\aleph) \subseteq F'(\aleph)\cup H'(\aleph)=L(\aleph)$ for all $\aleph\in Z\cap C$. Thus, $(G,Z) \overset{*}{*_{\varepsilon}} (H,C) \tilde{\subseteq} (F,Z) \overset{*}{*_{\varepsilon}} (H,C)$.

17) If $(G,Z) \overset{*}{*_{\varepsilon}} (H,C) \tilde{\subseteq} (F,Z) \overset{*}{*_{\varepsilon}} (H,C)$, then $(F,Z)\tilde{\subseteq}(G,Z)$ need not to hold. That is, the converse of Theorem 3.3. (16) is not true.

***Proof*:** Let us give an example to show that the converse of Theorem 3.3. (16) is not true. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, $A=\{e_1,e_3\}$, $C=\{e_1,e_3,e_5\}$ be the subset of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set.

Let $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,Z)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$, $H,C)= \{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_2,h_3,h_4\})\}$ be soft sets over U. Let $(G,Z) \overset{*}{*_{\varepsilon}} (H,C)=(L,Z\cup C)$, then $(L,Z\cup C)= (L,Z\cup C)=\{(e_1,U),(e_3,U),(e_5,\{h_1,h_5\})\}$ and let $(F,Z) \overset{*}{*_{\varepsilon}} (H,C)=(K,Z\cup C)$, thus $(K,Z\cup C)=\{ (K,Z\cup C)=\{(e_1,U),(e_3,U),(e_5,\{h_1,h_5\})\}$. Hence, $(G,Z) \overset{*}{*_{\varepsilon}} (H,C) \tilde{\subseteq} (F,Z) \overset{*}{*_{\varepsilon}} (H,C)$ but (F,Z) is not a soft subnset of (G,Z).

18) If (F,Z) $\cong$ (G,Z) and (K,C) $\cong$ (L,C), then (G,Z) $\overset{*}{*_\varepsilon}$(L,C) $\cong$(F,Z) $\overset{*}{*_\varepsilon}$(K,C).

***Proof*:** Let (F,Z)$\cong$ (G,Z) and (K,C) $\cong$ (L,C). Hence, Z⊆C and for all ℵ∈Z, F(ℵ)⊆ G(ℵ) and G'(ℵ)⊆ F′(ℵ) and for all ℵ∈C, K(ℵ)⊆ L(ℵ) and L'(ℵ)⊆ K′(ℵ) Let (G,Z) $\overset{*}{*_\varepsilon}$(L,C) = (W,Z∪C). Thus, for all ℵ∈ Z∪C,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in Z-C \\ L'(\aleph), & \aleph\in C-Z \\ G'(\aleph)\cup L'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let (F,Z) $\overset{*}{*_\varepsilon}$(K,C) = (S,Z∪C). Thus, for all ℵ∈Z∪C,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ K'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup K'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Hence, since for all ℵ∈ Z-C, W(ℵ)=G'(ℵ)⊆F'(ℵ)=S(ℵ),for all ℵ∈ C-Z, W(ℵ)=L'(ℵ)⊆K'(ℵ)=S(ℵ),for all ℵ∈ Z∩C, W(ℵ)=G'(ℵ)∪L'(ℵ) ⊆ F'(ℵ)∪K'(ℵ)=S(ℵ) , (G,Z) $\overset{*}{*_\varepsilon}$(L,C) $\cong$ (F,Z) $\overset{*}{*_\varepsilon}$(K,C).

Theorem 3.4. The complementary extended star operation has the following distributions over other soft set operations:

Theorem 3.4.1. The complementary extended star operation has the following distributions over restricted soft set operations:

i) LHS Distributions of the Complementary Extended Star Operation on Restricted Soft Set Operations:

1) If Z'∩C∩R= Z∩C∩R=∅ then (F,Z)$\overset{*}{*_\varepsilon}$ [(G,C)$\cap_R$(H,R)]=[(F,Z)$\overset{*}{*_\varepsilon}$ (G,C)]$\cap_R$[(F,Z)$\overset{*}{*_\varepsilon}$(H,R)].

Proof: Consider the LHS first. Let (G,C)$\cap_R$(H,R)=(M,C∩R), where for all ℵ∈C∩R, M(ℵ) = G(ℵ) ∩ H(ℵ). Let (F,Z)$\overset{*}{*_\varepsilon}$ (M,C∩R)=(N,Z∪(C∩R)), where for all ℵ∈Z∪(C∩R),

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cap R) \\ M'(\aleph), & \aleph\in (C\cap R)-Z \\ F'(\aleph)\cup M'(\aleph), & \aleph\in Z\cap(C\cap R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cap R)=Z-(C\cap R) \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cup(G'(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now let',s handle the RHS i.e. [(F,Z)$\overset{*}{*_\varepsilon}$ (G,C)]$\cap_R$[(F,Z)$\overset{*}{*_\varepsilon}$ (H,R)]. Let (F,Z)$\overset{*}{*_\varepsilon}$ (G,C)=(V,Z∪C), where for all ℵ∈Z∪C,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Assume that (F,Z) $\overset{*}{*_\varepsilon}$ (H,R)=(W,Z∪R), where for all ℵ∈Z∪R,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\cap_R(W,Z\cup R)=(T,(Z\cup C)\cap(Z\cup R)))$, where for all $\aleph\in Z\cup(C\cap R)$, $T(\aleph)=V(\aleph)\cap W(\aleph)$. Hence,

$$T(\aleph)=\begin{cases} F'(\aleph)\cap F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in(Z-C)\cap(R-Z)=\emptyset \\ F'(\aleph)\cap(F'(\aleph)\cup H'(\aleph)) & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap F'(\aleph), & \aleph\in(C-Z)\cap(Z-R)=\emptyset \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cap(F'(\aleph)\cup H'(\aleph)), & \aleph\in(C-Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cap F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cap H'(\aleph), & \aleph\in(Z\cap C)\cap(R-Z)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cap(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph) & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(\aleph) & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cap(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, when considering $Z-(C\cap R)$ in the function N, since $Z-(C\cap R)=Z\cap(C\cap R)'$, if an element is in the complement of $(C\cap R)$, it is either in C-R, in R-C, or in $(C\cup R)'$. Thus, if $\aleph\in Z-(C\cap R)$, then $\aleph\in Z\cap C\cap R'$ or $\aleph\in Z\cap C'\cap R$ or $\aleph\in Z\cap C'\cap R'$. Thus, N=T under the condition $Z'\cap C\cap R=\emptyset$.

2) $(F,Z)\overset{*}{*_{\varepsilon}}[(G,C)\cup_R(H,R)]=[(F,Z)\overset{*}{*_{\varepsilon}}(G,C)]\cap_R[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]$.

3) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{*_{\varepsilon}}[(G,C)*_R(H,R)]=[(F,Z)\overset{*}{+_{\varepsilon}}(G,C)]\cap_R[(F,Z)\overset{*}{+_{\varepsilon}}(H,R)]$.

4) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{*_{\varepsilon}}[(G,C)\theta_R(H,R)]=[(F,Z)\overset{*}{+_{\varepsilon}}(G,C)]\cap_R[(F,Z)\overset{*}{+_{\varepsilon}}(H,R)]$.

ii) RHS Distribution of Complementary Extended Star Operation on Restricted Soft Set Operations

1) $[(F,Z)\cup_R(G,C)]\overset{*}{*_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]\cap_R[(G,C)\overset{*}{*_{\varepsilon}}(H,R)]$.

Proof: Consider the LHS first. Let $(F,Z)\cup_R(G,C)=(M,Z\cap C)$, where for all $\aleph\in Z\cap C$, $M(\aleph)=F(\aleph)\cup G(\aleph)$. Let $(M,Z\cap C)\overset{*}{*_{\varepsilon}}(H,R)=(N,(Z\cap C)\cup R))$, where $\aleph\in(Z\cap C)\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in(Z\cap C)-R \\ H'(\aleph), & \aleph\in R-(Z\cap C) \\ M'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap(C\cap R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph)\cap G'(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-(Z\cap C)=R-(Z\cap C) \\ (F'(\aleph)\cap G'(\aleph))\cup H'(\aleph), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS. i.e. $[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]\cap_R[(G,C)\overset{*}{*_{\varepsilon}}(H,R)]$. Let $(F,Z)\overset{*}{*_{\varepsilon}}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ∈Z-R \\ H'(ℵ), & ℵ∈R-Z \\ F'(ℵ)∪H'(ℵ), & ℵ∈Z∩R \end{cases}$$

Now, let $(G,C)\overset{*}{*_ε}(H,R)=(W, C∪R)$, where for all ℵ∈C∪R,

$$W(ℵ)=\begin{cases} G'(ℵ), & ℵ∈C-R \\ H'(ℵ), & ℵ∈R-C \\ G'(ℵ)∪H'(ℵ), & ℵ∈C∩R \end{cases}$$

Let $(V,Z∪R) ∩_R (W,C∪R)=(T,(Z∪R)∩(C∪R))$. Here, for all ℵ∈(Z∩C)∪R, T(ℵ)=V(ℵ)∩W(ℵ).Thus,

$$T(ℵ)=\begin{cases} F'(ℵ)∩G'(ℵ), & ℵ∈(Z-R)∩(C-R)=Z∩C∩R' \\ F'(ℵ)∩H'(ℵ), & ℵ∈(Z-R)∩(R-C)=∅ \\ F'(ℵ)∩(G'(ℵ)∪H'(ℵ)), & ℵ∈(Z-R)∩(C∩R)=∅ \\ H'(ℵ)∩G'(ℵ), & ℵ∈(R-Z)∩(C-R)=∅ \\ H'(ℵ)∩H'(ℵ), & ℵ∈(R-Z)∩(R-C)=Z'∩C'∩R \\ H'(ℵ)∩(G'(ℵ)∪H'(ℵ)), & ℵ∈(R-Z)∩(C∩R)=Z'∩C∩R \\ (F'(ℵ)∪H'(ℵ))∩G'(ℵ), & ℵ∈(Z∩R)∩(C-R)=∅ \\ (F'(ℵ)∪H'(ℵ))∩H'(ℵ) & ℵ∈(Z∩R)∩(R-C)=Z∩C'∩R \\ (F'(ℵ)∪H'(ℵ))∩(G'(ℵ)∪H'(ℵ)), & ℵ∈(Z∩C)∩(Z∩R)=Z∩C∩R \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F'(ℵ)∩G'(ℵ), & ℵ∈(Z-R)∩(C-R)=Z∩C∩R' \\ H'(ℵ) & ℵ∈(R-Z)∩(R-C)=Z'∩C'∩R \\ H'(ℵ), & ℵ∈(R-Z)∩(C∩R)=Z'∩C∩R \\ H'(ℵ), & ℵ∈(Z∩R)∩(R-C)=Z∩C'∩R \\ (F'(ℵ)∪H(ℵ))∩(G'(ℵ)∪H(ℵ)), & ℵ∈(Z∩C)∩(Z∩R)=Z∩C∩R \end{cases}$$

Here, if we consider R-(Z∩C) in the function N, since R-(Z∩C)=R∩(Z∩C)', if an element is in the complement of (Z∩C), it is either in Z-C, in C-Z, or in $(Z∪C)'$. Thus, if ℵ ∈ R −(Z∩C), ℵ ∈R∩Z∩C' or ℵ ∈R∩Z'∩C or ℵ ∈R∩Z'∩C'. Hence, N=T is satisfied under the condition Z'∩C∩R=Z∩C'∩R= ∅. It is obvious that the condition Z'∩C∩R=Z∩C'∩R=∅ implies that (ZΔC)∩R=∅

2) If Z∩C∩R'=Z∩C∩R=∅,then $[(F,Z)∩_R(G,C)]\overset{*}{*_ε}(H,R)=[(F,Z)\overset{*}{*_ε}(H,R)]∩_R[(G,C)\overset{*}{*_ε}(H,R)]$.

3) If Z∩C∩R'=∅ , then $[(F,Z)*_R(G,C)]\overset{*}{*_ε}(H,R)=[(F,Z)\overset{*}{λ_ε}(H,R)]∩_R[(G,C)\overset{*}{λ_ε}(H,R)]$.

4) If Z∩C∩R'=Z∩C∩R=∅ , then $[(F,Z)θ_R(G,C)]\overset{*}{*_ε}(H,R)=[(F,Z)\overset{*}{λ_ε}(H,R)]∩_R[(G,C)\overset{*}{λ_ε}(H,R)]$.

Theorem 3.4.2. The following distributions of the complementary extended star operation over extended soft set operations hold:

i)LHS Distributions of the Complementary Extended Star Operation on Extended Soft Set Operations

1)If (ZΔC)∩R=Z∩C∩R'= ∅, then $(F,Z)\overset{*}{*_ε}[(G,C)*_ε(H,R)]=[(F,Z)\overset{*}{+_ε}(G,C)]∩_ε[(F,ZR\overset{*}{+_ε}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)*_ε(H,R)=(M,C∪R)$, where for all ℵ ∈ C∪R,

$$M(ℵ)=\begin{cases} G(ℵ), & ℵ∈C-R \\ H(ℵ), & ℵ∈R-C \\ G'(ℵ)∪H'(ℵ), & ℵ∈C∩R \end{cases}$$

Let $(F,Z)\overset{*}{*_ε}(M,C∪R)=(N,Z∪(C∪R))$, where for all ℵ∈ Z∪C∪R,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R) \\ M'(\aleph), & \aleph\in(C\cup R)-Z \\ F'(\aleph)\cup M'(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R)=Z\cap C'\cap R' \\ G'(\aleph), & \aleph\in(C-R)-Z=Z'\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-C)-Z=Z\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in(C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap(R-C)=Z\cap C'\cap R \\ F'(\aleph)\cup(G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{+}_{\varepsilon}(G,C)]\cap_{\varepsilon}[(F,Z)\overset{*}{+}_{\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{+}_{\varepsilon}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\overset{*}{+}_{\varepsilon}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cup H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\cap_{\varepsilon}(W,Z\cup R)=(T,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup C)-(Z\cup R) \\ W(\aleph), & \aleph\in(Z\cup R)-(Z\cup C) \\ V(\aleph)\cap W(\aleph), & \aleph\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-C)-(Z\cup R)=\emptyset \\ G'(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)-(Z\cup R)=\emptyset \\ F'(\aleph), & \aleph\in(Z-R)-(Z\cup C)=\emptyset \\ H'(\aleph), & \aleph\in(R-Z)-(Z\cup C)=Z'\cap C\cap R \\ F'(\aleph)\cup H(\aleph), & \aleph\in(Z\cap R)-(Z\cup C)=\emptyset \\ F'(\aleph)\cap F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in(Z-C)\cap(R-Z)=\emptyset \\ F'(\aleph)\cap(F'(\aleph)\cup H(\aleph)), & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap F'(\aleph), & \aleph\in(C-Z)\cap(Z-R)=\emptyset \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cap(F'(\aleph)\cup H(\aleph)), & \aleph\in(C-Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cup G(\aleph))\cap F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G(\aleph))\cap H'(\aleph), & \aleph\in(Z\cap C)\cap(R-Z)=\emptyset \\ (F'(\aleph)\cup G(\aleph))\cap(F'(\aleph)\cup H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Hence,

$$T(ℵ)=\begin{cases} G'(ℵ), & ℵ\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ H'(ℵ), & ℵ\in(R-Z)-(Z\cup C)=Z'\cap C'\cap R \\ F'(ℵ), & ℵ\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(ℵ)\cap H(ℵ) & ℵ\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cap H'(ℵ), & ℵ\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(ℵ)\cap G(ℵ) & ℵ\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(ℵ)\cup G(ℵ))\cap(F'(ℵ)\cup H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that N=T is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. I is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equal to $(Z\Delta C)\cap R=\emptyset$.

2) $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\cap_\varepsilon(H,R)]=[(F,Z)\overset{*}{*_\varepsilon}(G,C)]\cup_\varepsilon[(F,Z)\overset{*}{*_\varepsilon}(H,R)]$.

3) $Z\cap(C\Delta R)=\emptyset$, $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\cup_\varepsilon(H,R)]=[(F,Z)\overset{*}{*_\varepsilon}(G,C)]\cap_\varepsilon[(F,Z)\overset{*}{*_\varepsilon}(H,R)]$.

4) $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\theta_\varepsilon(H,R)]=[(F,Z)\overset{*}{+_\varepsilon}(G,C)]\cup_\varepsilon[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Star Operation over Extended Soft Set Operations

1)If $Z\cap C\cap R'=Z\cap C\cap R=\emptyset$, then $[(F,Z)\cup_\varepsilon(G,C)]\overset{*}{*_\varepsilon}(H,R)=[(F,Z)\overset{*}{*_\varepsilon}(H,R)]\cup_\varepsilon[(G,C)\overset{*}{*_\varepsilon}(H,R)]$.

Proof: Consider first the LHS. Let $(F,Z)\cup_\varepsilon(G,C)=(M,Z\cup C)$, where for all $ℵ\in Z\cup C$,

$$M(ℵ)=\begin{cases} F(ℵ), & ℵ\in Z-C \\ G(ℵ), & ℵ\in C-Z \\ F(ℵ)\cup G(ℵ), & ℵ\in Z\cap C \end{cases}$$

Let $(M,Z\cup C)\overset{*}{*_\varepsilon}(H,R)=(N,(Z\cup C)\cup R)$, where for all $ℵ\in Z\cup C\cup R$,

$$N(ℵ)=\begin{cases} M'(ℵ), & ℵ\in(Z\cup C)-R \\ H'(ℵ), & ℵ\in R-(Z\cup C) \\ M'(ℵ)\cup H'(ℵ), & ℵ\in(Z\cup C)\cap R \end{cases}$$

Thus,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-C)-R=Z\cap C'\cap R' \\ G'(ℵ), & ℵ\in(C-Z)-R=Z'\cap C\cap R' \\ F'(ℵ)\cap G'(ℵ), & ℵ\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(ℵ), & ℵ\in R-(Z\cup C)=Z'\cap C'\cap R \\ F'(ℵ)\cup H'(ℵ), & ℵ\in(Z-C)\cap R=Z\cap C'\cap R \\ G'(ℵ)\cup H'(ℵ), & ℵ\in(C-Z)\cap R=Z'\cap C\cap R \\ (F'(ℵ)\cap G'(ℵ))\cup H'(ℵ), & ℵ\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{*_\varepsilon}(H,R)]\cup_\varepsilon[(G,C)\overset{*}{*_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{*_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $ℵ\in Z\cup R$,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ F'(ℵ)\cup H'(ℵ) & ℵ\in Z\cap R \end{cases}$$

Let $(G,C)\overset{*}{*_\varepsilon}(H,R)=(W,C\cup R)$, where for all $ℵ\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G'(\aleph)\cup H'(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\cup_{\varepsilon}(W,C\cup R)=(T,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup R)-(C\cup R) \\ W(\aleph), & \aleph\in(C\cup R)-(Z\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ H'(\aleph), & \aleph\in(R-Z)-(C\cup R)=\emptyset \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)-(C\cup R)=\emptyset \\ G'(\aleph), & \aleph\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-C)-(Z\cup R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)-(Z\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-R)\cap(R-C)=\emptyset \\ F'(\aleph)\cup(G'(\aleph)\cup H'(\aleph)), & \aleph\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R-Z)\cap(C-R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G'(\aleph)\cup H'(\aleph)\big) & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ (F'(\aleph)\cup H'(\aleph))\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C-R)=\emptyset \\ (F'(\aleph)\cup H'(\aleph))\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(\aleph)\cup H'(\aleph))\cup(G'(\aleph)\cup H'(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ G'(\aleph), & \aleph\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph) & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ F'(\aleph)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(\aleph)\cup H'(\aleph))\cup(G'(\aleph)\cup H'(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Hence, under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$, N=T

2) $[(F,Z)\cap_{\varepsilon}(G,C)]\overset{*}{*_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]\cap_{\varepsilon}[(G,C)\overset{*}{*_{\varepsilon}}(H,R)]$.

3) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\theta_{\varepsilon}(G,C)]\overset{*}{*_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{\lambda_{\varepsilon}}(H,R)]\cup_{\varepsilon}[(G,C)\overset{*}{\lambda_{\varepsilon}}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)*_{\varepsilon}(G,C)]\overset{*}{*_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{\lambda_{\varepsilon}}(H,R)]\cap_{\varepsilon}[(G,C)\overset{*}{\lambda_{\varepsilon}}(H,R)]$.

Theorem 3.4.3. The following distributions of the complementary extended star operation over complementary extended operations hold:

i) LHS Distributions of Complementary Extended Star Operations over Complementary Extended Soft Set Operations

1) If $Z\cap(C\Delta R)=\emptyset$, then $(F,Z)\overset{*}{*_{\varepsilon}}[(G,C)\overset{*}{\cap_{\varepsilon}}(H,R)]=[(F,Z)\overset{*}{*_{\varepsilon}}(G,C)]\overset{*}{\cup_{\varepsilon}}[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)\overset{*}{\cap_{\varepsilon}}((H,R)=(M,C\cup R)$, where for all $\aleph\in C\cup R$,

$$M(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z)\overset{*}{*_{\varepsilon}}(M,C\cup R)=(N,Z\cup(C\cup R))$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R) \\ M'(\aleph), & \aleph\in (C\cup R)-Z \\ F'(\aleph)\cup M'(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R)=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in (C-R)-Z=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in (R-C)-Z=Z'\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cup G(\aleph), & \aleph\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup H(\aleph), & \aleph\in Z\cap(R-C)=Z\cap C'\cap R \\ F'(\aleph)\cup(G'(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{*_{\varepsilon}}(G,C)]\overset{*}{\cup_{\varepsilon}}[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]$. Let $(F,Z)\overset{*}{*_{\varepsilon}}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\overset{*}{*_{\varepsilon}}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\overset{*}{\cup_{\varepsilon}}(W,Z\cup R)=(T,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in (Z\cup C)-(Z\cup R) \\ W'(\aleph), & \aleph\in (Z\cup R)-(Z\cup C) \\ V(\aleph)\cup W(\aleph), & \aleph\in (Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in (Z-C)-(Z\cup R)=\emptyset \\ G(\aleph), & \aleph\in (C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F(\aleph)\cap G(\aleph), & \aleph\in (Z\cap C)-(Z\cup R)=\emptyset \\ F(\aleph), & \aleph\in (Z-R)-(Z\cup C)=\emptyset \\ H(\aleph), & \aleph\in (R-Z)-(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cap H(\aleph), & \aleph\in (Z\cap R)-(Z\cup C)=\emptyset \\ F'(\aleph)\cup F'(\aleph), & \aleph\in (Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in (Z-C)\cap(R-Z)=\emptyset \\ F'(\aleph)\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in (Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup F'(\aleph), & \aleph\in (C-Z)\cap(Z-R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in (C-Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cup F'(\aleph), & \aleph\in (Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cup H'(\aleph), & \aleph\in (Z\cap C)\cap(R-Z)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in (Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} G(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R-Z)-(Z\cup C)=Z'\cap C'\cap R \\ F'(\aleph) & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph) & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(\aleph)\cup G'(\aleph) & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cap(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

N=T is satisfied under the condition $Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z\cap C\cap R'=Z\cap C'\cap R=\emptyset$ is equivalent to $(Z\cap C)\,\Delta R=\emptyset$.

2) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\overset{*}{\cup_\varepsilon}(H,R)]=[(F,Z)\overset{*}{*_\varepsilon}(G,C)]\overset{*}{\cup_\varepsilon}[(F,Z)\overset{*}{*_\varepsilon}(H,R)]$.

3) If $Z'\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\overset{*}{*_\varepsilon}(H,R)]=[(F,Z)\overset{*}{+_\varepsilon}(G,C)]\overset{*}{\cap_\varepsilon}[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

4) If $Z'\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\overset{*}{\theta_\varepsilon}(H,R)]=[(F,Z)\overset{*}{+_\varepsilon}(G,C)]\overset{*}{\cup_\varepsilon}[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Star Operation over Complementary Extended Operations

1) If $Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\overset{*}{*_\varepsilon}(H,R)=[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\overset{*}{\cup_\varepsilon}[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\overset{*}{\theta_\varepsilon}(G,C)=(M,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$M(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z\cup C)\overset{*}{*_\varepsilon}(H,R)=(N,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in(Z\cup C)-R \\ H'(\aleph), & \aleph\in R-(Z\cup C) \\ M'(\aleph)\cup H'(\aleph), & \aleph\in(Z\cup C)\cap R \end{cases}$$

Hence,

$$N(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C-Z)-R=Z'\cap C\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ G(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap R=Z'\cap C\cap R \\ (F(\aleph)\cup G(\aleph))\cup H'(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\overset{*}{\cup_\varepsilon}[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F(\aleph)\cup H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(G,C)\overset{*}{\lambda_\varepsilon}(H,R)=(W,C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cup H'(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\overset{*}{\cup}_{\varepsilon}(W,C\cup R)=(T,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in(Z\cup R)-(C\cup R) \\ W'(\aleph), & \aleph\in(C\cup R)-(Z\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C\cap R' \\ H(\aleph), & \aleph\in(R-Z)-(C\cup R)=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z\cap R)-(C\cup R)=\emptyset \\ G(\aleph), & \aleph\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R-C)-(Z\cup R)=\emptyset \\ G'(\aleph)\cap H(\aleph), & \aleph\in(C\cap R)-(Z\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-R)\cap(R-C)=\emptyset \\ F'(\aleph)\cup(G(\aleph)\cup H'(\aleph)), & \aleph\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R-Z)\cap(C-R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G(\aleph)\cup H'(\aleph)\big) & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ (F(\aleph)\cup H'(\aleph))\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C-R)=\emptyset \\ \big(F(\aleph)\cup H'(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F(\aleph)\cup H'(\aleph))\cup(G(\aleph)\cup H'(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ G(\aleph)\cup H'(\aleph), & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F(\aleph)\cup H'(\aleph))\cup(G(\aleph)\cup H'(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that N=T under the condition $Z\cap C\cap R'=\emptyset$.

2) If $Z\cap(C\,\Delta R)=\emptyset$, then $[(F,Z)\overset{*}{\cup}_{\varepsilon}(G,C)]\overset{*}{*}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{*}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{*}_{\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{*}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{*}_{\varepsilon}(H,R)]\overset{*}{\cup}_{\varepsilon}[(G,C)\overset{*}{*}_{\varepsilon}(H,R)]$.

4) If $Z\cap C\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\overset{*}{*}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\lambda}_{\varepsilon}(H,R)]\overset{*}{\cup}_{\varepsilon}[(G,C)\overset{*}{\lambda}_{\varepsilon}(H,R)]$.

Theorem 3.4.4. The following distributions of the complementary extended star operation over soft binary piecewise operations hold:

i) LHS Distributions of the Complementary Extended Star Operation on Soft Binary Pievewise Operations

1) If $Z\cap C'\cap R=\emptyset$, then $(F,Z)\overset{*}{*}_{\varepsilon}[(G,C)\overset{\sim}{\cap}(H,R)]=[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\overset{\sim}{\cup}[(F,Z)\overset{*}{*}(H,R)]$.

Proof *:* Consider first the LHS. Let $(G,C)\overset{\sim}{\cap}(H,R)=(M,C)$. Hence for all $\aleph\in C$,

$$M(\aleph)=\begin{cases} G(\aleph), & \aleph\in C\text{-}R \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z)\overset{*}{*_{\varepsilon}}(M,C)=(N,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ M'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cup M'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in(C\text{-}R)\text{-}Z=Z'\cap C\cap R' \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cup(G'(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the rhs, i.e. $[(F,Z)\overset{*}{*_{\varepsilon}}(G,C)]\tilde{\cup}[(F,Z)\overset{*}{*_{\varepsilon}}(H,R)]$. Let $(F,Z)\overset{*}{*_{\varepsilon}}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\overset{*}{*_{\varepsilon}}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\tilde{\cup}(W,Z\cup R)=(T,(Z\cup C))$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup C)\text{-}(Z\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G'(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)\text{-}(Z\cup R)=\emptyset \\ F'(\aleph)\cup F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}C)\cap(R\text{-}Z)=\emptyset \\ F'(\aleph)\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup F'(\aleph), & \aleph\in(C\text{-}Z)\cap(Z\text{-}R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ G'(\aleph)\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(C\text{-}Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cup F'(\aleph), & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cup H'(\aleph), & \aleph\in(Z\cap C)\cap(R\text{-}Z)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(ℵ)=\begin{cases} G'(ℵ), & ℵ\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F'(ℵ), & ℵ\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(ℵ)\cup H'(ℵ), & ℵ\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cup H'(ℵ), & ℵ\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(ℵ)\cup G'(ℵ), & ℵ\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(ℵ)\cup G'(ℵ))\cup(F'(ℵ)\cup H'(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider Z-C in the function N, since Z-C=Z∩C', if an element is in the complement of C, it is either in R-C or (C ∪ R)' . Thus, if ℵ ∈ Z- C, ℵ ∈Z∩C'∩R or ℵ ∈Z∩C'∩R'. Thus, it is seen that N=T under the condition Z∩C'∩R=∅.

2) If Z∩C∩R'= ∅ , then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\,\tilde{\cup}\,(H,R)]=[(F,Z)\overset{*}{*_\varepsilon}(G,C)]\,\tilde{\cap}\,[(F,Z)\overset{*}{*_\varepsilon}(H,R)]$.

3) If (ZΔR)∩C=∅ , then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\,\tilde{*}\,(H,R)]=[(F,Z)\overset{*}{+_\varepsilon}(G,C)]\,\tilde{\cap}\,[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

4) If (ZΔC)∩R=Z∩C'∩R=∅ , then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\,\tilde{\theta}\,(H,R)]=[(F,Z)\overset{*}{+}(G,C)]\,\tilde{\cup}\,[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

ii) RHS Distributions of the Complementary Extended Star Operation over Soft Binary Piecewise Operations

1) If (ZΔC)∩R=Z∩C∩R'=∅ , then $[(F,Z)\,\tilde{\theta}\,(G,C)]\overset{*}{*_\varepsilon}(H,R)=[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\,\tilde{\cup}\,[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\,\tilde{\theta}\,(G,C)=(M,Z)$, where for all ℵ∈Z,

$$M(ℵ)=\begin{cases} F(ℵ), & ℵ\in Z-C \\ F'(ℵ)\cap G'(ℵ), & ℵ\in Z\cap C \end{cases}$$

Let $(M,Z)\overset{*}{*_\varepsilon}($ $(H,R)=(N,Z\cup R)$, where for all ℵ∈Z∪R,

$$N(ℵ)=\begin{cases} M'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ M'(ℵ)\cup H'(ℵ), & ℵ\in Z\cap R \end{cases}$$

Thus,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-C)-R=Z\cap C'\cap R' \\ F(ℵ)\cup G(ℵ), & ℵ\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(ℵ), & ℵ\in R-Z \\ F'(ℵ)\cup H'(ℵ), & ℵ\in(Z-C)\cap R=Z\cap C'\cap R \\ (F(ℵ)\cup G(ℵ))\cup H'(ℵ), & ℵ\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, that is, $[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\,\tilde{\cup}\,[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)=(V,Z\cup R)$, where for all ℵ∈Z∪R,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ F(ℵ)\cup H'(ℵ), & ℵ\in Z\cap R \end{cases}$$

Now let $(G,C)\overset{*}{\lambda_\varepsilon}(H,R)=(W,C\cup R)$, where for all ℵ∈CUR,

$$W(ℵ)=\begin{cases} G'(ℵ), & ℵ\in C-R \\ H'(ℵ), & ℵ\in R-C \\ G(ℵ)\cup H'(ℵ), & ℵ\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\,\tilde{\cup}\,(W,C\cup R)=(T,(Z\cup R))$, where for all ℵ∈Z∪R,

$$T(ℵ)=\begin{cases} V(ℵ), & ℵ\in(Z\cup R)-(C\cup R) \\ V(ℵ)\cup W(ℵ), & ℵ\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,,

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ H'(ℵ), & ℵ\in(R-Z)-(C\cup R)=\emptyset \\ F(ℵ)\cup H'(ℵ), & ℵ\in(Z\cap R)-(C\cup R)=\emptyset \\ F'(ℵ)\cup G'(ℵ), & ℵ\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(ℵ)\cup H'(ℵ), & ℵ\in(Z-R)\cap(R-C)=\emptyset \\ F'(ℵ)\cup\big(G(ℵ)\cup H'(ℵ)\big), & ℵ\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(ℵ)\cup G'(ℵ), & ℵ\in(R-Z)\cap(C-R)=\emptyset \\ H'(ℵ)\cup H'(ℵ), & ℵ\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(ℵ)\cup\big(G(ℵ)\cup H'(ℵ)\big) & ℵ\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \big(F(ℵ)\cup H'(ℵ)\big)\cup G'(ℵ), & ℵ\in(Z\cap R)\cap(C-R)=\emptyset \\ \big(F(ℵ)\cup H'(ℵ)\big)\cup H'(ℵ) & ℵ\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ \big(F'(ℵ)\cup H'(ℵ)\big)\cup\big(G(ℵ)\cup H'(ℵ)\big), & ℵ\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Hence

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ F'(ℵ)\cup G'(ℵ), & ℵ\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(ℵ), & ℵ\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ G(ℵ)\cup H'(ℵ), & ℵ\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(ℵ)\cup H'(ℵ), & ℵ\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ \big(F(ℵ)\cup H'(ℵ)\big)\cup\big(G(ℵ)\cup H'(ℵ)\big), & ℵ\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Under the condition Z'∩C∩R=Z∩C'∩R=Z∩C∩R'=∅, it can be seen that N=T. It is obvious that the condition Z'∩C∩R= Z∩C'∩R=∅ is equivalent to the condition (ZΔC)∩R=∅.

2) If Z∩C'∩R=∅, then $[(F,Z)\tilde{\cup}(G,C)]\overset{*}{*_\varepsilon}(H,R)=[(F,Z)\overset{*}{*_\varepsilon}(H,R)]\tilde{\cap}[(G,C)\overset{*}{*_\varepsilon}(H,R)]$.

3)If Z'∩C∩R= ∅, then $[(F,Z)\tilde{\cap}(G,C)]\overset{*}{*_\varepsilon}(H,R)=[(F,Z)\overset{*}{*_\varepsilon}(H,R)]\tilde{\cup}[(G,C)\overset{*}{*_\varepsilon}(H,R)]$.

4) If Z∩(CΔ*R*)=∅, then $[(F,Z)\tilde{*}(G,C)]\overset{*}{*}(H,R)=[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\tilde{\cap}[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$.

Theorem 3.4.5. The following distributions of the complementary extended star operation over the complementary soft binary piecewise operations exist:

i) LHS Distribution of the Complementary Extended Star Operation on Complementary Soft Binary Piecewise Operations

1)If Z∩(CΔ*R*)=∅, then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\overset{*}{\underset{*}{\underset{\cap}{\sim}}}(H,R)]=[(F,Z)\overset{*}{*_\varepsilon}(G,C)]\overset{*}{\underset{\cup}{\sim}}[(F,Z)\overset{*}{*_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)\underset{\cap}{\sim}(H,R)=(M,C)$, where for all ℵ∈ C,

$$M(ℵ)=\begin{cases} G'(ℵ), & ℵ\in C-R \\ G(ℵ)\cap H(ℵ), & ℵ\in C\cap R \end{cases}$$

Let $(F,Z)\overset{*}{*_\varepsilon}(M,C)=(N,Z\cup C)$, where for all ℵ∈Z∪C,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-C \\ M'(ℵ), & ℵ\in C-Z \\ F'(ℵ)\cup M'(ℵ), & ℵ\in Z\cap C \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G(\aleph), & \aleph\in(C-R)-Z=Z'\cap C\cap R' \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cup G(\aleph), & \aleph\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup(G'(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider RHS, i.e. $[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\underset{\cup}{\overset{*}{\sim}}[(F,Z)\overset{*}{*}_{\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{*}_{\varepsilon}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\overset{*}{*}_{\varepsilon}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cup H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\underset{\cup}{\overset{*}{\sim}}(W,Z\cup R)=(T,(Z\cup C))$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in(Z\cup C)-(Z\cup R) \\ V(\aleph)\cap W(\aleph), & \aleph\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-C)-(Z\cup R)=\emptyset \\ G(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)-(Z\cup R)=\emptyset \\ F'(\aleph)\cup F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-C)\cap(R-Z)=\emptyset \\ F'(\aleph)\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup F'(\aleph), & \aleph\in(C-Z)\cap(Z-R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(C-Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cup F'(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cup H'(\aleph), & \aleph\in(Z\cap C)\cap(R-Z)=\emptyset \\ (F'(\aleph)\cup G'(\aleph))\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Hence,

$$T(\aleph)=\begin{cases} G(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph) & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(\aleph)\cup G'(\aleph) & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cup G'(\aleph))\cup(F'(\aleph)\cup H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider Z-C in the function N, since $Z-C=Z\cap C'$, if an element is in the complement of C, it is either in R-C or $(C\cup R)'$. Thus, if $\aleph\in Z-C$, $\aleph\in Z\cap C'\cap R$ or $\aleph\in Z\cap C'\cap R'$. Hence, it is seen that N=T under the condition $Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$ is equivalent to the condition $Z\cap(C\Delta R)=\emptyset$.

2) If $Z\cap C\cap R'=\emptyset$, then $(F,Z)\overset{*}{*}_{\varepsilon}[(G,C)\underset{\cup}{\overset{*}{\sim}}(H,R)]=[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\underset{\cap}{\overset{*}{\sim}}[(F,Z)\overset{*}{*}_{\varepsilon}(H,R)]$.

3)If (Z∆R)∩C=∅ , then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\overset{*}{\sim}(H,R)]=[(F,Z)\overset{*}{+_\varepsilon}(G,C)]\underset{\cap}{\overset{*}{\sim}}[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

4)If (Z∆C)∩R=∅, then $(F,Z)\overset{*}{*_\varepsilon}[(G,C)\underset{\theta}{\overset{*}{\sim}}(H,C)]=[(F,Z)\overset{*}{+_\varepsilon}(G,C)]\underset{\cup}{\overset{*}{\sim}}[(F,Z)\overset{*}{+_\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Star Operation over Complementary Soft Binary Piecewise Operations

1) If (Z∆R)∩C=∅ then $[(F,Z)\underset{\theta}{\overset{*}{\sim}}(G,C)]\overset{*}{*_\varepsilon}(H,R)=[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\underset{\cup}{\overset{*}{\sim}}[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\underset{\theta}{\overset{*}{\sim}}(G,C)=(M,Z)$, where for all ℵ∈Z,

$$M(ℵ)=\begin{cases}F'(ℵ), & ℵ\in Z-C\\ F'(ℵ)\cap G'(ℵ), & ℵ\in Z\cap C\end{cases}$$

Let $(M,Z)\overset{*}{*_\varepsilon}(H,R)=(N,Z\cup R)$, where for all ℵ∈Z∪R,

$$N(ℵ)=\begin{cases}M'(ℵ), & ℵ\in Z-R\\ H'(ℵ), & ℵ\in R-Z\\ M'(ℵ)\cup H'(ℵ), & ℵ\in Z\cap R\end{cases}$$

Thus,

$$N(ℵ)=\begin{cases}F(ℵ), & ℵ\in(Z-C)-R=Z\cap C'\cap R'\\ F(ℵ)\cup G(ℵ), & ℵ\in(Z\cap C)-R=Z\cap C\cap R'\\ H'(ℵ), & ℵ\in R-Z\\ F(ℵ)\cup H'(ℵ), & ℵ\in(Z-C)\cap R=Z\cap C'\cap R\\ (F(ℵ)\cup G(ℵ))\cup H'(ℵ), & ℵ\in(Z\cap C)\cap R=Z\cap C\cap R\end{cases}$$

Now consider the RHS, that is, $[(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)]\underset{\cup}{\overset{*}{\sim}}[(G,C)\overset{*}{\lambda_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\lambda_\varepsilon}(H,R)=(V,Z\cup R)$, where for all ℵ∈Z∪R,

$$V(ℵ)=\begin{cases}F'(ℵ), & ℵ\in Z-R\\ H'(ℵ), & ℵ\in R-Z\\ F(ℵ)\cup H'(ℵ), & ℵ\in Z\cap R\end{cases}$$

Now let $(G,C)\overset{*}{\lambda_\varepsilon}(H,R)=(W,C\cup R)$, where for all ℵ∈C∪R,

$$W(ℵ)=\begin{cases}G'(ℵ), & ℵ\in C-R\\ H'(ℵ), & ℵ\in R-C\\ G(ℵ)\cup H'(ℵ), & ℵ\in C\cap R\end{cases}$$

Let $(V,Z\cup R)\underset{\cup}{\overset{*}{\sim}}(W,C\cup R)=(T,(Z\cup R))$, where for all ℵ∈Z∪R,

$$T(ℵ)=\begin{cases}V'(ℵ), & ℵ\in(Z\cup R)-(C\cup R)\\ V(ℵ)\cup W(ℵ), & ℵ\in(Z\cup R)\cap(C\cup R)\end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ H(\aleph), & \aleph\in(R-Z)-(C\cup R)=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z\cap R)-(C\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z-R)\cap(R-C)=\emptyset \\ F'(\aleph)\cup\big(G(\aleph)\cup H'(\aleph)\big), & \aleph\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R-Z)\cap(C-R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G(\aleph)\cup H'(\aleph)\big) & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \big(F(\aleph)\cup H'(\aleph)\big)\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C-R)=\emptyset \\ \big(F(\aleph)\cup H'(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ \big(F(\aleph)\cup H'(\aleph)\big)\cup\big(G(\aleph)\cup H'(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ G(\aleph)\cup H'(\aleph), & \aleph\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ \big(F(\aleph)\cup H(\aleph)\big)\cup\big(G(\aleph)\cup H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Under the condition $Z'\cap C\cap R= Z\cap C\cap R'=\emptyset$, N=T is satisfied. It is obvious that the condition $Z'\cap C\cap R= Z\cap C\cap R'= \emptyset$ is equivalent to $(Z\Delta R)\cap C= \emptyset$

2) If $Z\cap C'\cap R=\emptyset$, then $[(F,Z)\underset{\cup}{\overset{*}{\sim}}(G,C)]\overset{*}{*}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{*}_{\varepsilon}(H,R)]\underset{\cap}{\overset{*}{\sim}}[(G,C)\overset{*}{*}_{\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\underset{\cap}{\overset{*}{\sim}}(G,C)]\overset{*}{*}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{*}_{\varepsilon}(H,R)]\underset{\cup}{\overset{*}{\sim}}[(G,C)\overset{*}{*}_{\varepsilon}(H,R)]$.

4) If $Z\cap(C\Delta R)=\emptyset$, then $[(F,Z)\underset{*}{\overset{*}{\sim}}(G,C)]\overset{*}{*}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\lambda}_{\varepsilon}(H,R)]\underset{\cap}{\sim}[(G,C)\overset{*}{\lambda}_{\varepsilon}(H,R)]$.

4. CONCLUSION

Soft set operations are at the heart of soft set theory, providing a foundational structure for managing uncertainty in both data analysis and decision-making processes. This paper introduces a novel soft set operation named complementary extended star operation and explores its algebraic properties. Additionally, we examine how complementary extended star distributes over various other operations on soft sets. It is our aim that this study will serve as a guiding framework for future investigations into soft set operations. Further study may examine various kinds of complementary extended soft set operations and their distributions and characteristics to find out what algebraic structures arise in the collection of soft sets together with complementary extended star operation of soft sets.

SIMILARITY RATE: 19%

AUTHOR CONTRIBUTION

Aslıhan Sezgin: Conceptualization, methodology, data curation, editing, Murat Sarıalioğlu: Conceptualization, writing, editing.

CONFLICT of INTEREST

The authors declared that they have no known conflict of interest.

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