



## Optimizing Medical Waste Collection in Eskişehir by Using Multi-Objective Mathematical Model

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### Abstract

Nowadays, transportation, because of the intense need to be delivered to the market in terms of both products and commercial and tourist activity in our times, is one of the vital sectors. Especially in countries in which a large portion of the road transport and transport services such as Turkey, road transport has become very important. This study made an application related to collect the medical waste from hospitals in Eskişehir, Turkey. In order to collect waste regularly, the route is extremely key factor. In this study, a mathematical model is demonstrated to make an efficient route path. The locations of hospitals are defined and the distances are calculated in digital map. Subsequently, the monthly data are compared with the routes of the vehicle that are suggested by developed model. In this article, a generalized multi-objective vehicle routing model with collecting the medical wastes from hospitals in Eskişehir is proposed. Some methods have been used to combine the objective functions. Seven methods are applied to the mathematical model and the results are compared to determine the most effective method to combine the objective functions. The mathematical model has 2 objective functions. These are to collect the medical waste by covering a minimum distance and driving with minimum speed per hour.

## 1. INTRODUCTION

Waste management system has received wide attention from environmental planners as a reason of its complex coordination of various management strategies. Due to the temporal and spatial variations over social, economic and regional factors, waste management programs have to be recognized commonly to handle various issues. One issue is how to effectively distribute the collection crew size and vehicles in a growing metropolitan region. A methodology used in this case study combines an operations research method with systems engineering. It illustrates how waste transportation costs are minimized in a major urban area. Due to the complexity of a real size situation, a model is developed to consider a including transportation by vehicle is evaluated. Collector vehicle reside overnight in depot and daily evacuate accumulated waste to an incinerator or a transfer station where it is deposited before being shipped out to the incinerator. In this paper, medical waste collection is applied in Eskişehir, Turkey. A vehicle collect the medical wastes from 6 different hospitals and take them back to the depot to recycle. A mathematical model has developed to obtain a route to collect the medical waste. In next section, literature review has been presented. In third section and fourth sections, waste collection and vehicle routing problem are expounded. In fifth and sixth sections, an application and developed mathematical model are given.

## 2. LITERATURE REVIEW

There are many different studies in the literature about waste collection. Tung and Pinnoi [1]'s study is in Vietnam. In their study, vehicles are same and they can go to sites with time window constraints. In Apaydin and Gonullu [2]'s study, the environmental emission is added to the goal equation with travelling distances and times of trucks. Tavares et al. [3] presented that short routes do not guarantee minimum fuel consumption of vehicles. Fan et al. [4]'s study's objective is to minimize the travelling distance and

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maximize total heat value. Arribas et al. [5] aim to minimize collection time and operational and transport. Galante et al. [6] declared the model examined both initial investment and operation costs with transportation and transfer stations. In Larsen et al. [7]'s study, they created five scenarios. Tan et al. [8,9] defined the study with fuzzy in objective functions. In Faccio et al. [10]'s study, the objective function consists of the number of used vehicles and their travelling times and distances. Pires et al. [11] presented a literature review of models of possible overlapped. Tai et al. [12] presented a review of different methods of collection and transportation. Chatzouridis and Komilis [13] created a vehicle routing model that their goal is a non-linear equation that minimized total collection cost. Gunalay et al. [14] reflect that simulation-optimization modelling can be used to efficiently generate multiple policy alternatives. Hemmelmayr et al. [15] and Hemmelmayr et al [16] created combination of a vehicle routing and a box carrying problem. Mora et al. [17] presented a model for an integrated waste management system. Schneider et al. [18] solve vehicle routing problem by using time windows and recharging stations. In this study, a generalized multi-objective vehicle routing model with collecting the medical wastes from hospitals in Eskişehir is proposed. Some scenarios have been created to combine the objective functions. Seven methods are applied to the mathematical model and results are compared to determine the most effective method to combine the objective functions.

### **3. WASTE COLLECTION**

Waste is defined as, production and use activities emerged as a result that are inconvenient to give buyer mediation directly or indirectly, resulting in human and environmental health. Waste management is also defined as, reduction in the source of waste, collection, transportation, temporary storage according to its characteristics, control after waste disposal and disposal, recovery etc. Is a form of structuring involving transactions. It is essential to prevent and reduce the harmfulness of waste production and waste. It is essential that the waste be recycled or used as a source of energy where the formation of the waste is inevitable. Different types of wastes must be collected separately at the source. It is essential to dispose of wastes in licensed recycling and disposal facilities using appropriate methods and technologies. For medical waste collecting, medical waste producers, carriers and disposers are responsible for environmental pollution caused by medical wastes and damage caused by intact wastes. It is forbidden to give buyer mediation directly or indirectly. It should not be mixed with dangerous and domestic wastes at its source, collected separately, transported and disposed of. According to the characteristics of the waste generated in the hospital environment, it is aimed to be collected separately in the source and removed so as not to damage the health. General wastes are wastes from healthy persons, non-sick sections, first aid areas, administrative units, cleaning services, kitchens, warehouses and workshops. All administrative units can be re-used due to kitchen, warehouse, workshop. The transport and disposal of infectious agents to prevent their spread is waste that requires special application. Cutting and puncture wastes are resistant to perforation, tearing, breaking and explosion apart from other medical wastes.

### **4. VEHICLE ROUTING PROBLEM**

The problem of vehicle locomotion is the problem of designing optimal distribution / collection routes of vehicles assigned to geographically dispersed customers to serve one or more depots. Vehicle Routing can be defined as one or more depots, product distribution or collection to specific customers. The vehicle may encounter some problems in the route. These problems concentrate on the effective use of vehicles with a certain capacity, distributing vehicle capacities and service life constraints that arise in the customer. The car locus problem has been studied for over 50 years. Dantzig and Ramster first worked on vehicle locating problem in 1959. Clarke and Wright developed the method of Dantzig and Ramster in 1964. VRP is concerned with building  $n$  vehicle routes. These routes start from the main warehouse and are sent to the lower warehouse or receivers, and the vehicle turns back to the main warehouse. Each customer must be on one of the  $n$  vehicle routes and the capacity of the vehicles must not be exceeded. The main goal in this problem is to minimize all of the constraints, minimize the number of vehicles to be used and minimize the total distance while minimizing the cost function.

## 5. COMBINING MULTI-OBJECTIVE MODELS

It is difficult to solve the mathematical model as the model has more than one objectives. So the best way is to convert the multi-objective problem to single objective problem. Principle of scalarization is to convert multi-objective problem to (parameterized single objective problem and solve repeatedly with different parameter values. Desirable properties of scalarization are correctness and completeness. Correctness means that optimal solutions are (weakly, properly) efficient. Completeness means that all (weakly, properly) efficient solutions can be found. Scalarization has three ideas. These are aggregate objectives, convert objectives to constraints and minimize distance to ideal point. There are 7 scalarization methods in the literature. These are the weighted sum method, the  $\epsilon$ -constraint method, the hybrid method, the elastic constraint method, the benson method, the augmented weighted tchebycheff function method and the conic scalarization method [19].

### 5.1. The Weighted Sum Method

The oldest and most widely used method of combining objectives. Ease of use and user preferences are advantages to reflect on the model. All effective solution to generate value in the field and has the condition of being convex objective functions.

Let  $\lambda \geq 0$

$$\text{Min } \left\{ \sum_{k=1}^p \lambda_k f_k(x) : x \in X \right\} \quad (1)$$

### 5.2. The $\epsilon$ -Constraint Method

Unlike the weighted sums instead of combining the objectives, this method adds to one of the others when optimizing the objective constraints. The solution that found is at least a weakly efficient solution. Convexity does not need the requirement. Determining the stage of the constraint values are cumbersome.

$$f_k(x) \leq \epsilon_k, \quad k=1, \dots, p, \quad j \neq k \quad (2)$$

$$\text{Min } f_j(x) \quad (3)$$

### 5.3. The Hybrid Method

In the hybrid method, the weighted sum of the objective function under constraints imposed on all objective function minimization. Convexity does not need the requirement. User preferences may reflect the model. But just reach the point that can be supported by a linear function.

$$f_k(x) \leq f_k(x^0), \quad k=1, \dots, p \quad (4)$$

$$\text{Min } \sum_{k=1}^p w_k f_k(x) \quad (5)$$

### 5.4. The Elastic-Constraint Method

Epsilon constraints set for inclusion in the methods of the constraints in the objective function is difficult to solve the problem. Elastic constraint method,  $\epsilon$ -constraint loosening the constraints allowed deviation due to a penalty. There are computational difficulties associated with penalty function. The decision-makers are not taken into account the weight of preference.

$$f_k(x) - s_k \leq \epsilon_k, \quad k=1, \dots, p, \quad j \neq k \quad (6)$$

$$s_k \geq 0, \quad k=1, \dots, p, \quad j \neq k \quad (7)$$

$$\text{Min } \sum_k f_j + \sum_{k \neq j} M_k s_k \quad (8)$$

### 5.5. The Benson Method

The basic idea is choosing a suitable solution  $x^0 \in X$  point of the arc to find pareto effective value depending on this point.  $l_k = f_k(x^0) - f_k(x)$  is added as non-negative deviation variables. The sum of the deviation variable is minimized. Benson method's constraints are more flexible than epsilon-constraint method. Its implementation is easy. This method needs an initial appropriate solution.

$$f_k(x^0) - l_k - f_k(x) = 0, \quad k=1, \dots, p \quad (9)$$

$$l_k \geq 0, \quad k=1, \dots, p \quad (10)$$

$$\text{Min } \sum_k l_k \quad (11)$$

### 5.6. The Augmented Weighted Tchebycheff Function Method

In which  $\rho$  is sufficiently small positive number.  $y_k^*$ :  $k^{\text{th}}$  reference value for the objective function (ideal point) and  $p$  is the number of objectives. In recent years, methods based on the tchebycheff metric has been the most popular multi-purpose tool used in solving the general problem.

$$\text{Min Max } [w_k (f_k(x) - y_k^*)] + \rho \sum_{k=1}^p (f_k(x) - y_k^*) \quad (12)$$

$$\alpha \geq w_k (f_k(x) - y_k^*), \quad k=1, \dots, p, \quad x \in X \quad (13)$$

$$\text{Min } \alpha + \rho \sum_{k=1}^p (f_k(x) - y_k^*) \quad (14)$$

### 5.7. The Conic Scalarization Method

The basic idea is to use the support cone instead of finding the pareto efficient support hyperplanes value. It does not require the condition of convexity over the objective functions and appropriate solution area. It was developed to solve the general problem multipurpose. Decision makers reflect the preferences and wishes of the mathematical model.

$$\text{Min } \alpha \sum_k^p |f_k(x) - a_k| + \sum_{k \neq j} w_k (f_k(x) - a_k) \quad (15)$$

## 6. APPLICATION

In this study, a route plan has developed to collect the medical wastes in hospitals in Eskişehir regularly. Eskişehir map is given in (Figure.1) below. There are 6 hospitals in Eskişehir. A vehicle collect the medical wastes from these 6 hospitals and carry them to the depot to recycle. 6 hospitals and the depot is shown in (Figure.1) with red spots.



Figure 1. Location of Hospitals and the Depot in Eskişehir

In order to collect the wastes, a mathematical model is used to make an optimal route path. In this mathematical model, there is no vehicle capacity while carrying the medical wastes. Route path can be determine with this mathematical model. This model has two objective functions. This makes this model different from many other mathematical models in the literature. In this study, 7 scalarization methods have been used to combine the objective functions. The model has been solved for each method. And the results are compared to decide the best scalarization method for vehicle routing problems.

### 6.1. Mathematical Model

The mathematical model has similarities with travelling salesman. There is no vehicle capacity in this mathematical model. There are two objective functions. These are; minimization of total carrying distance and minimizing of average speed at the hour.

$i, j$ : Transportation node index,  $i=1,2,3,\dots,7$  and  $j=1,2,3,\dots,7$  customer nodes and  $i, j=1$  is delivery center or depot node.

$d_{ij}$ : Carrying distance from node  $i$  to node  $j$ .

$R$ : A set of intermediate nodes.

$u_i$ : It has any real value of the node  $i$ .

$v_{ij}$ : The maximum speed limit can be entered from the way node  $i$  to node  $j$ .

$$\min F1 = \sum_{i=1}^7 \sum_{j=1}^7 d_{ij} x_{ij} \quad (16)$$

$$\min F2 = \sum_{i=1}^7 \sum_{j=1}^7 v_{ij} \cdot x_{ij} \quad (17)$$

$$\sum_{i=0}^{N_k} x_{ij} = 1 \quad \forall j=1,2,\dots,7 \quad (18)$$

$$\sum_{i=0}^{N_k} x_{ij} = 1 \quad \forall j=1,2,\dots,7 \quad (19)$$

$$u_i - u_j + n \cdot x_{ij} \leq n - 1 \quad \forall i \neq j \in R \quad (20)$$

The equations of (16) and (17) are the objective functions of the mathematical model. (18) and (19) constraints are specified to visit each nodes at least once. Equation (20) denotes the sub-tour elimination constraint. This type of sub-tour elimination constraints, using Gomory cutting planes approach developed by Miller, Tucker and Zemlin in 1960[20]. These constraints took the tour of the state model to inhibit the formation of approximately  $(n^2-3n+2)$  lead to the addition of one constraint[21]. In particular, the use of this Miller, Tucker and Zemlin constraints in oversized model is much easier and routing problems is one of the constraints to improved well-received tour blocking.

### 6.2. Computational Results

In this study, firstly, the weighted sum method is applied to the mathematical model. And it is solved by using GAMS 24.7.1 program. Weighted scale is chosen between 1-21. The mathematical model is solved for all weight values separately. The results for the weighted sum method is given in Table1.

**Table 1.** Results for the weighted sum method

<i>Solutions</i>	<i>Objective (F<sub>1</sub>,F<sub>2</sub>)</i>	<i>Route</i>
1	710,350	1-5-3-7-6-4-2-1
2-3	600,350	1-5-3-7-6-4-2-1
4-10	530,360	1-5-3-7-2-6-7-1
11-20	430,450	1-5-4-3-2-6-7-1

Second method to combine the objectives is the epsilon-constraint method. In the epsilon-constraint method, one of the objective function is written as a constraint. According to this situation, objective function and constraints have to be written below. Epsilon values are changing between 310-710. The

results for the epsilon-constraint method is given in Table2. Objective function for the epsilon-constraint method;

$$\min z = \sum_{i=1}^7 \sum_{j=1}^7 d_{ij} x_{ij} \quad (21)$$

Constraints for the epsilon-constraint method;

$$\sum_{i=1}^7 \sum_{j=1}^7 v_{ij} \cdot x_{ij} = \epsilon \quad (22)$$

$$\sum_{i=0}^{N_k} x_{ij} = 1 \quad \forall j=1,2,\dots,7 \quad (23)$$

$$\sum_{j=0}^{N_k} x_{ij} = 1 \quad \forall i=1,2,\dots,7 \quad (24)$$

$$u_i - u_j + n \cdot x_{ij} \leq n - 1 \quad \forall i \neq j \in R \quad (25)$$

**Table 2.** Results for the epsilon-constraint method

<b>Solutions</b>	<b>Objective (<math>F_1, F_2</math>)</b>	<b>Route</b>
1-8	430,450	1-5-4-3-2-6-7-1
9	460,470	1-5-2-3-4-6-7-1
10	470,530	1-7-4-3-2-6-5-1
11	470,570	1-6-7-4-3-2-5-1
12	470,540	1-5-6-2-3-4-7-1
13-14	470,570	1-6-7-4-3-2-5-1
15	500,590	1-4-3-2-5-6-7-1
16	510,670	1-7-4-3-2-5-6-1
17	510,640	1-7-4-3-6-2-5-1
18-19	510,670	1-7-4-3-2-5-6-1
20-21	550-720	1-7-4-5-2-3-6-1

The other method to combine the objectives is the hybrid method. The hybrid method is the combination of the weighted sum method and epsilon-constraint method. The results for the hybrid method are given in Table3.

**Table 3.** Results for the hybrid method

<b>Solutions</b>	<b>Objective (<math>F_1, F_2</math>)</b>	<b>Route</b>
1-21	550,720	1-5-3-7-6-4-2-1

In the elastic-constraint method, constraints in the mathematical model are relaxed and due to a penalty it can be allowed deviation. The results for the elastic-constraint method is given in Table4.

**Table 4.** Results for the elastic-constraint method

<b>Solutions</b>	<b>Objective (<math>F_1, F_2</math>)</b>	<b>Route</b>
1-10	430,450	1-5-4-3-2-6-7-1
11	510,440	1-5-2-6-3-4-7-1
12-15	530,360	1-5-3-4-2-6-7-1
16-20	600,350	1-5-3-4-6-7-2-1
21	610,350	1-2-6-4-3-7-5-1

In the benson scalarization method, an initial solution is needed. Therefore, the range of solution that found previously is selected while deviation variable is minimized. The results for the benson scalarization method is given in Table5.

**Table 5.** Results for the benson scalarization method

<i>Solutions</i>	<i>Objective (F<sub>1</sub>,F<sub>2</sub>)</i>	<i>Route</i>
1	510,440	1-5-2-6-3-4-7-1

The augmented weighted tchebycheff function method is based on the minimization of distance from ideal point. The results for the augmented weighted tchebycheff function method is given in Table6.

**Table 6.** Results for the augmented weighted tchebycheff function method

<i>Solutions</i>	<i>Objective (F<sub>1</sub>,F<sub>2</sub>)</i>	<i>Route</i>
1	600,350	1-5-3-4-6-7-2-1
2-6	530,360	1-5-3-4-2-6-7-1
7	510,440	1-5-2-6-3-4-7-1
8-21	430,450	1-5-4-3-2-6-7-1

The basic idea of the conic-scalarization method is the use of support cones instead of hyperplane support for finding effective pareto value. The results for the conic-scalarization method is given in Table7.

**Table 7.** Results for the conic-scalarization method

<i>Solutions</i>	<i>Objective (F<sub>1</sub>,F<sub>2</sub>)</i>	<i>Route</i>
1-21	500,590	1-5-3-7-6-4-2-1

## 7. CONCLUSION

In this study, firstly, a multi-objective mathematical model is created for the vehicle routing problem. The objectives of the problem are combined by scalarization methods. Then, it is examined which method is more effective. According to the results that calculated by GAMS 24.7.1 program, the most efficient scalarization method for the multi-objective vehicle routing problem is ‘the augmented weighted tchebycheff function method’. It has more pareto efficient points than the other methods have. And also ‘the epsilon-constraint method is very efficient scalarization method. It found more points than the other methods. So as seen from the results that it is the best way to combine the objective functions by using these two scalarization methods for this study. If the problem size is increased, these methods can be used for this study too. Objective functions are efficient on optimizing the route path.

## CONFLICT OF INTEREST

No conflict of interest was declared by the authors

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