

Dynamical Analysis and Solutions of Nonlinear Difference Equations of Thirty Order

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Abstract

Discrete-time systems are sometimes used to explain natural phenomena that happen in nonlinear sciences. We study the periodicity, boundedness, oscillation, stability, and certain exact solutions of nonlinear difference equations in this paper. Using the standard iteration method, exact solutions are obtained. Some well-known theorems are used to test the stability of the equilibrium points. Some numerical examples are also provided to confirm the theoretical work's validity. The numerical component is implemented with Wolfram Mathematica. The method presented may be simply applied to other rational recursive issues.

In this paper, we explore the dynamics of adhering to a rational difference formula

$$x_{n+1} = \frac{x_{n-29}}{\pm 1 \pm x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}},$$

where the initials are arbitrary nonzero real numbers.

1. Introduction

A particular natural phenomenon's evolution is frequently explained over a period of time employing differential equations. Nevertheless, in certain instances, numerous real-life issues can be modeled using discrete time intervals, resulting in difference equations. As a result, recursive equations play an influential and potent role in mathematics. They are effectively employed to explore various applications in engineering, physics, biology, economics, and other fields [1–5]. For example, recursive equations have been effectively employed in modeling various natural phenomena, including population size, the Fibonacci sequence, drug concentrations in the bloodstream, information transmission, pricing dynamics of certain commodities, propagation patterns of annual plants, and more [6–12]. Additionally, certain scholars have utilized difference equations to obtain numerical solutions for certain differential equations. In particular, discretizing a given differential equation produces a corresponding difference equation. For example, the Runge-Kutta scheme arises from discretizing a first-order differential equation. This prompts consideration regarding the convergence of the difference scheme to the solution of a differential equation. The study discussed in reference [13] is dedicated to investigating the preservation of a solution bounded on the entire axis during the transition from differential to difference equations and vice versa. In reference [14], analogous inquiries were undertaken to maintain the oscillatory nature of solutions to second-order equations. Advancements in technology have spurred the utilization of recurrence equations as approximations to partial differential equations. It's noteworthy that fractional-order difference equations are frequently employed to study certain real-life phenomena that arise in nonlinear sciences. Almatrafi et. al. in [15] aim to analyzed the asymptotic stability, global stability, periodicity of the solution of an eighth-order difference equation. Sanbo et. al. in [16], discussed the periodicity, stability, and some solutions of a fifth-order recursive equation. Yeniçerioglu et. al. in [17], examined the behavior of solutions of the neutral functional differential equations. Using a suitable real root of the corresponding characteristic equation, they explained the asymptotic behavior of the solutions and the stability of the trivial solution. Ahmed et al. [18] discovered new solutions and conducted a dynamical analysis for certain nonlinear difference relations of fifteenth order. Berkal et. al. in [19], have derived the forbidden set and

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determined the solutions of the difference equation that contains a quadratic term. Oğul et. al. in [20], examined solutions of the sixth-order difference equations.

The inspiration behind this article stems from the exploration of eighteenth-order difference equations outlined in [21]. As such, the objective of this study is to analyze various dynamical properties including equilibrium points, local and global behaviors, boundedness, and analytic solutions of the nonlinear recursive equations (1.1).

$$x_{n+1} = \frac{x_{n-29}}{\pm 1 \pm x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}. \tag{1.1}$$

Here, the initial values $x_{-29}, x_{-28}, x_{-27}, \dots, x_{-2}, x_{-1}, x_0$, are arbitrary non-zero real numbers. In this work, we also illustrate some 2D figures with the help of Wolfram Mathematica to validate the obtained results.

In this study, stability, periodicity and global asymptotic stability definitions and theorems in the [1] source were used.

2. Solution of the Difference Equation $x_{n+1} = \frac{x_{n-29}}{1+x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this section, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers.

$$x_{n+1} = \frac{x_{n-29}}{1 + x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}, \tag{2.1}$$

where,

$$\begin{aligned} x_{-29} &= A_{30}, & x_{-28} &= A_{29}, & x_{-27} &= A_{28}, & x_{-26} &= A_{27}, & x_{-25} &= A_{26}, & x_{-24} &= A_{25}, & x_{-23} &= A_{24}, & x_{-22} &= A_{23}, \\ x_{-21} &= A_{22}, & x_{-20} &= A_{21}, & x_{-19} &= A_{20}, & x_{-18} &= A_{19}, & x_{-17} &= A_{18}, & x_{-16} &= A_{17}, & x_{-15} &= A_{16}, & x_{-14} &= A_{15}, \\ x_{-13} &= A_{14}, & x_{-12} &= A_{13}, & x_{-11} &= A_{12}, & x_{-10} &= A_{11}, & x_{-9} &= A_{10}, & x_{-8} &= A_9, & x_{-7} &= A_8, & x_{-6} &= A_7, \\ x_{-5} &= A_6, & x_{-4} &= A_5, & x_{-3} &= A_4, & x_{-2} &= A_3, & x_{-1} &= A_2, & x_0 &= A_1. \end{aligned} \tag{2.2}$$

Theorem 2.1. Let $\{x_n\}_{n=-29}^\infty$ be a solution of (2.1). Then,

$$\begin{aligned} x_{30n+1} &= \frac{A_{30} \prod_{i=0}^n (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 + (5i + 1)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+2} &= \frac{A_{29} \prod_{i=0}^n (1 + 5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 + (5i + 1)A_5A_{11}A_{17}A_{23}A_{29})}, \\ x_{30n+3} &= \frac{A_{28} \prod_{i=0}^n (1 + 5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 + (5i + 1)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+4} &= \frac{A_{27} \prod_{i=0}^n (1 + 5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 + (5i + 1)A_3A_9A_{15}A_{21}A_{27})}, \\ x_{30n+5} &= \frac{A_{26} \prod_{i=0}^n (1 + 5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 + (5i + 1)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+6} &= \frac{A_{25} \prod_{i=0}^n (1 + 5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 + (5i + 1)A_1A_7A_{13}A_{19}A_{25})}, \\ x_{30n+7} &= \frac{A_{24} \prod_{i=0}^n (1 + (5i + 1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 + (5i + 2)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+8} &= \frac{A_{23} \prod_{i=0}^n (1 + (5i + 1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 + (5i + 2)A_5A_{11}A_{17}A_{23}A_{29})}, \\ x_{30n+9} &= \frac{A_{22} \prod_{i=0}^n (1 + (5i + 1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 + (5i + 2)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+10} &= \frac{A_{21} \prod_{i=0}^n (1 + (5i + 1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 + (5i + 2)A_3A_9A_{15}A_{21}A_{27})}, \\ x_{30n+11} &= \frac{A_{20} \prod_{i=0}^n (1 + (5i + 1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 + (5i + 2)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+12} &= \frac{A_{19} \prod_{i=0}^n (1 + (5i + 1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 + (5i + 2)A_1A_7A_{13}A_{19}A_{25})}, \\ x_{30n+13} &= \frac{A_{18} \prod_{i=0}^n (1 + (5i + 2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 + (5i + 3)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+14} &= \frac{A_{17} \prod_{i=0}^n (1 + (5i + 2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 + (5i + 3)A_5A_{11}A_{17}A_{23}A_{29})}, \\ x_{30n+15} &= \frac{A_{16} \prod_{i=0}^n (1 + (5i + 2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 + (5i + 3)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+16} &= \frac{A_{15} \prod_{i=0}^n (1 + (5i + 2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 + (5i + 3)A_3A_9A_{15}A_{21}A_{27})}, \\ x_{30n+17} &= \frac{A_{14} \prod_{i=0}^n (1 + (5i + 2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 + (5i + 3)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+18} &= \frac{A_{13} \prod_{i=0}^n (1 + (5i + 2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 + (5i + 3)A_1A_7A_{13}A_{19}A_{25})}, \\ x_{30n+19} &= \frac{A_{12} \prod_{i=0}^n (1 + (5i + 3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 + (5i + 4)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+20} &= \frac{A_{11} \prod_{i=0}^n (1 + (5i + 3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 + (5i + 4)A_5A_{11}A_{17}A_{23}A_{29})}, \\ x_{30n+21} &= \frac{A_{10} \prod_{i=0}^n (1 + (5i + 3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 + (5i + 4)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+22} &= \frac{A_9 \prod_{i=0}^n (1 + (5i + 3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 + (5i + 4)A_3A_9A_{15}A_{21}A_{27})}, \\ x_{30n+23} &= \frac{A_8 \prod_{i=0}^n (1 + (5i + 3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 + (5i + 4)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+24} &= \frac{A_7 \prod_{i=0}^n (1 + (5i + 3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 + (5i + 4)A_1A_7A_{13}A_{19}A_{25})}, \\ x_{30n+25} &= \frac{A_6 \prod_{i=0}^n (1 + (5i + 4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 + (5i + 5)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+26} &= \frac{A_5 \prod_{i=0}^n (1 + (5i + 4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 + (5i + 5)A_5A_{11}A_{17}A_{23}A_{29})}, \\ x_{30n+27} &= \frac{A_4 \prod_{i=0}^n (1 + (5i + 4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 + (5i + 5)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+28} &= \frac{A_3 \prod_{i=0}^n (1 + (5i + 4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 + (5i + 5)A_3A_9A_{15}A_{21}A_{27})}, \\ x_{30n+29} &= \frac{A_2 \prod_{i=0}^n (1 + (5i + 4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 + (5i + 5)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+30} &= \frac{A_1 \prod_{i=0}^n (1 + (5i + 4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 + (5i + 5)A_1A_7A_{13}A_{19}A_{25})}, \end{aligned}$$

where, x_0, \dots, x_{-29} defines as in (2.2).

Proof of Theorem 2.1. The proof of each formula are carried out in similar way. So, we will demonstrate proof using one of the formula. We will employ the mathematical induction method. Let's posit that, with n being greater than zero and supposing our assumption is true for $n = 1$. That is,

$$\begin{aligned}
 x_{30n-29} &= \frac{A_{30} \prod_{i=0}^{n-1} (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-28} &= \frac{A_{29} \prod_{i=0}^{n-1} (1 + 5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-27} &= \frac{A_{28} \prod_{i=0}^{n-1} (1 + 5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-26} &= \frac{A_{27} \prod_{i=0}^{n-1} (1 + 5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-25} &= \frac{A_{26} \prod_{i=0}^{n-1} (1 + 5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-24} &= \frac{A_{25} \prod_{i=0}^{n-1} (1 + 5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-23} &= \frac{A_{24} \prod_{i=0}^{n-1} (1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-22} &= \frac{A_{23} \prod_{i=0}^{n-1} (1 + (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-21} &= \frac{A_{22} \prod_{i=0}^{n-1} (1 + (5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-20} &= \frac{A_{21} \prod_{i=0}^{n-1} (1 + (5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-19} &= \frac{A_{20} \prod_{i=0}^{n-1} (1 + (5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-18} &= \frac{A_{19} \prod_{i=0}^{n-1} (1 + (5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-17} &= \frac{A_{18} \prod_{i=0}^{n-1} (1 + (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-16} &= \frac{A_{17} \prod_{i=0}^{n-1} (1 + (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-15} &= \frac{A_{16} \prod_{i=0}^{n-1} (1 + (5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-14} &= \frac{A_{15} \prod_{i=0}^{n-1} (1 + (5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-13} &= \frac{A_{14} \prod_{i=0}^{n-1} (1 + (5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-12} &= \frac{A_{13} \prod_{i=0}^{n-1} (1 + (5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-11} &= \frac{A_{12} \prod_{i=0}^{n-1} (1 + (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-10} &= \frac{A_{11} \prod_{i=0}^{n-1} (1 + (5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-9} &= \frac{A_{10} \prod_{i=0}^{n-1} (1 + (5i+3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-8} &= \frac{A_9 \prod_{i=0}^{n-1} (1 + (5i+3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-7} &= \frac{A_8 \prod_{i=0}^{n-1} (1 + (5i+3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-6} &= \frac{A_7 \prod_{i=0}^{n-1} (1 + (5i+3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-5} &= \frac{A_6 \prod_{i=0}^{n-1} (1 + (5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-4} &= \frac{A_5 \prod_{i=0}^{n-1} (1 + (5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-3} &= \frac{A_4 \prod_{i=0}^{n-1} (1 + (5i+4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-2} &= \frac{A_3 \prod_{i=0}^{n-1} (1 + (5i+4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-1} &= \frac{A_2 \prod_{i=0}^{n-1} (1 + (5i+4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n} &= \frac{A_1 \prod_{i=0}^{n-1} (1 + (5i+4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_1A_7A_{13}A_{19}A_{25})}.
 \end{aligned}$$

Now, using the main (2.1), one has

$$\begin{aligned}
 x_{30n+1} &= \frac{x_{30n-29}}{1 + x_{30n-5}x_{30n-11}x_{30n-17}x_{30n-23}x_{30n-29}} \\
 &= \frac{\frac{A_{30} \prod_{i=0}^{n-1} (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}}{1 + \frac{A_6 \prod_{i=0}^{n-1} (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30})} \frac{\prod_{i=0}^{n-1} (1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+2)A_6A_{12}A_{18}A_{24}A_{30})} \frac{\prod_{i=0}^{n-1} (1 + (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+3)A_6A_{12}A_{18}A_{24}A_{30})} \frac{\prod_{i=0}^{n-1} (1 + (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+4)A_6A_{12}A_{18}A_{24}A_{30})} \frac{\prod_{i=0}^{n-1} (1 + (5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 + (5i+5)A_6A_{12}A_{18}A_{24}A_{30})}} \\
 &= \frac{A_{30} \prod_{i=0}^{n-1} (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})}{(1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30}) \prod_{i=0}^{n-1} (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})} = A_{30} \prod_{i=0}^{n-1} \frac{1 + 5iA_6A_{12}A_{18}A_{24}A_{30}}{1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30}} \left(\frac{1}{1 + \frac{A_6A_{12}A_{18}A_{24}A_{30}}{1 + (5i-5)A_6A_{12}A_{18}A_{24}A_{30}}} \right) \\
 &= A_{30} \prod_{i=0}^{n-1} \frac{1 + 5iA_6A_{12}A_{18}A_{24}A_{30}}{1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30}} \left(\frac{1 + (5i-5)A_6A_{12}A_{18}A_{24}A_{30}}{1 + (5i-4)A_6A_{12}A_{18}A_{24}A_{30}} \right).
 \end{aligned}$$

Hence, we have

$$x_{30n+1} = A_{30} \prod_{i=0}^n \frac{1 + 5iA_6A_{12}A_{18}A_{24}A_{30}}{1 + (5i+1)A_6A_{12}A_{18}A_{24}A_{30}}.$$

Similarly,

$$\begin{aligned}
 x_{30n+2} &= \frac{x_{30n-28}}{1 + x_{30n-4}x_{30n-10}x_{30n-16}x_{30n-22}x_{30n-28}} \\
 &= \frac{\prod_{i=0}^{n-1} \frac{A_{29} \prod_{j=0}^{i-1} (1+5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{j=0}^{i-1} (1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1 + \frac{A_5 \prod_{i=0}^{n-1} (1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1+(5i+5)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{11} \prod_{i=0}^{n-1} (1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{17} \prod_{i=0}^{n-1} (1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{23} \prod_{i=0}^{n-1} (1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{29} \prod_{i=0}^{n-1} (1+5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}} \\
 &= \frac{A_{29} \prod_{i=0}^{n-1} \frac{(1+5iA_5A_{11}A_{17}A_{23}A_{29})}{(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1 + 5iA_5A_{11}A_{17}A_{23}A_{29} \prod_{i=0}^{n-1} \frac{1+5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i+5)A_5A_{11}A_{17}A_{23}A_{29}}} = A_{29} \prod_{i=0}^{n-1} \frac{1 + 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 + (5i + 1)A_5A_{11}A_{17}A_{23}A_{29}} \left(\frac{1}{1 + \frac{5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i-5)A_5A_{11}A_{17}A_{23}A_{29}}} \right) \\
 &= A_{29} \prod_{i=0}^{n-1} \frac{1 + 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 + (5i + 1)A_5A_{11}A_{17}A_{23}A_{29}} \left(\frac{1 + (5i - 5)A_5A_{11}A_{17}A_{23}A_{29}}{1 + (5i - 4)A_5A_{11}A_{17}A_{23}A_{29}} \right).
 \end{aligned}$$

Therefore, we have

$$x_{30n+2} = A_{29} \prod_{i=0}^{n-1} \frac{1 + 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 + (5i + 1)A_5A_{11}A_{17}A_{23}A_{29}}.$$

Additional relationships can be acquired in the same way, thereby completing the proof. □

Theorem 2.2. *The equation (2.1) has a unique equilibrium point which is the number zero and this equilibrium is not locally asymptotically stable. Also \bar{x} is non hyperbolic.*

Proof of Theorem 2.2. For the equilibriums of equation (2.1), we have

$$\bar{x} = \frac{\bar{x}}{1 + \bar{x}^5},$$

then

$$\bar{x} + \bar{x}^6 = \bar{x}, \quad \bar{x}^6 = 0.$$

In consequence, the equilibrium point of (2.1), is $\bar{x} = 0$. Consider $f : (0, \infty)^5 \rightarrow (0, \infty)$ as the function defined by

$$f(\xi, v, \rho, \chi, \kappa) = \frac{\xi}{1 + \xi v \rho \chi \kappa}.$$

Therefore, it is deduced that,

$$\begin{aligned}
 f_\xi(\xi, v, \rho, \chi, \kappa) &= \frac{1}{(1 + \xi v \rho \chi \kappa)^2}, & f_v(\xi, v, \rho, \chi, \kappa) &= \frac{-\xi^2 \rho \chi \alpha}{(1 + \xi v \rho \chi \kappa)^2}, & f_\rho(\xi, v, \rho, \chi, \kappa) &= \frac{-\xi^2 v \chi \kappa}{(1 + \xi v \rho \chi \kappa)^2}, \\
 f_\chi(\xi, v, \rho, \chi, \kappa) &= \frac{-\xi^2 v \rho \kappa}{(1 + \xi v \rho \chi \kappa)^2}, & f_\kappa(\xi, v, \rho, \chi, \kappa) &= \frac{-\xi^2 v \chi \rho}{(1 + \xi v \rho \chi \kappa)^2}.
 \end{aligned}$$

We see that,

$$f_\xi(\bar{x}, \bar{x}, \bar{x}, \bar{x}, \bar{x}) = 1, \quad f_v(\bar{x}, \bar{x}, \bar{x}, \bar{x}, \bar{x}) = 0, \quad f_\rho(\bar{x}, \bar{x}, \bar{x}, \bar{x}, \bar{x}) = 0, \quad f_\chi(\bar{x}, \bar{x}, \bar{x}, \bar{x}, \bar{x}) = 0, \quad f_\kappa(\bar{x}, \bar{x}, \bar{x}, \bar{x}, \bar{x}) = 0.$$

The proof now follows by using Theorem 2.1. □

3. Solution of the Difference Equation $x_{n+1} = \frac{x_{n-29}}{1 - x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this part, we furnish a specific pattern for the solutions of the difference equation given, assuming that the initial conditions are arbitrary real numbers, where, x_0, \dots, x_{-29} defines as in (2.2)

$$x_{n+1} = \frac{x_{n-29}}{1 - x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}. \tag{3.1}$$

Theorem 3.1. Let's $\{x_n\}_{n=-29}^\infty$ be a solution of equation (3.1). Accordingly,

$$\begin{aligned}
 x_{30n+1} &= \frac{A_{30} \prod_{i=0}^n (1 - 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+2} &= \frac{A_{29} \prod_{i=0}^n (1 - 5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n+3} &= \frac{A_{28} \prod_{i=0}^n (1 - 5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 + (5i+1)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+4} &= \frac{A_{27} \prod_{i=0}^n (1 - 5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 + (5i+1)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n+5} &= \frac{A_{26} \prod_{i=0}^n (1 - 5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 - (5i+1)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+6} &= \frac{A_{25} \prod_{i=0}^n (1 - 5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 - (5i+1)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n+7} &= \frac{A_{24} \prod_{i=0}^n (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 - (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+8} &= \frac{A_{23} \prod_{i=0}^n (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 - (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n+9} &= \frac{A_{22} \prod_{i=0}^n (1 - (5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 - (5i+2)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+10} &= \frac{A_{21} \prod_{i=0}^n (1 - (5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 - (5i+2)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n+11} &= \frac{A_{20} \prod_{i=0}^n (1 - (5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 - (5i+2)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+12} &= \frac{A_{19} \prod_{i=0}^n (1 - (5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 - (5i+2)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n+13} &= \frac{A_{18} \prod_{i=0}^n (1 - (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 - (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+14} &= \frac{A_{17} \prod_{i=0}^n (1 - (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 - (5i+3)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n+15} &= \frac{A_{16} \prod_{i=0}^n (1 - (5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 - (5i+3)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+16} &= \frac{A_{15} \prod_{i=0}^n (1 - (5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 - (5i+3)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n+17} &= \frac{A_{14} \prod_{i=0}^n (1 - (5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 - (5i+3)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+18} &= \frac{A_{13} \prod_{i=0}^n (1 - (5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 - (5i+3)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n+19} &= \frac{A_{12} \prod_{i=0}^n (1 - (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 - (5i+4)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+20} &= \frac{A_{11} \prod_{i=0}^n (1 - (5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 - (5i+4)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n+21} &= \frac{A_{10} \prod_{i=0}^n (1 - (5i+3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 - (5i+4)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+22} &= \frac{A_9 \prod_{i=0}^n (1 - (5i+3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 - (5i+4)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n+23} &= \frac{A_8 \prod_{i=0}^n (1 - (5i+3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 - (5i+4)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+24} &= \frac{A_7 \prod_{i=0}^n (1 - (5i+3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 - (5i+4)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n+25} &= \frac{A_6 \prod_{i=0}^n (1 - (5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 - (5i+5)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n+26} &= \frac{A_5 \prod_{i=0}^n (1 - (5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^n (1 - (5i+5)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n+27} &= \frac{A_4 \prod_{i=0}^n (1 - (5i+4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^n (1 - (5i+5)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n+28} &= \frac{A_3 \prod_{i=0}^n (1 - (5i+4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^n (1 - (5i+5)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n+29} &= \frac{A_2 \prod_{i=0}^n (1 - (5i+4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^n (1 - (5i+5)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n+30} &= \frac{A_1 \prod_{i=0}^n (1 - (5i+4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^n (1 - (5i+5)A_1A_7A_{13}A_{19}A_{25})},
 \end{aligned}$$

holds.

Proof of Theorem 3.1. Let's suppose that n is greater than 0, and our assumption remains valid for n=1. That is,

$$\begin{aligned}
 x_{30n-29} &= \frac{A_{30} \prod_{i=0}^{n-1} (1 - 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-28} &= \frac{A_{29} \prod_{i=0}^{n-1} (1 - 5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-27} &= \frac{A_{28} \prod_{i=0}^{n-1} (1 - 5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-26} &= \frac{A_{27} \prod_{i=0}^{n-1} (1 - 5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-25} &= \frac{A_{26} \prod_{i=0}^{n-1} (1 - 5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-24} &= \frac{A_{25} \prod_{i=0}^{n-1} (1 - 5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-23} &= \frac{A_{24} \prod_{i=0}^{n-1} (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-22} &= \frac{A_{23} \prod_{i=0}^{n-1} (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-21} &= \frac{A_{22} \prod_{i=0}^{n-1} (1 - (5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-20} &= \frac{A_{21} \prod_{i=0}^{n-1} (1 - (5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-19} &= \frac{A_{20} \prod_{i=0}^{n-1} (1 - (5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-18} &= \frac{A_{19} \prod_{i=0}^{n-1} (1 - (5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-17} &= \frac{A_{18} \prod_{i=0}^{n-1} (1 - (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-16} &= \frac{A_{17} \prod_{i=0}^{n-1} (1 - (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-15} &= \frac{A_{16} \prod_{i=0}^{n-1} (1 - (5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-14} &= \frac{A_{15} \prod_{i=0}^{n-1} (1 - (5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-13} &= \frac{A_{14} \prod_{i=0}^{n-1} (1 - (5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-12} &= \frac{A_{13} \prod_{i=0}^{n-1} (1 - (5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_1A_7A_{13}A_{19}A_{25})},
 \end{aligned}$$

$$\begin{aligned}
 x_{30n-11} &= \frac{A_{12} \prod_{i=0}^{n-1} (1 - (5i + 3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i + 4)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-10} &= \frac{A_{11} \prod_{i=0}^{n-1} (1 - (5i + 3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i + 4)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-9} &= \frac{A_{10} \prod_{i=0}^{n-1} (1 - (5i + 3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 - (5i + 4)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-8} &= \frac{A_9 \prod_{i=0}^{n-1} (1 - (5i + 3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 - (5i + 4)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-7} &= \frac{A_8 \prod_{i=0}^{n-1} (1 - (5i + 3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 - (5i + 4)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n-6} &= \frac{A_7 \prod_{i=0}^{n-1} (1 - (5i + 3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 - (5i + 4)A_1A_7A_{13}A_{19}A_{25})}, \\
 x_{30n-5} &= \frac{A_6 \prod_{i=0}^{n-1} (1 - (5i + 4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i + 5)A_6A_{12}A_{18}A_{24}A_{30})}, & x_{30n-4} &= \frac{A_5 \prod_{i=0}^{n-1} (1 - (5i + 4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i + 5)A_5A_{11}A_{17}A_{23}A_{29})}, \\
 x_{30n-3} &= \frac{A_4 \prod_{i=0}^{n-1} (1 - (5i + 4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1} (1 - (5i + 5)A_4A_{10}A_{16}A_{22}A_{28})}, & x_{30n-2} &= \frac{A_3 \prod_{i=0}^{n-1} (1 - (5i + 4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1} (1 - (5i + 5)A_3A_9A_{15}A_{21}A_{27})}, \\
 x_{30n-1} &= \frac{A_2 \prod_{i=0}^{n-1} (1 - (5i + 4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1} (1 - (5i + 5)A_2A_8A_{14}A_{20}A_{26})}, & x_{30n} &= \frac{A_1 \prod_{i=0}^{n-1} (1 - (5i + 4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1} (1 - (5i + 5)A_1A_7A_{13}A_{19}A_{25})}.
 \end{aligned}$$

Now, using the main equation (3.1), one has

$$\begin{aligned}
 x_{30n+1} &= \frac{x_{30n-29}}{1 - x_{30n-5}x_{30n-11}x_{30n-17}x_{30n-23}x_{30n-29}} \\
 &= \frac{\frac{A_{30} \prod_{i=0}^{n-1} (1 + 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}}{1 + \frac{A_6 \prod_{i=0}^{n-1} (1 - 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})} \frac{A_{11} \prod_{i=0}^{n-1} (1 - (5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_6A_{12}A_{18}A_{24}A_{30})} \frac{A_{12} \prod_{i=0}^{n-1} (1 - (5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_6A_{12}A_{18}A_{24}A_{30})} \frac{A_{13} \prod_{i=0}^{n-1} (1 - (5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+4)A_6A_{12}A_{18}A_{24}A_{30})} \frac{A_{14} \prod_{i=0}^{n-1} (1 - (5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1} (1 - (5i+5)A_6A_{12}A_{18}A_{24}A_{30})}} \\
 &= \frac{A_{30} \prod_{i=0}^{n-1} (1 - 5iA_6A_{12}A_{18}A_{24}A_{30})}{1 + iA_6A_{12}A_{18}A_{24}A_{30} \prod_{i=0}^{n-1} \frac{1 - 5iA_6A_{12}A_{18}A_{24}A_{30}}{1 - (5i+5)A_6A_{12}A_{18}A_{24}A_{30}}} = A_{30} \prod_{i=0}^{n-1} \frac{1 - 5iA_6A_{12}A_{18}A_{24}A_{30}}{1 - (5i + 1)A_6A_{12}A_{18}A_{24}A_{30}} \left(\frac{1}{1 - \frac{A_6A_{12}A_{18}A_{24}A_{30}}{1 - (5i-5)A_6A_{12}A_{18}A_{24}A_{30}}} \right) \\
 &= A_{30} \prod_{i=0}^{n-1} \frac{1 - 5iA_6A_{12}A_{18}A_{24}A_{30}}{1 - (5i + 1)A_6A_{12}A_{18}A_{24}A_{30}} \left(\frac{1 - (5i - 5)A_6A_{12}A_{18}A_{24}A_{30}}{1 - (5i - 4)A_6A_{12}A_{18}A_{24}A_{30}} \right).
 \end{aligned}$$

Hence, we have

$$x_{30n+1} = \frac{A_{30} \prod_{i=0}^n (1 - 5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^n (1 - (5i + 1)A_6A_{12}A_{18}A_{24}A_{30})}.$$

Similarly,

$$\begin{aligned}
 x_{30n+2} &= \frac{x_{30n-28}}{1 - x_{30n-4}x_{30n-10}x_{30n-16}x_{30n-22}x_{30n-28}} \\
 &= \frac{\frac{A_{29} \prod_{i=0}^{n-1} (1 - 5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1 + \frac{A_5 \prod_{i=0}^{n-1} (1 - 5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{11} \prod_{i=0}^{n-1} (1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+2)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{12} \prod_{i=0}^{n-1} (1 - (5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+3)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{13} \prod_{i=0}^{n-1} (1 - (5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+4)A_5A_{11}A_{17}A_{23}A_{29})} \frac{A_{14} \prod_{i=0}^{n-1} (1 - (5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1} (1 - (5i+5)A_5A_{11}A_{17}A_{23}A_{29})}} \\
 &= \frac{A_{29} \prod_{i=0}^{n-1} (1 - 5iA_5A_{11}A_{17}A_{23}A_{29})}{1 + 5iA_5A_{11}A_{17}A_{23}A_{29} \prod_{i=0}^{n-1} \frac{1 - 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 - (5i+5)A_5A_{11}A_{17}A_{23}A_{29}}} = A_{29} \prod_{i=0}^{n-1} \frac{1 - 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 - (5i + 1)A_5A_{11}A_{17}A_{23}A_{29}} \left(\frac{1}{1 - \frac{5iA_5A_{11}A_{17}A_{23}A_{29}}{1 - (5i+1)A_5A_{11}A_{17}A_{23}A_{29}}} \right) \\
 &= A_{29} \prod_{i=0}^{n-1} \frac{1 - 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 - (5i + 1)A_5A_{11}A_{17}A_{23}A_{29}} \left(\frac{1 - (5i - 5)A_5A_{11}A_{17}A_{23}A_{29}}{1 - (5i - 4)A_5A_{11}A_{17}A_{23}A_{29}} \right).
 \end{aligned}$$

Therefore, we have

$$x_{30n+2} = A_{29} \prod_{i=0}^n \frac{1 - 5iA_5A_{11}A_{17}A_{23}A_{29}}{1 - (5i + 1)A_5A_{11}A_{17}A_{23}A_{29}}.$$

In a similar way, it is readily achieved in extra relationships. □

Theorem 3.2. In (3.1) there is a unique equilibrium point located at $\bar{x} = 0$, yet it does not fulfill the criteria for local asymptotic stability.

Proof of Theorem 3.2. The proof follows the same procedure as the proof of Theorem 2.2, thus it is not detailed. □

4. Solution of the Difference Equation $x_{n+1} = \frac{x_{n-29}}{-1 + x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this case, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers,

$$x_{n+1} = \frac{x_{n-29}}{-1 + x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}, \tag{4.1}$$

where, x_0, \dots, x_{29} defines as in (2.2) with $x_{-5}x_{-11}x_{-17}x_{-23}x_{-29} \neq 1$, $x_{-4}x_{-10}x_{-16}x_{-22}x_{-28} \neq 1$, $x_{-3}x_{-9}x_{-15}x_{-21}x_{-27} \neq 1$, $x_{-2}x_{-8}x_{-14}x_{-20}x_{-26} \neq 1$, $x_{-1}x_{-7}x_{-13}x_{-19}x_{-25} \neq 1$, $x_0x_{-6}x_{-12}x_{-18}x_{-24} \neq 1$.

Theorem 4.1. Each solution $\{x_n\}_{n=-29}^{\infty}$ of equation (4.1) recurs every sixty units and has the structure,

$$\begin{aligned}
 x_{60n+1} &= \frac{A_{30}}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}, & x_{60n+2} &= \frac{A_{29}}{-1 + A_5A_{11}A_{17}A_{23}A_{29}}, & x_{60n+3} &= \frac{A_{28}}{-1 + A_4A_{10}A_{16}A_{22}A_{28}}, \\
 x_{60n+4} &= \frac{A_{27}}{-1 + A_3A_9A_{15}A_{21}A_{27}}, & x_{60n+5} &= \frac{A_{26}}{-1 + A_2A_8A_{14}A_{20}A_{26}}, & x_{60n+6} &= \frac{A_{25}}{-1 + A_1A_7A_{13}A_{19}A_{25}}, \\
 x_{60n+7} &= A_{24}(-1 + A_6A_{12}A_{18}A_{24}A_{30}), & x_{60n+8} &= A_{23}(-1 + A_5A_{11}A_{17}A_{23}A_{29}), & x_{60n+9} &= A_{22}(-1 + A_4A_{10}A_{16}A_{22}A_{28}), \\
 x_{60n+10} &= A_{21}(-1 + A_3A_9A_{15}A_{21}A_{27}), & x_{60n+11} &= A_{20}(-1 + A_2A_8A_{14}A_{20}A_{26}), & x_{60n+12} &= A_{19}(-1 + A_1A_7A_{13}A_{19}A_{25}), \\
 x_{60n+13} &= \frac{A_{18}}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}, & x_{60n+14} &= \frac{A_{17}}{-1 + A_5A_{11}A_{17}A_{23}A_{29}}, & x_{60n+15} &= \frac{A_{16}}{-1 + A_4A_{10}A_{16}A_{22}A_{28}}, \\
 x_{60n+16} &= \frac{A_{15}}{-1 + A_3A_9A_{15}A_{21}A_{27}}, & x_{60n+17} &= \frac{A_{14}}{-1 + A_2A_8A_{14}A_{20}A_{26}}, & x_{60n+18} &= \frac{A_{13}}{-1 + A_1A_7A_{13}A_{19}A_{25}}, \\
 x_{60n+19} &= A_{12}(-1 + A_6A_{12}A_{18}A_{24}A_{30}), & x_{60n+20} &= A_{11}(-1 + A_5A_{11}A_{17}A_{23}A_{29}), & x_{60n+21} &= A_{10}(-1 + A_4A_{10}A_{16}A_{22}A_{28}), \\
 x_{60n+22} &= A_9(-1 + A_3A_9A_{15}A_{21}A_{27}), & x_{60n+23} &= A_8(-1 + A_2A_8A_{14}A_{20}A_{26}), & x_{60n+24} &= A_7(-1 + A_1A_7A_{13}A_{19}A_{25}), \\
 x_{60n+25} &= \frac{A_6}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}, & x_{60n+26} &= \frac{A_5}{-1 + A_5A_{11}A_{17}A_{23}A_{29}}, & x_{60n+27} &= \frac{A_4}{-1 + A_4A_{10}A_{16}A_{22}A_{28}}, \\
 x_{60n+28} &= \frac{A_3}{-1 + A_3A_9A_{15}A_{21}A_{27}}, & x_{60n+29} &= \frac{A_2}{-1 + A_2A_8A_{14}A_{20}A_{26}}, & x_{60n+30} &= \frac{A_1}{-1 + A_1A_7A_{13}A_{19}A_{25}}, \\
 \\
 x_{60n+31} &= A_{30}, & x_{60n+32} &= A_{29}, & x_{60n+33} &= A_{28}, & x_{60n+34} &= A_{27}, & x_{60n+35} &= A_{26}, & x_{60n+36} &= A_{25}, & x_{60n+37} &= A_{24}, \\
 x_{60n+38} &= A_{23}, & x_{60n+39} &= A_{22}, & x_{60n+40} &= A_{21}, & x_{60n+41} &= A_{20}, & x_{60n+42} &= A_{19}, & x_{60n+43} &= A_{18}, & x_{60n+44} &= A_{17}, \\
 x_{60n+45} &= A_{16}, & x_{60n+46} &= A_{15}, & x_{60n+47} &= A_{14}, & x_{60n+48} &= A_{13}, & x_{60n+49} &= A_{12}, & x_{60n+50} &= A_{11}, & x_{60n+51} &= A_{10}, \\
 x_{60n+52} &= A_9, & x_{60n+53} &= A_8, & x_{60n+54} &= A_7, & x_{60n+55} &= A_6, & x_{60n+56} &= A_5, & x_{60n+57} &= A_4, & x_{60n+58} &= A_3, \\
 x_{60n+59} &= A_2, & x_{60n+60} &= A_1.
 \end{aligned}$$

The solutions consist of 60 periods.

Proof of Theorem 4.1. Suppose,

$$\begin{aligned}
 x_{60n-59} &= \frac{A_{30}}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}, & x_{60n-58} &= \frac{A_{29}}{-1 + A_5A_{11}A_{17}A_{23}A_{29}}, & x_{60n-57} &= \frac{A_{28}}{-1 + A_4A_{10}A_{16}A_{22}A_{28}}, \\
 x_{60n-56} &= \frac{A_{27}}{-1 + A_3A_9A_{15}A_{21}A_{27}}, & x_{60n-55} &= \frac{A_{26}}{-1 + A_2A_8A_{14}A_{20}A_{26}}, & x_{60n-54} &= \frac{A_{25}}{-1 + A_1A_7A_{13}A_{19}A_{25}}, \\
 x_{60n-53} &= A_{24}(-1 + A_6A_{12}A_{18}A_{24}A_{30}), & x_{60n-52} &= A_{23}(-1 + A_5A_{11}A_{17}A_{23}A_{29}), & x_{60n-51} &= A_{22}(-1 + A_4A_{10}A_{16}A_{22}A_{28}), \\
 x_{60n-50} &= A_{21}(-1 + A_3A_9A_{15}A_{21}A_{27}), & x_{60n-49} &= A_{20}(-1 + A_2A_8A_{14}A_{20}A_{26}), & x_{60n-48} &= A_{19}(-1 + A_1A_7A_{13}A_{19}A_{25}), \\
 x_{60n-47} &= \frac{A_{18}}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}, & x_{60n-46} &= \frac{A_{17}}{-1 + A_5A_{11}A_{17}A_{23}A_{29}}, & x_{60n-45} &= \frac{A_{16}}{-1 + A_4A_{10}A_{16}A_{22}A_{28}}, \\
 x_{60n-44} &= \frac{A_{15}}{-1 + A_3A_9A_{15}A_{21}A_{27}}, & x_{60n-43} &= \frac{A_{14}}{-1 + A_2A_8A_{14}A_{20}A_{26}}, & x_{60n-42} &= \frac{A_{13}}{-1 + A_1A_7A_{13}A_{19}A_{25}}, \\
 x_{60n-41} &= A_{12}(-1 + A_6A_{12}A_{18}A_{24}A_{30}), & x_{60n-40} &= A_{11}(-1 + A_5A_{11}A_{17}A_{23}A_{29}), & x_{60n-39} &= A_{10}(-1 + A_4A_{10}A_{16}A_{22}A_{28}), \\
 x_{60n-38} &= A_9(-1 + A_3A_9A_{15}A_{21}A_{27}), & x_{60n-37} &= A_8(-1 + A_2A_8A_{14}A_{20}A_{26}), & x_{60n-36} &= A_7(-1 + A_1A_7A_{13}A_{19}A_{25}), \\
 x_{60n-35} &= \frac{A_6}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}, & x_{60n-34} &= \frac{A_5}{-1 + A_5A_{11}A_{17}A_{23}A_{29}}, & x_{60n-33} &= \frac{A_4}{-1 + A_4A_{10}A_{16}A_{22}A_{28}}, \\
 x_{60n-32} &= \frac{A_3}{-1 + A_3A_9A_{15}A_{21}A_{27}}, & x_{60n-31} &= \frac{A_2}{-1 + A_2A_8A_{14}A_{20}A_{26}}, & x_{60n-30} &= \frac{A_1}{-1 + A_1A_7A_{13}A_{19}A_{25}}, \\
 \\
 x_{60n-29} &= A_{30}, & x_{60n-28} &= A_{29}, & x_{60n-27} &= A_{28}, & x_{60n-26} &= A_{27}, & x_{60n-25} &= A_{26}, & x_{60n-24} &= A_{25}, & x_{60n-23} &= A_{24}, \\
 x_{60n-22} &= A_{23}, & x_{60n-21} &= A_{22}, & x_{60n-20} &= A_{21}, & x_{60n-19} &= A_{20}, & x_{60n-18} &= A_{19}, & x_{60n-17} &= A_{18}, & x_{60n-16} &= A_{17}, \\
 x_{60n-15} &= A_{16}, & x_{60n-14} &= A_{15}, & x_{60n-13} &= A_{14}, & x_{60n-12} &= A_{13}, & x_{60n-11} &= A_{12}, & x_{60n-10} &= A_{11}, & x_{60n-9} &= A_{10}, \\
 x_{60n-8} &= A_9, & x_{60n-7} &= A_8, & x_{60n-6} &= A_7, & x_{60n-5} &= A_6, & x_{60n-4} &= A_5, & x_{60n-3} &= A_4, & x_{60n-2} &= A_3, \\
 x_{60n-1} &= A_2, & x_{60n} &= A_1.
 \end{aligned}$$

Now, it follows from equation (4.1) that

$$x_{60n+1} = \frac{x_{60n-29}}{-1 + x_{60n-5}x_{60n-11}x_{60n-17}x_{60n-23}x_{60n-29}} = \frac{A_{30}}{-1 + A_6A_{12}A_{18}A_{24}A_{30}}.$$

Then, we have

$$x_{60n+1} = \frac{A_{30}}{-1 + A_6 A_{12} A_{18} A_{24} A_{30}}.$$

Other relation can be given by the same way. □

Theorem 4.2. Equation (4.1) has three equilibrium points which $0, \pm\sqrt[5]{2}$, and these equilibrium points aren't locally asymptotically stable.

Proof of Theorem 4. The proof follows the same procedure as the proof of Theorem 2.2, thus it is not detailed. □

5. Solution of the Difference Equation $x_{n+1} = \frac{x_{n-29}}{-1 - x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this case, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers,

$$x_{n+1} = \frac{x_{n-29}}{-1 - x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}, \tag{5.1}$$

where, x_0, \dots, x_{-29} defines as in (2.2) with $x_{-5}x_{-11}x_{-17}x_{-23}x_{-29} \neq -1$, $x_{-4}x_{-10}x_{-16}x_{-22}x_{-28} \neq 1$, $x_{-3}x_{-9}x_{-15}x_{-21}x_{-27} \neq -1$, $x_{-2}x_{-8}x_{-14}x_{-20}x_{-26} \neq -1$, $x_{-1}x_{-7}x_{-13}x_{-19}x_{-25} \neq -1$, $x_0x_{-6}x_{-12}x_{-18}x_{-24} \neq -1$.

Theorem 5.1. Each solution $\{x_n\}_{n=-29}^\infty$ of equation (4.1) is periodic with period sixty and is of the form,

$x_{60n+1} = \frac{A_{30}}{-1 - A_6 A_{12} A_{18} A_{24} A_{30}}$	$x_{60n+2} = \frac{A_{29}}{-1 - A_5 A_{11} A_{17} A_{23} A_{29}}$	$x_{60n+3} = \frac{A_{28}}{-1 - A_4 A_{10} A_{16} A_{22} A_{28}}$
$x_{60n+4} = \frac{A_{27}}{-1 - A_3 A_9 A_{15} A_{21} A_{27}}$	$x_{60n+5} = \frac{A_{26}}{-1 - A_2 A_8 A_{14} A_{20} A_{26}}$	$x_{60n+6} = \frac{A_{25}}{-1 - A_1 A_7 A_{13} A_{19} A_{25}}$
$x_{60n+7} = A_{24}(-1 - A_6 A_{12} A_{18} A_{24} A_{30})$	$x_{60n+8} = A_{23}(-1 - A_5 A_{11} A_{17} A_{23} A_{29})$	$x_{60n+9} = A_{22}(-1 - A_4 A_{10} A_{16} A_{22} A_{28})$
$x_{60n+10} = A_{21}(-1 - A_3 A_9 A_{15} A_{21} A_{27})$	$x_{60n+11} = A_{20}(-1 - A_2 A_8 A_{14} A_{20} A_{26})$	$x_{60n+12} = A_{19}(-1 - A_1 A_7 A_{13} A_{19} A_{25})$
$x_{60n+13} = \frac{A_{18}}{-1 - A_6 A_{12} A_{18} A_{24} A_{30}}$	$x_{60n+14} = \frac{A_{17}}{-1 - A_5 A_{11} A_{17} A_{23} A_{29}}$	$x_{60n+15} = \frac{A_{16}}{-1 - A_4 A_{10} A_{16} A_{22} A_{28}}$
$x_{60n+16} = \frac{A_{15}}{-1 - A_3 A_9 A_{15} A_{21} A_{27}}$	$x_{60n+17} = \frac{A_{14}}{-1 - A_2 A_8 A_{14} A_{20} A_{26}}$	$x_{60n+18} = \frac{A_{13}}{-1 - A_1 A_7 A_{13} A_{19} A_{25}}$
$x_{60n+19} = A_{12}(-1 - A_6 A_{12} A_{18} A_{24} A_{30})$	$x_{60n+20} = A_{11}(-1 - A_5 A_{11} A_{17} A_{23} A_{29})$	$x_{60n+21} = A_{10}(-1 - A_4 A_{10} A_{16} A_{22} A_{28})$
$x_{60n+22} = A_9(-1 - A_3 A_9 A_{15} A_{21} A_{27})$	$x_{60n+23} = A_8(-1 - A_2 A_8 A_{14} A_{20} A_{26})$	$x_{60n+24} = A_7(-1 - A_1 A_7 A_{13} A_{19} A_{25})$
$x_{60n+25} = \frac{A_6}{-1 - A_6 A_{12} A_{18} A_{24} A_{30}}$	$x_{60n+26} = \frac{A_5}{-1 - A_5 A_{11} A_{17} A_{23} A_{29}}$	$x_{60n+27} = \frac{A_4}{-1 - A_4 A_{10} A_{16} A_{22} A_{28}}$
$x_{60n+28} = \frac{A_3}{-1 - A_3 A_9 A_{15} A_{21} A_{27}}$	$x_{60n+29} = \frac{A_2}{-1 - A_2 A_8 A_{14} A_{20} A_{26}}$	$x_{60n+30} = \frac{A_1}{-1 - A_1 A_7 A_{13} A_{19} A_{25}}$

$x_{60n+31} = A_{30}$	$x_{60n+32} = A_{29}$	$x_{60n+33} = A_{28}$	$x_{60n+34} = A_{27}$	$x_{60n+35} = A_{26}$	$x_{60n+36} = A_{25}$	$x_{60n+37} = A_{24}$
$x_{60n+38} = A_{23}$	$x_{60n+39} = A_{22}$	$x_{60n+40} = A_{21}$	$x_{60n+41} = A_{20}$	$x_{60n+42} = A_{19}$	$x_{60n+43} = A_{18}$	$x_{60n+44} = A_{17}$
$x_{60n+45} = A_{16}$	$x_{60n+46} = A_{15}$	$x_{60n+47} = A_{14}$	$x_{60n+48} = A_{13}$	$x_{60n+49} = A_{12}$	$x_{60n+50} = A_{11}$	$x_{60n+51} = A_{10}$
$x_{60n+52} = A_9$	$x_{60n+53} = A_8$	$x_{60n+54} = A_7$	$x_{60n+55} = A_6$	$x_{60n+56} = A_5$	$x_{60n+57} = A_4$	$x_{60n+58} = A_3$
$x_{60n+59} = A_2$	$x_{60n+60} = A_1$					

The solutions consist of 60 periods.

Proof. The proof mirrors the proof of Theorem 4.1, and hence, it is not elaborated upon. □

Theorem 5.2. Equation (5.1) has three equilibrium points which $0, \pm\sqrt[5]{-2}$, and these equilibrium points are not locally asymptotically stable.

Proof. The proof follows the same procedure as the proof of Theorem 2.2, thus it is not detailed. □

6. Numerical Investigation

We devote this section to verifying the theoretical work obtained in this article.

Example 6.1. For Eq. 2.1 and 3.1 we consider following initial conditions.

$x_{-29} = 3.2,$	$x_{-28} = 3.3,$	$x_{-27} = 3.4,$	$x_{-26} = 3.5,$	$x_{-25} = 3.6,$	$x_{-24} = 3.7,$
$x_{-23} = 3.8,$	$x_{-22} = 3.9,$	$x_{-21} = 4,$	$x_{-20} = 4.1,$	$x_{-19} = 4.2,$	$x_{-18} = 4.3,$
$x_{-17} = 4.4,$	$x_{-16} = 4.5,$	$x_{-15} = 4.6,$	$x_{-14} = 4.7,$	$x_{-13} = 4.8,$	$x_{-12} = 4.9,$
$x_{-11} = 5,$	$x_{-10} = 5.1,$	$x_{-9} = 5.2,$	$x_{-8} = 5.3,$	$x_{-7} = 5.4,$	$x_{-6} = 5.5,$
$x_{-5} = 5.6,$	$x_{-4} = 5.7,$	$x_{-3} = 5.8,$	$x_{-2} = 5.9,$	$x_{-1} = 6.1,$	$x_0 = 6.$

Example 6.2. For Eq. 4.1 and 5.1 we consider following initial conditions.

$x_{-29} = 0.32,$	$x_{-28} = 0.33,$	$x_{-27} = 0.34,$	$x_{-26} = 0.35,$	$x_{-25} = 0.36,$	$x_{-24} = 0.37,$
$x_{-23} = 0.38,$	$x_{-22} = 0.39,$	$x_{-21} = 0.4,$	$x_{-20} = 0.41,$	$x_{-19} = 0.42,$	$x_{-18} = 0.43,$
$x_{-17} = 0.44,$	$x_{-16} = 0.45,$	$x_{-15} = 0.46,$	$x_{-14} = 0.47,$	$x_{-13} = 0.48,$	$x_{-12} = 0.49,$
$x_{-11} = 0.5,$	$x_{-10} = 0.51,$	$x_{-9} = 0.52,$	$x_{-8} = 0.53,$	$x_{-7} = 0.54,$	$x_{-6} = 0.55,$
$x_{-5} = 0.56,$	$x_{-4} = 0.57,$	$x_{-3} = 0.58,$	$x_{-2} = 0.59,$	$x_{-1} = 0.61,$	$x_0 = 0.6.$

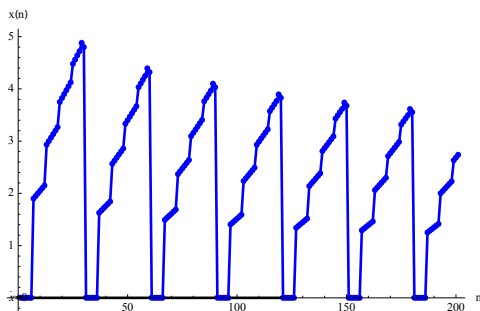


Figure 6.1: Plot illustrates the stability of Eq. 2.1

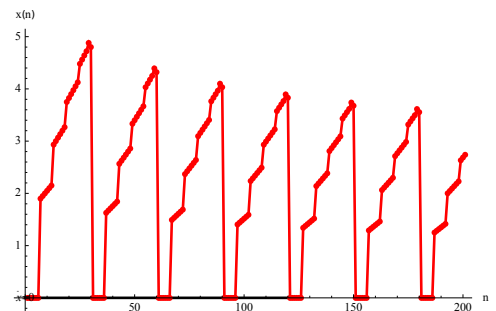


Figure 6.2: Plot illustrates the stability of Eq. 3.1

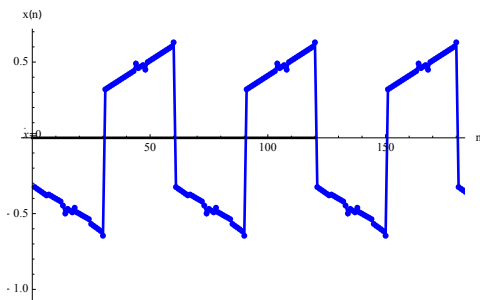


Figure 6.3: Plot illustrates the stability of Eq. 4.1

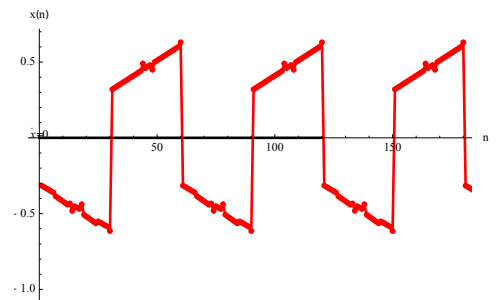


Figure 6.4: Plot illustrates the stability of Eq. 5.1

7. Conclusion

This article extensively explores the qualitative behaviors of difference equations. It effectively examines local stability, periodicity, oscillation, and solutions. Traditional iteration methods are employed to derive exact solutions for the relevant equations.

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