International Journal of Engineering and Geosciences, 2025, 10(1), 14-21

International Journal of Engineering and Geosciences

https://dergipark.org.tr/en/pub/ijeg

e-ISSN 2548-0960

Evaluation of the high-degree global gravity field models in the territory of Kazakhstan

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Cite this study: Kosarev, N., Goldobin, D., Sermiagin, R., Kemerbayev, N. & Sholomitskii, A. (2025). Evaluation of the high-degree global gravity field models in the territory of Kazakhstan, 10 (1), 14-21.

https://doi.org/10.26833/ijeg.1485621

Keywords
High-degree global gravity field model
height anomaly
deflection of the vertical (DOV)
EIGEN-6C4

Research/Review Article

Received:17.05.2024
1.Revised: 109.07.2024
2.Revised: 13.09.2024
Accepted:17.09.2024
Published:01.02.2025

Abstract

In the territory of the Republic of Kazakhstan, the national project is currently being implemented Development of a Geoid Model of the Republic of Kazakhstan as a Basis for an Integrated State System of Coordinates and Heights, according to which the relevant model of the geoid for the territory of Kazakhstan will be formed and the gravity calibration line will be established to calibrate relative gravimeters. In this regard, a relevant scientific problem aims to select the optimal high-degree global gravity field model, which should best describe the long-wave component of the geoid model for the Republic of Kazakhstan. To select the optimal model were made the comparisons of the calculated height anomalies and the components of deflection of the vertical (DOV) obtained from global geopotential models XGM2019e_2159, SGG-UGM-2, SGG-UGM-1, EGM2008, GECO and EIGEN-6C4 with terrestrial precision measurements. In total, 59 Laplace stations and 154 high-precision levelling stations were involved in the precision analysis. Modelling of the characteristics of the Earth's gravity was performed with software developed at the physical geodesy laboratory of the Siberian State University of Geosystems and Technologies.

The study results suggest that the high-degree global geopotential models have an approximately common modeling error, which has a negative average value from –0.092 to –0.123 m. It may indicate a regular drift in the system of normal heights and the difference between the actual W0 and the normal U0 of potentials. In addition to the constant shift, there is a positive drift from west to east and north to south, which may indicate the accumulation of systematic errors in the geometric leveling method. The standard deviation of the component of deflection of the vertical in the plane of meridian $\Delta\xi$ in the regions with the altitudes less than 500 m varies from 0.93 to 1.13 arcseconds, in the regions with the altitudes greater than 500 m, from 1.26 to 1.54 arcseconds, while the standard deviation of the component of the deflection of the vertical in the plane of the first vertical $\Delta\eta$ in the regions with the altitudes less than 500 m varies from 0.70 to 0.84 arcseconds, in the regions with the altitudes greater than 500 m, from 0.62 to 0.81 arcseconds.

Based on the results given in Table 4, with the selection criteria being standard deviations, range, and mean values, the model SGG-UGM-2 was chosen the optimal high-degree global geopotential model which best describes the long-wave component of the model of the geoid for the Republic of Kazakhstan.

1. Introduction

Most of the problems of geodesy, astronomy, geodynamics, and space monitoring require high-precision measurements of the gravity and topography which are associated with determining the spatial coordinates to the degree of the order of one centimeter and maintenance of this precision for long periods of time [1-4].

To meet the modern requirements for the degree of precision in measuring the spatial coordinates and to form the geodetic grids which are the implementation of the reference system, it is necessary to use new approaches and techniques which require joint precise determination of spatial coordinates considering the gravity field and their integration into a common spatial and temporal continuum [5].

The modern space methods of determining the gravity field of the Earth, namely, satellite gradientometry (CHAMP, GRACE, and GOCE), laser-based measurement of satellites' positions (LAGEOS-1 and LAGEOS-2), satellite altimetry (Topex/Poseidon, Jason 1 and 2) brought about essential improvements in the knowledge of long-wave and medium-wave components of the gravity field. The other fundamental datasets used in developing the geopotential models are gravity measurements, including the data collected from moving platforms [6]. The main achievement of these projects consists in detailed maps of the gravity anomalies the precision of which is approximately 1000 times higher than that of the previous generation of maps.

Based on the results of the space gravimetric missions, specialists developed a series of global high-resolution geopotential models with improved precision characteristics of harmonic coefficients, such as XGM2019e_2159, SGG-UGM-2, SGG-UGM-1, EGM2008, GECO, and EIGEN-6C4, the information about which is shown in Table 1 [6].

Table 1. High-degree global gravity field models

Name	Year	Deg.	Data	Authors
SGG-UGM-2	2020	2190	A, EGM2008, S(Go), S(Gr)	Liang et al. [7]
XGM2019e_2159	2019	2190 5540	A, G, S(GOCO06s), T	Zingerle et al. [8]
SGG-UGM-1	2018	2159	EGM2008, S(Go)	Liang et al. [9, 10]
GECO	2015	2190	EGM2008, S(Go)	Gilardoni et al. [11]
EIGEN-6C4	2014	2190	A, G, S(Go), S(Gr), S(La)	Förste et al. [12]
EGM2008	2008	2190	A, G, S(Gr)	Pavlis et al. [13]

A – altimetry, S – satellite (Go – GOCE, Gr – GRACE, La – LAGEOS), G – surface, T – topo

The global gravity field models have limited value without determining their precision. Comparison of the model results with independent data is the main method of their validation. For example, the calculated models are often compared with the geopotential transformants, such as height anomalies [14-19], deflection of the vertical [20-22], gravity anomalies [23-24], and vertical gradients with their measured values [25-26].

In the territory of the Republic of Kazakhstan, the national project is currently being implemented Development of a Geoid Model of the Republic of Kazakhstan as a Basis for an Integrated State System of Coordinates and Heights, according to which the relevant model of the geoid for the territory of Kazakhstan will be formed and the gravity calibration line will be established to calibrate relative gravimeters. In this regard, a relevant scientific problem arises, aimed at selecting the optimal high-degree global gravity field model, which should best describe the long-wave component of the geoid model for the Republic of Kazakhstan.

After the first introductory part, the Section 2 describes the source data that was used by the authors to select the optimal high-degree global gravity field model, and also provides algorithms for calculating height anomalies and the components of deflection of the vertical (DOV) obtained by harmonic coefficients of high-degree global gravity field models. Section 3 presents the results of comparisons of the calculated height anomalies and the components of deflection of the vertical (DOV) obtained from global geopotential models with terrestrial precision measurements. Discussion and the rationale for choosing the optimal high-degree global geopotential model, which would best describe the long-wave component of the geoid model for the Republic of Kazakhstan, are given in Section 4. The results obtained were summarized in conclusion.

2. Methods

2.1. Source Data

The results of astronomic measurements made at 59 Laplace stations and the results of the GNSS measurements carried out at 154 high-precision levelling in the framework of an investment project of the National Spatial Data Infrastructure of the Republic of Kazakhstan served as basic information for the study[1].

Measurements at Laplace stations were carried out during the creation of the astrogeodetic network of the USSR, which was the frame of the SC-42 coordinate system. At these stations, longitudes, latitudes and azimuths were determined using astronomical measurements. Longitudes and latitudes were usually determined using the Zinger and Talcott methods, respectively, and astronomical azimuths - mainly based on observations of the Polar Star. The standard error of the direction azimuth, according to some skeptical estimates, was 0.6-1.2 arc seconds. For astronomical observations, astronomic theodolites were used, and mechanical chronometers were used to record the moments of observations. The chronometers were calibrated using precise time radio signals using the «eye-ear» method.

The GNSS measurements were made at the national geodetic control network stations, and the SC-42 reference system was implemented to ensure its connection with the new geodetic reference system named Qazaqstan Terrestrial Reference System (QTRF). A detailed description of the SC-42 reference system

[1] https://qazgeodesy.kz/o-predpriyatii/realizuemye-proekty

may be found in [27]. The QTRF is developed on the basis of the ITRF standards and of the other modern reference systems [28].

The measurements were static 6-hour GNSS observations with reference to permanent stations implementing the new reference system. Communication with the permanent stations was carried out via temporary base stations, which were continuously operated during all the observations within a range of 70 km.

Thus, measurements were made at more than 40,000 stations of the state geodetic grid. An adjustment was carried out using BERNESE software, whereas the coordinates at the geodetic and levelling networks stations were calculated using several commercial software programs for GNSS processing. A minor part of these stations had normal height determined with high-precision levelling. After comparing the high-precision levelling database with the database of GNSS measurements, 154 stations of high-precision levelling were selected.

Shown in Fig. 1 are the locations of the stations involved in the analysis of the precision of high-degree global geopotential models.

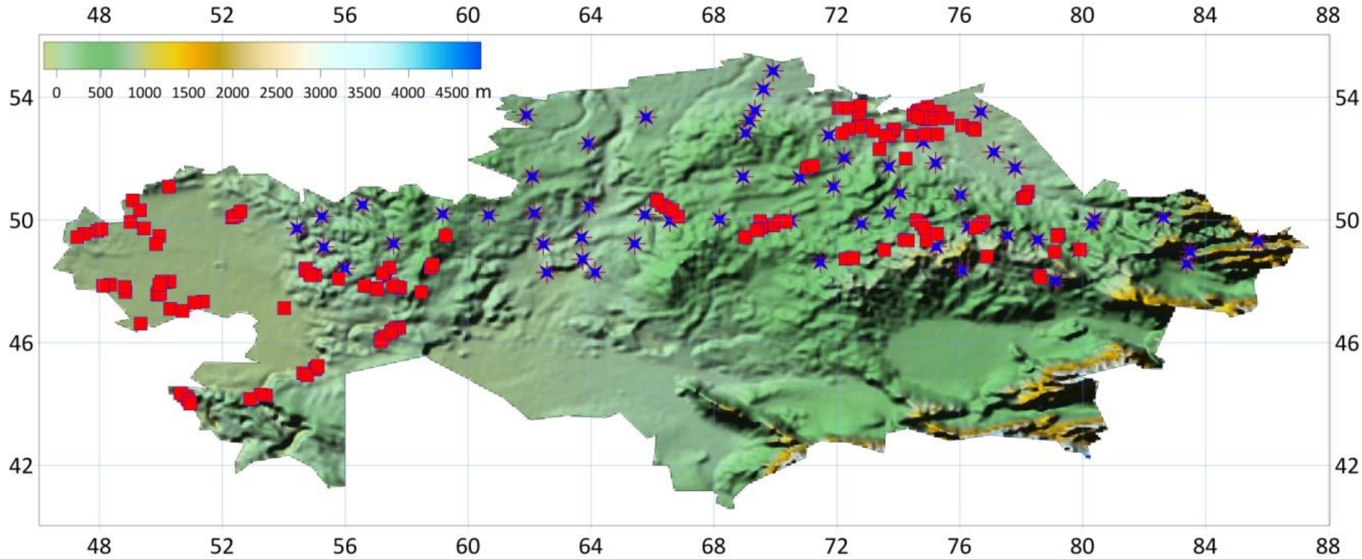


Figure 1. Locations of the stations involved in the evaluation of precision of high-degree global geopotential models (Laplace stations are in blue, and high-precision levelling stations are shown in red).

2.2. The technique of calculating height anomalies by the data of global geopotential models

To evaluate the resolution capability of the global gravity field models in question, information has been used contained in normalized harmonic coefficients $\bar{C}_{nm}$, $\bar{S}_{nm}$ of geopotential resolution into a number of spherical functions of geocentric coordinates [29-30]:

$$V(\varphi,\lambda,r)=\frac{fM}{r}\left[1+\sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n\sum_{m=0}^{n}(\bar{C}_{nm}\,cos\,m\,\lambda+\bar{S}_{nm}\,sin\,m\,\lambda)\bar{P}_{nm}(sin\,\varphi)\right], \quad (1)$$

where fM – is the geocentric gravitational constant; φ – is the geographic latitude; λ – is the geographic longitude; r – is the vector radius of the station; a_e – is the major semi-axis; $\bar{C}_{nm}$, $\bar{S}_{nm}$ – are dimensionless normalized harmonic coefficients of the geopotential model of degree n and order m; $\bar{P}_{nm}(\sin\varphi)$ are associated Legendre functions.

Series (1) represents spectral resolution of a gravity field by waves of the wavelength approximately equal to $360°/n$, corresponding to spatial resolution of about $180°/n$. In this case, indices n and m – the degree and order of the spherical function in series (1) – may be interpreted as frequencies.

Resolution into a series of spherical functions for a height of a quasi-geoid over an ellipsoid is as follows [29-30]:

$$\zeta(\varphi,\lambda,r)=\frac{fM}{r\gamma}\left[1+\sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}_{nm}\cos m\lambda+\Delta\bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\varphi)\right], \quad (2)$$

where $\Delta\bar{C}_{nm}$ – is the difference of coefficients of normalized spherical functions of the real and normal gravity fields; γ is the normal gravity value.

The absence of spherical functions $n = 0$ and $n = 1$ in formulae (1, 2) is accounted for by the choice of the geocentric reference system.

2.3. The technique of calculating the components of deflection of the vertical by the data of the global geopotential models

To calculate the components of deflection of the vertical (DOV), harmonic coefficients of the disturbing potential are determined ($\Delta\overline{C}_{n,m}$, $\Delta\overline{S}_{n,m}$), obtained by subtracting harmonic coefficients of a normal gravity field from the harmonic geopotential coefficients.

Determining the derivatives of the disturbing potential by harmonic synthesis from coefficients, one can calculate the components of deflection of the vertical in the meridian plane (ξ) and of the first vertical (η) at the station with polar spatial coordinates φ, λ, r by the following formulae:

$$\eta''=-\left[fM\sum_{n=2}^{N_0}\frac{a_e^n}{r^{n+1}}\sum_{m=0}^{n}m\left(-\Delta\overline{C}_{n,m}\sin m\lambda+\Delta\overline{S}_{n,m}\cos m\lambda\right)\overline{P}_{n,m}(\sin\phi)\right]\sec\phi\,\frac{\rho''}{N}, \quad (3)$$

$$\xi'' = -\left[fM\sum_{n=2}^{N_0}\frac{a_e^n}{r^{n+1}}\sum_{m=0}^{n}\left(\Delta\overline{C}_{n,m}\cos m\lambda + \Delta\overline{S}_{n,m}\sin m\lambda\right)\frac{d\overline{P}_{n,m}(\sin\phi)}{d\phi}\right]\frac{\rho''}{M}, \quad (4)$$

where $\rho'' = 206265$; η'' and ξ'' are components of deflection of the vertical in the first vertical and in the meridional plane, accordingly; $\Delta\overline{C}_{nm}$ and $\Delta\overline{S}_{nm}$ are normalized coefficients of the spherical functions of the gravity field; N is the curvature radius in the first vertical; M is the curvature radius in the meridian.

3. Results

Modelling of the characteristics of the Earth's gravity field according to high-degree global geopotential models was performed with software developed at the physical geodesy laboratory of Siberian State University of Geosystems and Technologies [31]. The developed software package allows you to calculate: quasi-geoid heights and Bruns' correction, free-air gravity anomalies (mixed and pure), attractive force, gravity, centrifugal acceleration, the total gravitational potential and its derivatives (first, second with respect to the radius vector, latitude and longitude), centrifugal potential and its derivatives (first, second with respect to the radius vector and latitude), attraction potential and its derivatives (first, second with respect to the radius vector, latitude and longitude), DOV (in the meridian plane, in the plane of the first vertical), abnormal vertical and horizontal gradients, radius-vector of the reference surface (complete and above the reference ellipsoid) and its derivatives (first, second latitude and longitude), radius of curvature.

The accuracy of modeling the height anomalies was evaluated by comparing the model values of the height anomalies with those obtained by terrestrial measurements at the GNSS levelling stations:

$$\Delta\zeta = (H^{geod} - H^{norm}) - \zeta^{mod}, \quad (5)$$

where H^{geod} – geodetic height obtained by GNSS measurements, H^{norm} – normal height obtained by geometric leveling, ζ^{mod} – anomaly height obtained from global geopotential models presented in Table 1.

Table 2 – Statistical characteristics of evaluating the analysis of precision height anomalies in the territory of Kazakhstan (in m)

Parameters	EGM2008	SGG-UGM-1	EIGEN-6C4	GECO	XGM2019e (2159)	SGG-UGM-2
Number of values	154	154	154	154	154	154
Min	-0.348	-0.355	-0.349	-0.387	-0.344	-0.352
Max	0.337	0.359	0.345	0.314	0.374	0.369
Range	0.686	0.714	0.693	0.700	0.718	0.721
Mean	-0.121	-0.115	-0.115	-0.123	-0.092	-0.097
Standard Deviation	0.124	0.120	0.117	0.115	0.123	0.118

In total, 154 stations were involved in the precision analysis. Table 2 represents the results of the evaluation.

Analysis of the data provided in Table 2 shows that:

– the high-degree global geopotential models in question have approximately the same modeling error, a standard deviation of about 11-12 cm, with the range of deviations of the model values of height anomalies obtained by terrestrial measurements at the GNSS-levelling stations being 68-72 cm.

– the studied high-degree global geopotential models have a negative mean difference value and are within the range from -0.092 to -0.123 m, indicating a systematic shift in the system of normal heights and the difference of potential of real W_0 and normal U_0 at the start of calculating the heights not being equal to zero [32].

The accuracy of modeling the components of deflection of the vertical was evaluated by comparing their model values with the values obtained from terrestrial measurements at Laplace stations.

To ensure the correct use of the results of terrestrial measurements made at Laplace stations, the geodetic coordinates (B, L) referred to the Krasovsky reference ellipsoid were transformed into the WGS-84 reference coordinate system [20]:

$$\Delta\xi'' = \xi_{WGS-84} + (B_{WGS-84} - B_{SC-95}) - \xi_{SC-95}, \quad (6)$$

$$\Delta\eta'' = \eta_{WGS-84} + (L_{WGS-84} - L_{SC-95}) \times \cos B_{WGS-84} - \xi_{CS-95}. \quad (7)$$

In formulae (6-7): B_{WGS-84} and L_{WGS-84} are geodetic latitude and longitude of the Laplace point in the reference coordinate system WGS-84; B_{SC-95} and L_{SC-95} are geodetic latitude and longitude of the Laplace point in the coordinate system SC-95; ξ_{WGS-84} and η_{WGS-84} are the DOV components in WGS-84 obtained from global geopotential models, presented in Table 1; ξ_{SC-95} and η_{SC-95} are the DOV components in SC-95 obtained from the terrestrial data.

Transformation of coordinates from WGS-84 to SC-95 was carried out according to standard parameters EPSG 4284[2].

In total, 59 Laplace stations were involved in the precision analysis. The control stations were divided into two groups concerning the fact that in the mountainous regions modelling of DOV is known to yield rough results: with altitudes exceeding 500 meters (H>500 m) and with altitudes less than 500 meters (H<500 m). The calculation results are provided in Tables 3 and 4.

Analysis of the data provided in Tables 3 and 4 demonstrates that:

[2] https://epsg.io/4284. Accessed 2024-05-01

– the standard deviation of the component of deflection of the vertical in the plane of meridian $\Delta\xi$ in the regions with altitudes less than 500 m varies from 0.93 to 1.13 arcseconds, and in the regions with altitudes greater than 500 m, from 1.26 to 1.54 arcseconds;

– the standard deviation of the component of deflection of the vertical in the plane of the first vertical $\Delta\eta$ in the regions with altitudes less than 500 m varies from 0.70 to 0.84 arcseconds, and in the regions with altitudes greater than 500 m, from 0.62 to 0.81 arcseconds.

Table 3 – Statistical characteristics of the analysis of precision of modeling the DOV component for the Republic of Kazakhstan (in arcseconds)

Parameters	EGM2008		EIGEN-6C4		SGG-UGM-1		XGM2019e (2159)		GECO		SGG-UGM-2	
H<500	Δξ	Δη	Δξ	Δη	Δξ	Δη	Δξ	Δη	Δξ	Δη	Δξ	Δη
Number of values	46	46	46	46	46	46	46	46	46	46	46	46
Min	-2.55	-1.79	-2.60	-1.80	-3.02	-1.73	-3.05	-1.57	-2.59	-2.08	-2.60	-1.90
Max	2.65	1.67	2.68	1.75	2.28	1.98	2.60	1.89	2.65	1.71	2.68	1.71
Range	5.20	3.46	5.28	3.55	5.30	3.72	5.65	3.46	5.23	3.80	5.28	3.62
Mean	0.33	0.16	0.34	0.13	0.32	0.07	0.34	0.07	0.35	0.12	0.34	0.13
SD	0.93	0.70	0.94	0.70	1.13	0.84	0.94	0.77	0.93	0.71	0.94	0.70

Table 4 – Statistical characteristics of the analysis of precision of modeling the DOV component for the Republic of Kazakhstan (in arcseconds)

Parameters	EGM2008		EIGEN-6C4		SGG-UGM-1		XGM2019e (2159)		GECO		SGG-UGM-2	
H>500	Δξ	Δη	Δξ	Δη	Δξ	Δη	Δξ	Δη	Δξ	Δη	Δξ	Δη
Number of values	13	13	13	13	13	13	13	13	13	13	13	13
Min	-2.99	-0.32	-3.06	-0.29	-3.33	-1.24	-2.86	-0.67	-3.29	-0.32	-2.89	-0.24
Max	2.89	2.24	2.89	1.88	2.47	1.81	2.99	1.99	2.71	2.08	2.95	1.95
Range	5.88	2.57	5.95	2.16	5.80	3.05	5.84	2.66	6.00	2.41	5.84	2.18
Mean	0.23	0.71	0.26	0.68	0.27	0.72	0.43	0.65	0.35	0.71	0.27	0.67
Standart Deviation	1.31	0.76	1.29	0.62	1.54	0.74	1.44	0.81	1.37	0.65	1.26	0.62

4. Discussion

Based on the results of the study conducted, the following conclusions may be made:

– the high-degree global geopotential models under study have an approximately common modelling error, a standard deviation of around 11-12 cm, while a range of deviations in the model values of the height anomalies from those obtained by terrain measurements at the GNSS stations is 68-72 cm;

– the high-degree global geopotential models under study have negative mean values and lie within the range from -0.092 to -0.123 m, indicating a possible systematic shift in the system of normal heights and the difference of potential of real W_0 and normal U_0 at the start of calculating the heights not being equal to zero [32].

– in addition to the constant shift, the distribution of $\Delta\zeta$ contains a positive drift from west to east and from north to south, which may indicate the accumulation of a systematic error in the geometric leveling method. This shift and drift generally matches the results of the studies in [33-34].

The distribution plot of $\Delta\zeta$ according to the data of this work and the data that was used to estimate the global GAO2012 model [33] is presented in Fig. 2. A similar drift but with a different displacement was obtained by the authors of the article [34].

The distribution trends of $\Delta\zeta$ on the territory of Kazakhstan and Russia coincide since the national leveling networks of these two countries were inherited from the common leveling network of the USSR. The standardized network of the USSR developed from its starting point in the city of Kronstadt on the Kotlin island in the Gulf of Finland of the Baltic Sea. The rate of change $\Delta\zeta$ is at least 0.1 mm/km, which, taking into account the extent of the territory of Kazakhstan from west to east, gives 25 cm, and for Russia – about 60 cm. These are approximate estimates; more detailed research is required to obtain more accurate characteristics.

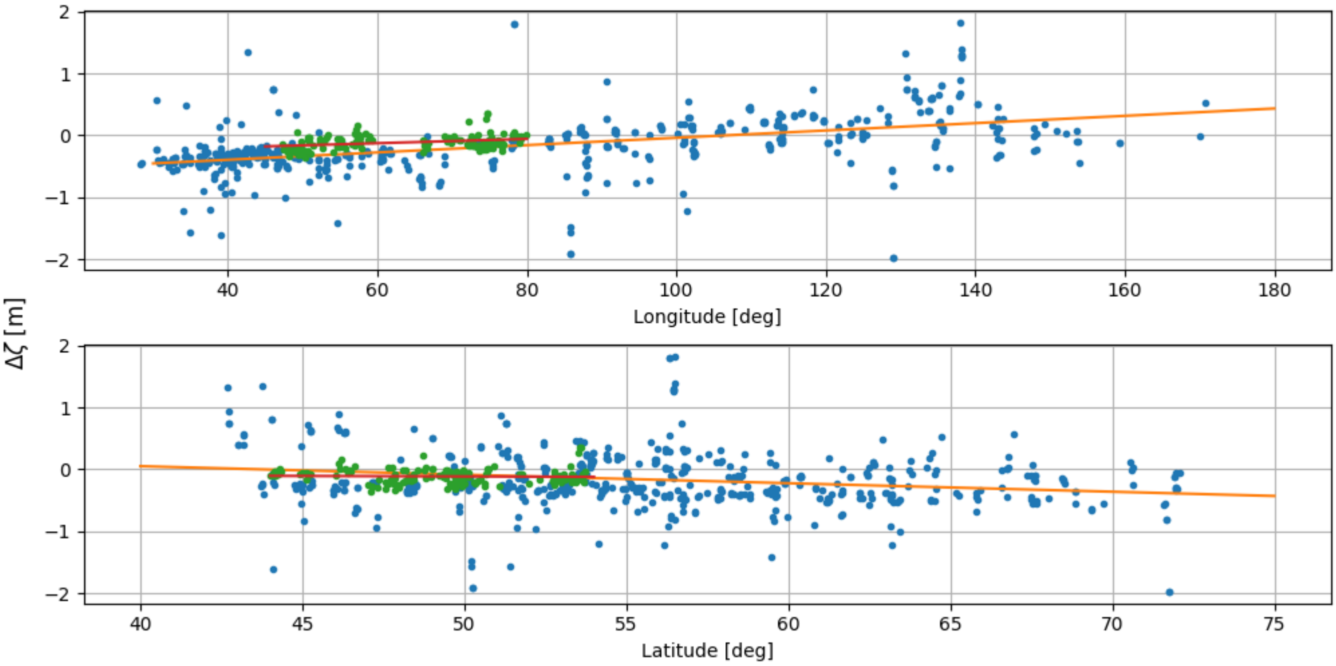
The obtained statistical characteristics of the analysis of precision in modeling the anomalies of the heights shown in Table 2 are generally in agreement with the evaluation provided in [35]. In Shoganbekova's studies, terrestrial measurements were used at the GNSS levelling stations. The accuracy of determining both geodetic and normal heights did not meet the requirements for selecting the optimal high-degree global geopotential model which would best describe the long-wave component of the model of the geoid for the Republic of Kazakhstan.

– the standard deviation of the component of deflection of the vertical in the plane of meridian $\Delta\xi$ in the regions with altitudes less than 500 m varies from 0.93 to 1.13 arcseconds, and in the regions with altitudes greater than 500 m, from 1.26 to 1.54 arcseconds;

– the standard deviation of the component of deflection of the vertical in the plane of the first vertical $\Delta\eta$ in the regions with altitudes less than 500 m varies from 0.70 to 0.84 arcseconds, and in the regions with altitudes greater than 500 m, from 0.62 to 0.81 arcseconds.

The evaluation of the accuracy of modeling the components of deflection of the vertical obtained by the authors for the territory of the Republic of Kazakhstan is in good agreement with the studies presented in [20-22] and may be used for selecting the optimal high-degree global geopotential model which would best describe the long-wave component of the model of the geoid for the Republic of Kazakhstan.

Figure 2. Distribution of $\Delta\zeta$ values by longitude and latitude: blue – the territory of Russia, green – the territory of Kazakhstan.



Following the statistical characteristics of analysis of precision of modeling height anomalies in the territory of Kazakhstan obtained at 154 stations of high-precision levelling and of the components of deflection of the vertical obtained at 59 Laplace stations, it is difficult to choose the optimal high-degree global geopotential model which best describes the long-wave component of the model of the geoid for the Republic of Kazakhstan because the standard deviations for height anomalies and components of DOV shown in Tables 2 and 3 have almost the same values. In this regard, the authors decided to determine the optimal model based on the results given in Table 4, with the selection criteria being standard deviations, range, and mean values. Based on the selected criteria, the SGG-UGM-2 is the optimal high-degree global geopotential model that best describes the long-wave component of the model of the geoid for the Republic of Kazakhstan.

5. Conclusion

In the territory of the Republic of Kazakhstan, the national project is currently being implemented Development of a Geoid Model of the Republic of Kazakhstan as a Basis for an Integrated State System of Coordinates and Heights, according to which the relevant model of the geoid for the territory of Kazakhstan will be formed and the gravity calibration line will be established to calibrate relative gravimeters. In this regard, a relevant scientific problem arises aimed at selecting the optimal high-degree global gravity field model, which should best describe the long-wave component of the geoid model for the Republic of Kazakhstan. To select the optimal model were made the comparisons of the calculated height anomalies and the components of deflection of the vertical (DOV) obtained from global geopotential models XGM2019e_2159, SGG-UGM-2, SGG-UGM-1, EGM2008, GECO and EIGEN-6C4 with terrestrial precision measurements. In total, 59 Laplace stations and 154 high-precision levelling stations were involved in the analysis of precision. Modelling of the characteristics of the Earth's gravity was performed with software developed at the physical geodesy laboratory of the Siberian State University of Geosystems and Technologies.

The study results suggest that the high-degree global geopotential models have an approximately common modelling error with a negative average value from −0.092 to −0.123 m. It may indicate a regular drift in the system of normal heights and the difference between the actual W_0 and the normal U_0 of potentials. In addition to the constant shift, there is a positive drift from west to east and north to south, which may indicate the accumulation of systematic errors in the geometric leveling method. The rate of change $\Delta\zeta$ is at least 0.1 mm/km, which, taking into account the extent of the territory of Kazakhstan from west to east, gives 25 cm.

The standard deviation of the component of deflection of the vertical in the plane of meridian $\Delta\xi$ in the regions with the altitudes less than 500 m varies from 0.93 to 1.13 arcseconds, in the regions with the altitudes greater than 500 m, from 1.26 to 1.54 arcseconds, while the standard deviation of the component of the deflection of the vertical in the plane of the first vertical $\Delta\eta$ in the regions with the altitudes less than 500 m varies from 0.70 to 0.84 arcseconds, in the regions with the altitudes greater than 500 m, from 0.62 to 0.81 arcseconds.

Based on the results given in Table 4, with the selection criteria being standard deviations, range, and mean values, the model SGG-UGM-2 was chosen as the optimal high-degree global geopotential model which best describes the long-wave component of the model of the geoid for the Republic of Kazakhstan.

This model will be used as the basic model in the framework of implementation of the national project Development of a Geoid Model for the Republic of Kazakhstan as a Basis for an Integrated State System of Coordinates and Heights, supported by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan.

Acknowledgement

We express our sincere gratitude to everyone who contributed to the realization of this research. Special thanks to the scientific team for the project «Development of the Geoid Model of the Republic of Kazakhstan as the Basis of a Unified State Coordinate and Height System» for their invaluable contribution at all stages of the work on the article.

Author contributions

Nikolai Kosarev: Conceptualization, Methodology, Data curation, Writing-Original draft preparation. **Denis Goldobin:** Software, Field study, Validation. **Roman Sermiagin:** Visualization, Investigation, Writing-Reviewing and Editing. **Nurgan Kemerbayev:** Investigation, Writing-Reviewing and Editing. **Andrei Sholomitskii:** Writing-Reviewing and Editing

Funding

The research is funded by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. BR21882366)

Conflicts of interest

The authors declare no conflicts of interest.

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