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A New Soft Set Operation: Complementary Extended Gamma Operation

Research Article

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Abstract

Since its inception, soft set theory has shown to be a useful mathematical framework for addressing problems involving uncertainty, proving its usefulness in a variety of academic and practical disciplines. The operations of soft sets are at the very core concept of this theory. In this regard, a new kind of soft set operation known as the complementary extended gamma operation for soft sets is presented in order to improve the theory and theoretically contribute to it in this study. To shed light on the relationship between the complementary extended gamma operation and other soft set operations, a thorough analysis of this operation's attributes, including its distributions across other soft set operations, has been conducted. Additionally, this paper aims to contribute to the literature on soft sets by examining the algebraic structure of soft sets from the perspective of soft set operations, which provides a thorough grasp of their use as well as an appreciation of the ways in which soft sets can be applied to both classical and nonclassical logical thought.
Keywords: Soft Set, Soft Set Operations, Complementary Extended Soft Set Operations.

1. INTRODUCTION

In our daily lives, we often encounter subjective concepts which lack the objectivity of scientific knowledge and vary from person to person. To address the complexities of the uncertainty we face, people have sought various solutions over time. However, existing methods have shown discrepancies in tackling new complex problems arising from changing conditions. Among the theories proposed to handle uncertain situations, Zadeh's theory of fuzzy sets stands out as the most prominent. Fuzzy sets are defined by their membership functions. As fuzzy set theory rapidly developed, certain structural issues came to light. In response, Molodtsov [1] introduced soft set theory as a solution to these structural problems.

The soft set has been applied in many theoretical and practical situations since its debut, and several additional studies have been published in the literature. Maji et al. [2] paved the path for more study on the subject of soft set theory by defining the equality of two soft sets, the subset and superset of a soft set, the complement of a soft set, and soft binary operations like and/or, union, and intersection operations for soft sets. Set theory concepts led Pei and Miao [3] to redefine the concepts of "soft subset" and "intersection of two soft sets." Ali et al. [4] then suggested a number of other soft set operations, which Sezgin and Atagün [5] and Ali et al. [6] carefully investigated. Sezgin et al. [7] and Stojanovic [8] described the extended difference and extended symmetric difference of soft sets, respectively, and their properties were carefully investigated in relation to other operations on soft sets.

Soft set operations may be broadly classified into two types: restricted soft set operations and extended soft set operations, according to an analysis of the research conducted thus far. Eren and Çalışıcı [9] developed and evaluated the soft binary piecewise difference operation for soft sets, and Sezgin and Çalışıcı [10] carried out an in-depth investigation of the properties of this soft set operation. The inclusive and exclusive complement of sets is a novel concept in set theory that was presented in 2021 by Çağman [11]. Sezgin et al. [12] provided five new concepts pertaining to binary complement operations that were already described by Çağman [11]. Aybek [13] explored the properties of numerous more restricted and extended soft set operations with the motivation of [11,12]. Moreover, the soft binary piecewise operation form-of which Eren and Çağman [9] were the pioneers-was modified somewhat by taking the complement of the image set in the first row. Consequently, several researchers have studied the complementary soft binary piecewise operations in great detail [14–22]. On the other hand, Akbulut [23] and Demirci [24] altered the form of the existing extended soft set operations in the literature by taking the complement of the image set in the first and second rows, and defining the complementary extended difference, lambda and union, plus and theta, respectively, and giving their algebraic properties and relations with other soft set operations. For more on soft equality, please see [25-32] and for other applications of soft sets to algebraic structures, see the following [32-46].

Analyzing the characteristics of designated operations on sets as well as the sets themselves is a crucial component of algebraic structures, which is to categorize mathematical structures. This analysis is important within the context of algebra. There are two main kinds of soft set collections to be aware of when thinking about soft sets as algebraic structures: collections with a fixed set of parameters and collections with changing parameter sets. Depending on the extra actions performed, these collections exhibit different behaviors. Concepts related to soft set operations are just as essential as the operations of classical set theory, which form the basis of soft sets.

In this study, a novel soft set operation named "complementary extended gamma" is introduced and its properties are thoroughly examined, with the goal of advancing the theory of soft sets. In addition, an analysis is conducted to investigate the relationship between different kinds of soft set operations and the complemented extended gamma operation in order to clarify them. This topic is significant within the framework because knowledge of the algebraic structures of soft sets in relation to novel operations is necessary to comprehend their applications.

2. PRELIMINARIES

Definition 2.1. Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let $D \subseteq E$. A pair (F, D) is called a soft set on U. Here, F is a function given by $F: D \rightarrow P(U)$ [1].

The notation of the soft set (F,D) is also shown as F_D, however, we prefer to use the notation of (F,D) as is used by Molodtsov [1] and Maji et al. [2].
The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$. In other words, in the collection $S_K(U)$, only soft sets with the parameter set K are included, while in the collection $S_E(U)$, soft sets over U with any parameter set can be included. Clearly, the set $S_K(U)$ is a subset of the set $S_E(U)$.
Definition 2.2. Let (F,D) be a soft set over U. If F(ℵ)=∅ for all ℵ∈D, then the soft set (F,D) is called a null soft set with respect to D, denoted by $\emptyset_D$. Similarly, let (F,E) be a soft set over U. If F(e)=∅ for all ℵ∈E, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ [4].
A soft set can be defined as F: ∅ ⟶ P(U), where U is a universal set. Such a soft set is called an empty soft set and is denoted as $\emptyset_\emptyset$. Thus, $\emptyset_\emptyset$ is the only soft set with an empty parameter set [6].

Definition 2.3. Let (F,D) be a soft set over U. If F(ℵ)=U for all ℵ∈D, then the soft set (F,K) is called a relative whole soft set with respect to D, denoted by U_D. Similarly, let (F,E) be a soft set over U. If F(e)=U for all ℵ∈E, then the soft set (F,E) is called a whole soft set with respect to E, denoted by U_E [4].
Definition 2.4. Let (F,D) and (G,Y) be soft sets over U. If D⊆Y and F(ℵ) ⊆G(ℵ) for all ℵ∈D, then (F,D) is said to be a soft subset of (G,Y), denoted by $(F,D)\tilde{\subseteq}(G,Y)$. If (G,Y) is a soft subset of (F,D), then (F,D) is said to be a soft superset of (G,Y), denoted by $(F,D)\tilde{\supseteq}(G,Y)$. If $(F,D)\tilde{\subseteq}(G,Y)$ and $(G,Y)\tilde{\subseteq}(F,D)$, then (F,D) and (G,Y) are called soft equal sets [3].
Definition 2.5. Let (F,D) be a soft set over U. The soft complement of (F,D), denoted by $(F,D)^r=(F^r,D)$, is defined as follows: for all ℵ∈D, $F^r(ℵ)$=U-F(ℵ) [4].
Çağman [11] introduced two new complements as a novel concept in set theory, termed as the inclusive complement and exclusive complement. For ease of representation, we denote these binary operations as + and θ, respectively. For two sets D and Y, these binary operations are defined as D+Y=D'∪Y, DθY=D'∩Y'. Sezgin et al. [12] examined the relations between these two operations and also defined three new binary operations and analyzed their relations with each other. Let D and Y be two sets D*Y=D'∪Y', DγY= D'∩Y, and DλY=D∪Y'.

As a summary for soft set operations, we can categorize all types of soft set operations as follows: Let "⍟" be used to represent the set operations (i.e., here ⍟ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), then all type of soft set operations are defined as follows:

Definition 2.6. Let $(F,D),(G,Y)\in S_E(U)$. The restricted ⍟ operation of (F,D) and (G,Y) is the soft set (H,P), denoted to be $(F,D)\circledast_{\mathfrak{R}}(G,Y)=(H,P)$, where $P=D\cap Y\neq\emptyset$ and for all ℵ∈P, H(ℵ) = F(ℵ)⍟G(ℵ). Here, if P=D∩Y=∅, then $(F,D)\circledast_R(G,Y)=\emptyset_\emptyset$ [4,5,6,13]
Definition 2.7. Let $(F,D),(G,Y)\in S_E(U)$. The extended ⍟ operation (F, D) and (G, Y) is the soft set (H, P), denoted by $(F,D)\circledast_\varepsilon(G,Y)=(H,P)$, where $P=D\cup Y$ and for all $\aleph\in P$,

$$H(\aleph)=\begin{cases}F(\aleph), & \aleph\in D-Y\\ G(\aleph), & \aleph\in Y-D\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[2,4,6,7,8,13]
Definition 2.8. Let $(F,D),(G,Y)\in S_E(U)$. The complementary extended ⍟ operation (F,D) and (G,Y) is the soft set (H,P), denoted by $(F,D)\overset{*}{\circledast}_\varepsilon(G,Y)=(H,P)$, where $P=D\cup Y$ and for all $\aleph\in P$,

$$H(\aleph)=\begin{cases}F'(\aleph), & \aleph\in D-Y\\ G'(\aleph), & \aleph\in Y-D\\ F(\aleph)\circledast G(\aleph), & \aleph\in D\cap Y\end{cases}$$

[23,24]
Definition 2.9. Let $(F,D),(G,Y) \in S_E(U)$. The soft binary piecewise ⍟ of (F,D) and (G,Y) is the soft set (H,D), denoted by $(F,D)\underset{⍟}{\tilde{}}(G,Y)=(H,D)$, where for all $\aleph \in D$,

$$H(\aleph)=\begin{cases} F(\aleph), & \aleph \in D-Y \\ F(\aleph)⍟G(\aleph), & \aleph \in D\cap Y \end{cases}$$

[9,10,47,48]
Definition 2.10. Let $(F,D),(G,Y) \in S_E(U)$. The complementary soft binary piecewise ⍟ of (F,D) and (G,Y) is the soft set (H,D), denoted by $(F,D)\overset{*}{\underset{⍟}{\sim}}(G,Y)=(H,D)$, where for all $\aleph \in D$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph \in D-Y \\ F(\aleph)⍟G(\aleph), & \aleph \in D\cap Y \end{cases}$$

[14-22].

Definition 2.11. Let (S, ⋆) be an algebraic structure. An element s ∈S is called idempotent if $s^2=s$. If $s^2=s$ for all s∈S, then the algebraic structure (S,⋆) is said to be idempotent. An idempotent semigroup is called a band, an idempotent and commutative semigroup is called a semilattice, an idempotent and commutative monoid is called a bounded semilattice [49].
In a monoid, although the identity element is unique, a semigroup/groupoid can have one or more left identities, however, if it has more than one left identity, it does not have a right identity element, thus it does not have an identity element. Similarly, a semigroup/groupoid can have one or more right identities, however, if it has more than one right identity, it does not have a left identity element, thus it does not have an identity element [50].
Similarly, in a group, although each element has a unique inverse, in a monoid, an element can have one or more left inverses, however, if an element has more than one left inverse, it does not have a right inverse, thus it does not have an inverse. Similarly, in a monoid, an element can have one or more right inverses, however, if an element has more than one right inverse, it does not have a left inverse, thus it does not have an inverse [50]. We refer to [51] for the potential future graph applications and network analysis with respect to soft sets.

3. COMPLEMENTARY EXTENDED GAMMA OPERATION

In this section, a new soft set operation called the complementary extended gamma operation of soft sets is introduced with its example, and its full algebraic properties are analyzed.
Definition 3.1. Let (F, Z), (G,C) be soft sets over U. The complementary extended gamma operation (γ) of (F, Z) and (G, C) is the soft set (H, K), denoted by $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(G,C)=(H,K)$, where for all ℵ∈K=Z∪C,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F(\aleph)\gamma G(\aleph), & \aleph\in Z\cap C \end{cases}$$

where $F(\aleph)\gamma G(\aleph)=F'(\aleph)\cap G(\aleph)$ for all ℵ∈Z∩C.
Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set, and $Z=\{e_1,e_3\}$ and $C=\{e_2,e_3,e_4\}$ be two subsets of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Assume that $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,C)=\{(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$ be two soft sets over U. Let $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(G,C)=(L,Z\cup C)$, where for all ℵ ∈Z∪C,

$$L(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Here, since $Z\cup C=\{e_1,e_2,e_3,e_4\}$, $Z-C=\{e_1\}$, $C-Z=\{e_2,e_4\}$, $Z\cap C=\{e_3\}$, thus $L(e_1)=F'(e_1)=\{h_1,h_3,h_4\}$, $L(e_2)=G'(e_2)=\{h_2,h_3\}$, $L(e_4)=G'(e_4)=\{h_1,h_2,h_4\}$, $L(e_3)=F'(e_3)\cap G(e_3)=\{h_3,h_4\}\cap\{h_2,h_3,h_4\}=\{h_3,h_4\}$. Hence,

$$(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_2,h_3\}),(e_3,\{h_3,h_4\}),(e_4,\{h_1,h_2,h_4\})\}.$$

Theorem 3.3. (Algebraic Properties of Operation)

1) The set $S_E(U)$ is closed under $\overset{*}{\gamma_\varepsilon}$.

Proof: It is clear that $\overset{*}{\gamma_\varepsilon}$ is a binary operation on $S_E(U)$. Indeed,

$$\overset{*}{\gamma_\varepsilon}: S_E(U)\times S_E(U)\rightarrow S_E(U)$$

$$((F,Z),(G,C))\rightarrow (F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=(L,Z\cup C)$$

Similarly,

$$\overset{*}{\gamma_\varepsilon}: S_Z(U)\times S_Z(U)\rightarrow S_Z(U)$$

$$((F,Z),(G,Z))\rightarrow (F,Z)\overset{*}{\gamma_\varepsilon}(G,Z)=(T,Z\cup Z)=(T,Z)$$

That is, when Z is a fixed subset of the set E and (F,Z) and (G,Z) are elements of $S_Z(U)$, then so is $(F,Z)\overset{*}{\gamma_\varepsilon}(G,Z)$. Namely, $S_Z(U)$ is closed under $\overset{*}{\gamma_\varepsilon}$, too.

2) $[(F,Z)\overset{*}{\gamma_\varepsilon}G,C)]\overset{*}{\gamma_\varepsilon}(H,R)\neq(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Firstly, let's handle the left hand side (LHS). Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=(T,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(T,Z\cup C)\overset{*}{\gamma_\varepsilon}(H,R)=(M,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in (Z\cup C)-R \\ H'(\aleph), & \aleph\in R-(Z\cup C) \\ T'(\aleph)\cap H(\aleph), & \aleph\in (Z\cup C)\cap R \end{cases}$$

Thus,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}C)\text{-}R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C\text{-}Z)\text{-}R=Z'\cap C\cap R' \\ F(\aleph)\cup G'(\aleph), & \aleph\in(Z\cap C)\text{-}R=Z\cap C\cap R' \\ H'(\aleph) & \aleph\in R\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap R=Z\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in(C\text{-}Z)\cap R=Z'\cap C\cap R \\ \big(F(\aleph)\cup G'(\aleph)\big)\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now let's handle the right hand side (RFS) of the equation, Let $(G,C)\overset{*}{\gamma_\varepsilon}(H,R)=(K,C\cup R)$. Here, for all $\aleph\in C\cup R$,

$$K(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}C \\ G'(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Assume that $(F,Z)\overset{*}{\gamma_\varepsilon}(K,C\cup R)=(S,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cup R) \\ K'(\aleph), & \aleph\in(C\cup R)\text{-}Z \\ F'(\aleph)\cap K(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cup R)=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}Z=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R\text{-}C)\text{-}Z=Z'\cap C'\cap R \\ G(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap(R\text{-}C)=Z\cap C'\cap R \\ F'(\aleph)\cap(G(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that $M\neq S$. That is, in the set $S_E(U)$, $\overset{*}{\gamma_\varepsilon}$ does not have associative property.

3) $[(F,Z)\overset{*}{\gamma_\varepsilon}(G,Z)]\overset{*}{\gamma_\varepsilon}(H,Z)\neq(F,Z)\overset{*}{\gamma_\varepsilon}[(G,Z)\overset{*}{\gamma_\varepsilon}(H,Z)]$

Proof : Firstly, let's look at the LHS. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z\cup Z=Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ G'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Let $(T,Z)\overset{*}{\gamma_\varepsilon}(H,Z)=(M,Z\cup Z)$, where for all $\aleph\in Z$,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ T'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Thus,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ (F(\aleph)\cup G'(\aleph))\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Now let's handle RHS. Let $(G,Z) \underset{\gamma_\varepsilon}{*} (H,Z)=(L, Z\cup Z)$, where for all $\aleph\in Z$,

$$L(\aleph)=\begin{cases} G'(\aleph), & \aleph\in Z-Z=\emptyset \\ H'(\aleph), & \aleph\in Z-Z=\emptyset \\ G'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Let $(F,Z) \underset{\gamma_\varepsilon}{*} (L,Z) = (N,Z\cup Z)$, where for all $\aleph\in Z$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ L'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cap L(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ L'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cap(G'(\aleph)\cap H(\aleph)), & \aleph\in Z\cap Z=Z \end{cases}$$

It is seen that $M\neq N$. That is, in the set $S_Z(U)$, $\underset{\gamma_\varepsilon}{*}$ does not have associative property.

4) $(F,Z) \underset{\gamma_\varepsilon}{*} (G,C) \neq (G,C) \underset{\gamma_\varepsilon}{*} (F,Z)$.

Proof: Firstly, we observe that the parameter set of the soft set on both sides of the equation is $Z\cup C$, and thus the first condition of the soft equality is satisfied. Now let us look at the LHS. Let $(F,Z) \underset{\gamma_\varepsilon}{*} (G,C)=(H,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Now let's handle the RHS. Assume that $(G,C) \underset{\gamma_\varepsilon}{*} (F,Z)=(T,C\cup Z)$, where for all $\aleph\in C\cup Z$,

$$T(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-Z \\ F'(\aleph), & \aleph\in Z-C \\ G'(\aleph)\cap F(\aleph), & \aleph\in C\cap Z \end{cases}$$

Thus, it is seen that $H\neq T$. Similarly, it is easily seen that $(F,Z) \underset{\gamma_\varepsilon}{*} (G,Z) \neq (G,Z) \underset{\gamma_\varepsilon}{*} (F,Z)$. That is, $\underset{\gamma_\varepsilon}{*}$ is commutative neither in $S_E(U)$ nor in $S_Z(U)$.

5) $(F,Z) \underset{\gamma_\varepsilon}{*} (F,Z)= \emptyset_Z$

***Proof*:** Let $(F,Z) \underset{\gamma_\varepsilon}{*} (F,Z)=(H,Z\cup Z)$, where for all $\aleph\in Z$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph), & \aleph\in Z-Z=\emptyset \\ F'(\aleph)\cap F(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence, for all $\aleph\in Z$, $H(\aleph)= F'(\aleph)\cap F(\aleph)=\emptyset$, and so $(H,Z)= \emptyset_Z$. That is, $\underset{\gamma_\varepsilon}{*}$ is not idempotent in $S_E(U)$.

6) $\emptyset_Z \underset{\gamma_\varepsilon}{*} (F,Z)= (F,Z)$

Proof: Let $\emptyset_Z$=(T,Z). Thus, for all ℵ∈Z, T(ℵ)=∅. Let $(T, Z) \underset{\gamma_\varepsilon}{*} (F, Z)$ =(H,Z∪Z), where for all ℵ∈Z,

$$H(\aleph)=\begin{cases} T'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ T'(\aleph)\cap F(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)= T'(ℵ)∩F(ℵ)= U∩F(ℵ)=F(ℵ) and (H,Z)=(F,Z). That is, in $S_Z(U)$, the left identity element of $\underset{\gamma_\varepsilon}{*}$ is the soft set.

7) (F,Z) $\underset{\gamma_\varepsilon}{*}\emptyset_Z=\emptyset_Z$.

Proof: Let $\emptyset_Z$ =(S,Z). Thus, for all ℵ∈Z, S(ℵ)= ∅. Let (F,Z) $\underset{\gamma_\varepsilon}{*}$ (S,Z)=(H,Z∪Z), where for all ℵ∈Z,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ S'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap S(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Hence, for all ℵ∈Z, H(ℵ)= F'(ℵ)∩S(ℵ)= F'(ℵ)∩∅=∅ and (H,Z)= $\emptyset_Z$. That is, the right absorbing element of $\underset{\gamma_\varepsilon}{*}$ in $S_Z(U)$ is the soft set $\emptyset_Z$.

8) (F,Z) $\underset{\gamma_\varepsilon}{*}$ $\emptyset_\emptyset$= $\emptyset_\emptyset$ $\underset{\gamma_\varepsilon}{*}$(F,Z) = $(F,Z)^r$.

Proof: Let $\emptyset_\emptyset$= (K, ∅) and (F,Z) $\underset{\gamma_\varepsilon}{*}$ (K, ∅) = (Q,Z ∪ ∅) = (Q,Z), where for all ℵ∈Z,

$$Q(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}\emptyset=Z \\ K'(\aleph), & \aleph\in \emptyset\text{-}Z=\emptyset \\ F'(\aleph)\cap K(\aleph), & \aleph\in Z\cap\emptyset=\emptyset \end{cases}$$

Hence, for all ℵ∈Z, Q(ℵ)= F'(ℵ) and thus (Q,Z) = $(F,Z)^r$.

Similarly, $\emptyset_\emptyset$ $\underset{\gamma_\varepsilon}{*}$(F,Z)= (W, ∅ ∪ Z) = (W,Z). where for all ℵ∈Z,

$$W(\aleph)=\begin{cases} K'(\aleph), & \aleph\in \emptyset\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}\emptyset=Z \\ K'(\aleph)\cap F(\aleph), & \aleph\in \emptyset\cap Z=\emptyset \end{cases}$$

Hence, for all ℵ∈Z, W(ℵ)= F'(ℵ) and thus (W,Z) = $(F,Z)^r$.

9) U_Z $\underset{\gamma_\varepsilon}{*}$(F,Z)= $\emptyset_Z$

Proof: Let U_Z =(H,Z), where for all ℵ∈Z, H(ℵ)=U. Let (H,Z) $\underset{\gamma_\varepsilon}{*}$(F,Z)=(T,Z∪Z), where for all ℵ∈Z,

$$T(\aleph)=\begin{cases} H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph)\cap F(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all ℵ∈Z, T(ℵ)= H'(ℵ)∩F(ℵ)= ∅ ∩F(ℵ)=∅, and thus (T,Z)= $\emptyset_Z$.

10) (F,Z) $\underset{\gamma_\varepsilon}{*}$ U_Z=$(F,Z)^r$.

Proof: Let U_Z =(H,Z), where for all ℵ∈Z, H(ℵ)=U. Let (F,Z) $\underset{\gamma_\varepsilon}{*}$(H,Z)=(T,Z∪Z), where for all ℵ∈Z,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= F'(\aleph)\cap H(\aleph)= F'(\aleph)\cap U=F'(\aleph)$, and thus $(T,Z)= (F,Z)^r$

11) $U_E \overset{*}{\gamma_\varepsilon}(F,Z) =\emptyset_E$

Proof: Let $U_E =(H,E)$, where for all $\aleph\in Z$, $H(\aleph)=U$. Let $(H,E) \overset{*}{\gamma_\varepsilon}(F,Z)=(T,E\cup Z)$, where for all $\aleph\in E$,

$$T(\aleph)=\begin{cases} H'(\aleph), & \aleph\in E\text{-}Z=Z' \\ F'(\aleph), & \aleph\in Z\text{-}E=\emptyset \\ H'(\aleph)\cap F(\aleph), & \aleph\in E\cap Z=Z \end{cases}$$

Here for all $\aleph\in E$, $T(\aleph)= H'(\aleph)\cap F(\aleph)= \emptyset \cap F(\aleph)=\emptyset$, and

$$T(\aleph)=\begin{cases} \emptyset, & \aleph\in E\text{-}Z=Z' \\ F'(\aleph), & \aleph\in Z\text{-}E=\emptyset \\ \emptyset, & \aleph\in Z\cap E=Z \end{cases}$$

Thus, for all $\aleph\in E$, $T(\aleph)=\emptyset$, therefore $(T,E)= \emptyset_E$.

12) $(F,Z) \overset{*}{\gamma_\varepsilon}(F,Z)^r = (F,Z)^r$.

Proof: Let $(F,Z)^r =(H,Z)$, where for all $\aleph\in Z$, $H(\aleph)=F'(\aleph)$. Let $(F,Z) \overset{*}{\gamma_\varepsilon}(H,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= F'(\aleph)\cap H(\aleph)= F'(\aleph) \cap F'(\aleph)=F'(\aleph)$, and thus $(T,Z)= (F,Z)^r$

That is, the right absorbing element of $\overset{*}{\gamma_\varepsilon}$ in $S_Z(U)$ is the soft set $(F,Z)^r$.

13) $(F,Z)^r \overset{*}{\gamma_\varepsilon}(F,Z)= (F,Z)$.

Proof: Let $(F,Z)^r =(H,Z)$, where for all $\aleph\in Z$, $H(\aleph)=F'(\aleph)$. Let $(H,Z) \overset{*}{\gamma_\varepsilon}(F,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} H'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ H'(\aleph)\cap F(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Here for all $\aleph\in Z$, $T(\aleph)= H'(\aleph)\cap F(\aleph)= F(\aleph)\cap F'(\aleph)=F(\aleph)$, and thus $(T,Z)= (F,Z)$

That is, the left unit element of $\overset{*}{\gamma_\varepsilon}$ in $S_Z(U)$ is the soft set $(F,Z)^r$.

14) $[(F,Z) \overset{*}{\gamma_\varepsilon}(G,C)]^r =(F,Z)\, \lambda_\varepsilon(G,C)$.

Proof: Let $(F,Z) \overset{*}{\gamma_\varepsilon} (G,C)=(H,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(H,Z\cup C)^r =(T,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in Z\text{-}C \\ G(\aleph), & \aleph\in C\text{-}Z \\ F(\aleph)\cup G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Hence, $(T, Z\cup C) = (F,Z)\ \lambda_{\varepsilon}(G,C)$.

15) $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z)=U_Z \Leftrightarrow (F,Z)=\emptyset_Z$ and $(G,Z)=U_Z$

Proof: Let $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z)=(T,Z\cup Z)$, where for all $\aleph\in Z$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ G'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Since $(T,Z)=U_Z$, $T(\aleph)=U$ for all $\aleph\in Z$ and thus, $F'(\aleph)\cap G(\aleph)=U$ for all $\aleph\in Z$. So, $F'(\aleph)=G(\aleph)=U$ for all $\aleph\in Z$. Thus, $(F,Z)=\emptyset_Z$, and $(G,Z)=U_Z$.

16) $\emptyset_Z \mathrel{\tilde{\subseteq}} (F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C)$, $\emptyset_C \mathrel{\tilde{\subseteq}} (F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C)$, $\emptyset_{Z\cup C} \mathrel{\tilde{\subseteq}} (F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C)$, $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C) \mathrel{\tilde{\subseteq}} U_{Z\cup C}$.

17) $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z) \mathrel{\tilde{\subseteq}} (F,Z)^r$ and $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z) \mathrel{\tilde{\subseteq}} (G,Z)$.

Proof: Let $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z)=(H, Z\cup Z)$, where for all $\aleph\in Z$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ G'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Since $H(\aleph)=F'(\aleph)\cap G(\aleph)\subseteq F'(\aleph)$ for all $\aleph\in Z$, $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z) \mathrel{\tilde{\subseteq}} (F,Z)^r$. Similarly, $H(\aleph)=F'(\aleph)\cap G(\aleph)\subseteq G(\aleph)$ for all $\aleph\in Z$. Thus, $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,Z) \mathrel{\tilde{\subseteq}} (G,Z)$.

18) If $(F,Z) \mathrel{\tilde{\subseteq}} (G,Z)$, then $(G,Z)\overset{*}{\gamma_{\varepsilon}}(H,C) \mathrel{\tilde{\subseteq}} (F,Z)\overset{*}{\gamma_{\varepsilon}}(H,C)$.

Proof: Let $(F,Z) \mathrel{\tilde{\subseteq}} (G,Z)$ then, $F(\aleph)\subseteq G(\aleph)$ for all $\aleph\in Z$ and so $G'(\aleph)\subseteq F'(\aleph)$. Let $(G,Z)\overset{*}{\gamma_{\varepsilon}}(H,C)=(W,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in Z\text{-}C \\ H'(\aleph), & \aleph\in C\text{-}Z \\ G'(\aleph)\cap H(\aleph), & \aleph\in Z\cap C \end{cases}$$

$(F,Z)\overset{*}{\gamma_{\varepsilon}}(H,C)=(L,Z\cup C)$. where for all $\aleph\in Z\cup C$,

$$L(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ H'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus, $W(\aleph)=G'(\aleph)\subseteq G'(\aleph)=L(\aleph)$ for all $\aleph\in C\text{-}Z$...

Thus, $W(\aleph)=G'(\aleph)\subseteq G'(\aleph)=L(\aleph)$ for all $\aleph\in Z\text{-}C$, $W(\aleph)=H'(\aleph)\subseteq H'(\aleph)=L(\aleph)$ for all $\aleph\in C\text{-}Z$, and $W(\aleph)=H'(\aleph)\cap F(\aleph)\subseteq H'(\aleph)\cap G(\aleph)=L(\aleph)$ for all $\aleph\in Z\cap C$. Thus, $(G,Z)\overset{*}{\gamma_{\varepsilon}}(H,C) \mathrel{\tilde{\subseteq}} (F,Z)\overset{*}{\gamma_{\varepsilon}}(H,C)$.

19) If $(G,Z)\overset{*}{\gamma_{\varepsilon}}(H,C) \mathrel{\tilde{\subseteq}} (F,Z)\overset{*}{\gamma_{\varepsilon}}(H,C)$, then $(F,Z) \mathrel{\tilde{\subseteq}} (G,Z)$ need not have to be true. That is, the converse of Theorem 3.3. (18) is not true.

Proof: Let us give an example to show that the converse of Theorem 3.3. (18) is not true. Let E=$\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, A=$\{e_1,e_3\}$, C=$\{e_1,e_3,e_5\}$ be the subset of E, and U=$\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set.
Let (F,Z)=$\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, (G,Z)=$\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$, (H,C)=$\{(e_1,\emptyset),(e_3,\emptyset),(e_5,U)\}$ be soft sets over U.
Let $(G,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,C)$=(L,Z∪C), then (L,Z∪C)=$\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\emptyset)\}$ and let $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,C)$=(K,Z∪C) (K,Z∪C)=$\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\emptyset)\}$. Hence, $(G,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,C) \,\tilde{\subseteq}\, (F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,C)$ but (F,Z) is not a soft subset of (G,Z).

20) If (G,Z) $\tilde{\subseteq}$ (F,C) and (K,Z) $\tilde{\subseteq}$ (L,C), then $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(K,Z) \,\tilde{\subseteq}\, (G,C)\overset{*}{\underset{\gamma_\varepsilon}{}}(L,C)$.

Proof: Let (G,Z)$\tilde{\subseteq}$ (F,C) and (K,Z) $\tilde{\subseteq}$ (L,C). Hence, Z⊆C and for all ℵ∈Z, G(ℵ)⊆ F(ℵ) and K(ℵ)⊆ L(ℵ). Let $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(K,Z)$ = (W,Z). Thus, for all ℵ∈Z,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ K'(\aleph), & \aleph\in Z\text{-}Z=\emptyset \\ F'(\aleph)\cap K(\aleph), & \aleph\in Z\cap Z=Z \end{cases}$$

Let $(G,C)\overset{*}{\underset{\gamma_\varepsilon}{}}(L,C)$ = (S,C). Thus, for for all ℵ∈C,

$$S(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}C=\emptyset \\ L'(\aleph), & \aleph\in C\text{-}C=\emptyset \\ G'(\aleph)\cap L(\aleph), & \aleph\in C\cap C=C \end{cases}$$

Hence, sinec for all ℵ∈Z, G(ℵ)⊆ F(ℵ) since F'(ℵ)⊆ G'(ℵ) W(ℵ)=F'(ℵ)∩K(ℵ) ⊆ G'(ℵ)∩L(ℵ)=S(ℵ), $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(K,Z) \,\tilde{\subseteq}\, (G,C)\overset{*}{\underset{\gamma_\varepsilon}{}}(L,C)$.

Theorem 3.4. The complementary extended gamma operation has the following distributions over other soft set operations:

Theorem 3.4.1. The complementary extended gamma operation has the following distributions over restricted soft set operations:

i) LHS Distributions of the Complementary Extended Gamma Operation on Restricted Soft Set Operations:

1) If Z∩C∩R=∅ then $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}[(G,C)\cap_R(H,R)]=[(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(G,C)]\cup_R[(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,R)]$.

Proof: Consider first the LHS. Let $(G,C)\cap_R(H,R)$=(M,C∩R), where for all ℵ∈C∩R, M(ℵ) = G(ℵ) ∩ H(ℵ). Let $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}$(M,C∩R)=(N,Z∪(C∩R)), where for all ℵ∈Z∪(C∩R),

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cap R) \\ M'(\aleph), & \aleph\in (C\cap R)\text{-}Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap(C\cap R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}(C\cap R)=Z\text{-}(C\cap R) \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cap(G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now lets handle the RHS i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=(V,Z\cup C)$, where, for all ℵ∈Z∪C,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-C \\ G'(ℵ), & ℵ\in C-Z \\ F'(ℵ)\cap G(ℵ), & ℵ\in Z\cap C \end{cases}$$

Assume that $(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(W,Z\cup R)$, where for all ℵ∈Z∪R,

$$W(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ F'(ℵ)\cap H(ℵ), & ℵ\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\cup_R(W,Z\cup R)=(T,(Z\cup C)\cap(Z\cup R)))$, where, for all $ℵ\in Z\cup(C\cap R)$, $T(ℵ)=V(ℵ)\cap W(ℵ)$. Hence,

$$T(ℵ)=\begin{cases} F'(ℵ)\cup F'(ℵ), & ℵ\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(ℵ)\cup H'(ℵ), & ℵ\in(Z-C)\cap(R-Z)=\emptyset \\ F'(ℵ)\cup(F'(ℵ)\cap H(ℵ)) & ℵ\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cup F'(ℵ), & ℵ\in(C-Z)\cap(Z-R)=\emptyset \\ G'(ℵ)\cup H'(ℵ), & ℵ\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ G'(ℵ)\cup\left(F'(ℵ)\cap H(ℵ)\right), & ℵ\in(C-Z)\cap(Z\cap R)=\emptyset \\ (F'(ℵ)\cap G(ℵ))\cup F'(ℵ), & ℵ\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(ℵ)\cap G(ℵ))\cup H'(ℵ), & ℵ\in(Z\cap C)\cap(R-Z)=\emptyset \\ (F'(ℵ)\cap G(ℵ))\cup(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(ℵ) & ℵ\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(ℵ)\cup H'(ℵ), & ℵ\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(ℵ), & ℵ\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(ℵ)\cap G(ℵ))\cup(F'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, when considering $Z-(C\cap R)$ in the function N, since $Z-(C\cap R)=Z\cap(C\cap R)'$, if an element is in the complement of $(C\cap R)$, it is either in C-R, in R-C, or in $(C\cup R)'$. Thus, if $ℵ\in Z-(C\cap R)$, then $ℵ\in Z\cap C\cap R'$ or $ℵ\in Z\cap C'\cap R$ or $ℵ\in Z\cap C'\cap R'$. Thus, N=T under $Z\cap C\cap R=\emptyset$.

2) If $Z'\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\cup_R(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $Z'\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)*_R(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

4) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$ then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\theta_R(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\cup_R[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$

ii) RHS Distribution of Complementary Extended Gamma Operation on Restricted Soft Set Operations

1) If $(Z\Delta C)\cap R=\emptyset$ if $[(F,Z)\cup_R(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cap_R[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\cup_R(G,C)=(M,Z\cap C)$, where for all $ℵ\in Z\cap C$, $M(ℵ)=F(ℵ)\cup G(ℵ)$. Let $(M,Z\cap C)\overset{*}{\cap_\varepsilon}(H,R)=(N,(Z\cap C)\cup R))$, where $ℵ\in(Z\cap C)\cup R$,

$$N(ℵ)=\begin{cases} M'(ℵ), & ℵ\in(Z\cap C)-R \\ H'(ℵ), & ℵ\in R-(Z\cap C) \\ M'(ℵ)\cap H(ℵ), & ℵ\in Z\cap(C\cap R) \end{cases}$$

Thus,

$$N(ℵ)=\begin{cases} F'(ℵ)\cap G'(ℵ), & ℵ\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(ℵ), & ℵ\in R-(Z\cap C)=R-(Z\cap C) \\ (F'(ℵ)\cap G'(ℵ))\cap H(ℵ), & ℵ\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider RHS. i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cap_R[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $ℵ\in Z\cup R$,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ F'(ℵ)\cap H(ℵ), & ℵ\in Z\cap R \end{cases}$$

Now, let $(G,C)\overset{*}{\gamma_\varepsilon}(H,R)=(W, C\cup R)$, where for all $ℵ\in C\cup R$,

$$W(ℵ)=\begin{cases} G'(ℵ), & ℵ\in C-R \\ H'(ℵ), & ℵ\in R-C \\ G'(ℵ)\cap H(ℵ), & ℵ\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\cap_R(W,C\cup R)=(T,(Z\cup R)\cap(C\cup R))$. Here, for all $ℵ\in(Z\cap C)\cup R$, $T(ℵ)=V(ℵ)\cap W(ℵ)$. . Thus,

$$T(ℵ)=\begin{cases} F'(ℵ)\cap G'(ℵ), & ℵ\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(ℵ)\cap H'(ℵ), & ℵ\in(Z-R)\cap(R-C)=\emptyset \\ F'(ℵ)\cap\big(G'(ℵ)\cap H(ℵ)\big), & ℵ\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(ℵ)\cap G'(ℵ), & ℵ\in(R-Z)\cap(C-R)=\emptyset \\ H'(ℵ)\cap H'(ℵ), & ℵ\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ H'(ℵ)\cap(G'(ℵ)\cap H(ℵ)), & ℵ\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \big(F'(ℵ)\cap H(ℵ)\big)\cap G'(ℵ), & ℵ\in(Z\cap R)\cap(C-R)=\emptyset \\ \big(F'(ℵ)\cap H(ℵ)\big)\cap H'(ℵ) & ℵ\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(ℵ)\cap H(ℵ))\cap(G'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F'(ℵ)\cap G'(ℵ), & ℵ\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(ℵ) & ℵ\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ \emptyset, & ℵ\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \emptyset, & ℵ\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(ℵ)\cap H(ℵ))\cap(G'(ℵ)\cap H(ℵ)), & ℵ\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider $R-(Z\cap C)$ in the function N, since $R-(Z\cap C)=R\cap(Z\cap C)'$, if an element is in the complement of $(Z\cap C)$, it is either in Z-C, in C-Z, or in $(Z\cup C)'$. Thus, if $ℵ\in R-(Z\cap C)$, then $ℵ\in R\cap Z\cap C'$ or $ℵ\in R\cap Z'\cap C$ or $ℵ\in R\cap Z'\cap C'$. Hence, N=T is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$. The condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ implies that $(Z\Delta C)\cap R=\emptyset$ is obvious

2) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\cap_R(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cup_R[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\theta_R(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\cup_R[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

4) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)*_R (G,C)] \underset{\gamma_\varepsilon}{\overset{*}{}} (H,R)=[(F,Z) \underset{\cap_\varepsilon}{\overset{*}{}} (H,R)] \cap_R [(G,C) \underset{\cap_\varepsilon}{\overset{*}{}} (H,R)]$.

Theorem 3.4.2. The following distributions of the complementary extended gamma operation over extended soft set operations hold:

i)LHS Distributions of the Complementary Extended Gamma Operation on Extended Soft Set Operations

1)If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $(F,Z) \underset{\gamma_\varepsilon}{\overset{*}{}} [(G,C)*_\varepsilon(H,R)]=[(F,Z) \underset{\theta_\varepsilon}{\overset{*}{}} (G,C)]\cup_\varepsilon[(F,Z) \underset{\theta_\varepsilon}{\overset{*}{}} (H,R)]$.

Proof: Consider first LHS. Let $(G,C)*_\varepsilon (H,R)=(M,C\cup R)$, where for all $\aleph \in C\cup R$,

$$M(\aleph)=\begin{cases} G(\aleph), & \aleph\in C-R \\ H(\aleph), & \aleph\in R-C \\ G'(\aleph)\cup H'(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z) \underset{\gamma_\varepsilon}{\overset{*}{}} (M,C\cup R)=(N,Z\cup(C\cup R))$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R) \\ M'(\aleph), & \aleph\in(C\cup R)-Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R)=Z\cap C'\cap R' \\ G'(\aleph), & \aleph\in(C-R)-Z=Z'\cap C\cap R' \\ H'(\aleph), & \aleph\in(R-C)-Z=Z'\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in(C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap(R-C)=Z\cap C'\cap R \\ F'(\aleph)\cap(G'(\aleph)\cup H'(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z) \underset{\theta_\varepsilon}{\overset{*}{}} (G,C)]\cup_\varepsilon[(F,Z) \underset{\theta_\varepsilon}{\overset{*}{}} (H,R)]$. Let $(F,Z) \underset{\theta_\varepsilon}{\overset{*}{}} (G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z) \underset{\theta_\varepsilon}{\overset{*}{}} (H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H'(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C) \cup_\varepsilon(W,Z\cup R)=(T,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup C)-(Z\cup R) \\ W(\aleph), & \aleph\in(Z\cup R)-(Z\cup C) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G'(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cap G'(\aleph), & \aleph\in(Z\cap C)\text{-}(Z\cup R)=\emptyset \\ F'(\aleph), & \aleph\in(Z\text{-}R)\text{-}(Z\cup C)=\emptyset \\ H'(\aleph), & \aleph\in(R\text{-}Z)\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F'(\aleph)\cap H'(\aleph), & \aleph\in(Z\cap R)\text{-}(Z\cup C)=\emptyset \\ F'(\aleph)\cup F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}C)\cap(R\text{-}Z)=\emptyset \\ F'(\aleph)\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup F'(\aleph), & \aleph\in(C\text{-}Z)\cap(Z\text{-}R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ G'(\aleph)\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(C\text{-}Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cap G'(\aleph))\cup F'(\aleph), & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G'(\aleph))\cup H'(\aleph), & \aleph\in(Z\cap C)\cap(R\text{-}Z)=\emptyset \\ (F'(\aleph)\cap G'(\aleph))\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Hence,

$$T(\aleph)=\begin{cases} G'(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ H'(\aleph), & \aleph\in(R\text{-}Z)\text{-}(Z\cup C)=Z\cap C'\cap R \\ F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ F'(\aleph), & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R \\ (F'(\aleph)\cap G'(\aleph))\cup(F'(\aleph)\cap H'(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that N=T is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equal to $(Z\Delta C)\cap R=\emptyset$.

2) If $Z'\cap C\cap R=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\cap_\varepsilon(H,R)]=[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\cap_\varepsilon[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$.

3) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\cup_\varepsilon(H,R)]=[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\cup_\varepsilon[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$.

4) If $Z\cap C\cap R'=(Z\Delta C)\cap R=\emptyset$, then $(F,Z)\underset{\gamma_\varepsilon}{*}[(G,C)\theta_\varepsilon(H,R)]=[(F,Z)\underset{\theta_\varepsilon}{*}(G,C)]\cap_\varepsilon[(F,Z)\underset{\theta_\varepsilon}{*}(H,R)]$.

ii) RHS Distributions of Complementary Extended Gamma Operation over Extended Soft Set Operations

1) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\cup_\varepsilon(G,C)]\underset{\gamma_\varepsilon}{*}(H,R)=[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]\cap_\varepsilon[(G,C)\underset{\gamma_\varepsilon}{*}(H,R)]$.

Proof: Consider first the LHS. Let $(F,Z)\cup_\varepsilon(G,C)=(M,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in Z\text{-}C \\ G(\aleph), & \aleph\in C\text{-}Z \\ F(\aleph)\cup G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z\cup C)\underset{\gamma_\varepsilon}{*}(H,R)=(N,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$N(ℵ)=\begin{cases} M'(ℵ), & ℵ\in(Z\cup C)-R \\ H'(ℵ), & ℵ\in R-(Z\cup C) \\ M'(ℵ)\cap H(ℵ), & ℵ\in(Z\cup C)\cap R \end{cases}$$

Thus,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-C)-R=Z\cap C'\cap R' \\ G'(ℵ), & ℵ\in(C-Z)-R=Z'\cap C\cap R' \\ F'(ℵ)\cap G'(ℵ), & ℵ\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(ℵ), & ℵ\in R-(Z\cup C)=Z'\cap C'\cap R \\ F'(ℵ)\cap H(ℵ), & ℵ\in(Z-C)\cap R=Z\cap C'\cap R \\ G'(ℵ)\cap H(ℵ), & ℵ\in(C-Z)\cap R=Z'\cap C\cap R \\ (F'(ℵ)\cap G'(ℵ))\cap H(ℵ), & ℵ\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\cap_\varepsilon[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $ℵ\in Z\cup R$,

$$V(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-R \\ H'(ℵ), & ℵ\in R-Z \\ F'(ℵ)\cap H(ℵ), & ℵ\in Z\cap R \end{cases}$$

Let $(G,C)\overset{*}{\gamma_\varepsilon}(H,R)=(W,C\cup R)$, where for all $ℵ\in C\cup R$,

$$W(ℵ)=\begin{cases} G'(ℵ), & ℵ\in C-R \\ H'(ℵ), & ℵ\in R-C \\ G'(ℵ)\cap H(ℵ), & ℵ\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\cap_\varepsilon(W,C\cup R)=(T,Z\cup C\cup R)$, where for all $ℵ\in Z\cup C\cup R$,

$$T(ℵ)=\begin{cases} V(ℵ), & ℵ\in(Z\cup R)-(C\cup R) \\ W(ℵ), & ℵ\in(C\cup R)-(Z\cup R) \\ V(ℵ)\cap W(ℵ), & ℵ\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ H'(ℵ), & ℵ\in(R-Z)-(C\cup R)=\emptyset \\ F'(ℵ)\cap H(ℵ), & ℵ\in(Z\cap R)-(C\cup R)=\emptyset \\ G'(ℵ), & ℵ\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ H'(ℵ), & ℵ\in(R-C)-(Z\cup R)=\emptyset \\ G'(ℵ)\cap H(ℵ), & ℵ\in(C\cap R)-(Z\cup R)=\emptyset \\ F'(ℵ)\cap G'(ℵ), & ℵ\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ F'(ℵ)\cap H'(ℵ), & ℵ\in(Z-R)\cap(R-C)=\emptyset \\ F'(ℵ)\cap(G'(ℵ)\cap H(ℵ)), & ℵ\in(Z-R)\cap(C\cap R)=\emptyset \\ H'(ℵ)\cap G'(ℵ), & ℵ\in(R-Z)\cap(C-R)=\emptyset \\ H'(ℵ)\cap H'(ℵ), & ℵ\in(R-Z)\cap(R-C)=Z\cap C'\cap R \\ H'(ℵ)\cap\big(G'(ℵ)\cap H(ℵ)\big) & ℵ\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ (F'(ℵ)\cap H(ℵ))\cap G'(ℵ), & ℵ\in(Z\cap R)\cap(C-R)=\emptyset \\ (F'(ℵ)\cap H(ℵ))\cap H'(ℵ) & ℵ\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(ℵ)\cap H(ℵ))\cap(G'(ℵ)\cap H(ℵ)) & ℵ\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F'(ℵ), & ℵ\in(Z-R)-(C\cup R)=Z\cap C'\cap R' \\ G'(ℵ), & ℵ\in(C-R)-(Z\cup R)=Z'\cap C\cap R' \\ F'(ℵ)\cap G'(ℵ), & ℵ\in(Z-R)\cap(C-R)=Z\cap C\cap R' \\ H'(ℵ), & ℵ\in(R-Z)\cap(R-C)=Z'\cap C'\cap R \\ \emptyset & ℵ\in(R-Z)\cap(C\cap R)=Z'\cap C\cap R \\ \emptyset & ℵ\in(Z\cap R)\cap(R-C)=Z\cap C'\cap R \\ (F'(ℵ)\cap H(ℵ))\cap(G'(ℵ)\cap H(ℵ)) & ℵ\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Hence, under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$, N=T is satisfied. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\cap_{\varepsilon}(G,C)]\overset{*}{\gamma_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{\gamma_{\varepsilon}}(H,R)]\cup_{\varepsilon}[(G,C)\overset{*}{\gamma_{\varepsilon}}(H,R)]$.

3) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\theta_{\varepsilon}(G,C)]\overset{*}{\gamma_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{\cap_{\varepsilon}}(H,R)]\cup_{\varepsilon}[(G,C)\overset{*}{\cap_{\varepsilon}}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)*_{\varepsilon}(G,C)]\overset{*}{\gamma_{\varepsilon}}(H,R)=[(F,Z)\overset{*}{\cap_{\varepsilon}}(H,R)]\cap_{\varepsilon}[(G,C)\overset{*}{\cap_{\varepsilon}}(H,R)]$.

Theorem 3.4.3. The following distributions of the complementary extended gamma operation over complementary extended operations hold:

i) LHS Distributions of Complementary Extended Gamma Operations over Complementary Extended Soft Set Operations

1) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\overset{*}{\gamma_{\varepsilon}}[(G,C)\overset{*}{\cap_{\varepsilon}}(H,R)]=[(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C)]\overset{*}{\cap_{\varepsilon}}[(F,Z)\overset{*}{\gamma_{\varepsilon}}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)\overset{*}{\cap_{\varepsilon}}((H,R)=(M,C\cup R)$, where for all $ℵ\in C\cup R$,

$$M(ℵ)=\begin{cases} G'(ℵ), & ℵ\in C-R \\ H'(ℵ), & ℵ\in R-C \\ G(ℵ)\cap H(ℵ), & ℵ\in C\cap R \end{cases}$$

Let $(F,Z)\overset{*}{\gamma_{\varepsilon}}(M,C\cup R)=(N,Z\cup(C\cup R))$, where for all $ℵ\in Z\cup C\cup R$,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-(C\cup R) \\ M'(ℵ), & ℵ\in(C\cup R)-Z \\ F'(ℵ)\cap M(ℵ), & ℵ\in Z\cap(C\cup R) \end{cases}$$

Thus,

$$N(ℵ)=\begin{cases} F'(ℵ), & ℵ\in Z-(C\cup R)=Z\cap C'\cap R' \\ G(ℵ), & ℵ\in(C-R)-Z=Z'\cap C\cap R' \\ H(ℵ), & ℵ\in(R-C)-Z=Z'\cap C'\cap R \\ G'(ℵ)\cup H'(ℵ), & ℵ\in(C\cap R)-Z=Z'\cap C\cap R \\ F'(ℵ)\cap G'(ℵ), & ℵ\in Z\cap(C-R)=Z\cap C\cap R' \\ F'(ℵ)\cap H'(ℵ), & ℵ\in Z\cap(R-C)=Z\cap B'\cap R \\ F'(ℵ)\cap(G(ℵ)\cap H(ℵ)), & ℵ\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C)]\overset{*}{\cap_{\varepsilon}}[(F,Z)\overset{*}{\gamma_{\varepsilon}}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_{\varepsilon}}(G,C)=(V,Z\cup C)$, where for all $ℵ\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z) \overset{*}{\gamma_\varepsilon} (H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C) \overset{*}{\cap_\varepsilon} (W,Z\cup R)=(T,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup C\cup R$

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in(Z\cup C)\text{-}(Z\cup R) \\ W'(\aleph), & \aleph\in(Z\cup R)\text{-}(Z\cup C) \\ V(\aleph)\cap W(\aleph), & \aleph\in(Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F(\aleph)\cup G'(\aleph), & \aleph\in(Z\cap C)\text{-}(Z\cup R)=\emptyset \\ F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(Z\cup C)=\emptyset \\ H(\aleph), & \aleph\in(R\text{-}Z)\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\text{-}(Z\cup C)=\emptyset \\ F'(\aleph)\cap F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in(Z\text{-}C)\cap(R\text{-}Z)=\emptyset \\ F'(\aleph)\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap F'(\aleph), & \aleph\in(C\text{-}Z)\cap(Z\text{-}R)=\emptyset \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ G'(\aleph)\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(C\text{-}Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap F'(\aleph), & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap H'(\aleph), & \aleph\in(Z\cap C)\cap(R\text{-}Z)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} G(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R\text{-}Z)\text{-}(Z\cup C)=Z'\cap C'\cap R \\ F'(\aleph) & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph)) & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

$N=T$ is satisfied under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z) \overset{*}{\gamma_\varepsilon} [(G,C) \overset{*}{\cup_\varepsilon} (H,R)]=[(F,Z) \overset{*}{\gamma_\varepsilon} (G,C)] \overset{*}{\cup_\varepsilon} [(F,Z) \overset{*}{\gamma_\varepsilon} (H,R)]$.

3) If $Z'\cap C\cap R=Z\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma}_{\varepsilon}[(G,C)\overset{*}{*}_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\theta}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}[(F,Z)\overset{*}{\theta}_{\varepsilon}(H,R)]$.

4) If $Z'\cap C\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma}_{\varepsilon}[(G,C)\overset{*}{\theta}_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\theta}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}[(F,Z)\overset{*}{\theta}_{\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Gamma Operation over Complementary Extended Operations

1) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{\theta}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\overset{*}{\cup}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\overset{*}{\theta}_{\varepsilon}(G,C)=(M,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$M(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z\cup C)\overset{*}{\gamma}_{\varepsilon}(H,R)=(N,(Z\cup C)\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in(Z\cup C)-R \\ H'(\aleph), & \aleph\in R-(Z\cup C) \\ M'(\aleph)\cap H(\aleph), & \aleph\in(Z\cup C)\cap R \end{cases}$$

Hence,

$$N(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C-Z)-R=Z'\cap C\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-(Z\cup C)=Z'\cap C'\cap R \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ G(\aleph)\cap H(\aleph), & \aleph\in(C-Z)\cap R=Z'\cap C\cap R \\ (F(\aleph)\cup G(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, i.e. $[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\overset{*}{\cup}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)=(W,C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\overset{*}{\cup}_{\varepsilon}(W,C\cup R)=(T,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in(Z\cup R)-(C\cup R) \\ W'(\aleph), & \aleph\in(C\cup R)-(Z\cup R) \\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R' \\ H(\aleph), & \aleph\in(R\text{-}Z)\text{-}(C\cup R)=\emptyset \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\text{-}(C\cup R)=\emptyset \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in(R\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G'(\aleph)\cup H'(\aleph), & \aleph\in(C\cap R)\text{-}(Z\cup R)=\emptyset \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}R)\cap(R\text{-}C)=\emptyset \\ F'(\aleph)\cup(G(\aleph)\cap H(\aleph)), & \aleph\in(Z\text{-}R)\cap(C\cap R)=\emptyset \\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R\text{-}Z)\cap(C\text{-}R)=\emptyset \\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ H'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big) & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ (F(\aleph)\cap H(\aleph))\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C\text{-}R)=\emptyset \\ \big(F(\aleph)\cap H(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ (F(\aleph)\cap H(\aleph))\cup(G(\aleph)\cap H(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap Z' \\ G(\aleph), & \aleph\in(C\text{-}R)\text{-}(Z\cup R)=Z'\cap C\cap Z \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap Z' \\ H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R \\ H'(\aleph)\cup G(\aleph), & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R \\ F(\aleph)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R \\ (F(\aleph)\cap H(\aleph))\cup(G(\aleph)\cap H(\aleph)) & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that N=T under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\overset{*}{\cup}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\overset{*}{\gamma}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

Theorem 3.4.4. The following distributions of the complementary extended gamma operation over soft binary piecewise operations hold:

i) LHS Distributions of the Complementary Extended Gamma Operation on Soft Binary Pievewise Operations

1) If $(Z\Delta C)\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma}_{\varepsilon}[(G,C)\overset{\sim}{\cap}(H,R)]=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(G,C)]\overset{\sim}{\cap}[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

Proof *:* Consider first the LHS. Let $(G,C)\overset{\sim}{\cap}(H,R)=(M,C)$. Hence for all $\aleph\in C$,

$$M(\aleph)=\begin{cases} G(\aleph), & \aleph\in C\text{-}R \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(F,Z)\underset{\gamma_\varepsilon}{*}(M,C)=(N,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ M'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in (C\text{-}R)\text{-}Z=Z'\cap C\cap R' \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)\text{-}Z=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap(C\text{-}R)=Z\cap C\cap R' \\ F'(\aleph)\cap(G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap(C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider the rhs, i.e. $[(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)]\tilde{\cap}[(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)]$. Let $(F,Z)\underset{\gamma_\varepsilon}{*}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ G'(\aleph), & \aleph\in C\text{-}Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\underset{\gamma_\varepsilon}{*}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\tilde{\cap}(W,Z\cup R)=(T,(Z\cup C))$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in (Z\cup C)\text{-}(Z\cup R) \\ V(\aleph)\cap W(\aleph), & \aleph\in (Z\cup C)\cap(Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in (Z\text{-}C)\text{-}(Z\cup R)=\emptyset \\ G'(\aleph), & \aleph\in (C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph)\cap G(\aleph), & \aleph\in (Z\cap C)\text{-}(Z\cup R)=\emptyset \\ F'(\aleph)\cap F'(\aleph), & \aleph\in (Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in (Z\text{-}C)\cap(R\text{-}Z)=\emptyset \\ F'(\aleph)\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in (Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap F'(\aleph), & \aleph\in (C\text{-}Z)\cap(Z\text{-}R)=\emptyset \\ G'(\aleph)\cap H'(\aleph), & \aleph\in (C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ G'(\aleph)\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in (C\text{-}Z)\cap(Z\cap R)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap F'(\aleph), & \aleph\in (Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap H'(\aleph), & \aleph\in (Z\cap C)\cap(R\text{-}Z)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in (Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Therefore,

$$T(\aleph)=\begin{cases} G'(\aleph), & \aleph\in(C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph), & \aleph\in(Z-C)\cap(Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C-Z)\cap(R-Z)=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)\cap(Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider Z-C in the function N, since Z-C=Z∩C' if an element is in the complement of C, it is either in R-C or (C ∪ R)'. Thus, if $\aleph\in Z-C$, then $\aleph\in Z\cap C'\cap R$ or $\aleph\in Z\cap C'\cap R'$. Thus, it is seen that N=T under the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta R)\cap C=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\tilde{\cup}\,(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\,\tilde{\cup}\,[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\tilde{*}\,(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\,\tilde{\cup}\,[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C'\cap R=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\,\tilde{\theta}\,(H,R)]=[(F,Z)\overset{*}{\theta_\varepsilon}(G,C)]\,\tilde{\cap}\,[(F,Z)\overset{*}{\theta_\varepsilon}(H,R)]$.

ii) RHS Distributions of the Complementary Extended Gamma Operation over Soft Binary Piecewise Operations

1) If $(Z\Delta C)\cap R=Z\cap C'\cap R=\emptyset$, then $[(F,Z)\,\tilde{\theta}\,(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\,\tilde{\cup}\,[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\,\tilde{\theta}\,(G,C)=(M,Z)$, where for all $\aleph\in Z$,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in Z-C \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(N,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ M'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ (F(\aleph)\cup G(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, that is, $[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\,\tilde{\cup}\,[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\cap_\varepsilon}(H,R)=(V,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Now let $(G,C)\overset{*}{\cap_\varepsilon}(H,R)=(W,C\cup R)$, where for all $\aleph\in C\cup R$,

$$W(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Let $(V,Z\cup R)\,\underset{\cup}{\sim}\,(W,C\cup R)=(T,(Z\cup R))$, where for all $\aleph\in Z\cup R$,

$$T(\aleph)=\begin{cases} V(\aleph), & \aleph\in(Z\cup R)-(C\cup R)\\ V(\aleph)\cup W(\aleph), & \aleph\in(Z\cup R)\cap(C\cup R)\end{cases}$$

Thus,,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R'\\ H'(\aleph), & \aleph\in(R\text{-}Z)\text{-}(C\cup R)=\emptyset\\ F(\aleph)\cap H(\aleph), & \aleph\in(Z\cap R)\text{-}(C\cup R)=\emptyset\\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R'\\ F'(\aleph)\cup H'(\aleph), & \aleph\in(Z\text{-}R)\cap(R\text{-}C)=\emptyset\\ F'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\text{-}R)\cap(C\cap R)=\emptyset\\ H'(\aleph)\cup G'(\aleph), & \aleph\in(R\text{-}Z)\cap(C\text{-}R)=\emptyset\\ H'(\aleph)\cup H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R\\ H'(\aleph)\cup\big(G(\aleph)\cap H(\aleph)\big) & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R\\ \big(F(\aleph)\cap H(\aleph)\big)\cup G'(\aleph), & \aleph\in(Z\cap R)\cap(C\text{-}R)=\emptyset\\ \big(F(\aleph)\cap H(\aleph)\big)\cup H'(\aleph) & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R\\ \big(F'(\aleph)\cap H(\aleph)\big)\cup\big(G(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R\end{cases}$$

Hence

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in(Z\text{-}R)\text{-}(C\cup R)=Z\cap C'\cap R'\\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\text{-}R)\cap(C\text{-}R)=Z\cap C\cap R'\\ H'(\aleph), & \aleph\in(R\text{-}Z)\cap(R\text{-}C)=Z'\cap C'\cap R\\ H'(\aleph)\cup G(\aleph), & \aleph\in(R\text{-}Z)\cap(C\cap R)=Z'\cap C\cap R\\ F(\aleph)\cup H'(\aleph), & \aleph\in(Z\cap R)\cap(R\text{-}C)=Z\cap C'\cap R\\ \big(F'(\aleph)\cap H(\aleph)\big)\cup\big(G'(\aleph)\cap H(\aleph)\big), & \aleph\in(Z\cap R)\cap(C\cap R)=Z\cap C\cap R\end{cases}$$

Under the condition $Z'\cap C\cap R=Z\cap C'\cap R=Z\cap C\cap R'=\emptyset$, it can be seen that N=T. It is obvious that the condition $Z'\cap C\cap R=Z\cap C'\cap R=\emptyset$ is equivalent to the condition $(Z\Delta C)\cap R=\emptyset$.

2) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\underset{\cup}{\sim}(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\underset{\cap}{\sim}[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

3) If $(Z\Delta C)\cap R=\emptyset$, then $[(F,Z)\underset{\cap}{\sim}(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]\underset{\cup}{\sim}[(G,C)\overset{*}{\gamma_\varepsilon}(H,R)]$.

4) If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $[(F,Z)\underset{*}{\sim}(G,C)]\overset{*}{\gamma_\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]\underset{\cap}{\sim}[(G,C)\overset{*}{\cap_\varepsilon}(H,R)]$.

Theorem 3.4.5. The following distributions of the complementary extended gamma operation over the complementary soft binary piecewise operations exist:

i) LHS Distribution of the Complementary Extended Gamma Operation on Complementary Soft Binary Piecewise Operations

1)If $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$, then $(F,Z)\overset{*}{\gamma_\varepsilon}[(G,C)\overset{*}{\underset{\cap}{\sim}}(H,R)]=[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\overset{*}{\underset{\cap}{\sim}}[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$.

Proof: Consider first LHS. Let $(G,C)\overset{*}{\underset{\cap}{\sim}}(H,R)=(M,C)$, where for all $\aleph\in C$,

$$M(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C\text{-}R\\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R\end{cases}$$

Let $(F,Z)\overset{*}{\gamma_\varepsilon}(M,C)=(N,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ M'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap M(\aleph), & \aleph\in Z\cap C \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G(\aleph), & \aleph\in (C-R)-Z=Z'\cap C\cap R' \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)-Z=Z'\cap C\cap R \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap (C-R)=Z\cap C\cap R' \\ F'(\aleph)\cap (G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap (C\cap R)=Z\cap C\cap R \end{cases}$$

Now consider RHS, i.e. $[(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)]\underset{\cap}{\overset{*}{\sim}}[(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)]$. Let $(F,Z)\overset{*}{\gamma_\varepsilon}(G,C)=(V,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F'(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(F,Z)\overset{*}{\gamma_\varepsilon}(H,R)=(W,Z\cup R)$, where for all $\aleph\in Z\cup R$,

$$W(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-R \\ H'(\aleph), & \aleph\in R-Z \\ F'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Let $(V,Z\cup C)\underset{\cap}{\overset{*}{\sim}}(W,Z\cup R)=(T,(Z\cup C))$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} V'(\aleph), & \aleph\in (Z\cup C)-(Z\cup R) \\ V(\aleph)\cap W(\aleph), & \aleph\in (Z\cup C)\cap (Z\cup R) \end{cases}$$

Thus,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph\in (Z-C)-(Z\cup R)=\emptyset \\ G(\aleph), & \aleph\in (C-Z)-(Z\cup R)=Z'\cap C\cap R' \\ F(\aleph)\cup G'(\aleph), & \aleph\in (Z\cap C)-(Z\cup R)=\emptyset \\ F'(\aleph)\cap F'(\aleph), & \aleph\in (Z-C)\cap (Z-R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H'(\aleph), & \aleph\in (Z-C)\cap (R-Z)=\emptyset \\ F'(\aleph)\cap (F'(\aleph)\cap H(\aleph)), & \aleph\in (Z-C)\cap (Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap F'(\aleph), & \aleph\in (C-Z)\cap (Z-R)=\emptyset \\ G'(\aleph)\cap H'(\aleph), & \aleph\in (C-Z)\cap (R-Z)=Z'\cap C\cap R \\ G'(\aleph)\cap (F'(\aleph)\cap H(\aleph)), & \aleph\in (C-Z)\cap (Z\cap R)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap F'(\aleph), & \aleph\in (Z\cap C)\cap (Z-R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap H'(\aleph), & \aleph\in (Z\cap C)\cap (R-Z)=\emptyset \\ (F'(\aleph)\cap G(\aleph))\cap (F'(\aleph)\cap H(\aleph)), & \aleph\in (Z\cap C)\cap (Z\cap R)=Z\cap C\cap R \end{cases}$$

Hence,

$$T(\aleph)=\begin{cases} G(\aleph), & \aleph\in(C\text{-}Z)\text{-}(Z\cup R)=Z'\cap C\cap R' \\ F'(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\text{-}R)=Z\cap C'\cap R' \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap(Z\cap R)=Z\cap C'\cap R \\ G'(\aleph)\cap H'(\aleph), & \aleph\in(C\text{-}Z)\cap(R\text{-}Z)=Z'\cap C\cap R \\ F'(\aleph)\cap G(\aleph), & \aleph\in(Z\cap C)\cap(Z\text{-}R)=Z\cap C\cap R' \\ (F'(\aleph)\cap G(\aleph))\cap(F'(\aleph)\cap H(\aleph)), & \aleph\in(Z\cap C)\cap(Z\cap R)=Z\cap C\cap R \end{cases}$$

Here, if we consider Z-C in the function N, since Z-C=Z∩C', if an element in the complement of C it is either in R-C or $(C \cup R)'$. Thus, if $\aleph \in Z - C$, then $\aleph \in Z\cap C'\cap R$ or $\aleph \in Z\cap C'\cap R'$. Hence, it is seen that N=T under the condition Z'∩C∩R=Z∩C'∩R=Z∩C∩R'=∅. It is obvious that the condition Z'∩C∩R=Z∩C'∩R=∅ is equivalent to the condition (ZΔC)∩R=∅.

2) If (ZΔR)∩C=∅, then $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}[(G,C)\overset{*}{\underset{\cup}{\sim}}(H,R)]=[(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(G,C)]\overset{*}{\underset{\cup}{\sim}}[(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,R)]$.

3)If (ZΔR)∩C= ∅, then $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}[(G,C)\overset{*}{\sim}(H,R)]=[(F,Z)\overset{*}{\underset{\theta_\varepsilon}{}}(G,C)]\overset{*}{\underset{\cup}{\sim}}[(F,Z)\overset{*}{\underset{\theta_\varepsilon}{}}(H,R)]$.

4)If (ZΔC)∩R=∅, then $(F,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}[(G,C)\overset{*}{\underset{\theta}{\sim}}(H,R)]=[(F,Z)\overset{*}{\underset{\theta_\varepsilon}{}}(G,C)]\overset{*}{\underset{\cap}{\sim}}[(F,Z)\overset{*}{\underset{\theta_\varepsilon}{}}(H,R)]$.

ii) RHS Distributions of Complementary Extended Gamma Operation over Complementary Soft Binary Piecewise Operations

1) If (ZΔC)∩R=Z∩C∩R'=∅, then $[(F,Z)\overset{*}{\underset{\theta}{\sim}}(G,C)]\overset{*}{\underset{\gamma_\varepsilon}{}}(H,R)=[(F,Z)\overset{*}{\underset{\cap_\varepsilon}{}}(H,R)]\overset{*}{\underset{\cup}{\sim}}[(G,C)\overset{*}{\underset{\cap_\varepsilon}{}}(H,R)]$.

Proof: Consider first LHS. Let $(F,Z)\overset{*}{\underset{\theta}{\sim}}(G,C)=(M,Z)$, where for all ℵ∈Z,

$$M(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}C \\ F'(\aleph)\cap G'(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(M,Z)\overset{*}{\underset{\gamma_\varepsilon}{}}(H,R)=(N,Z\cup R)$, where for all ℵ∈Z∪R,

$$N(\aleph)=\begin{cases} M'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ M'(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Thus,

$$N(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z\text{-}C)\text{-}R=Z\cap C'\cap R' \\ F(\aleph)\cup G(\aleph), & \aleph\in(Z\cap C)\text{-}R=Z\cap C\cap R' \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F(\aleph)\cap H(\aleph), & \aleph\in(Z\text{-}C)\cap R=Z\cap C'\cap R \\ (F(\aleph)\cup G(\aleph))\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now consider the RHS, that is, $[(F,Z)\overset{*}{\underset{\cap_\varepsilon}{}}(H,R)]\overset{*}{\underset{\cup}{\sim}}[(G,C)\overset{*}{\underset{\cap_\varepsilon}{}}(H,R)]$. Let $(F,Z)\overset{*}{\underset{\cap_\varepsilon}{}}(H,R)=(V,Z\cup R)$, where for all ℵ∈Z∪R,

$$V(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z\text{-}R \\ H'(\aleph), & \aleph\in R\text{-}Z \\ F(\aleph)\cap H(\aleph), & \aleph\in Z\cap R \end{cases}$$

Now let $(G,C)\overset{*}{\underset{\cap_\varepsilon}{}}(H,R)=(W,C\cup R)$, where for all ℵ∈CUR,

$$W(ℵ)=\begin{cases} G'(ℵ), & ℵ∈C\text{-}R \\ H'(ℵ), & ℵ∈R\text{-}C \\ G(ℵ)∩H(ℵ), & ℵ∈C∩R \end{cases}$$

Let $(V,Z∪R) \underset{∪}{\overset{*}{\sim}} (W,C∪R)=(T,(Z∪R))$, where for all $ℵ∈Z∪R$,

$$T(ℵ)=\begin{cases} V'(ℵ), & ℵ∈(Z∪R)\text{-}(C∪R) \\ V(ℵ)∪W(ℵ), & ℵ∈(Z∪R)∩(C∪R) \end{cases}$$

Thus,

$$T(ℵ)=\begin{cases} F(ℵ), & ℵ∈(Z\text{-}R)\text{-}(C∪R)=Z∩C'∩R' \\ H(ℵ), & ℵ∈(R\text{-}Z)\text{-}(C∪R)=∅ \\ F'(ℵ)∪H'(ℵ), & ℵ∈(Z∩R)\text{-}(C∪R)=∅ \\ F'(ℵ)∪G'(ℵ), & ℵ∈(Z\text{-}R)∩(C\text{-}R)=Z∩C∩R' \\ F'(ℵ)∪H'(ℵ), & ℵ∈(Z\text{-}R)∩(R\text{-}C)=∅ \\ F'(ℵ)∪\big(G(ℵ)∩H(ℵ)\big), & ℵ∈(Z\text{-}R)∩(C∩R)=∅ \\ H'(ℵ)∪G'(ℵ), & ℵ∈(R\text{-}Z)∩(C\text{-}R)=∅ \\ H'(ℵ)∪H'(ℵ), & ℵ∈(R\text{-}Z)∩(R\text{-}C)=Z'∩C'∩R \\ H'(ℵ)∪\big(G(ℵ)∩H(ℵ)\big) & ℵ∈(R\text{-}Z)∩(C∩R)=Z'∩C∩R \\ \big(F(ℵ)∩H(ℵ)\big)∪G'(ℵ), & ℵ∈(Z∩R)∩(C\text{-}R)=∅ \\ \big(F(ℵ)∩H(ℵ)\big)∪H'(ℵ) & ℵ∈(Z∩R)∩(R\text{-}C)=Z∩C'∩R \\ \big(F(ℵ)∩H(ℵ)\big)∪\big(G(ℵ)∩H(ℵ)\big), & ℵ∈(Z∩R)∩(C∩R)=Z∩C∩R \end{cases}$$

Therefore,

$$T(ℵ)=\begin{cases} F(ℵ), & ℵ∈(Z\text{-}R)\text{-}(C∪R)=Z∩C'∩R' \\ F'(ℵ)∪G'(ℵ), & ℵ∈(Z\text{-}R)∩(C\text{-}R)=Z∩C∩R' \\ H'(ℵ), & ℵ∈(R\text{-}Z)∩(R\text{-}C)=Z'∩C'∩R \\ H'(ℵ)∪G(ℵ), & ℵ∈(R\text{-}Z)∩(C∩R)=Z'∩C∩R \\ F(ℵ)∪H'(ℵ), & ℵ∈(Z∩R)∩(R\text{-}C)=Z∩C'∩R \\ \big(F(ℵ)∩H(ℵ)\big)∪\big(G(ℵ)∩H(ℵ)\big), & ℵ∈(Z∩R)∩(C∩R)=Z∩C∩R \end{cases}$$

Under the condition $Z'∩C∩R=Z∩C'∩R= Z∩C∩R'=∅$, N=T is satisfied. It is obvious that the condition $Z'∩C∩R= Z∩C'∩R= ∅$ is equivalent to $(ZΔC)∩R= ∅$.

2) If $(ZΔC)∩R=Z∩C∩R'=∅$, then $[(F,Z)\underset{∪}{\overset{*}{\sim}} (G,C)] \overset{*}{\gamma_ε} (H,R)=[(F,Z) \overset{*}{\gamma_ε} (H,R)]\underset{∩}{\overset{*}{\sim}}[(G,C) \overset{*}{\gamma_ε} (H,R)]$.

3)I

f $(ZΔC)∩R=∅$, then $[(F,Z)\underset{∩}{\overset{*}{\sim}} (G,C)] \overset{*}{\gamma_ε} (H,R) =[(F,Z) \overset{*}{\gamma_ε} (H,R)]\underset{∪}{\overset{*}{\sim}}[(G,C) \overset{*}{\gamma_ε} (H,R)]$.

4) If $(ZΔC)∩R=Z∩C∩R'=∅$, then $[(F,Z)\underset{*}{\overset{*}{\sim}} (G,C)] \overset{*}{\gamma_ε}(H,R) =[(F,Z) \overset{*}{∩_ε} (H,R)]\underset{∩}{\overset{*}{\sim}}[(G,C) \overset{*}{∩_ε}(H,R)]$.

4. CONCLUSION

Soft set operations play a central role in soft set theory, offering a soft structure for addressing uncertainty in data analysis and decision-making. This study investigates the algebraic features of a new soft set operation called complementary extended gamma operation. We also study the distribution of complementary extended gamma over several more soft set operations. Our hope is that this work will provide a foundation for further research on soft set operations. In order to determine what algebraic structures emerge in the collection of soft sets together with complementary extended gamma operations of soft sets, more research may look at different types of complementary extended soft set operations, as well as their distributions and properties.

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