Universal Journal of Mathematics and Applications, 7 (3) (2024) 111-120
Research paper

Universal Journal of Mathematics and Applications

Journal Homepage: www.dergipark.gov.tr/ujma
ISSN 2619-9653
DOI: https://doi.org/10.32323/ujma.1483974

Dynamical Analysis and Solutions of Nonlinear Difference Equations of Thirty Order

Burak Ogul[1*] and Dağıstan Şimşek[2]

[1]*Department of Management Information Systems, Faculty of Applied Sciences,Istanbul Aydin University,Istanbul, Turkey*
[2]*Department of Engineering Basic Scieneces, Faculty of Engineering and Natural Sciences, Konya Technical University, Konya, Turkey*
[*]*Corresponding author*

Article Info

Keywords: *Difference equation, Local stability, Periodicity*
2010 AMS: *39A10, 39A30*
Received: *14 May 2024*
Accepted: *31 July 2024*
Available online: *25 August 2024*

Abstract

Discrete-time systems are sometimes used to explain natural phenomena that happen in nonlinear sciences. We study the periodicity, boundedness, oscillation, stability, and certain exact solutions of nonlinear difference equations in this paper. Using the standard iteration method, exact solutions are obtained. Some well-known theorems are used to test the stability of the equilibrium points. Some numerical examples are also provided to confirm the theoretical work's validity. The numerical component is implemented with Wolfram Mathematica. The method presented may be simply applied to other rational recursive issues.
In this paper, we explore the dynamics of adhering to a rational difference formula

$$x_{n+1} = \frac{x_{n-29}}{\pm 1 \pm x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}},$$

where the initials are arbitrary nonzero real numbers.

1. Introduction

A particular natural phenomenon's evolution is frequently explained over a period of time employing differential equations. Nevertheless, in certain instances, numerous real-life issues can be modeled using discrete time intervals, resulting in difference equations. As a result, recursive equations play an influential and potent role in mathematics. They are effectively employed to explore various applications in engineering, physics, biology, economics, and other fields [1–5]. For example, recursive equations have been effectively employed in modeling various natural phenomena, including population size, the Fibonacci sequence, drug concentrations in the bloodstream, information transmission, pricing dynamics of certain commodities, propagation patterns of annual plants, and more [6–12]. Additionally, certain scholars have utilized difference equations to obtain numerical solutions for certain differential equations. In particular, discretizing a given differential equation produces a corresponding difference equation. For example, the Runge-Kutta scheme arises from discretizing a first-order differential equation. This prompts consideration regarding the convergence of the difference scheme to the solution of a differential equation. The study discussed in reference [13] is dedicated to investigating the preservation of a solution bounded on the entire axis during the transition from differential to difference equations and vice versa. In reference [14], analogous inquiries were undertaken to maintain the oscillatory nature of solutions to second-order equations. Advancements in technology have spurred the utilization of recurrence equations as approximations to partial differential equations. It's noteworthy that fractional-order difference equations are frequently employed to study certain real-life phenomena that arise in nonlinear sciences. Almatrafi et. al. in [15] aim to analyzed the asymptotic stability, global stability, periodicity of the solution of an eighth-order difference equation. Sanbo et. al. in [16], discussed the periodicity, stability, and some solutions of a fifth-order recursive equation. Yeniçerioğlu et. al. in [17], examined the behavior of solutions of the neutral functional differential equations. Using a suitable real root of the corresponding characteristic equation, they explained the asymptotic behavior of the solutions and the stability of the trivial solution. Ahmed et al. [18] discovered new solutions and conducted a dynamical analysis for certain nonlinear difference relations of fifteenth order. Berkal et. al. in [19], have derived the forbidden set and

***Email addresses and ORCID numbers*: burakogul@aydin.edu.tr, 0000-0002-3264-4340 (B. Oğul), dsimsek@ktun.edu.tr, 0000-0003-3003-807X (D. Şimşek)**
Cite as: B. Ogul, D. Şimşek, *Dynamical analysis and solutions of nonlinear difference equations of thirty order*, Univers. J. Math. Appl., 7(3) (2024), 111-120.

determined the solutions of the difference equation that contains a quadratic term. Oğul et. al. in [20], examined soluttions of the sixth-order difference equations.
The inspiration behind this article stems from the exploration of eighteenth-order difference equations outlined in [21]. As such, the objective of this study is to analyze various dynamical properties including equilibrium points, local and global behaviors, boundedness, and analytic solutions of the nonlinear recursive equations (1.1).

$$x_{n+1}=\frac{x_{n-29}}{\pm 1\pm x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}. \tag{1.1}$$

Here, the initial values $x_{-29},x_{-28},x_{-27},\dots,x_{-2},x_{-1},x_0$, are arbitrary non-zero real numbers. In this work, we also illustrate some 2D figures with the help of Wolfram Mathematica to validate the obtained results.
In this study, stability, periodicity and global asymptotic stability definitions and theorems in the [1] source were used.

2. Solution of the Difference Equation $x_{n+1}=\frac{x_{n-29}}{1+x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this section, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers.

$$x_{n+1}=\frac{x_{n-29}}{1+x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}, \tag{2.1}$$

where,

$$\begin{array}{llllllll}
x_{-29}=A_{30}, & x_{-28}=A_{29}, & x_{-27}=A_{28}, & x_{-26}=A_{27}, & x_{-25}=A_{26}, & x_{-24}=A_{25}, & x_{-23}=A_{24}, & x_{-22}=A_{23},\\
x_{-21}=A_{22}, & x_{-20}=A_{21}, & x_{-19}=A_{20}, & x_{-18}=A_{19}, & x_{-17}=A_{18}, & x_{-16}=A_{17}, & x_{-15}=A_{16}, & x_{-14}=A_{15},\\
x_{-13}=A_{14}, & x_{-12}=A_{13}, & x_{-11}=A_{12}, & x_{-10}=A_1 1, & x_{-9}=A_{10}, & x_{-8}=A_9, & x_{-7}=A_8, & x_{-6}=A_7,\\
x_{-5}=A_6, & x_{-4}=A_5, & x_{-3}=A_4, & x_{-2}=A_3, & x_{-1}=A_2, & x_0=A_1. & &
\end{array} \tag{2.2}$$

Theorem 2.1. *Let $\{x_n\}_{n=-29}^{\infty}$ be a solution of (2.1). Then,*

$$x_{30n+1}=\frac{A_{30}\prod_{i=0}^{n}(1+5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n+2}=\frac{A_{29}\prod_{i=0}^{n}(1+5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+3}=\frac{A_{28}\prod_{i=0}^{n}(1+5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1+(5i+1)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n+4}=\frac{A_{27}\prod_{i=0}^{n}(1+5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1+(5i+1)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+5}=\frac{A_{26}\prod_{i=0}^{n}(1+5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1+(5i+1)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n+6}=\frac{A_{25}\prod_{i=0}^{n}(1+5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1+(5i+1)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+7}=\frac{A_{24}\prod_{i=0}^{n}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1+(5i+2)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n+8}=\frac{A_{23}\prod_{i=0}^{n}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+9}=\frac{A_{22}\prod_{i=0}^{n}(1+(5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1+(5i+2)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n+10}=\frac{A_{21}\prod_{i=0}^{n}(1+(5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1+(5i+2)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+11}=\frac{A_{20}\prod_{i=0}^{n}(1+(5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1+(5i+2)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n+12}=\frac{A_{19}\prod_{i=0}^{n}(1+(5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1+(5i+2)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+13}=\frac{A_{18}\prod_{i=0}^{n}(1+(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1+(5i+3)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n+14}=\frac{A_{17}\prod_{i=0}^{n}(1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+15}=\frac{A_{16}\prod_{i=0}^{n}(1+(5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1+(5i+3)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n+16}=\frac{A_{15}\prod_{i=0}^{n}(1+(5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1+(5i+3)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+17}=\frac{A_{14}\prod_{i=0}^{n}(1+(5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1+(5i+3)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n+18}=\frac{A_{13}\prod_{i=0}^{n}(1+(5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1+(5i+3)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+19}=\frac{A_{12}\prod_{i=0}^{n}(1+(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1+(5i+4)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n+20}=\frac{A_{11}\prod_{i=0}^{n}(1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+21}=\frac{A_{10}\prod_{i=0}^{n}(1+(5i+3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1+(5i+4)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n+22}=\frac{A_9\prod_{i=0}^{n}(1+(5i+3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1+(5i+4)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+23}=\frac{A_8\prod_{i=0}^{n}(1+(5i+3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1+(5i+4)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n+24}=\frac{A_7\prod_{i=0}^{n}(1+(5i+3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1+(5i+4)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+25}=\frac{A_6\prod_{i=0}^{n}(1+(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1+(5i+5)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n+26}=\frac{A_5\prod_{i=0}^{n}(1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+5)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+27}=\frac{A_4\prod_{i=0}^{n}(1+(5i+4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1+(5i+5)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n+28}=\frac{A_3\prod_{i=0}^{n}(1+(5i+4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1+(5i+5)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+29}=\frac{A_2\prod_{i=0}^{n}(1+(5i+4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1+(5i+5)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n+30}=\frac{A_1\prod_{i=0}^{n}(1+(5i+4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1+(5i+5)A_1A_7A_{13}A_{19}A_{25})},$$

where, $x_0,\dots,x_{-29}$ defines as in (2.2).

Proof of Theorem 2.1. The proof of each formula are carried out in similar way. So, we will demonstrate proof using one of the formula. We will employ the mathematical induction method. Let's posit that, with n being greater than zero and supposing our assumption is true for $n=1$. That is,

$$x_{30n-29}=\frac{A_{30}\prod_{i=0}^{n-1}(1+5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}, \quad x_{30n-28}=\frac{A_{29}\prod_{i=0}^{n-1}(1+5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-27}=\frac{A_{28}\prod_{i=0}^{n-1}(1+5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1+(5i+1)A_4A_{10}A_{16}A_{22}A_{28})}, \quad x_{30n-26}=\frac{A_{27}\prod_{i=0}^{n-1}(1+5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1+(5i+1)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-25}=\frac{A_{26}\prod_{i=0}^{n-1}(1+5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1+(5i+1)A_2A_8A_{14}A_{20}A_{26})}, \quad x_{30n-24}=\frac{A_{25}\prod_{i=0}^{n-1}(1+5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1+(5i+1)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-23}=\frac{A_{24}\prod_{i=0}^{n-1}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}, \quad x_{30n-22}=\frac{A_{23}\prod_{i=0}^{n-1}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-21}=\frac{A_{22}\prod_{i=0}^{n-1}(1+(5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1+(5i+2)A_4A_{10}A_{16}A_{22}A_{28})}, \quad x_{30n-20}=\frac{A_{21}\prod_{i=0}^{n-1}(1+(5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1+(5i+2)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-19}=\frac{A_{20}\prod_{i=0}^{n-1}(1+(5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1+(5i+2)A_2A_8A_{14}A_{20}A_{26})}, \quad x_{30n-18}=\frac{A_{19}\prod_{i=0}^{n-1}(1+(5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1+(5i+2)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-17}=\frac{A_{18}\prod_{i=0}^{n-1}(1+(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}, \quad x_{30n-16}=\frac{A_{17}\prod_{i=0}^{n-1}(1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-15}=\frac{A_{16}\prod_{i=0}^{n-1}(1+(5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1+(5i+3)A_4A_{10}A_{16}A_{22}A_{28})}, \quad x_{30n-14}=\frac{A_{15}\prod_{i=0}^{n-1}(1+(5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1+(5i+3)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-13}=\frac{A_{14}\prod_{i=0}^{n-1}(1+(5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1+(5i+3)A_2A_8A_{14}A_{20}A_{26})}, \quad x_{30n-12}=\frac{A_{13}\prod_{i=0}^{n-1}(1+(5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1+(5i+3)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-11}=\frac{A_{12}\prod_{i=0}^{n-1}(1+(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}, \quad x_{30n-10}=\frac{A_{11}\prod_{i=0}^{n-1}(1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-9}=\frac{A_{10}\prod_{i=0}^{n-1}(1+(5i+3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1+(5i+4)A_4A_{10}A_{16}A_{22}A_{28})}, \quad x_{30n-8}=\frac{A_9\prod_{i=0}^{n-1}(1+(5i+3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1+(5i+4)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-7}=\frac{A_8\prod_{i=0}^{n-1}(1+(5i+3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1+(5i+4)A_2A_8A_{14}A_{20}A_{26})}, \quad x_{30n-6}=\frac{A_7\prod_{i=0}^{n-1}(1+(5i+3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1+(5i+4)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-5}=\frac{A_6\prod_{i=0}^{n-1}(1+(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+5)A_6A_{12}A_{18}A_{24}A_{30})}, \quad x_{30n-4}=\frac{A_5\prod_{i=0}^{n-1}(1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+5)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-3}=\frac{A_4\prod_{i=0}^{n-1}(1+(5i+4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1+(5i+5)A_4A_{10}A_{16}A_{22}A_{28})}, \quad x_{30n-2}=\frac{A_3\prod_{i=0}^{n-1}(1+(5i+4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1+(5i+5)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-1}=\frac{A_2\prod_{i=0}^{n-1}(1+(5i+4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1+(5i+5)A_2A_8A_{14}A_{20}A_{26})}, \quad x_{30n}=\frac{A_1\prod_{i=0}^{n-1}(1+(5i+4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1+(5i+5)A_1A_7A_{13}A_{19}A_{25})}.$$

Now, using the main (2.1), one has

$$\begin{aligned}
x_{30n+1} &= \frac{x_{30n-29}}{1+x_{30n-5}x_{30n-11}x_{30n-17}x_{30n-23}x_{30n-29}} \\
&= \frac{\frac{A_{30}\prod_{i=0}^{n-1}(1+5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}}{1+A_6A_{12}A_{18}A_{24}A_{30}\frac{\prod_{i=0}^{n-1}(1+5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1+(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1+(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1+(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1+(5i+5)A_6A_{12}A_{18}A_{24}A_{30})}} \\
&= \frac{A_{30}\prod_{i=0}^{n-1}\frac{(1+5iiA_6A_{12}A_{18}A_{24}A_{30})}{(1+(5i+1)iA_6A_{12}A_{18}A_{24}A_{30})}}{1+5iA_6A_{12}A_{18}A_{24}A_{30}\prod_{i=0}^{n-1}\frac{1+5iA_6A_{12}A_{18}A_{24}A_{30}}{1+(5i+5)A_6A_{12}A_{18}A_{24}A_{30}}} = A_{30}\prod_{i=0}^{n-1}\frac{1+5iA_6A_{12}A_{18}A_{24}A_{30}}{1+(5i+1)iA_6A_{12}A_{18}A_{24}A_{30}}\left(\frac{1}{1+\frac{A_6A_{12}A_{18}A_{24}A_{30}}{1+(5i-5)A_6A_{12}A_{18}A_{24}A_{30}}}\right) \\
&= A_{30}\prod_{i=0}^{n-1}\frac{1+5iA_6A_{12}A_{18}A_{24}A_{30}}{1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30}}\left(\frac{1+(5i-5)A_6A_{12}A_{18}A_{24}A_{30}}{1+(5i-4)A_6A_{12}A_{18}A_{24}A_{30}}\right).
\end{aligned}$$

Hence, we have

$$x_{30n+1}=A_{30}\prod_{i=0}^{n}\frac{1+5iA_6A_{12}A_{18}A_{24}A_{30}}{1+(5i+1)A_6A_{12}A_{18}A_{24}A_{30}}.$$

Similarly,

$$x_{30n+2} = \frac{x_{30n-28}}{1+x_{30n-4}x_{30n-10}x_{30n-16}x_{30n-22}x_{30n-28}}$$
$$= \frac{\frac{A_{29}\prod_{i=0}^{n-1}(1+5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1+\frac{A_5\prod_{i=0}^{n-1}(1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+5)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{11}\prod_{i=0}^{n-1}(1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{17}\prod_{i=0}^{n-1}(1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{23}\prod_{i=0}^{n-1}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1+(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{29}\prod_{i=0}^{n-1}(1+5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}$$
$$= \frac{A_{29}\prod_{i=0}^{n-1}\frac{(1+5iA_5A_{11}A_{17}A_{23}A_{29})}{(1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1+5iA_5A_{11}A_{17}A_{23}A_{29}\prod_{i=0}^{n-1}\frac{1+5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i+5)A_5A_{11}A_{17}A_{23}A_{29}}} = A_{29}\prod_{i=0}^{n-1}\frac{1+5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29}}\left(\frac{1}{1+\frac{5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i-5)A_5A_{11}A_{17}A_{23}A_{29}}}\right)$$
$$= A_{29}\prod_{i=0}^{n-1}\frac{1+5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i+1)A_5A_{11}A_{17}A_{23}A_{29}}\left(\frac{1+(5i-5)A_5A_{11}A_{17}A_{23}A_{29}}{1+(5i-4)A_5A_{11}A_{17}A_{23}A_{29}}\right).$$

Therefore, we have

$$x_{30n+2} = A_{29}\prod_{i=0}^{n}\frac{1+5iA_5A_{11}A_{17}A_{23}A_{29}}{1+(5i+1)iA_5A_{11}A_{17}A_{23}A_{29}}.$$

Additional relationships can be acquired in the same way, thereby completing the proof. □

Theorem 2.2. *The equation (2.1) has a unique equilibrium point which is the number zero and this equilibrium is not locally asymptotically stable. Also $\bar{x}$ is non hyperbolic.*

Proof of Theorem 2.2. For the equilibriums of equation (2.1), we have

$$\bar{x} = \frac{\bar{x}}{1+\bar{x}^5},$$

then

$$\bar{x}+\bar{x}^6 = \bar{x}, \quad \bar{x}^6 = 0.$$

In consequence, the equilibrium point of (2.1), is $\bar{x} = 0$.
Consider $f : (0,\infty)^5 \to (0,\infty)$ as the function defined by

$$f(\xi,\nu,\rho,\chi,\kappa) = \frac{\xi}{1+\xi\nu\rho\chi\kappa}.$$

Therefore, it is deduced that,

$$f_\xi(\xi,\nu,\rho,\chi,\kappa) = \frac{1}{(1+\xi\nu\rho\chi\kappa)^2}, \qquad f_\nu(\xi,\nu,\rho,\chi,\kappa) = \frac{-\xi^2\rho\chi\alpha}{(1+\xi\nu\rho\chi\kappa)^2}, \qquad f_\rho(\xi,\nu,\rho,\chi,\kappa) = \frac{-\xi^2\nu\chi\kappa}{(1+\xi\nu\rho\chi\kappa)^2},$$
$$f_\chi(\xi,\nu,\rho,\chi,\kappa) = \frac{-\xi^2\nu\rho\kappa}{(1+\xi\nu\rho\chi\kappa)^2}, \qquad f_\kappa(\xi,\nu,\rho,\chi,\kappa) = \frac{-\xi^2\nu\chi\rho}{(1+\xi\nu\rho\chi\kappa)^2}.$$

We see that,

$$f_\xi(\bar{x},\bar{x},\bar{x},\bar{x},\bar{x}) = 1, \quad f_\nu(\bar{x},\bar{x},\bar{x},\bar{x},\bar{x}) = 0, \quad f_\rho(\bar{x},\bar{x},\bar{x},\bar{x},\bar{x}) = 0, \quad f_\chi(\bar{x},\bar{x},\bar{x},\bar{x},\bar{x}) = 0, \quad f_\kappa(\bar{x},\bar{x},\bar{x},\bar{x},\bar{x}) = 0.$$

The proof now follows by using Theorem 2.1. □

3. Solution of the Difference Equation $x_{n+1} = \frac{x_{n-29}}{1-x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this part, we furnish a specific pattern for the solutions of the difference equation given, assuming that the initial conditions are arbitrary real numbers, where, $x_0,\ldots,x_{-29}$ defines as in (2.2)

$$x_{n+1} = \frac{x_{n-29}}{1-x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}. \tag{3.1}$$

Theorem 3.1. *Let's $\{x_n\}_{n=-29}^{\infty}$ be a solution of equation (3.1). Accordingly,*

$$x_{30n+1}=\frac{A_{30}\prod_{i=0}^{n}(1-5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n+2}=\frac{A_{29}\prod_{i=0}^{n}(1-5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+3}=\frac{A_{28}\prod_{i=0}^{n}(1-5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1+(5i+1)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n+4}=\frac{A_{27}\prod_{i=0}^{n}(1-5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1+(5i+1)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+5}=\frac{A_{26}\prod_{i=0}^{n}(1-5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1-(5i+1)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n+6}=\frac{A_{25}\prod_{i=0}^{n}(1-5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1-(5i+1)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+7}=\frac{A_{24}\prod_{i=0}^{n}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1-(5i+2)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n+8}=\frac{A_{23}\prod_{i=0}^{n}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+2)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+9}=\frac{A_{22}\prod_{i=0}^{n}(1-(5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1-(5i+2)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n+10}=\frac{A_{21}\prod_{i=0}^{n}(1-(5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1-(5i+2)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+11}=\frac{A_{20}\prod_{i=0}^{n}(1-(5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1-(5i+2)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n+12}=\frac{A_{19}\prod_{i=0}^{n}(1-(5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1-(5i+2)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+13}=\frac{A_{18}\prod_{i=0}^{n}(1-(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1-(5i+3)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n+14}=\frac{A_{17}\prod_{i=0}^{n}(1-(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+3)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+15}=\frac{A_{16}\prod_{i=0}^{n}(1-(5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1-(5i+3)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n+16}=\frac{A_{15}\prod_{i=0}^{n}(1-(5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1-(5i+3)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+17}=\frac{A_{14}\prod_{i=0}^{n}(1-(5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1-(5i+3)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n+18}=\frac{A_{13}\prod_{i=0}^{n}(1-(5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1-(5i+3)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+19}=\frac{A_{12}\prod_{i=0}^{n}(1-(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1-(5i+4)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n+20}=\frac{A_{11}\prod_{i=0}^{n}(1-(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+4)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+21}=\frac{A_{10}\prod_{i=0}^{n}(1-(5i+3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1-(5i+4)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n+22}=\frac{A_9\prod_{i=0}^{n}(1-(5i+3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1-(5i+4)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+23}=\frac{A_8\prod_{i=0}^{n}(1-(5i+3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1-(5i+4)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n+24}=\frac{A_7\prod_{i=0}^{n}(1-(5i+3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1-(5i+4)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n+25}=\frac{A_6\prod_{i=0}^{n}(1-(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1-(5i+5)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n+26}=\frac{A_5\prod_{i=0}^{n}(1-(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+5)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n+27}=\frac{A_4\prod_{i=0}^{n}(1-(5i+4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n}(1-(5i+5)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n+28}=\frac{A_3\prod_{i=0}^{n}(1-(5i+4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n}(1-(5i+5)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n+29}=\frac{A_2\prod_{i=0}^{n}(1-(5i+4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n}(1-(5i+5)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n+30}=\frac{A_1\prod_{i=0}^{n}(1-(5i+4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n}(1-(5i+5)A_1A_7A_{13}A_{19}A_{25})},$$

holds.

Proof of Theorem 3.1. Let's suppose that n is greater than 0, and our assumption remains valid for n=1. That is,

$$x_{30n-29}=\frac{A_{30}\prod_{i=0}^{n-1}(1-5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n-28}=\frac{A_{29}\prod_{i=0}^{n-1}(1-5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-27}=\frac{A_{28}\prod_{i=0}^{n-1}(1-5iA_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1-(5i+1)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n-26}=\frac{A_{27}\prod_{i=0}^{n-1}(1-5iA_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1-(5i+1)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-25}=\frac{A_{26}\prod_{i=0}^{n-1}(1-5iA_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1-(5i+1)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n-24}=\frac{A_{25}\prod_{i=0}^{n-1}(1-5iA_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1-(5i+1)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-23}=\frac{A_{24}\prod_{i=0}^{n-1}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+2)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n-22}=\frac{A_{23}\prod_{i=0}^{n-1}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+2)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-21}=\frac{A_{22}\prod_{i=0}^{n-1}(1-(5i+1)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1-(5i+2)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n-20}=\frac{A_{21}\prod_{i=0}^{n-1}(1-(5i+1)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1-(5i+2)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-19}=\frac{A_{20}\prod_{i=0}^{n-1}(1-(5i+1)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1-(5i+2)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n-18}=\frac{A_{19}\prod_{i=0}^{n-1}(1-(5i+1)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1-(5i+2)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-17}=\frac{A_{18}\prod_{i=0}^{n-1}(1-(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+3)A_6A_{12}A_{18}A_{24}A_{30})},\quad x_{30n-16}=\frac{A_{17}\prod_{i=0}^{n-1}(1-(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+3)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-15}=\frac{A_{16}\prod_{i=0}^{n-1}(1-(5i+2)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1-(5i+3)A_4A_{10}A_{16}A_{22}A_{28})},\quad x_{30n-14}=\frac{A_{15}\prod_{i=0}^{n-1}(1-(5i+2)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1-(5i+3)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-13}=\frac{A_{14}\prod_{i=0}^{n-1}(1-(5i+2)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1-(5i+3)A_2A_8A_{14}A_{20}A_{26})},\quad x_{30n-12}=\frac{A_{13}\prod_{i=0}^{n-1}(1-(5i+2)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1-(5i+3)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-11}=\frac{A_{12}\prod_{i=0}^{n-1}(1-(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+4)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n-10}=\frac{A_{11}\prod_{i=0}^{n-1}(1-(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+4)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-9}=\frac{A_{10}\prod_{i=0}^{n-1}(1-(5i+3)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1-(5i+4)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n-8}=\frac{A_9\prod_{i=0}^{n-1}(1-(5i+3)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1-(5i+4)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-7}=\frac{A_8\prod_{i=0}^{n-1}(1-(5i+3)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1-(5i+4)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n-6}=\frac{A_7\prod_{i=0}^{n-1}(1-(5i+3)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1-(5i+4)A_1A_7A_{13}A_{19}A_{25})},$$

$$x_{30n-5}=\frac{A_6\prod_{i=0}^{n-1}(1-(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+5)A_6A_{12}A_{18}A_{24}A_{30})},\qquad x_{30n-4}=\frac{A_5\prod_{i=0}^{n-1}(1-(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+5)A_5A_{11}A_{17}A_{23}A_{29})},$$

$$x_{30n-3}=\frac{A_4\prod_{i=0}^{n-1}(1-(5i+4)A_4A_{10}A_{16}A_{22}A_{28})}{\prod_{i=0}^{n-1}(1-(5i+5)A_4A_{10}A_{16}A_{22}A_{28})},\qquad x_{30n-2}=\frac{A_3\prod_{i=0}^{n-1}(1-(5i+4)A_3A_9A_{15}A_{21}A_{27})}{\prod_{i=0}^{n-1}(1-(5i+5)A_3A_9A_{15}A_{21}A_{27})},$$

$$x_{30n-1}=\frac{A_2\prod_{i=0}^{n-1}(1-(5i+4)A_2A_8A_{14}A_{20}A_{26})}{\prod_{i=0}^{n-1}(1-(5i+5)A_2A_8A_{14}A_{20}A_{26})},\qquad x_{30n}=\frac{A_1\prod_{i=0}^{n-1}(1-(5i+4)A_1A_7A_{13}A_{19}A_{25})}{\prod_{i=0}^{n-1}(1-(5i+5)A_1A_7A_{13}A_{19}A_{25})}.$$

Now, using the main equation (3.1), one has

$$\begin{aligned}
x_{30n+1}&=\frac{x_{30n-29}}{1-x_{30n-5}x_{30n-11}x_{30n-17}x_{30n-23}x_{30n-29}}\\
&=\frac{\frac{A_{30}\prod_{i=0}^{n-1}(1+5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}}{1+A_6A_{12}A_{18}A_{24}A_{30}\frac{\prod_{i=0}^{n-1}(1-5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1-(5i+2)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1-(5i+3)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}\frac{\prod_{i=0}^{n-1}(1-(5i+4)A_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n-1}(1-(5i+5)A_6A_{12}A_{18}A_{24}A_{30})}}\\
&=\frac{A_{30}\prod_{i=0}^{n-1}\frac{(1-5iA_6A_{12}A_{18}A_{24}A_{30})}{(1-(5i+1)iA_6A_{12}A_{18}A_{24}A_{30})}}{1+iA_6A_{12}A_{18}A_{24}A_{30}\prod_{i=0}^{n-1}\frac{1-5iA_6A_{12}A_{18}A_{24}A_{30}}{1-(5i+5)A_6A_{12}A_{18}A_{24}A_{30}}}=A_{30}\prod_{i=0}^{n-1}\frac{1-5iA_6A_{12}A_{18}A_{24}A_{30}}{1-(5i+1)iA_6A_{12}A_{18}A_{24}A_{30}}\left(\frac{1}{1-\frac{A_6A_{12}A_{18}A_{24}A_{30}}{1-(5i-5)A_6A_{12}A_{18}A_{24}A_{30}}}\right)\\
&=A_{30}\prod_{i=0}^{n-1}\frac{1-5iA_6A_{12}A_{18}A_{24}A_{30}}{1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30}}\left(\frac{1-(5i-5)A_6A_{12}A_{18}A_{24}A_{30}}{1-(5i-4)A_6A_{12}A_{18}A_{24}A_{30}}\right).
\end{aligned}$$

Hence, we have

$$x_{30n+1}=\frac{A_{30}\prod_{i=0}^{n}(1-5iA_6A_{12}A_{18}A_{24}A_{30})}{\prod_{i=0}^{n}(1-(5i+1)A_6A_{12}A_{18}A_{24}A_{30})}.$$

Similarly,

$$\begin{aligned}
x_{30n+2}&=\frac{x_{30n-28}}{1-x_{30n-4}x_{30n-10}x_{30n-16}x_{30n-22}x_{30n-28}}\\
&=\frac{\frac{A_{29}\prod_{i=0}^{n-1}(1-5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1+\frac{A_5\prod_{i=0}^{n-1}(1-(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+5)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{11}\prod_{i=0}^{n-1}(1-(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+4)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{17}\prod_{i=0}^{n-1}(1-(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+3)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{23}\prod_{i=0}^{n-1}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n-1}(1-(5i+2)A_5A_{11}A_{17}A_{23}A_{29})}\frac{A_{29}\prod_{i=0}^{n-1}(1-5iA_5A_{11}A_{17}A_{23}A_{29})}{\prod_{i=0}^{n}(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}\\
&=\frac{A_{29}\prod_{i=0}^{n-1}\frac{(1-5iA_5A_{11}A_{17}A_{23}A_{29})}{(1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29})}}{1+5iA_5A_{11}A_{17}A_{23}A_{29}\prod_{i=0}^{n-1}\frac{1-5iA_5A_{11}A_{17}A_{23}A_{29}}{1-(5i+5)A_5A_{11}A_{17}A_{23}A_{29}}}=A_{29}\prod_{i=0}^{n-1}\frac{1-5iA_5A_{11}A_{17}A_{23}A_{29}}{1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29}}\left(\frac{1}{1-\frac{5iA_5A_{11}A_{17}A_{23}A_{29}}{1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29}}}\right)\\
&=A_{29}\prod_{i=0}^{n-1}\frac{1-5iA_5A_{11}A_{17}A_{23}A_{29}}{1-(5i+1)A_5A_{11}A_{17}A_{23}A_{29}}\left(\frac{1-(5i-5)A_5A_{11}A_{17}A_{23}A_{29}}{1-(5i-4)A_5A_{11}A_{17}A_{23}A_{29}}\right).
\end{aligned}$$

Therefore, we have

$$x_{30n+2}=A_{29}\prod_{i=0}^{n}\frac{1-5iA_5A_{11}A_{17}A_{23}A_{29}}{1-(5i+1)iA_5A_{11}A_{17}A_{23}A_{29}}.$$

In a similar way, it is readily achieved in extra relationships. □

Theorem 3.2. *In (3.1) there is a unique equilibrium point located at* $\bar{x}=0$*, yet it does not fulfill the criteria for local asymptotic stability.*

Proof of Theorem 3.2. The proof follows the same procedure as the proof of Theorem 2.2, thus it is not detailed. □

4. Solution of the Difference Equation $x_{n+1}=\frac{x_{n-29}}{-1+x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this case, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers,

$$x_{n+1}=\frac{x_{n-29}}{-1+x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}, \tag{4.1}$$

where, $x_0,\ldots,x_{-29}$ defines as in (2.2) with $x_{-5}x_{-11}x_{-17}x_{-23}x_{-29}\neq 1$, $x_{-4}x_{-10}x_{-16}x_{-22}x_{-28}\neq 1$, $x_{-3}x_{-9}x_{-15}x_{-21}x_{-27}\neq 1$, $x_{-2}x_{-8}x_{-14}x_{-20}x_{-26}\neq 1$, $x_{-1}x_{-7}x_{-13}x_{-19}x_{-25}\neq 1$, $x_0x_{-6}x_{-12}x_{-18}x_{-24}\neq 1$.

Theorem 4.1. *Each solution $\{x_n\}_{n=-29}^{\infty}$ of equation (4.1) recurs every sixty units and has the structure,*

$$x_{60n+1}=\frac{A_{30}}{-1+A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n+2}=\frac{A_{29}}{-1+A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n+3}=\frac{A_{28}}{-1+A_4A_{10}A_{16}A_{22}A_{28}},$$

$$x_{60n+4}=\frac{A_{27}}{-1+A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n+5}=\frac{A_{26}}{-1+A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n+6}=\frac{A_{25}}{-1+A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n+7}=A_{24}(-1+A_6A_{12}A_{18}A_{24}A_{30}),\quad x_{60n+8}=A_{23}(-1+A_5A_{11}A_{17}A_{23}A_{29}),\quad x_{60n+9}=A_{22}(-1+A_4A_{10}A_{16}A_{22}A_{28}),$$

$$x_{60n+10}=A_{21}(-1+A_3A_9A_{15}A_{21}A_{27}),\quad x_{60n+11}=A_{20}(-1+A_2A_8A_{14}A_{20}A_{26}),\quad x_{60n+12}=A_{19}(-1+A_1A_7A_{13}A_{19}A_{25}),$$

$$x_{60n+13}=\frac{A_{18}}{-1+A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n+14}=\frac{A_{17}}{-1+A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n+15}=\frac{A_{16}}{-1+A_4A_{10}A_{16}A_{22}A_{28}},$$

$$x_{60n+16}=\frac{A_{15}}{-1+A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n+17}=\frac{A_{14}}{-1+A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n+18}=\frac{A_{13}}{-1+A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n+19}=A_{12}(-1+A_6A_{12}A_{18}A_{24}A_{30}),\quad x_{60n+20}=A_{11}(-1+A_5A_{11}A_{17}A_{23}A_{29}),\quad x_{60n+21}=A_{10}(-1+A_4A_{10}A_{16}A_{22}A_{28}),$$

$$x_{60n+22}=A_9(-1+A_3A_9A_{15}A_{21}A_{27}),\quad x_{60n+23}=A_8(-1+A_2A_8A_{14}A_{20}A_{26}),\quad x_{60n+24}=A_7(-1+A_1A_7A_{13}A_{19}A_{25}),$$

$$x_{60n+25}=\frac{A_6}{-1+A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n+26}=\frac{A_5}{-1+A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n+27}=\frac{A_4}{-1+A_4A_{10}A_{16}A_{22}A_{28}},$$

$$x_{60n+28}=\frac{A_3}{-1+A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n+29}=\frac{A_2}{-1+A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n+30}=\frac{A_1}{-1+A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n+31}=A_{30},\quad x_{60n+32}=A_{29},\quad x_{60n+33}=A_{28},\quad x_{60n+34}=A_{27},\quad x_{60n+35}=A_{26},\quad x_{60n+36}=A_{25},\quad x_{60n+37}=A_{24},$$

$$x_{60n+38}=A_{23},\quad x_{60n+39}=A_{22},\quad x_{60n+40}=A_{21},\quad x_{60n+41}=A_{20},\quad x_{60n+42}=A_{19},\quad x_{60n+43}=A_{18},\quad x_{60n+44}=A_{17},$$

$$x_{60n+45}=A_{16},\quad x_{60n+46}=A_{15},\quad x_{60n+47}=A_{14},\quad x_{60n+48}=A_{13},\quad x_{60n+49}=A_{12},\quad x_{60n+50}=A_{11},\quad x_{60n+51}=A_{10},$$

$$x_{60n+52}=A_9,\quad x_{60n+53}=A_8,\quad x_{60n+54}=A_7,\quad x_{60n+55}=A_6,\quad x_{60n+56}=A_5,\quad x_{60n+57}=A_4,\quad x_{60n+58}=A_3,$$

$$x_{60n+59}=A_2,\quad x_{60n+60}=A_1.$$

The solutions consist of 60 periods.

Proof of Theorem 4.1. Suppose,

$$x_{60n-59}=\frac{A_{30}}{-1+A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n-58}=\frac{A_{29}}{-1+A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n-57}=\frac{A_{28}}{-1+A_4A_{10}A_{16}A_{22}A_{28}},$$

$$x_{60n-56}=\frac{A_{27}}{-1+A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n-55}=\frac{A_{26}}{-1+A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n-54}=\frac{A_{25}}{-1+A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n-53}=A_{24}(-1+A_6A_{12}A_{18}A_{24}A_{30}),\quad x_{60n-52}=A_{23}(-1+A_5A_{11}A_{17}A_{23}A_{29}),\quad x_{60n-51}=A_{22}(-1+A_4A_{10}A_{16}A_{22}A_{28}),$$

$$x_{60n-50}=A_{21}(-1+A_3A_9A_{15}A_{21}A_{27}),\quad x_{60n-49}=A_{20}(-1+A_2A_8A_{14}A_{20}A_{26}),\quad x_{60n-48}=A_{19}(-1+A_1A_7A_{13}A_{19}A_{25}),$$

$$x_{60n-47}=\frac{A_{18}}{-1+A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n-46}=\frac{A_{17}}{-1+A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n-45}=\frac{A_{16}}{-1+A_4A_{10}A_{16}A_{22}A_{28}},$$

$$x_{60n-44}=\frac{A_{15}}{-1+A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n-43}=\frac{A_{14}}{-1+A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n-42}=\frac{A_{13}}{-1+A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n-41}=A_{12}(-1+A_6A_{12}A_{18}A_{24}A_{30}),\quad x_{60n-40}=A_{11}(-1+A_5A_{11}A_{17}A_{23}A_{29}),\quad x_{60n-39}=A_{10}(-1+A_4A_{10}A_{16}A_{22}A_{28}),$$

$$x_{60n-38}=A_9(-1+A_3A_9A_{15}A_{21}A_{27}),\quad x_{60n-37}=A_8(-1+A_2A_8A_{14}A_{20}A_{26}),\quad x_{60n-36}=A_7(-1+A_1A_7A_{13}A_{19}A_{25}),$$

$$x_{60n-35}=\frac{A_6}{-1+A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n-34}=\frac{A_5}{-1+A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n-33}=\frac{A_4}{-1+A_4A_{10}A_{16}A_{22}A_{28}},$$

$$x_{60n-32}=\frac{A_3}{-1+A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n-31}=\frac{A_2}{-1+A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n-30}=\frac{A_1}{-1+A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n-29}=A_{30},\quad x_{60n-28}=A_{29},\quad x_{60n-27}=A_{28},\quad x_{60n-26}=A_{27},\quad x_{60n-25}=A_{26},\quad x_{60n-24}=A_{25},\quad x_{60n-23}=A_{24},$$

$$x_{60n-22}=A_{23},\quad x_{60n-21}=A_{22},\quad x_{60n-20}=A_{21},\quad x_{60n-19}=A_{20},\quad x_{60n-18}=A_{19},\quad x_{60n-17}=A_{18},\quad x_{60n-16}=A_{17},$$

$$x_{60n-15}=A_{16},\quad x_{60n-14}=A_{15},\quad x_{60n-13}=A_{14},\quad x_{60n-12}=A_{13},\quad x_{60n-11}=A_{12},\quad x_{60n-10}=A_{11},\quad x_{60n-9}=A_{10},$$

$$x_{60n-8}=A_9,\quad x_{60n-7}=A_8,\quad x_{60n-6}=A_7,\quad x_{60n-5}=A_6,\quad x_{60n-4}=A_5,\quad x_{60n-3}=A_4,\quad x_{60n-2}=A_3,$$

$$x_{60n-1}=A_2,\quad x_{60n}=A_1.$$

Now, it follows from equation (4.1) that

$$x_{60n+1}=\frac{x_{60n-29}}{-1+x_{60n-5}x_{60n-11}x_{60n-17}x_{60n-23}x_{60n-29}}=\frac{A_{30}}{-1+A_6A_{12}A_{18}A_{24}A_{30}}.$$

Then, we have

$$x_{60n+1}=\frac{A_{30}}{-1+A_6A_{12}A_{18}A_{24}A_{30}}.$$

Other relation can be given by the same way. □

Theorem 4.2. *Equation (4.1) has three equilibrium points which* $0,\pm\sqrt[5]{2}$*, and these equilibrium points aren't locally asymptotically stable.*

Proof of Theorem 4. The proof follows the same procedure as the proof of Theorem 2.2, thus it is not detailed. □

5. Solution of the Difference Equation $x_{n+1}=\frac{x_{n-29}}{-1-x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}$

In this case, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers,

$$x_{n+1}=\frac{x_{n-29}}{-1-x_{n-5}x_{n-11}x_{n-17}x_{n-23}x_{n-29}}, \tag{5.1}$$

where, $x_0,\dots,x_{-29}$ defines as in (2.2) with $x_{-5}x_{-11}x_{-17}x_{-23}x_{-29}\neq-1,\quad x_{-4}x_{-10}x_{-16}x_{-22}x_{-28}\neq 1,\quad x_{-3}x_{-9}x_{-15}x_{-21}x_{-27}\neq-1,$ $x_{-2}x_{-8}x_{-14}x_{-20}x_{-26}\neq-1,\quad x_{-1}x_{-7}x_{-13}x_{-19}x_{-25}\neq-1,\quad x_0x_{-6}x_{-12}x_{-18}x_{-24}\neq-1.$

Theorem 5.1. *Each solution* $\{x_n\}_{n=-29}^{\infty}$ *of equation (4.1) is periodic with period sixty and is of the form,*

$$x_{60n+1}=\frac{A_{30}}{-1-A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n+2}=\frac{A_{29}}{-1-A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n+3}=\frac{A_{28}}{-1-A_4A_{10}A_{16}A_{22}A_{28}},$$
$$x_{60n+4}=\frac{A_{27}}{-1-A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n+5}=\frac{A_{26}}{-1-A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n+6}=\frac{A_{25}}{-1-A_1A_7A_{13}A_{19}A_{25}},$$
$$x_{60n+7}=A_{24}(-1-A_6A_{12}A_{18}A_{24}A_{30}),\quad x_{60n+8}=A_{23}(-1-A_5A_{11}A_{17}A_{23}A_{29}),\quad x_{60n+9}=A_{22}(-1-A_4A_{10}A_{16}A_{22}A_{28}),$$
$$x_{60n+10}=A_{21}(-1-A_3A_9A_{15}A_{21}A_{27}),\quad x_{60n+11}=A_{20}(-1-A_2A_8A_{14}A_{20}A_{26}),\quad x_{60n+12}=A_{19}(-1-A_1A_7A_{13}A_{19}A_{25}),$$
$$x_{60n+13}=\frac{A_{18}}{-1-A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n+14}=\frac{A_{17}}{-1-A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n+15}=\frac{A_{16}}{-1-A_4A_{10}A_{16}A_{22}A_{28}},$$
$$x_{60n+16}=\frac{A_{15}}{-1-A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n+17}=\frac{A_{14}}{-1-A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n+18}=\frac{A_{13}}{-1-A_1A_7A_{13}A_{19}A_{25}},$$
$$x_{60n+19}=A_{12}(-1-A_6A_{12}A_{18}A_{24}A_{30}),\quad x_{60n+20}=A_{11}(-1-A_5A_{11}A_{17}A_{23}A_{29}),\quad x_{60n+21}=A_{10}(-1-A_4A_{10}A_{16}A_{22}A_{28}),$$
$$x_{60n+22}=A_9(-1-A_3A_9A_{15}A_{21}A_{27}),\quad x_{60n+23}=A_8(-1-A_2A_8A_{14}A_{20}A_{26}),\quad x_{60n+24}=A_7(-1-A_1A_7A_{13}A_{19}A_{25}),$$
$$x_{60n+25}=\frac{A_6}{-1-A_6A_{12}A_{18}A_{24}A_{30}},\quad x_{60n+26}=\frac{A_5}{-1-A_5A_{11}A_{17}A_{23}A_{29}},\quad x_{60n+27}=\frac{A_4}{-1-A_4A_{10}A_{16}A_{22}A_{28}},$$
$$x_{60n+28}=\frac{A_3}{-1-A_3A_9A_{15}A_{21}A_{27}},\quad x_{60n+29}=\frac{A_2}{-1-A_2A_8A_{14}A_{20}A_{26}},\quad x_{60n+30}=\frac{A_1}{-1-A_1A_7A_{13}A_{19}A_{25}},$$

$$x_{60n+31}=A_{30},\quad x_{60n+32}=A_{29},\quad x_{60n+33}=A_{28},\quad x_{60n+34}=A_{27},\quad x_{60n+35}=A_{26},\quad x_{60n+36}=A_{25},\quad x_{60n+37}=A_{24},$$
$$x_{60n+38}=A_{23},\quad x_{60n+39}=A_{22},\quad x_{60n+40}=A_{21},\quad x_{60n+41}=A_{20},\quad x_{60n+42}=A_{19},\quad x_{60n+43}=A_{18},\quad x_{60n+44}=A_{17},$$
$$x_{60n+45}=A_{16},\quad x_{60n+46}=A_{15},\quad x_{60n+47}=A_{14},\quad x_{60n+48}=A_{13},\quad x_{60n+49}=A_{12},\quad x_{60n+50}=A_{11},\quad x_{60n+51}=A_{10},$$
$$x_{60n+52}=A_9,\quad x_{60n+53}=A_8,\quad x_{60n+54}=A_7,\quad x_{60n+55}=A_6,\quad x_{60n+56}=A_5,\quad x_{60n+57}=A_4,\quad x_{60n+58}=A_3,$$
$$x_{60n+59}=A_2,\quad x_{60n+60}=A_1.$$

The solutions consist of 60 periods.

Proof. The proof mirrors the proof of Theorem 4.1, and hence, it is not elaborated upon. □

Theorem 5.2. *Equation (5.1) has three equilibrium points which* $0,\pm\sqrt[5]{-2}$*, and these equilibrium points are not locally asymptotically stable.*

Proof. The proof follows the same procedure as the proof of Theorem 2.2, thus it is not detailed. □

6. Numerical Investigation

We devote this section to verifying the theoretical work obtained in this article.

Example 6.1. *For Eq. 2.1 and 3.1 we consider following initial conditions.*

$$\begin{array}{cccccc}
x_{-29}=3.2, & x_{-28}=3.3, & x_{-27}=3.4, & x_{-26}=3.5, & x_{-25}=3.6, & x_{-24}=3.7, \\
x_{-23}=3.8, & x_{-22}=3.9, & x_{-21}=4, & x_{-20}=4.1, & x_{-19}=4.2, & x_{-18}=4.3, \\
x_{-17}=4.4, & x_{-16}=4.5, & x_{-15}=4.6, & x_{-14}=4.7, & x_{-13}=4.8, & x_{-12}=4.9, \\
x_{-11}=5, & x_{-10}=5.1, & x_{-9}=5.2, & x_{-8}=5.3, & x_{-7}=5.4, & x_{-6}=5.5, \\
x_{-5}=5.6, & x_{-4}=5.7, & x_{-3}=5.8, & x_{-2}=5.9, & x_{-1}=6.1, & x_{0}=6.
\end{array}$$

Example 6.2. *For Eq. 4.1 and 5.1 we consider following initial conditions.*

$$\begin{array}{cccccc}
x_{-29}=0.32, & x_{-28}=0.33, & x_{-27}=0.34, & x_{-26}=0.35, & x_{-25}=0.36, & x_{-24}=0.37, \\
x_{-23}=0.38, & x_{-22}=0.39, & x_{-21}=0.4, & x_{-20}=0.41, & x_{-19}=0.42, & x_{-18}=0.43, \\
x_{-17}=0.44, & x_{-16}=0.45, & x_{-15}=0.46, & x_{-14}=0.47, & x_{-13}=0.48, & x_{-12}=0.49, \\
x_{-11}=0.5, & x_{-10}=0.51, & x_{-9}=0.52, & x_{-8}=0.53 & x_{-7}=0.54, & x_{-6}=0.55, \\
x_{-5}=0.56, & x_{-4}=0.57, & x_{-3}=0.58, & x_{-2}=0.59, & x_{-1}=0.61, & x_{0}=0.6.
\end{array}$$

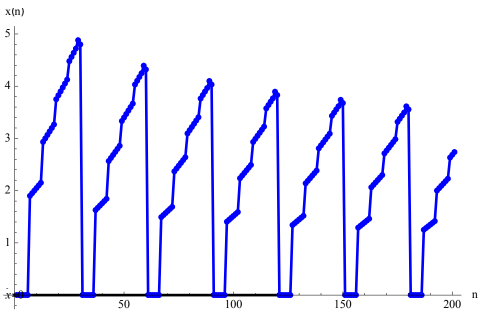

Figure 6.1: Plot illustrates the stability of Eq. 2.1

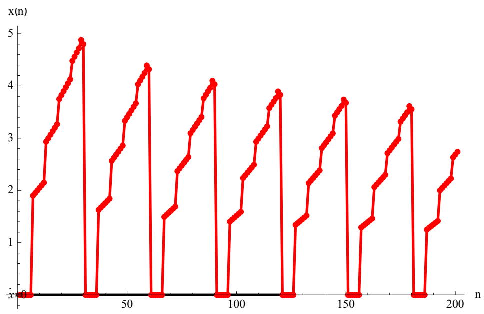

Figure 6.2: Plot illustrates the stability of Eq. 3.1

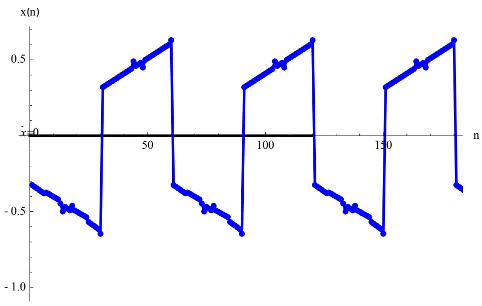

Figure 6.3: Plot illustrates the stability of Eq. 4.1

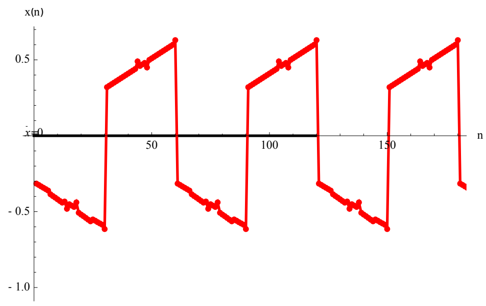

Figure 6.4: Plot illustrates the stability of Eq. 5.1

7. Conclusion

This article extensively explores the qualitative behaviors of difference equations. It effectively examines local stability, periodicity, oscillation, and solutions. Traditional iteration methods are employed to derive exact solutions for the relevant equations.

Article Information

Acknowledgements: The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Author's Contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflict of Interest Disclosure: No potential conflict of interest was declared by the authors.

Copyright Statement: Authors own the copyright of their work published in the journal and their work is published under the CC BY-NC 4.0 license.

Supporting/Supporting Organizations: No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent: It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

Plagiarism Statement: This article was scanned by the plagiarism program.

References

[1] V.L. Kocic, G. Ladas, *Global Behavior of Nonlinear Difference Equations of Higher Order with Applications*, Mathematics and Its Applications, Kluwer Academic Publishers Group, Dordrecht (1993).

[2] M.R.S Kulenovic, G. Ladas, G., W.S. Sizer, *On the recursive sequence* $\frac{\alpha x_n+\beta x_{n-1}}{\chi x_n+\beta x_{n-1}}$, Math. Sci. Res.Hot-Line, **2**(5) (1998), 1-16.

[3] R. DeVault, G. Ladas, S.W. Schultz, *On the recursive sequence* $x_{n+1}=\frac{A}{x_n}+\frac{1}{x_{n-2}}$, Proc.Amer. Math. Soc., **126**(11) (1998), 3257-3261.

[4] A.M. Amleh, G.A. Grove, G. Ladas, Georgiou, D.A. *On the recursive sequence* $x_{n+1}=\alpha+\frac{x_{n-1}}{x_n}$, J. of Math. Anal. App., *233* (1999), 790-798.

[5] C.H. Gibbons, M.R.S. Kulenovic, G. Ladas, *On the recursive sequence* $\frac{\alpha+\beta xn-1}{\chi+\beta xn-1}$, Math. Sci. Res. Hot-Line, **4**(2) (2000), 1-11.

[6] S. Elaydi, An Introduction to Difference Equations, 3rd Ed., Springer, USA, (2005).

[7] R. Karatas, C. Cinar, D. Simsek, *On positive solutions of the difference equation* $x_{n+1}=\frac{x_{n-5}}{1+x_{n-2}x_{n-5}}$, Int. J. Contemp. Math. Sci, **10**(1) (2006), 495-500 .

[8] M. Aloqeili, *Dynamics of a rational difference equation*, Appl. Math. Comput., **176**(2) (2006), 768-774.

[9] S. Stevic, *A note on periodic character of a higher order difference equation*, Rostock. Math. Kolloq., **61** (2006), 2-30.

[10] E.M. Elsayed, *On the difference equation* $x_{n+1}=\frac{x_{n-5}}{-1+x_{n-2}x_{n-5}}$, Inter. J. Contemp. Math. Sci., **3**(33) (2008), 1657-1664.

[11] R.P. Agarwal, E.M. Elsayed, *On the solution of fourth-order rational recursive sequence*, Adv. Stud. Contemp. Math. , **20**(2) (2010), 525-545.

[12] S. Stevic, B. Iricanin, Z. Smarda, *On a product-type system of difference equations of second order solvable in closed form*, J. Inequal. Appl., **2015**(1) (2012), 327-334.

[13] O. Karpenko, O. Stanzhytskyi, *The relation between the existence of bounded solutions of differential equations and the corresponding difference equations*, J. Difference Equ. Appl., **19**(12) (2013), 1967-1982. https://doi.org/10.1080/10236198.2013.794795

[14] M. Bohner, O. Karpenko, O. Stanzhytskyi, *Oscillation of solutions of second-order linear differential equations and corresponding difference equations*, J. Difference Equ. Appl., **20**(7) (2014), 1112-1126. https://doi.org/10.1080/10236198.2014.893297

[15] M.B. Almatrafi, M.M. Alzubaidi, *Analysis of the qualitative behaviour of an eighth-order fractional difference equation*, Open J. Discrete Appl. Math., **2**(1), (2019), 41-47.

[16] A. Sanbo, A. E.M. Elsayed, *Some properties of the solutions of the difference equation* $x_{n+1}=\alpha x_n+(bx_nx_{n-4})/(cx_{n-3}+dx_{n-4})$, Open J. Discrete Appl. Math., **2**(2) (2019), 31–47.

[17] A. F. Yeniçerioğlu, C. Yazıcı, & V. Yazıcı, *Stability behaviour in functional differential equations of the neutral type*, Univers. J. Math. Appl., **4**(1) (2021), 33-40. https://doi.org/10.32323/ujma.711881

[18] A.M. Ahmed, A.M. Samir, and L.S. Aljoufi, *Expressions and dynamical behavior of solutions of a class of rational difference equations of fifteenth-order*, J. Math. Computer Sci., **25** (2022), 10-22.

[19] M. Berkal, R. Abo-zeid, *On a rational (P+1)th order difference equation with quadratic term*, Univers. J. Math. Appl., **5**(4) (2022), 136-144. https://doi.org/10.32323/ujma.1198471

[20] B. Oğul, D. Şimşek, T.F. Ibrahim, *A qualitative investigation of the solution of the difference equation* $\Psi_{m+1}=\frac{\Psi_{m-3}\Psi_{m-5}}{\Psi_{m-1}(\pm1\pm\Psi_{m-3}\Psi_{m-5})}$, Commun. Adv. Math. Sci., **6**(2) (2023), 78-85.

[21] B. Oğul, D. Şimşek, H. Öğünmez, A.S. Kurbanlı, *Dynamical behavior of rational difference equation* $x_{n+1}=\frac{x_{n-17}}{\pm1\pm x_{n-2}x_{n-5}x_{n-8}x_{n-11}x_{n-14}x_{n-17}}$, Bol. Soc. Mat. Mex., **27**(2) (2021), 1-20.