Maltepe Journal of Mathematics
ISSN:2667-7660, URL:http://dergipark.org.tr/tr/pub/mjm
Volume VI Issue 2 (2024), Pages 50-60, Doi: https://doi.org/10.47087/mjm.1482207

ON A WEAK FORM OF SEMI-OPEN FUNCTION BY NEUTROSOPHICATION

AHU ACIKGOZ[1] AND FERHAT ESENBEL[2]
[1,2] DEPARTMENT OF MATHEMATICS, BALIKESIR UNIVERSITY,10145 BALIKESIR, TURKEY
[1]ORCID ID: 0000-0003-1468-8240
[2]ORCID ID: 0000-0001-5581-3621

Abstract. In this study, a new perspective in neutrosophic topology is brought to some open set definitions in general topology and previously interpreted in different ways in some other non-standard topological spaces. Then, some types of functions and continuity are introduced using these new open sets. In addition to these, the relations between the types of open sets, which we have brought different perspectives, with the continuity and function types that we reinterpreted in neutrosophic topology, are examined and these relations are clarified by giving examples and counterexamples.

1. Introduction

The concept of neutrosophic sets was introduced by Smarandache in his classical paper [10]. After the discovery of the neutrosophic subsets, much attention has been paid to generalize the basic concepts of classical topology in neutrosophic setting and thus a modern theory of neutrosophic topology is developed. The notion of neutrosophic subsets naturally plays a significant role in the study of neutrosophic topology which was introduced by Salama and Alblowi [9] in 2012. In 2021, Acikgoz et. all [1], introduced the concepts of neutrosophic quasi-coincidence and neutrosophic q-neighbourhoods. As in [2, 3], these new concepts were used very effectively and gave some mathematicians the opportunity to reconsider some of the cornerstones of the world of topology in neutrosophic setting. In 1985, Rose [8] defined weakly open functions in a topological spaces. In 1997 J.H. Park et. all [7] introduced the notion of weakly open functions for a fuzzy topological space. In [4], Caldas et. all, introduced and discussed the concept of fuzzy weakly semiopen function which is weaker than fuzzy weakly open and fuzzy almost open functions introduced by [7] and Nanda [6] respectively and obtained several properties and characterizations of these functions comparing with the other functions. In this

2020 *Mathematics Subject Classification.* Primary 54A05; Secondary 54C10, 54D30, 54D101.
Key words and phrases. Neutrosophic semiopen set; Neutrosophic regular open set; Neutrosophic β-open set; Neutrosophic weakly open function; Neutrosophic almost open function; Neutrosophic weakly semiopen function; Neutrosophic θ-closure; neutrosophic hyperconnected space.

Submitted on May 11th, 2024. Accepted on October 3rd, 2024.
Communicated by Huseyin CAKALLI and Nazlım Deniz ARAL.

study, using the concepts of neutrosophic quasi-coincidence and neutrosophic q-neighbourhoods, we reinterpreted this function and open set variants and got reality of that neutrosophic semiopenness implied neutrosophic weakly semiopenness but converse was not true. We also showed that the reverse statement was also true when certain conditions were met.

2. Preliminaries

In this section, we present the basic definitions related to neutrosophic set theory.

Definition 2.1. [9]*A neutrosophic set A on the universe set X is defined as:*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

where $T,\ I,\ F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Scientifically, membership functions, indeterminacy functions and non-membership functions of a neutrosophic set take value from real standart or nonstandart subsets of $]^-0, 1^+[$. However, these subsets are sometimes inconvenient to be used in real life applications such as economical and engineering problems. On account of this fact, we consider the neutrosophic sets, whose membership function, indeterminacy functions and non-membership functions take values from subsets of $[0, 1]$.

Definition 2.2. [5] *Let X be a nonempty set. If $r,\ t,\ s$ are real standard or non standard subsets of $]^-0, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point in X given by*

$$x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1), & \text{if } x \neq x_p \end{cases}$$

For $x_p \in X$, it is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Definition 2.3. [8] *Let A be a neutrosophic set over the universe set X. The complement of A is denoted by A^c and is defined by:*
$A^c = \Big\{\Big\langle x, F_{\tilde{F}(e)}(x), 1 - I_{\tilde{F}(e)}(x), T_{\tilde{F}(e)}(x)\Big\rangle : x \in X\Big\}$.
It is obvious that $[A^c]^c = A$.

Definition 2.4. [8] *Let A and B be two neutrosophic sets over the universe set X. A is said to be a neutrosophic subset of B if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$, every x in X. It is denoted by $A \subseteq B$. A is said to be neutrosophic soft equal to B if $A \subseteq B$ and $B \subseteq A$. It is denoted by $A = B$.*

Definition 2.5. [8] *Let F_1 and F_2 be two neutrosophic soft sets over the universe set X. Then their union is denoted by $F_1 \cup F_2 = F_3$ is defined by:*

$$F_3 = \{\langle x, T_{F_3}(x), I_{F_3}(x), F_{F_3}(x) : x \in X\rangle\},$$

where

$$T_{F_3(x)} = \max\{T_{F_1(x)}, T_{F_2}(x)\},$$

$$I_{F_3(x)} = \max\{I_{F_1(x)}, I_{F_2}(x)\},$$

$$F_{F_3(x)} = \min\{F_{F_1(x)}, F_{F_2}(x)\}.$$

Definition 2.6. [8] *Let F_1 and F_2 be two neutrosophic soft sets over the universe set X. Then their intersection is denoted by $F_1 \cap F_2 = F_4$ is defined by:*

$$F_4 = \{\langle x, T_{F_4}(x), I_{F_4}(x), F_{F_4}(x) : x \in X\rangle\},$$

where

$$T_{F_4(x)} = \min\{T_{F_1(x)}, T_{F_2}(x)\},$$

$$I_{F_4(x)} = \min\{I_{F_1(x)}, I_{F_2}(x)\},$$

$$F_{F_4(x)} = \max\{F_{F_1(x)}, F_{F_2}(x)\}.$$

Definition 2.7. [8]*A neutrosophic set F over the universe set X is said to be a null neutrosophic set if $T_F(x) = 0$, $I_F(x) = 0$, $F_F(x) = 1$, every $x \in X$. It is denoted by 0_X.*

Definition 2.8. [8] *A neutrosophic set F over the universe set X is said to be an absolute neutrosophic set if $T_F(x) = 1$, $I_F(x) = 1$, $F_F(x) = 0$, every $x \in X$. It is denoted by 1_X.*

Clearly $0_X^c = 1_X$ and $1_X^c = 0_X$.

Definition 2.9. [8] *Let $NS(X)$ be the family of all neutrosophic sets over the universe the set X and $\tau \subset NS(X)$. Then τ is said to be a neutrosophic topology on X if:*

1) 0_X and 1_X belong to τ;

2) The union of any number of neutrosophic soft sets in τ belongs to τ;

3) The intersection of a finite number of neutrosophic soft sets in τ belongs to τ.

Then (X, τ) is said to be a neutrosophic topological space over X. Each member of τ is said to be a neutrosophic open set [8].

Definition 2.10. [8] *Let (X, τ) be a neutrosophic topological space over X and F be a neutrosophic set over X. Then F is said to be a neutrosophic closed set iff its complement is a neutrosophic open set.*

Definition 2.11. [2] *A neutrosophic point $x_{r,t,s}$ is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with F, denoted by $x_{r,t,s}\ q\ F$ if and only if $x_{r,t,s} \nsubseteq F^c$. If $x_{r,t,s}$ is not neutrosophic quasi-coincident with F, we denote by $x_{r,t,s}\ \tilde{q}\ F$.*

Definition 2.12. [2] *A neutrosophic set F in a neutrosophic topological space (X, τ) is said to be a neutrosophic q-neighborhood of a neutrosophic point $x_{r,t,s}$ if and only if there exists a neutrosophic open set G such that*
$x_{r,t,s}\ q\ G \subset F$.

Definition 2.13. [2] *A neutrosophic set G is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with F, denoted by $G\ q\ F$ if and only if $G \nsubseteq F^c$. If G is not neutrosophic quasi-coincident with F, we denote by $G\ \tilde{q}\ F$.*

Definition 2.14. [3] *A neutrosophic point $x_{r,t,s}$ is said to be a neutrosophic interior point of a neutrosophic set F if and only if there exists a neutrosophic open q-neighborhood G of $x_{r,t,s}$ such that $G \subset F$. The union of all neutrosophic interior points of F is called the neutrosophic interior of F and denoted by F°.*

Definition 2.15. [2] *A neutrosophic point $x_{r,t,s}$ is said to be a neurosophic cluster point of a neutrosophic set F if and only if every neutrosophic open q-neighborhood G of $x_{r,t,s}$ is q-coincident with F. The union of all neutrosophic cluster points of F is called the neutrosophic closure of F and denoted by $\overline{F}$.*

Definition 2.16. [2] *Let f be a function from X to Y. Let B be a neutrosophic set in Y with members hip function $T_B(y)$, indeterminacy function $I_B(y)$ and non-membership function $F_B(y)$. Then, the inverse image of B under f, written as $f^{-1}(B)$, is a neutrosophic subset of X whose membership function, indeterminacy function and non-membership function are defined as $T_{f^{-1}(B)}(x) = T_B(f(x))$, $I_{f^{-1}(B)}(x) = I_B(f(x))$ and $F_{f^{-1}(B)}(x) = F_B(f(x))$ for all x in X, respectively. Conversely, let A be a neutrosophic set in X with membership function $T_A(x)$, indeterminacy function $I_A(x)$ and non-membership function $F_A(x)$. The image of A under f, written as $f(A)$, is a neutrosophic subset of Y whose membership function, indeterminacy function and non-membership function are defined as*

$$T_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{T_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty, \\ 0, & if\ f^{-1}(y)\ is\ empty, \end{cases}$$

$$I_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{I_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty, \\ 0, & if\ f^{-1}(y)\ is\ empty, \end{cases}$$

$$F_{f(A)}(y) = \begin{cases} sup_{z\in f^{-1}(y)}\{F_A(z)\}, & if\ f^{-1}(y)\ is\ not\ empty, \\ 0, & if\ f^{-1}(y)\ is\ empty, \end{cases}$$

for all y in Y, where $f^{-1}(y) = \{x : f(x) = y\}$, respectively.

3. Some Definitions

This section provides some new definitions that form the cornerstones of the sections that follow.

Definition 3.1. *A neutrosophic set F in a neutrosophic topological space (X, τ) is said to be*

a) Neutrosophic semiopen, if $F \subseteq \overline{(F^\circ)}$,
b) Neutrosophic semiclosed, if $\left(\overline{F}\right)^\circ \subseteq F$,
c) Neutrosophic preopen, $F \subseteq \left(\overline{F}\right)^\circ$,
d) Neutrosophic regular open, if $F = \left(\overline{F}\right)^\circ$,
e) Neutrosophic α-open, if $F \subseteq \left(\overline{(F^\circ)}\right)^\circ$,
f) Neutrosophic β-open, $F \subseteq \overline{\left(\overline{F}\right)^\circ}$. Equivalently, if there exists a neutrosophic preopen set A such that $A \subseteq F \subseteq \overline{A}$.

Definition 3.2. *If, F be a neutrosophic set in neutrosophic topological space (X, τ) then, $\overline{F}_s = \bigcap\{F : F \subseteq A,\ A\ is\ neutrosophic\ semiclosed\}$ (resp. $F_s^\circ = \bigcup\{F : F \subseteq A,\ A\ is\ neutrosophic\ semiopen\}$) is called a neutrosophic semiclosure of F (resp. neutrosophic semi-interior of F).*

A neutrosophic set F in a neutrosophic topological space (X, τ) is neutrosophic semiclosed (resp. neutrosophic semiopen) if and only if $F = \overline{F}_s$ (resp. $F = F_s^\circ$).

Definition 3.3. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function from a neutrosophic topological space (X,τ) into a neutrosophic topological space (Y,σ). The function f is said to be:*

a) Neutrosophic semiopen, if $f(F)$ is a neutrosophic semiopen set of Y for each neutrosophic open set F in X.
b) Neutrosophic weakly open, if $f(F)\subseteq\left(f(\overline{F})\right)^{\circ}$ is for each neutrosophic open set F in X.
c) Neutrosophic almost open, if $f(F)$ is a neutrosophic open set of Y for each neutrosophic regular open set F in X.
d) Neutrosophic β-open, if $f(F)$ is a neutrosophic β-open set of Y for each neutrosophic open set F in X.

Definition 3.4. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function from a neutrosophic topological space (X,τ) into a neutrosophic topological space (Y,σ). The function f is said to be neutrosophic semicontinuous, if $f^{-1}(A)$ is a neutrosophic semiopen set of X, for each $A\in\sigma$.*

4. Neutrosophic Weakly Semiopen Functions

Since the concepts of neutrosophic semicontinuity and neutrosophic semiopenness are indispensable for each other, how the concepts of neutrosophic weak semiopenness and the neutrosophic weak semicontinuity is clarified in this study.

Definition 4.1. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function from a neutrosophic topological space (X,τ) into a neutrosophic topological space (Y,σ). The function f is said to be neutrosophic weakly semiopen, if $f(A)\subseteq\left(f\left(\overline{A}\right)\right)^{\circ}_{s}$, for each $A\in\sigma$.*

Obviously, every neutrosophic weakly open function is neutrosophic weakly semiopen and every neutrosophic semiopen function is also neutrosophic weakly semiopen.

Example 4.1. *Let $X=x,y,z$, $Y=a,b,c$ and neutrosophic sets λ and μ are defined as: $\lambda=\{\langle x,0,0,1\rangle,\langle y,0.3,0.3,0.7\rangle,\langle z,0.2,0.2,0.8\rangle\}$*
$\mu=\{\langle a,0,0,1\rangle,\langle b,0.2,0.2,0.8\rangle,\langle c,0.1,0.1,0.9\rangle\}$
Let $\tau=\{0,\lambda,1\}$ and $\sigma=\{0,\mu,1\}$. Then the mapping $f:(X,\tau)\to(Y,\sigma)$ defined by $f(x)=a$, $f(y)=b$ and $f(z)=c$ is neutrosophic weakly semiopen but neither neutrosophic semiopen nor neutrosophic weakly open.

Definition 4.2. *A neutrosophic point $x_{r,t,s}$ is said to be a neutrosophic θ-cluster point of a neutrosophic set λ, if, for every neutrosophic open q-nbd μ of $x_{r,t,s}$, $\overline{\mu}$ is q-coincident with λ. The set of all neutrosophic θ-cluster points of λ is called the neutrosophic θ-closure of λ and will be denoted by $\overline{\lambda}_\theta$. A neutrosophic λ will be called neutrosophic θ-closed if and only if $\lambda=\overline{\lambda}_\theta$. The complement of a neutrosophic θ-closed set is called of neutrosophic θ-open and the neutrosophic θ-interior of λ denoted by λ°_{θ} is defined as $\lambda^{\circ}_{\theta}=\{x_{r,t,s}\mid for\ some\ neutrosophic\ open\ q-$ nbd, β of $x_{r,t,s},\overline{\beta}\subseteq\lambda\}$.*

Lemma 4.1. *Let λ be a neutrosophic set in a neutrosophic topological space X, then:*

1) λ is a neutrosophic θ-open if and only if $\lambda=\lambda^{\circ}_{\theta}$.
2) $\left(\lambda^{\circ}_{\theta}\right)^{c}=\overline{(\lambda^{c})}_{\theta}$ and $\left(\lambda^{c}\right)^{\circ}_{\theta}=\left(\overline{\lambda}_{\theta}\right)^{c}$.

3) $\overline{\lambda}_\theta$ is a neutrosophic closed set but not necessarily is a neutrosophic θ-closed set.

Theorem 4.2. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function from a neutrosophic topological space (X,τ) into a neutrosophic topological space (Y,σ). Then, following conditions are equivalent:*

(i) f is neutrosophic weakly semiopen;
(ii) $f\big(\lambda^\circ_\theta\big)\subseteq\big(f(\lambda)\big)^\circ_s$ for every neutrosophic subset λ of X;
(iii) $\Bigg(f^{-1}\big(\beta\big)\Bigg)^\circ_\theta\subseteq f^{-1}\big(\beta^\circ_s\big)$ for every neutrosophic subset β of Y;
(iv) $f^{-1}\Bigg(\overline{\beta_s}\Bigg)\subseteq\overline{\Bigg(f^{-1}\big(\beta\big)\Bigg)}_\theta$ for every neutrosophic subset β of Y;
(v) For each neutrosophic θ-open set λ in X, $f(\lambda)$ is neutrosophic semiopen in Y;
(vi) For any neutrosophic set β of Y and any neutrosophic θ-closed set λ in X containing $f^{-1}(\beta)$, where X is a neutrosophic regular space, there exists a neutrosophic semiclosed set δ in Y containing β such that $f^{-1}(\delta)\subseteq\lambda$.

Proof. (i) $\Rightarrow$ (ii): Let λ be any neutrosophic subset of X and $x_{r,t,s}$ be a neutrosophic point in λ°_θ. Then, there exists a neutrosophic open $q-nbd$ γ of $x_{r,t,s}$ such that $\gamma\subseteq\overline{\gamma}\subseteq\lambda$. Then, $f(\gamma)\subseteq f(\overline{\gamma})\subseteq f(\lambda)$. Since f is neutrosophic weakly semiopen, $f(\gamma)\subseteq(f(\overline{\gamma}))^\circ_s\subseteq(f(\lambda))^\circ_s$. It implies that $f(x_{r,t,s})$ is a point in $(f(\lambda))^\circ_s$. This shows that $x_{r,t,s}\in f^{-1}((f(\lambda))^\circ_s)$. Thus $\lambda^\circ_\theta\subseteq f^{-1}((f(\lambda))^\circ_s)$, and so $f(\lambda^\circ_\theta)\subseteq(f(\lambda))^\circ_s$.
(ii)$\Rightarrow$ (i): Let μ be a neutrosophic open set in X. As $\mu\subseteq(\overline{\mu})^\circ_\theta$ implies,
$f(\mu)\subseteq f((\overline{\mu})^\circ_\theta)\subseteq(f(\overline{\mu}))^\circ_s$. Hence f is neutrosophic weakly semiopen.
(ii) $\Rightarrow$ (iii): Let β be any neutrosophic subset of Y. Then by (ii), $f((f^{-1}(\beta))^\circ_\theta)\subseteq\beta^\circ_s$. Therefore, $(f^{-1}(\beta))^\circ_\theta\subseteq f^{-1}(\beta^\circ_s)$.
(iii) $\Rightarrow$ (ii): This is obvious.
(iii) $\Rightarrow$ (iv): Let β be any neutrosophic subset of Y. Using (iii), we have

$$\Bigg[\overline{\big(f^{-1}(\beta)\big)}_\theta\Bigg]^c=$$
$$\Bigg[\big(f^{-1}(\beta)\big)^c\Bigg]^\circ_\theta=$$
$$\Bigg[f^{-1}(\beta^c)\Bigg]^\circ_\theta\subseteq f^1\Bigg(\big(\beta^c\big)^\circ_s\Bigg)=$$
$$f^{-1}\Bigg(\overline{(\beta_s)}\Bigg)^c=\Bigg[f^{-1}\overline{(\beta_s)}\Bigg]^c.$$

Therefore, we obtain $f^{-1}(\overline{\beta_s})\subseteq\overline{(f^{-1}(\beta))}_\theta$.
(iv) $\Rightarrow$ (iii): Similary we obtain, $[f^{-1}(\beta^\circ_s)]^c\subseteq[(f^{-1}(\beta))^\circ_\theta]^c$, for every neutrosophic subset β of Y, i.e., $(f^{-1}(\beta))^\circ_\theta\subseteq f^{-1}(\beta^\circ_s)$.
(iv) $\Rightarrow$ (v): Let λ be a neutrosophic θ-open set in X. Then, $(f(\lambda))^c$ is a neutrosophic set in Y and by (iv),

$$f^{-1}\Bigg(\overline{\big((f(\lambda))^c\big)_s}\Bigg)\subseteq\overline{f^{-1}\Bigg(\big(f(\lambda)\big)^c\Bigg)}_\theta.$$

Therefore, $\Bigg[f^{-1}\Bigg(\big(f(\lambda)\big)^\circ_s\Bigg)\Bigg]^c\subseteq\overline{(\lambda^c)}_\theta=\lambda^c$. Then, we have $\lambda\subseteq f^{-1}(\big(f(\lambda)\big)^\circ_s)$ which implies $f(\lambda)\subseteq(f(\lambda))^\circ_s$. Hence $f(\lambda)$ is neutrosophic semiopen in Y .

(v) $\Rightarrow$ (vi): Let β be any neutrosophic set in Y and λ be a neutrosophic θ-closed set in X such that $f^{-1}(\beta) \subseteq \lambda$. Since λ^c is neutrosophic θ-open in X, by (v), $f(\lambda^c)$ is neutrosophic semiopen in Y. Let $\delta = \left[f(\lambda^c)\right]^c$. Then δ is neutrosophic semiclosed and also $\beta \subseteq \delta$. Now, $f^{-1}(\delta) = f^{-1}([f(\lambda^c)]^c) = [f^{-1}(f(\lambda))]^c \subseteq \lambda$.
(vi) $\Rightarrow$ (iv): Let β be any neutrosophic set in Y. Then, $\lambda = \overline{f^{-1}(\beta)}_\theta$ is neutrosophic θ-closed set in X and neutrosophic $f^{-1}(\beta) \subseteq \lambda$. Then, there exists a neutrosophic semiclosed set δ in Y containing β such that $f^{-1}(\delta) \subseteq \lambda$. Since δ is neutrosophic semiclosed $f^{-1}(\overline{\beta}_s) \subseteq f^{-1}(\delta) \subseteq \overline{f^{-1}(\beta)}_\theta$. □

Furthermore, we can prove the following.

Theorem 4.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following statements are equivalent:*

(i) f is neutrosophic weakly semiopen.
(ii) $\overline{f(\lambda)}_s \subseteq f(\overline{\lambda})$ for each neutrosophic open set λ in X.
(iii) $\overline{f(\beta^\circ)}_s \subseteq f(\beta)$ for each neutrosophic closed set β in X.

Proof. (i) $\Rightarrow$ (iii): Let β be a neutrosophic closed set in X. Then we have

$$f(\beta^c) = \big(f(\beta)\big)^c \subseteq \big(f\big(\overline{\beta^c}\big)\big)^\circ_s$$

and so $\big(f(\beta)\big)^c \subseteq \big(\overline{f(\beta^\circ)_s}\big)^c$. Hence $\overline{f(\beta^\circ)_s} \subseteq f(\beta)$.
(iii) $\Rightarrow$ (ii): Let λ be a neutrosophic open set in X. Since $\overline{\lambda}$ is a neutrosophic closed set and $\lambda \subseteq \left(\overline{\lambda}\right)^\circ$ by (iii) we have $\overline{f\big(\lambda\big)}_s \subseteq \overline{f\big(\big(\overline{\lambda}\big)^\circ\big)}_s \subseteq f\big(\overline{\lambda}\big)$.
(ii) $\Rightarrow$ (iii): Similar to (iii) $\to$ (ii).
(iii) $\Rightarrow$ (i) : Clear. □

For the following theorem, the proof is mostly straightforward and is omitted.

Theorem 4.4. *For a function $f : (X, \tau) \to (Y, \sigma)$ the following conditions are equivalent: We define one additional near neutrosophic semiopen condition. This condition when combined with neutrosophic weak semiopenness imply neutrosophic semiopenness.*

(i) f is neutrosophic weakly semiopen;
(ii) For each neutrosophic closed subset β of X, $f(\beta^\circ) \subseteq (f(\beta))^\circ_s$;
(iii) For each neutrosophic open subset λ of X, $f\big(\big(\overline{\lambda}\big)^\circ\big) \subseteq \big(f\big(\overline{\lambda}\big)\big)^\circ_s$;
(iv) For each neutrosophic regular open subset λ of X, $f\big(\lambda\big) \subseteq \big(f\big(\overline{\lambda}\big)\big)^\circ_s$;
(v) For every neutrosophic preopen subset λ of X, $f\big(\lambda\big) \subseteq \big(f\big(\overline{\lambda}\big)\big)^\circ_s$;
(vi) For every neutrosophic α-open subset λ of X, $f\big(\lambda\big) \subseteq \big(f\big(\overline{\lambda}\big)\big)^\circ_s$.

We define one additional near neutrosophic semiopen condition. This condition when combined with neutrosophic weak semiopenness imply neutrosophic semiopenness.

Definition 4.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to satisfy the neutrosophic weakly semiopen interiority condition if $\big(f\big(\overline{\lambda}\big)\big)^\circ_s \subseteq f\big(\lambda\big)$ for every neutrosophic open subset λ of X.*

Definition 4.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be neutrosophic strongly continuous, if for every neutrosophic subset λ of X, $f\big(\overline{\lambda}\big) \subseteq f\big(\lambda\big)$.*

Obviously, every neutrosophic strongly continuous function satisfies the neutrosophic weakly semiopen interiority condition but the converse does not hold as is shown by the following example.

Example 4.2. *Let $X = \{a, b\}$, $Y = \{x, y\}$ and neutrosophic sets λ and μ are defined as: $\lambda = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.4, 0.4, 0.6\rangle\}$ $\mu = \{\langle x, 0.7, 0.7, 0.3\rangle, \langle y, 0.8, 0.8, 0.2\rangle\}$. Let $\tau = \{0, \lambda, 1\}$ and $\sigma = \{0, \mu, 1\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ satisfies neutrosophic weakly semiopen interiority but is not neutrosophic strongly continuous.*

Theorem 4.5. *Every function that satisfies the neutrosophic weakly semiopen interiority condition into a neutrosophic discrete topological space is neutrosophic strongly continuous.*

Theorem 4.6. *If $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic weakly semiopen and satisfies the neutrosophic weakly semiopen interiority condition, then f is neutrosophic semiopen.*

Proof. Let λ be a neutrosophic open subset of X. Since f is neutrosophic weakly semiopen $f\big(\lambda\big) \subseteq \big(f\big(\overline{\lambda}\big)\big)^{\circ}_{s}$. However, because f satisfies the neutrosophic weakly semiopen interiority condition, $f\big(\lambda\big) = \big(f\big(\overline{\lambda}\big)\big)^{\circ}_{s}$ and therefore $f(\lambda)$ is neutrosophic semiopen. □

Corollary 4.7. *If $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic weakly semiopen and neutrosophic strongly continuous, then f is neutrosophic semiopen.*

The following example shows that neither of this neutrosophic interiority condition yield a decomposition of neutrosophic semiopenness.

Example 4.3. *Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$. Define λ and μ as follows : $\lambda = \{\langle a, 0, 0, 1\rangle, \langle b, 0.2, 0.2, 0.8\rangle, \langle c, 0.7, 0.7, 0.3\rangle\}$, $\mu = \{\langle x, 0, 0, 1\rangle, \langle y, 0.2, 0.2, 0.8\rangle, \langle z, 0.2, 0.2, 0.8\rangle\}$. Let $\tau = \{0, \lambda, 1\}$ and $\sigma = \{0, \mu, 1\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is neutrosophic semiopen but not neutrosophic weakly semiopen interiority .*

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be neutrosophic contra-open (resp. neutrosophic contra-closed) if $f(\lambda)$ is a neutrosophic closed set (resp. neutrosophic open set) of Y for each neutrosophic open (resp. neutrosophic closed) set λ in X.

Theorem 4.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be a neutrosophic function. Then, the following statements hold.*

(i) If $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic preopen and neutrosophic contra-open, then f is a neutrosophic weakly semiopen function.
(ii) If $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic contra-closed, then f is a neutrosophic weakly semiopen function.

Proof. (i) Let λ be a neutrosophic open subset of X. Since f is neutrosophic preopen $f(\lambda) \subseteq (\overline{f(\lambda)})^{\circ}$ and since f is neutrosophic contra-open, $f(\lambda)$ is neutrosophic closed. Therefore $f(\lambda) \subseteq (\overline{f(\lambda)})^{\circ} = (f(\lambda))^{\circ} \subseteq (f(\overline{\lambda}))^{\circ}$.
(ii) Let λ be an neutrosophic open subset of X. Then, we have $f(\lambda) \subseteq \overline{f(\overline{\lambda})} \subseteq (f(\overline{\lambda}))^{\circ}_{s}$. □

The converse of Theorem 4.6 does not hold. Can be seen in Example 4.1.

Theorem 4.9. *Let X be a neutrosophic regular space. Then, $f:(X,\tau)\to(Y,\sigma)$ is neutrosophic weakly semiopen if and only if f is neutrosophic semiopen.*

Proof. The sufficiency is clear.
Necessity. Let λ be a non-null neutrosophic open subset of X. For each $x_{r,s,t}$ neutrosophic point in λ, let $\mu_{x_{r,s,t}}$ be a neutrosophic open set such that $x_{r,s,t}\in\mu_{x_{r,s,t}}\subseteq\overline{x_{r,s,t}}\subseteq\lambda$. Hence, we obtain that

$$\lambda=\bigcup\{\mu_{x_{r,s,t}}:x_{r,s,t}\in\lambda\}=\bigcup\{\overline{x_{r,s,t}}:x_{r,s,t}\in\lambda\}$$

and

$$f(\lambda)=\bigcup\{f(\mu_{x_{r,s,t}}):x_{r,s,t}\in\lambda\}\subseteq\bigcup\{(f(\overline{x_{r,s,t}}))^\circ_s:x_{r,s,t}\in\lambda\}\subseteq (f(\bigcup\{\overline{x_{r,s,t}}:x_{r,s,t}\in\lambda\}))^\circ_s=(f(\lambda))^\circ_s.$$

Thus, f is semiopen. □

Note that, $f:(X,\tau)\to(Y,\sigma)$ is said to be neutrosophic contra-pre-semiclosed provide that $f(\lambda)$ is neutrosophic semi-open for each neutrosophic semi-closed subset λ of X.

Theorem 4.10. *If $f:(X,\tau)\to(Y,\sigma)$ is neutrosophic weakly semiopen and Y has the property that union of neutrosophic semi-closed sets is neutrosophic semi-closed and if for each neutrosophic semi-closed subset β of X and each fiber*

$$f^{-1}(y_{r,s,t})\subseteq\beta^c$$

there exists a neutrosophic open subset μ of X for which $\beta\subseteq\mu$ and $f^{-1}(y_{r,s,t})\tilde{q}\overline{\mu}$, then f is neutrosophic contra-pre-semiclosed.

Proof. Assume β is a neutrosophic semi-closed subset of X and let $y_{r,s,t}\in(f(\beta))^c$. Thus, $f^{-1}(y_{r,s,t})\subseteq\beta^c$ and hence there exists a neutrosophic open subset μ of X for which $\beta\subseteq\mu$ and $f^{-1}(y_{r,s,t}))\tilde{q}\overline{\mu}$. Therefore, $y_{r,s,t}\in(f(\overline{\mu}))^c\subseteq(f(\beta))^c$. Since f is neutrosophic weakly semiopen $f(\mu)\subseteq(f(\overline{\mu}))^\circ_s$. By complement, we obtain

$$y_{r,s,t}\in\overline{(f(\overline{\mu}))^c}_s\subseteq(f(\beta))^c$$

Let $\delta_{y_{r,s,t}}=\overline{(f(\overline{\mu}))^c}_s$. Then $\delta_{y_{r,s,t}}$ is a neutrosophic semi-closed subset of Y containing $y_{r,s,t}$. Hence $(f(\beta))^c=\bigcup\{\delta_{y_{r,s,t}}:y_{r,s,t}\in(f(\beta))^c\}$ is neutrosophic semi-closed and therefore $f(\beta)$ is neutrosophic semi-open. □

Theorem 4.11. *If $f:(X,\tau)\to(Y,\sigma)$ is an neutrosophic almost open function, then it is neutrosophic weakly semiopen. The converse is not generally true.*

Proof. Let λ be a neutrosophic open set in X. Since f is neutrosophic almost open and $(\overline{\lambda})^\circ$ is neutrosophic regular open, $f((\overline{\lambda})^\circ)$ is neutrosophic open in Y and hence $f(\lambda)\subseteq f((\overline{\lambda})^\circ)\subseteq(f(\overline{\lambda}))^\circ\subseteq(f(\overline{\lambda}))^\circ_s$. This shows that f is neutrosophic weakly semiopen. □

Example 4.4. *The function f defined in Example 4.3 is neutrosophic weakly semiopen but not neutrosophic almost open.*

Lemma 4.12. *If $f:(X,\tau)\to(Y,\sigma)$ is a neutrosophic continuous function, then for any neutrosophic subset λ of X, $f(\overline{\lambda})\subseteq\overline{(f(\lambda))}$.*

Theorem 4.13. *If $f:(X,\tau)\to(Y,\sigma)$ is a neutrosophic weakly semiopen and neutrosophic continuous function, then f is a neutrosophic β-open function.*

Proof. Let λ be a neutrosophic open set in X. Then, by neutrosophic weak semiopenness of f , $f(\lambda) \subseteq (f(\overline{\lambda}))^{\circ}_{s}$. Since f is neutrosophic continuous $f(\overline{\lambda}) \subseteq \overline{f(\lambda)}$ and since for any neutrosophic subset β of X, $(\overline{\beta})^{\circ}_{s} \subseteq \beta \cap \overline{((\overline{\beta})^{\circ})}$, we obtain that, $f(\lambda) \subseteq (f(\overline{\lambda}))^{\circ}_{s} \subseteq \overline{((\overline{f(\lambda)})^{\circ}_{s}} \subseteq \overline{(((\overline{f(\lambda)})^{\circ})}$. Thus, $f(\lambda) \subseteq \overline{(((\overline{f(\lambda)})^{\circ})}$ which shows that $f(\lambda)$ is a neutrosophic β-open set in Y. Hence, f is a neutrosophic β-open function. □

Corollary 4.14. *If $f : (X, \tau) \to (Y, \sigma)$ is a neutrosophic weakly semiopen and neutrosophic strongly continuous function. Then f is a neutrosophic β-open function.*

Definition 4.5. *A neutrosophic topological space (X, τ) is said to be a neutrosophic connected space, if there don't exist neutrosophic clopen sets λ and β such that $\lambda\tilde{q}\beta$ and $\lambda^c\tilde{q}\beta^c$.*

Definition 4.6. *A neutrosophic topological space (X, τ) is said to be a neutrosophic semiconnected space, if there don't exist neutrosophic semiclopen sets λ and β such that $\lambda\tilde{q}\beta$ and $\lambda^c\tilde{q}\beta^c$.*

Theorem 4.15. *If $f : (X, \tau) \to (Y, \sigma)$ is a neutrosophic weakly semiopen of a space X onto a neutrosophic semiconnected space Y, then X is neutrosophic connected.*

Proof. Let X be not connected. Then there exist neutrosophic open sets β and γ in X such that $\beta\tilde{q}\gamma$ and $\beta^c\tilde{q}\gamma^c$. This implies that $f(\beta)\tilde{q}f(\gamma)$ and $f(\beta^c)\tilde{q}f(\gamma^c)$. Since f is neutrosophic weakly semiopen, we have $f(\beta) \subseteq (f(\overline{\beta}))^{\circ}_{s}$ and $f(\gamma) \subseteq (f(\overline{\gamma}))^{\circ}_{s}$ and since β and γ are neutrosophic open and also neutrosophic closed, we have $f(\overline{\beta}) = f(\beta)$, $f(\overline{\gamma}) = f(\gamma)$. Hence $f(\beta)$ and $f(\gamma)$ are neutrosophic semiopen and semiclosed in (Y, σ) such that $f(\beta)\tilde{q}f(\gamma)$ and $((f\beta))^c\tilde{q}(f(\gamma))^c$. Hence, this contrary to the fact that Y is neutrosophic semi-connected. Thus X is neutrosophic connected. □

Definition 4.7. *A space X is said to be neutrosophic hyperconnected if every non-null neutrosophic open subset of X is neutrosophic dense in X.*

Theorem 4.16. *If X is a neutrosophic hyperconnected space, then a function $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic weakly semiopen if and only if $f(X)$ is neutrosophic semi-open in Y.*

Proof. The necessity is clear.
For the sufficiency observe that for any neutrosophic open subset λ of X,
$f(\lambda) \subseteq f(X) = (f(X))^{\circ}_{s} = (f(\overline{\lambda}))^{\circ}_{s}$. □

5. Conclusion

In this study, after presenting the factors that inspired us to focus on this subject and giving the necessary preliminary information, in the third section, we adapted some open set types previously defined in the general topology to neutrosophic spaces, and then we defined some types of functions and continuity using these open set types. In the fourth section, we continued to present new open set and continuity types and illustrated the relationships between them by enriching them with examples. Additionally, we examined the properties of these new types of continuities and open sets and brought a new perspective to the concept of connected space. Our expectation from this study is that it will be one of the cornerstones of various studies to be carried out in the world of mathematics and that it will encourage scientists to conduct various research related to our study.

Conflict of interests

The authors declare that there is no conflict of interests.

Acknowledgment

The authors would like to thank the referees for the valueble suggestions, which improved the paper.

References

[1] A. Acikgoz, H. Çakalli, F. Esenbel, and Lj. D.R. Kočinac, A quest of G-continuity in neutrosophic spaces, Math Meth Appl Sci. **44** (9) (2021) 7834-7844.
[2] A. Acikgoz and F. Esenbel, Neutrosophic Connected topological spaces, Fasciculi Math. **66** (2023) 5-22.
[3] A. Acikgoz and F. Esenbel, Neutrosophic seperation axioms, Maltepe J. Math. **5** (2) (2023) 32-40.
[4] M. Caldas, G. Navalagi and R. Saraf, On Fuzzy Weakly Open Functions, Proyec. **21** (1) (2002) 51-63.
[5] R. Dhavaseelan, S. Jafari, C. Ozel and M. A Al-Shumrani, Generalized neutrosophic contra-continuity, New Trend Neut. Theory Appl. **II** (2018) 365-380.
[6] S. Nanda, On fuzzy topological spaces, Fuzzy Sets Syst. **19** (1986) 193-197.
[7] J. H. Park, Y. B. Park and J. S. Park, Fuzzy weakly open mappings and fuzzy RS-compact sets, Far East J. Math. Sci. **II** (1997) 201-212.
[8] D. A. Rose, On weak openness and almost openness, Internt. J. Math. I. Math. Sci. **7** (1984) 35-40.
[9] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR J. Math. **3** (2012) 31–35.
[10] F. Smarandache, Neutrosophic set, A generalisation of the intuitionistic fuzzy sets, Int J Pure Appl Math. **24** (2005) 287-297.

Ahu Acikgoz,
Department of Mathematics, Balikesir University,10145 Balikesir, Turkey
Email address: `ahuacikgoz@gmail.com`

Ferhat Esenbel,
Department of Mathematics, Balikesir University,10145 Balikesir, Turkey
Email address: `fesenbel@gmail.com`