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Research Article

Decision-making using the correlation coefficient measures of intuitionistic fuzzy rough graph

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Abstract

The hybrid approach produced by combining mathematical representations of intuitionistic fuzzy sets and rough sets is called an intuitionistic fuzzy rough framework. This novel approach addresses vagueness and soft computation by using the lower and upper approximation spaces. The degree of connection between intuitionistic fuzzy rough preference relations is assessed in this study using the correlation coefficient method. An improved comprehension of the link between fuzzy elements is made possible by the superior features of the suggested correlation coefficient measure over the current one. An intuitionistic fuzzy rough environment in which attribute decision-making is based on integrated with the correlation coefficient measure. Additionally, a novel method for determining expert weights based on intuitionistic fuzzy rough preference relations uncertainty and the degree of each intuitionistic fuzzy rough preference relations's correlation coefficient is proposed in the paper. The correlation coefficient measurements between each option and the optimal choice are used in the study to calculate the ranking order of the alternatives. Finally, we introduce a cooperative decision-making method in a cotton seed; this concept may be developed in several advantageous cotton seedlings.

Mathematics Subject Classification (2020). 05C72, 15B15

Keywords. Intuitionistic fuzzy rough graph, correlation coefficient, energy, intuitionistic fuzzy rough preference relation, decision-making problem

1. Introduction

Pawlak [28] initially presented rough set theory (RST) as a formal mathematical tool for resolving ambiguity and inconsistency in information systems. The rough set method is advantageous as it doesn't require additional data, unlike fuzzy set or probability theory, and its foundation is the inability to discriminate objects with identical descriptions [29]. RST categorizes knowledge into various domains, including decision analysis, expert systems, machine learning, pattern recognition, and knowledge discovery. However, it encounters challenges when dealing with inconsistencies in domains that involve preferences and orders. The idea of linear Diophantine fuzzy rough sets was then introduced by [7], who also provided its application to decision-making (DM) problems. The combined notion of rough sets (RSs) and fuzzy sets (FSs) theory was explored by several scholars; one

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such idea is fuzzy rough sets (FRSs), which were first proposed by [17]. Roughness in the quantale module and the generalized rough fuzzy ideals of quantale were first proposed by [31]. Al Shumrani et al. [37] present three novel topologies that provide more accurate information and issue representations: covering-based rough fuzzy, covering-based rough intuitionistic fuzzy, and covering-based rough neutrosophic nanotopology. The generalization for FRSs is the concept of intuitionistic fuzzy rough sets (IFRSs), which was put out by [15]. The IFR concept was applied by [14] to identify breast cancer. Through lower and upper approximations, the IFRSs address imprecision and ambiguity in real-world data by offering a granular representation of idea boundaries. The IFRSs model based on operators was examined by [47]. Haq et al. [18] tackle semantic issues with incomplete information by classifying types, introducing a complete system, proposing a fuzzy decision table, and developing a rule extraction method. Singh and Som [38] discuss IFSs and RST used to study their combination for real-world applications in artificial intelligence. Wang and Zhang [44] explore IFRSs, intuitionistic fuzzy β-covering approximation spaces, and intuitionistic fuzzy covering rough set models, proposing a new method for multiple criteria group decision making (MCGDM). Nazeer et al. [26] delves into the properties of intuitionistic fuzzy incidence graphs (IFIGs), including their applications in the textile industry and discusses methods for computing IFIG degree. Bashir et al. [10] explore the topological properties of IFRSs, exploring various topologies and the T-similarity class, finding interesting lattices for real-life problem modeling in intuitionistic fuzzy logic. Mahmood et al. [21] present innovative techniques for precise disease diagnosis using IFRSs, including confidence-level operators, an algorithm, and a medical diagnosis, demonstrating their effectiveness through comparative analysis. Mazarbhuiya and Shenify [24] introduces a hybrid approach that combines RST and IFS theory for anomaly detection in computer networks and databases, achieving high true positive rates. Ali et al. [5] present an enhanced version of the Decision-Theoretic Rough Set model, GI-DTRS, which combines Bayesian theory principles with intuitionistic fuzzy sets, demonstrating its practical efficacy through experimentation and comparative analysis.

Navigating decisions in competitive settings becomes intricate due to the socioeconomic context. The complexity is further heightened by uncertainties and a dearth of factual knowledge, making it challenging to make informed choices. In this scenario, the incorporation of MCGDM becomes imperative for the assessment of conflicting criteria. Recognizing the need for group decision-making models is crucial to addressing the intricate nature of decision-making in such environments. Zadeh [50] introduced fuzzy sets (FSs) in 1965 to address classical set theory deficits, with applications in clustering, data collection, medical diagnostics, artificial intelligence, and medicine. Atanassov [6] studied an IFSs, a hybrid structure defined by membership grade (MG) and non-membership grade (Non-MG) grade functions, gaining academic attention for its unique value interval. A notion of intuitionistic fuzzy-weighted averaging (IFWA) aggregation operators was first studied by [46]. Xu and Yager [48] introduce the idea of intuitionistic fuzzy - weighted geometric (IFWG) operators. The score and accuracy function ranking graphical method was created by [4]. He et al. [19] explored geometric interaction averaging operators and intuitionistic fuzzy neutral averaging operators, demonstrating their application in decision-making. Furthermore, Xiao [45] presented an IFS - based distance measure and used it to solve the pattern classification issue. Furthermore, academics have constructed several additional theories, such as aggregation operators (AOs) and similarity measures, based on IFSs. Some IFS-based Dombi aggregation operators were suggested by [36]. In order to determine COVID-19 trade deficits in developing nations, this research [1] investigates the application of directed rough fuzzy networks and makes a comparison between the findings and current approaches.

Computer science, social networks, optimization, and other fields benefit from the study of graph features, pairwise connections, and mathematical structures that are explored in graph theory. Initially, Kaufmann created fuzzy graphs. Later, Rosenfeld explored fuzzy graphs (FGs) and discovered analogies for several graph-theoretical issues. After that, Bhattacharya [11] offered some ideas on fuzzy graphs. Malik and Akram [23] investigate the use of IFR models in graphs, presenting construction methods and developing an efficient decision-making algorithm. Zhan et al. [51] introduce intuitionistic fuzzy rough graphs (IFRGs), combining FSs and RSs for flexible, expressive modeling in information systems, presenting applications in decision-making problems, and developing efficient algorithms. Yang and Mao [49] introduces intuitive fuzzy threshold graphs, alternating 4-cycles, and effectively control water and power resources by managing uncertainty. Tiwari et al. [41] presents a novel idea for intuitionistic fuzzy (IF)-aided mutual information, which improves the prediction of phospholipidosis-positive molecules by efficiently managing noise, uncertainty, and ambiguity in real-valued datasets. Mishra et al. [25] offers a framework for decision-making when assessing environmentally friendly wastewater treatment methods, highlighting the most efficient sustainable elements as sludge generation, odor impacts, maintenance, and operation. Mahmood et al. [22] introduce an EDAS technique for robotics data handling, utilizing intuitionistic fuzzy rough numbers and new aggregation operators, and showcases its effectiveness through an algorithm and comparative analysis. The traffic control system using the neutrosophic sets, rough sets, graph theory, fuzzy sets and its extension by [34]. By integrating kernelized intuitionistic fuzzy C-means with an intuitionistic fuzzy rough set model, the study [40] presents a unique method for reducing high-dimensional data and improving the prediction of animal toxic peptides.

This concept has several implications in computer science, physics, chemistry, and other mathematical domains. According to [9], the energy of a simple graph G is the total of the graph's eigenvalues' true values. The adjacency matrix of the fuzzy graph is defined, and the two boundaries of its energy are found. The idea of an intuitionistic fuzzy graph's energy was initially put out by [30] in 2014. The notion of fuzzy graph energy was expanded to encompass the energy of an intuitionistic fuzzy graph, and a lower and upper bound for its energy was established. Reddy and Basha [33] research implement a correlation coefficient measure (CCM) to assess the strength of the association between hesitancy fuzzy graphs (HFGs). Akula and Shaik [3] IFG are used in DM, offering a novel approach to computing comparative position loads and a cooperative way of DM for schemes including money investments. Chinram et al. [13] introduce an intuitionistic fuzzy rough-EDA method for MCFDM, introducing new score and accuracy functions and presenting a numerical example. Tripati et al. [42] In order to rank and evaluate medical remedies utilizing intuitionistic fuzzy sets with multi-criteria issues, the paper presents the IF-CoCoSo technique. In order to get around current problems and offer a practical solution, the study [16] presents a unique multi-attribute decision-making (MADM) technique for intuitionistic fuzzy numbers that makes use of advanced possibility degree measure (APDM). Akram and Zahid [2], with the use of TOPSIS, Analytic Hierarchy Process, and Pythagorean fuzzy rough numbers, the study presents a novel approach to effectively assess design concepts and support intricate decision-making procedures. Rao et al. [32] explores the definitions, properties, and applications of Cayley fuzzy graphs (CFGs) and pseudo-Cayley fuzzy graphs (PCFGs) in computer science and semigroup theory. Noorjahan and Shariefbasha [27] utilizes intuitionistic fuzzy rough models to handle complex uncertainty in graph-based models, integrates attribute decision-making, calculates Laplacian energy, and introduces a new ranking approach. Bozanic et al. [12] proposes a sensitivity analysis and a multi-criteria decision-making model for rating Lean organization systems management methodologies. It uses the DIBR II and MABAC procedures, engaging four experts. Sivaprakasam and Angamuthu [39] With an emphasis on decision analysis, this work presents a unique MAGDM model utilizing rough matrices based on generalized Z-fuzzy soft-covering. The best applicant for an associate professor position is found using an algorithm that is based on the AHP approach; a numerical example is given. Bajaj and Kumar [8] suggests a novel way for determining correlation coefficients in Intuitionistic Fuzzy Sets, proving its usefulness in pattern recognition and medical diagnosis, as well as its superiority over current techniques. Jahanshaloo [20] introduces a new method called TOPSIS, which uses triangular fuzzy numbers for rating and weighting complex and fuzzy data, demonstrating its effectiveness in numerical experiments. The bipolar fuzzy extended TOPSIS method addresses multi-criteria decision-making problems using bipolar measurements, addressing interactions between criteria and applications in medical treatments and food webs [35]. Wang et al. [43] introduces an enhanced TOPSIS model for q-rung orthopair hesitant fuzzy sets, enhancing accuracy in expressing fuzzy and ambiguous information through improved distance and similarity measures. Zulqarnain et al. [52] examines current approaches and presents interval-valued intuitionistic fuzzy soft sets for decision-making using prioritizing strategies, weighted correlation coefficients, and weighted average operators.

Adopting a single type of uncertainty method to handle such challenges is very difficult because human competence is limited in complex situations. As a result, the requirement to develop hybrid models for managing uncertainty emerges, combining the best aspects of many different mathematical models. The intuitionistic fuzzy rough model is more flexible and practical than previous models. Nevertheless, to the best of our knowledge, no literature-based study has examined the correlation coefficient in an IFRG situation. In the present study, we propose a method for resolving group decision-making problems in which the alternatives are solely determined by the IFRG and the weights of the criteria are unknown. We determine the relative weights for each decision matrix using the energy measure in order to handle confusing information demands. In order to satisfy the overall weight vector criterion, we aggregate all of the received energy weights. We determine which IFRG alternatives are ideal by computing the correlation degree for each ranking of the alternatives after evaluating them using the correlation coefficient metric.

1.1. Motivation for the research

The creation of several mathematical frameworks has been prompted by the need to manage ambiguity and uncertainty in decision-making processes. Although valuable, current approaches might not fully tackle the intricacies present in practical issues where intuitionistic fuzzy and rough set theory might be employed. Specifically, the intuitionistic fuzzy rough framework was developed to improve these models' accuracy and resilience in situations when decision-making is made based on imprecise, ambiguous, or insufficient data.

1.2. Novelties of the work

- The study addresses the shortcomings of conventional approaches in handling ambiguity by introducing a hybrid technique that combines the advantages of intuitionistic fuzzy sets and rough sets.
- We offer an improved measure of the correlation coefficient that works better than current techniques for determining the degree of association between intuitionistic fuzzy rough preference relations (IFRPRs).
- A novel technique for calculating expert weights based on correlation coefficients and IFRPR uncertainty is presented in the study, providing a more precise means of incorporating expert opinions into decision-making.
- The suggested framework's usefulness in agricultural contexts is demonstrated by applying it to the decision-making process for cotton seed.

1.3. Primary goals for this article are to make the following contributions

- To use the recently suggested correlation coefficient metric to give a greater understanding of the connections between fuzzy elements.
- To combine the correlation coefficient measure and the intuitionistic fuzzy rough framework to provide a more accurate and reliable decision-making tool.
- In order to increase the precision and dependability of multi-expert decision-making processes, a unique technique for calculating expert weights is presented.
- Applying the suggested techniques to cotton seed selection decision-making will serve as an example of their usefulness.

1.4. Structure of the paper

The remaining sections of the article follow this structure: Section 2 introduces the core concepts of IFRG, covering covariance and correlation coefficient measurements. Section 3 illustrates group decision-making through IFRG's developed approach involving energy and correlation coefficients. The relevant application and comparative analysis can be found in Section 4. Finally, Section 5 provides the conclusion of the article.

2. Preliminaries

Definition 2.1. [23] An intuitionistic fuzzy rough graph is defined as $G = (C, CE, D, DF)$ where C is intuitionistic fuzzy relation to F, $CE = (C\check{}E,\ C\hat{}E)$ represents an intuitionistic fuzzy rough set to F, D is intuitionistic fuzzy relation to $H \subseteq F$ x F and $DF = (D\check{}F,\ D\hat{}F)$ represents an intuitionistic fuzzy rough relation to F.

Thus $G = (G\check{},\ G\hat{}) = (CE, DF)$ is an intuitionistic fuzzy rough graph, $G\check{} = (C\check{}E,\ C\hat{}E)$ and $G\hat{} = (D\check{}F,\ D\hat{}F)$ are lower and upper approximations of intuitionistic fuzzy rough graph G. $\forall\ x, y \in F$.

$$(D\check{}F)^{+}(xy) \ \le\ min\ \left\{(C\check{}E)^{+}(x),\ (C\check{}E)^{+}(y)\right\},$$

$$(D\check{}F)^{-}(xy) \ \le\ max\ \left\{(C\check{}E)^{-}(x),\ (C\check{}E)^{-}(y)\right\},$$

$$(D\hat{}F)^{+}(xy) \ \le\ min\ \left\{(C\hat{}E)^{+}(x),\ (C\hat{}E)^{+}(y)\right\},$$

$$(D\hat{}F)^{-}(xy) \ \le\ max\ \left\{(C\hat{}E)^{-}(x),\ (C\hat{}E)^{-}(y)\right\}.$$

Definition 2.2. The following defines the energy of two IFRGs i.e $IFRG_1$ and $IFRG_2$

$$E\ (IFRG_1) = \sum_{i=1}^{n}\left[\mu^2_{IFRG_1}(x_i) + \nu^2_{IFRG_1}(x_i)\right] = \sum_{i=1}^{n}\lambda_i^2(IFRG_1)$$

$$E\ (IFRG_2) = \sum_{i=1}^{n}\left[\mu^2_{IFRG_2}(x_i) + \nu^2_{IFRG_2}(x_i)\right] = \sum_{i=1}^{n}\beta_i^2(IFRG_2)$$

The covariance of $IFRG_1$ and $IFRG_2$ is defined as follows:

$$Cov\ (IFRG_1,\ IFRG_2) = \sum_{i=1}^{n}\left[\left(\mu_{IFRG_1}(x_i)\,\mu_{IFRG_2}(x_i)\right)\ +\ \left(\nu_{IFRG_1}(x_i)\,\nu_{IFRG_2}(x_i)\right)\right]$$

Therefore, the correlation coefficient measure of IFRGs $IFRG_1$ and $IFRG_2$ are given by the equation

$$CC\ (IFRG_1, IFRG_2) = \frac{Cov\ (IFRG_1,\ IFRG_2)}{\sqrt{E(IFRG_1)}\ \sqrt{E(IFRG_2)}}$$

$$= \frac{\sum_{i=1}^{n}\left[\left(\mu_{IFRG_1}(x_i)\,\mu_{IFRG_2}(x_i)\right) + \left(\nu_{IFRG_1}(x_i)\,\nu_{IFRG_2}(x_i)\right)\right]}{\sqrt{\sum_{i=1}^{n}\left[\mu^2_{IFRG_1}(x_i)+\nu^2_{IFRG_1}(x_i)\right]}\sqrt{\sum_{i=1}^{n}\left[\mu^2_{IFRG_2}(x_i)+\nu^2_{IFRG_2}(x_i)\right]}}$$

Definition 2.3. Xu et al.[47] proposed an alternative formula for the KPCCMS of an intuitionistic fuzzy graph IFG_1 *and* IFG_2, so the same form can be converted on an intuitionistic fuzzy rough graph $IFRG_1$ *and* $IFRG_2$ as defined as follows:

$$CC\ (IFRG_1, IFRG_2) = \frac{Cov\ (IFRG_1,\ IFRG_2)}{max\left\{\sqrt{E(IFRG_1)}\sqrt{E(IFRG_2)}\right\}}$$

$$= \frac{\sum_{i=1}^{n}\left[\left(\mu_{IFRG_1}(x_i)\,\mu_{IFRG_2}(x_i)\right) + \left(\nu_{IFRG_1}(x_i)\,\nu_{IFRG_2}(x_i)\right)\right]}{max\left\{\sqrt{\sum_{i=1}^{n}\left[\mu^2_{IFRG_1}(x_i)+\nu^2_{IFRG_1}(x_i)\right]}\sqrt{\sum_{i=1}^{n}\left[\mu^2_{IFRG_2}(x_i)+\nu^2_{IFRG_2}(x_i)\right]}\right\}}$$

The correlation coefficient function $CC\ (IFRG_1, IFRG_2)$ satisfies the following conditions.

(i) $0 \leq CC\ (IFRG_1, IFRG_2) \leq 1$.

(ii) $CC\ (IFRG_1, IFRG_2) = CC\ (IFRG_2, IFRG_1)$.

(iii) $CC\ (IFRG_1, IFRG_2) = 1$, if $IFRG_1 = IFRG_2$.

Definition 2.4. An intuitionistic fuzzy rough adjacency matrix (IFRAM) is well defined for an IFRG by $A\,(G_i) = [a_{ij}]$, where $a_{ij} = (\mu_{ij}, \nu_{ij})$. It is worth nothing that μ_{ij} donates the strength of the membership between μ_i and μ_j and ν_{ij} denotes the strength of the non-membership among both ν_i and ν_j.

If IFRAM can be represented by two matrices, one carrying membership values as well as the other carrying non-membership values. So that we represent this matrix as $A\,(G_i) = [A_\mu(G_i),\ A_\nu(G_i)\ ]$, where $A_\mu(G_i)$ is the intuitionistic fuzzy rough membership matrix and $A_\nu(G_i)$ is the intuitionistic fuzzy rough non-membership matrix.

Definition 2.5. Let an IFRG, then the energy of IFRG is denoted as $E\ (IFRG)$ and is defined as $E\ (IFRG) = (E\ (\mu(IFRG), E\ (\nu(IFRG)) = \left(\sum_{i=1}^{n}|\alpha|\,,\ \sum_{i=1}^{n}|\beta|\right)$, where $(E\ (\mu(IFRG)), E\ (\nu(IFRG))$ represent the energy of intuitionistic fuzzy rough membership, non-membership matrix.

3. Group decision-making based on intuitionistic fuzzy rough graphs energy and correlation coefficient

3.1. Algorithm

With an emphasis on intuitionistic fuzzy rough preference relations, a functional process for group decision-making valid situations is being developed.

Consider $w = \{w_1,\ w_2,\ w_3,\ \cdots,\ w_n\}$ is a subjective scoring vector of experts for the decision-making issues based on intuitionistic fuzzy rough preference relations, where $w_m > 0$, $m = 1, 2, 3, \cdots, l$ and the total of all the scoring values of the experts is equal to one is written as $\sum_{i=1}^{l} w_i = 1$.

Step (1). Calculate the energy of an adjacency matrix's $E\left(M^{(k)}\right)$ using the following equation

$$E\left(M^{(k)}\right) = \left|\sum_{i=1}^{n} k_i\right|. \tag{3.1}$$

Step (2). Compute the weight w_k^1, determined by energy of an adjacency matrix, of the expert e_k using the equation

$$w_k^1 = \left((w_\mu)_i,\ (w_\nu)_i\right) = \left[\frac{E\left((D_\mu)_i\right)}{\sum\limits_{r=1}^{l} E\left((D_\mu)_r\right)},\ \frac{E\left((D_\nu)_i\right)}{\sum\limits_{r=1}^{l} E\left((D_\nu)_r\right)}\right]. \tag{3.2}$$

Step (3). Estimate Karl Pearsons correlation coefficient measure $CC\left(M^{(k)},\ M^{(d)}\right)$ between $M^{(k)}$ and $M^{(d)}$ for every $k \neq d$, using the equation

$$CC\left(M^{(k)},\ M^{(d)}\right) = \sum_{i=1}^{n} \frac{\left[\mu_{M^{(k)}}(t_i)\mu_{M^{(d)}}(t_i) + \nu_{M^{(k)}}(t_i)\ \nu_{M^{(d)}}(t_i)\right]}{\sqrt{\mu^2_{M^{(k)}}(t_i) + \nu^2_{M^{(k)}}(t_i)}\ \sqrt{\mu^2_{M^{(d)}}(t_i) + \nu^2_{M^{(d)}}(t_i)}}. \tag{3.3}$$

The average correlation coefficient degree $CC\left(M^{(k)}\right)$ of $M^{(k)}$ to the others is calculated by

$$CC\left(M^{(k)}\right) = \sum_{i=1,\ k\neq d}^{n} \left(\frac{1}{m-1}\left[CC\left(M^{(k)},\ M^{(d)}\right)\right]\right), k = 1, 2, 3, \cdots, l. \tag{3.4}$$

Step (4). Calculate the weight scores w_b^a, determined by $CC\left(M^{(k)}\right)$ of the expert e_k using the following formula

$$w_k^a = \frac{CC\left(M^{(k)}\right)}{\sum\limits_{i=1}^{l} CC\left(M^{(i)}\right)}, k = 1, 2, 3, \cdots, l. \tag{3.5}$$

Step (5). Compute the objective scores w_k^2 of the expert e_k using the following equation

$$w_k^2 = \eta w_k^1 + (1 - \eta) w_k^a, \forall\ \eta \in [0,\ 1], k = 1, 2, 3, \cdots, l. \tag{3.6}$$

Step (6). Include the subjective score w_k^1 and objective score w_k^2 into the weight w_k of the expert e_k is

$$w_k = \gamma w_k^1 + (1 - \gamma) w_k^2, \forall\ \gamma \in [0,\ 1],\ k = 1, 2, 3, \cdots, l. \tag{3.7}$$

Step (7). Measure the average intuitionistic fuzzy rough values (IFRVs) $r_i^{(k)}$ of replacements t_i to another replacement is

$$r_i^{(k)} = \frac{1}{n}\sum_{j=1}^{n} r_{ij}^{(k)}, i = 1, 2, 3, \cdots, n. \tag{3.8}$$

Step (8). Compute the values of $r_i^{(k)}$ $(i = 1, 2, 3, \cdots, n,\ k = 1, 2, 3, \cdots, l)$ equivalent to m experts into a collection of intuitionistic fuzzy rough values of the replacements t_i to another replacement is

$$r_i^{(k)} = \sum_{b=1}^{l} w_k r_{ij}^{(k)}. \tag{3.9}$$

Step (9). Calculate the rank function from the equation

$$CC\ (r_i)\ =\ \mu_i - \nu_i. \tag{3.10}$$

of r_i if the better value of $CC\ (r_i)$ is the better alternate r_i, then the alternates must be ranked in groups where the greater value of the replacement is used to generate the score functions, after which a ranking order is created. The aforementioned procedure weighs the opinions of experts based on both subjective and objective data. The decision-maker establishes the value of γ based on the factual and subjective weight information. The IFRPRs are then integrated to generate a collective IFRPR. The algorithm is existing for both of intuitionistic fuzzy rough graph for lower and upper approximation.

3.2. Flow chart

The Figure 1 illustrates how the technique would work to get the alternate rankings.

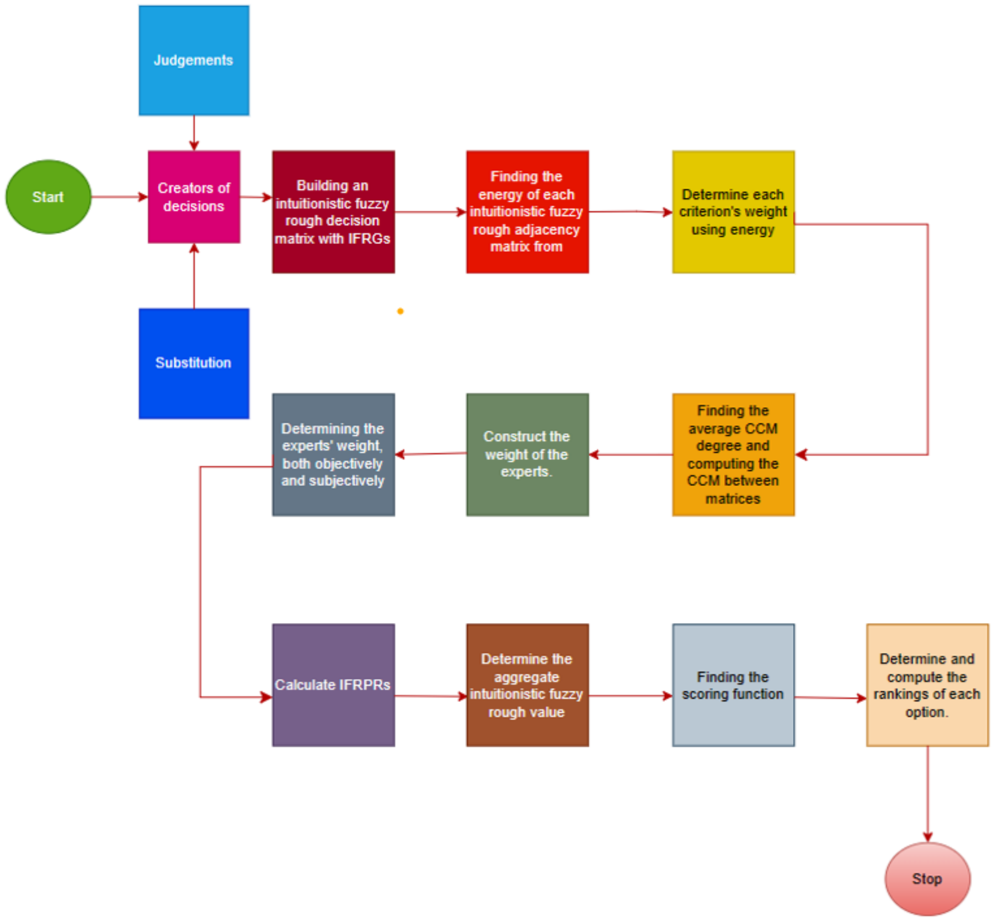

Figure 1. The measure of the correlation coefficient between intuitionistic fuzzy rough graph

4. Application: Finest selection of best seeds for cotton crop

Group decision-making is a collaborative process involving individuals collaborating to reach a consensus or make collective choices. It involves identifying shared goals, collecting information, and generating potential alternatives. However, challenges like conflicts and power dynamics can arise. Successful group decision-making requires effective communication, collaboration, and a structured process using techniques like brainstorming and consensus-building. Cotton cultivation is a global agricultural practice that produces fibres for the textile industry. Major cotton-producing countries include China, India, the USA, Pakistan, and Brazil. The crop requires well-drained soil and 180-200 days of growth. Cotton fibres are essential for clothing, linens, and textiles. However, challenges like pests and water usage necessitate pest management strategies. Sustainable practices, technological advancements, and market dynamics continue to shape cotton cultivation's landscape.

The selection of cotton seeds is a critical aspect of cotton cultivation, influencing the overall success of the crop. Farmers engage in a meticulous process to choose seeds based on factors such as yield, disease resistance, and adaptability to local growing conditions. Modern cotton farming often involves the use of hybrid and genetically modified (GM) cotton seeds, designed to enhance traits like pest resistance and improved fibre quality. Farmers consider the specific requirements of their region, including climate, soil type, and water availability, when selecting seeds. Additionally, the choice between conventional and genetically modified varieties is a decision that farmers weigh carefully, taking into account factors like pest management practices and market preferences. The goal of seed selection is to optimise yield and fibre. Quality and overall plant health, contributing to the economic viability of cotton cultivation. As sustainable agriculture gains importance, there is a growing emphasis on selecting seeds that align with environmentally friendly practices, promoting both productivity and ecological balance in cotton farming.

A cotton farmer in a specific region is planning the upcoming planting season. The farmer has access to several cotton seed varieties p_1, p_2, p_3, p_4 and needs to decide which seeds to plant. He can only pick one based on four criteria such as climate and soil condition (e_1), pest and disease resistance (e_2), fiber quality and cost (e_3), and maturity period (e_4). The decision-making process involves considering various factors to ensure a successful and productive cotton crop. Due to his inadequate expertise, he wanted to seek advice from experts who could offer the finest seed strategy. As a result, the experts will apply IFRGs to express their preference ratings in order to find the original ranking information, which is provided in the intuitionistic fuzzy rough decision matrices.

For the lower approximation of IFRG

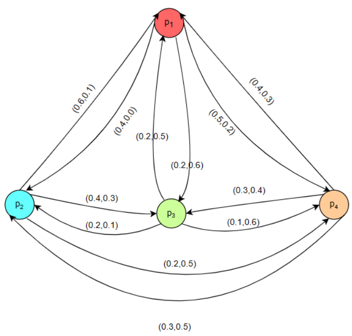

Figure 2. IFRLM ($G^{\hat{}}{}_1$) related to climate and soil condition

From Figure 2, the IFRLAM is defined as

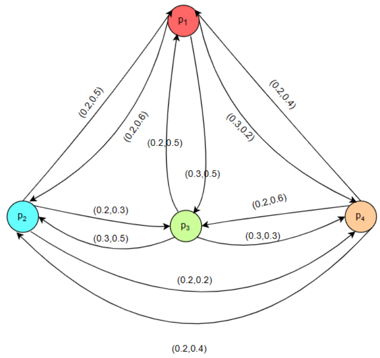

Figure 3. IFRLM ($G_{\hat{}2}$) related to pest and disease resistance

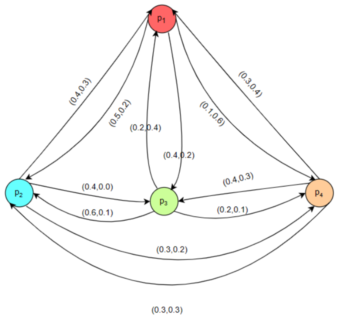

Figure 4. IFRLM ($G_{\hat{}3}$) related to fiber quality and cost

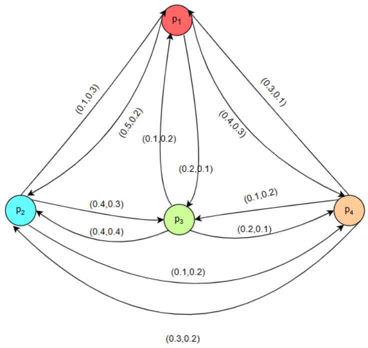

Figure 5. IFRLM ($G_{\hat{}4}$) related to maturity period

$$B^{(1)}_{\hat{}} = \begin{bmatrix} (0.0, 0.0) & (0.4, 0.0) & (0.2, 0.6) & (0.5, 0.2) \\ (0.6, 0.1) & (0.0, 0.0) & (0.4, 0.3) & (0.2, 0.5) \\ (0.2, 0.5) & (0.2, 0.1) & (0.0, 0.0) & (0.1, 0.6) \\ (0.4, 0.3) & (0.3, 0.5) & (0.3, 0.4) & (0.0, 0.0) \end{bmatrix}$$

From Figure 3, the IFRLAM is defined as

$$B^{(2)}_{\hat{}} = \begin{bmatrix} (0.0, 0.0) & (0.2, 0.6) & (0.3, 0.5) & (0.3, 0.2) \\ (0.2, 0.5) & (0.0, 0.0) & (0.2, 0.3) & (0.2, 0.2) \\ (0.2, 0.5) & (0.3, 0.5) & (0.0, 0.0) & (0.3, 0.3) \\ (0.2, 0.4) & (0.2, 0.4) & (0.2, 0.6) & (0.0, 0.0) \end{bmatrix}$$

From Figure 4, the IFRLAM is defined as

$$B_{\hat{}}^{(3)} = \begin{bmatrix} (0.0,0.0) & (0.5,0.2) & (0.4,0.2) & (0.1,0.6) \\ (0.4,0.3) & (0.0,0.0) & (0.4,0.0) & (0.3,0.2) \\ (0.2,0.4) & (0.6,0.1) & (0.0,0.0) & (0.2,0.1) \\ (0.3,0.4) & (0.3,0.3) & (0.4,0.3) & (0.0,0.0) \end{bmatrix}$$

From Figure 5, the IFRLAM is defined as

$$B_{\hat{}}^{(4)} = \begin{bmatrix} (0.0,0.0) & (0.5,0.2) & (0.2,0.1) & (0.3,0.4) \\ (0.1,0.3) & (0.0,0.0) & (0.4,0.3) & (0.1,0.2) \\ (0.1,0.2) & (0.4,0.4) & (0.0,0.0) & (0.2,0.1) \\ (0.4,0.3) & (0.3,0.2) & (0.1,0.2) & (0.0,0.0) \end{bmatrix}$$

For the upper approximation of IFRG

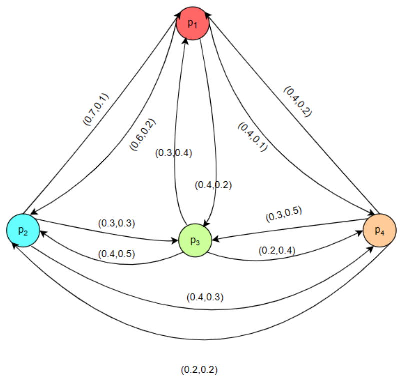

Figure 6. IFRUM ($G_1\hat{}$) related to climate and soil condition

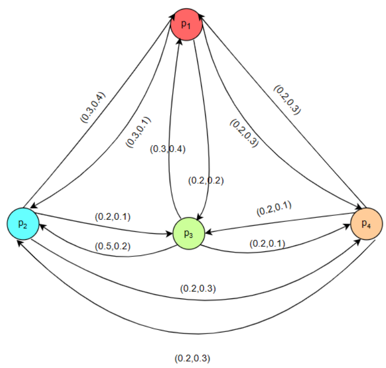

Figure 7. IFRUM ($G_2\hat{}$) related to pest and disease resistance

From Figure 6, the IFRUAM is defined as

$$B^{\hat{}(1)} = \begin{bmatrix} (0.0,0.0) & (0.6,0.2) & (0.4,0.2) & (0.4,0.1) \\ (0.7,0.1) & (0.0,0.0) & (0.3,0.3) & (0.4,0.3) \\ (0.3,0.4) & (0.4,0.5) & (0.0,0.0) & (0.2,0.4) \\ (0.4,0.2) & (0.2,0.2) & (0.3,0.5) & (0.0,0.0) \end{bmatrix}$$

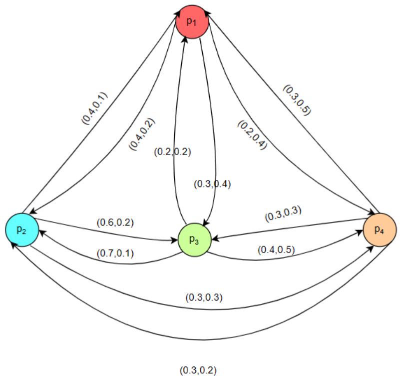

Figure 8. IFRUM (G_3^) related to fiber quality and cost

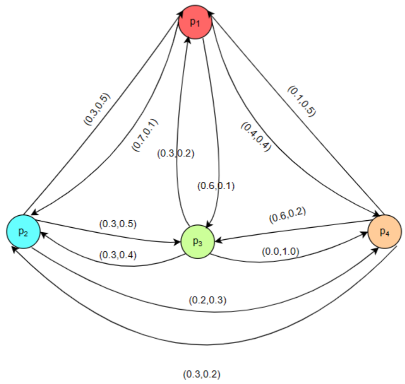

Figure 9. IFRUM (G_4^) related to maturity period

From Figure 7, the IFRUAM is defined as

$$B\hat{}^{(2)} = \begin{bmatrix} (0.0, 0.0) & (0.3, 0.1) & (0.2, 0.2) & (0.2, 0.3) \\ (0.3, 0.4) & (0.0, 0.0) & (0.2, 0.1) & (0.2, 0.3) \\ (0.3, 0.4) & (0.5, 0.2) & (0.0, 0.0) & (0.2, 0.1) \\ (0.2, 0.3) & (0.2, 0.3) & (0.2, 0.1) & (0.0, 0.0) \end{bmatrix}$$

From Figure 8, the IFRUAM is defined as

$$B\hat{}^{(3)} = \begin{bmatrix} (0.0, 0.0) & (0.4, 0.2) & (0.3, 0.4) & (0.2, 0.4) \\ (0.4, 0.1) & (0.0, 0.0) & (0.6, 0.2) & (0.3, 0.3) \\ (0.2, 0.2) & (0.7, 0.1) & (0.0, 0.0) & (0.4, 0.5) \\ (0.3, 0.5) & (0.3, 0.2) & (0.3, 0.3) & (0.0, 0.0) \end{bmatrix}$$

From Figure 9, the IFRUAM is defined as

$$B\hat{}^{(4)} = \begin{bmatrix} (0.0, 0.0) & (0.7, 0.1) & (0.6, 0.1) & (0.4, 0.4) \\ (0.3, 0.5) & (0.0, 0.0) & (0.3, 0.5) & (0.2, 0.3) \\ (0.3, 0.2) & (0.3, 0.4) & (0.0, 0.0) & (0.0, 1.0) \\ (0.1, 0.5) & (0.3, 0.2) & (0.6, 0.2) & (0.0, 0.0) \end{bmatrix}$$

Algorithm for lower approximation of IFRG

The energy of adjacency matrices $B_{\hat{}}^{(1)}$, $B_{\hat{}}^{(2)}$, $B_{\hat{}}^{(3)}$ and $B_{\hat{}}^{(4)}$ of IFRG for lower approximation using Equation (3.1) we get

From Figure 2 and $B_{\hat{}}^{(1)}$ we get, $E\left(B_{\hat{}}^{(1)}\right) = (1.9224, 2.3129)$,

From Figure 3 and $B_{\hat{}}^{(2)}$ we get, $E\left(B_{\hat{}}^{(2)}\right) = (1.3978, 2.4538)$,

From Figure 4 and $B_{\hat{}}^{(3)}$ we get, $E\left(B_{\hat{}}^{(3)}\right) = (2.0800, 1.6935)$,

From Figure 5 and $B_{\hat{}}^{(4)}$ we get, $E\left(B_{\hat{}}^{(4)}\right) = (1.4822, 1.4525)$.

Using Equation (3.2), we get the scores of each expert $G_{\hat{}i}$ determined with energy's as follows:

$$w_{\hat{}1}^{1} = (0.2793, 0.2923)\,, w_{\hat{}2}^{1} = (0.2031, 0.3101)\,,$$

$$w_{\hat{}3}^{1} = (0.3022, 0.2140)\,, w_{\hat{}4}^{1} = (0.2154, 0.1836)\,.$$

Calculate Karl Pearson's correlation coefficient measures $CC\left(B_{\hat{}}^{(K)}, B_{\hat{}}^{(d)}\right)$ between $B_{\hat{}}^{(K)}$ and $B_{\hat{}}^{(d)}$ for every $k \neq d$, using Equation (3.3) we get

$$CC\left(B_{\hat{}}^{(1)}, B_{\hat{}}^{(2)}\right) = 0.7896, CC\left(B_{\hat{}}^{(1)}, B_{\hat{}}^{(3)}\right) = 0.7794, CC\left(B_{\hat{}}^{(1)}, B_{\hat{}}^{(4)}\right) = 0.7537,$$

$$CC\left(B_{\hat{}}^{(2)}, B_{\hat{}}^{(3)}\right) = 0.7826, CC\left(B_{\hat{}}^{(2)}, B_{\hat{}}^{(4)}\right) = 0.8157, CC\left(B_{\hat{}}^{(3)}, B_{\hat{}}^{(4)}\right) = 0.8704.$$

By Equation (3.4), we get the average correlation coefficient degree $CC\left(B_{\hat{}}^{(k)}\right)$ of $B_{\hat{}}^{(k)}$ is obtained as below,

$$CC\left(B_{\hat{}}^{(1)}\right) = 0.7742, CC\left(B_{\hat{}}^{(2)}\right) = 0.7960, CC\left(B_{\hat{}}^{(3)}\right) = 0.8108, CC\left(B_{\hat{}}^{(4)}\right) = 0.8133.$$

By calculating the values of the scores $w_{\hat{}k}^{b}$ Equation (3.5), we have

$$w_{\hat{}1}^{b} = 0.2424, w_{\hat{}2}^{b} = 0.2492, w_{\hat{}3}^{b} = 0.2538, \ and\ w_{\hat{}4}^{b} = 0.2546.$$

Compute the objective scores $w_{\hat{}k}^{2}$ of the expert e_k,

Suppose $\eta = 0.5$, which means the objective weight is affected by half of the weight determined by the CCM. The objective weighting vector can be obtained by Equation (3.6), we have

$$w_{\hat{}1,\mu}^{2} = 0.2609 \ and\ w_{\hat{}1,\nu}^{2} = 0.2674,$$

$$w_{\hat{}2,\mu}^{2} = 0.2262 \ and\ w_{\hat{}2,\nu}^{2} = 0.2797,$$

$$w_{\hat{}3,\mu}^{2} = 0.2780 \ and\ w_{\hat{}3,\nu}^{2} = 0.2339,$$

$$w_{\hat{}4,\mu}^{2} = 0.2350 \ and\ w_{\hat{}4,\nu}^{2} = 0.2191.$$

So, weights of authorities are

$$w_{\hat{}1}^{2} = (0.2609, 0.2674), w_{\hat{}2}^{2} = (0.2262, 0.2797),$$

$$w^{2}_{\hat{}3} = (0.2780, 0.2339), w^{2}_{\hat{}4} = (0.2350, 0.2191).$$

Compute the subjective and objective scores $w^{1}_{\hat{}k}$ and $w^{2}_{\hat{}k}$ of the expert e_k.

Based on the decision-makers preferences for the objective and subjective weight vectors Equation (3.7) can integrate the subjective weighting vector $w^{1}_{\hat{}1}$, $w^{1}_{\hat{}2}$, $w^{1}_{\hat{}3}$, $w^{1}_{\hat{}4}$ and the objective weighting vector $w^{2}_{\hat{}1}$, $w^{2}_{\hat{}2}$, $w^{2}_{\hat{}3}$, $w^{2}_{\hat{}4}$ into the integrated weighting vector $w_{\hat{}1}$, $w_{\hat{}2}$, $w_{\hat{}3}$, and $w_{\hat{}4}$. In Equation (3.7), we assume that $\gamma = 0.5$ and calculate the integrated weighting vector $w_{\hat{}k}$, we have

$$w_{\hat{}1,\mu} = 0.2701 \textit{ and } w_{\hat{}1,\nu} = 0.2799,$$

$$w_{\hat{}2,\mu} = 0.2147 \textit{ and } w_{\hat{}2,\nu} = 0.2949,$$

$$w_{\hat{}3,\mu} = 0.2901 \textit{ and } w_{\hat{}3,\nu} = 0.2240,$$

$$w_{\hat{}4,\mu} = 0.2252 \textit{ and } w_{\hat{}4,\nu} = 0.2014.$$

So, impartial weights are

$$w_{\hat{}1} = (0.2701, 0.2799), w_{\hat{}2} = (0.2147, 0.2949),$$

$$w_{\hat{}3} = (0.2901, 0.2240), w_{\hat{}4} = (0.2252, 0.2014).$$

Compute the average intuitionistic fuzzy rough values (IFRVs) $r^{(k)}_{\hat{}i}$ of replacement t_i to replacement, using Equation (3.8), we have

From Figure 2 and $B^{(1)}_{\hat{}}$ we get

$$r^{(1)}_{\hat{}1} = (0.2750, 0.2000), r^{(1)}_{\hat{}2} = (0.3000, 0.2250),$$

$$r^{(1)}_{\hat{}3} = (0.1250, 0.3000), r^{(1)}_{\hat{}4} = (0.2500, 0.3000).$$

From Figure 3 and $B^{(2)}_{\hat{}}$ we get

$$r^{(2)}_{\hat{}1} = (0.2000, 0.3250), r^{(2)}_{\hat{}2} = (0.1500, 0.2500),$$

$$r^{(2)}_{\hat{}3} = (0.2000, 0.3250), r^{(2)}_{\hat{}4} = (0.1500, 0.3500).$$

From Figure 4 and $B^{(3)}_{\hat{}}$ we get

$$r^{(3)}_{\hat{}1} = (0.2500, 0.2500), r^{(3)}_{\hat{}2} = (0.2750, 0.1250),$$

$$r^{(3)}_{\hat{}3} = (0.2500, 0.1500), r^{(3)}_{\hat{}4} = (0.2500, 0.2500).$$

From Figure 5 and $B^{(4)}_{\hat{}}$ we get

$$r^{(4)}_{\hat{}1} = (0.2500, 0.1750), r^{(4)}_{\hat{}2} = (0.1500, 0.2000),$$

$$r^{(4)}_{\hat{}3} = (0.1750, 0.1750), r^{(4)}_{\hat{}4} = (0.2000, 0.1750).$$

By using the Equation (3.9), to find all $r^{(k)}_{\hat{}i}$, k is having $1, 2, \cdots, n$, we have

$$r_{\hat{}1,\mu} = 0.2460 \textit{ and } r_{\hat{}1,\nu} = 0.2431,$$

$$r\hat{}_{2,\mu} = 0.2268 \; and \; r\hat{}_{2,\nu} = 0.2050,$$

$$r\hat{}_{3,\mu} = 0.1886 \; and \; r\hat{}_{3,\nu} = 0.2487,$$

$$r\hat{}_{4,\mu} = 0.2173 \; and \; r\hat{}_{4,\nu} = 0.2784.$$

Therefore

$$r\hat{}_1 = (0.2460, 0.2431), \; r\hat{}_2 = (0.2268, 0.2050),$$

$$r\hat{}_3 = (0.1886, 0.2487), \; r\hat{}_4 = (0.2173, 0.2784).$$

By Equation (3.10), we have $CC\ (r\hat{}_i) = \mu_i - \nu_i$, we get

$$CC\ (r\hat{}_1) = 0.0029, CC\ (r\hat{}_2) = 0.0218, CC\ (r\hat{}_3) = -0.0601, CC\ (r\hat{}_4) = -0.0611.$$

Therefore

$$CC\ (r\hat{}_2) > CC\ (r\hat{}_1) > CC\ (r\hat{}_3) > CC\ (r\hat{}_4), as\ a\ result\ p_2 > p_1 > p_3 > p_4.$$

Hence, p_2 place the highest position, while p_4 place the last position, finally p_1 and p_3 place the center position orders and which are mentioned above.

Algorithm for upper approximation of IFRG

The energy of adjacency matrices $B\hat{}^{(1)}$, $B\hat{}^{(2)}$, $B\hat{}^{(3)}$ and $B\hat{}^{(4)}$ of IFRG for upper approximation using Equation (3.1) we get

From Figure 6 and $B\hat{}^{(1)}$ we get, $E\ (B\hat{}^{(1)}) = (2.3777, 1.7549)$,

From Figure 7 and $B\hat{}^{(2)}$ we get, $E\ (B\hat{}^{(2)}) = (1.4848, 1.3983)$,

From Figure 8 and $B\hat{}^{(3)}$ we get, $E\ (B\hat{}^{(3)}) = (2.0376, 1.8062)$,

From Figure 9 and $B\hat{}^{(4)}$ we get, $E\ (B\hat{}^{(4)}) = (2.0659, 2.0238)$.

Using Equation (3.2), we get the scores of each expert $G_i\hat{}$ determined with energy's as follows:

$$w_1\hat{}^1 = (0.2985, 0.2513)\,, w_2\hat{}^1 = (0.1864, 0.2002)\,,$$

$$w_3\hat{}^1 = (0.2558, 0.2586)\,, w_4\hat{}^1 = (0.2593, 0.2898)\,.$$

Calculate Karl Pearson's correlation coefficient measures $CC\ \left(B\hat{}^{(k)}, B\hat{}^{(d)}\right)$ between $B\hat{}^{(k)}$ and $B\hat{}^{(d)}$ for every $k \neq d$, using Equation (3.3) we get

$$CC\ \left(B\hat{}^{(1)}, B\hat{}^{(2)}\right) = 0.8407, CC\ \left(B\hat{}^{(1)}, B\hat{}^{(3)}\right) = 0.8523, CC\ \left(B\hat{}^{(1)}, B\hat{}^{(4)}\right) = 0.8183,$$

$$CC\ \left(B\hat{}^{(2)}, B\hat{}^{(3)}\right) = 0.8733, CC\ \left(B\hat{}^{(2)}, B\hat{}^{(4)}\right) = 0.7724, CC\ \left(B\hat{}^{(3)}, B\hat{}^{(4)}\right) = 0.8033.$$

By Equation (3.4), we get the average correlation coefficient degree $CC\ \left(B^{\wedge(k)}\right)$ of $B^{\wedge(k)}$ is obtained as below,

$$CC\ \left(B^{\wedge(1)}\right) = 0.8371, CC\ \left(B^{\wedge(2)}\right) = 0.8288, CC\ \left(B^{\wedge(3)}\right) = 0.8430, CC\ \left(B^{\wedge(4)}\right) = 0.7980.$$

By calculating the values of the scores ${w_k}^{\wedge b}$ Equation (3.5), we have

$$w_1{}^{\wedge b} = 0.2531, w_2{}^{\wedge b} = 0.2506, w_3{}^{\wedge b} = 0.2549,\ \textit{and}\ w_4{}^{\wedge b} = 0.2413.$$

Compute the objective scores ${w_k}^{\wedge 2}$ of the expert e_k,

Suppose $\eta = 0.5$, which means the objective weight is affected by half of the weight determined by the CCM. The objective weighting vector can be obtained by Equation (3.6), we have

$$w_{1,\mu}{}^{\wedge 2} = 0.2758\ \textit{and}\ w_{1,\nu}{}^{\wedge 2} = 0.2522,$$

$$w_{2,\mu}{}^{\wedge 2} = 0.2185\ \textit{and}\ w_{2,\nu}{}^{\wedge 2} = 0.2254,$$

$$w_{3,\mu}{}^{\wedge 2} = 0.2554\ \textit{and}\ w_{3,\nu}{}^{\wedge 2} = 0.2568,$$

$$w_{4,\mu}{}^{\wedge 2} = 0.2503\ \textit{and}\ w_{4,\nu}{}^{\wedge 2} = 0.2656.$$

So, the weights of authorities are

$$w_1{}^{\wedge 2} = (0.2758, 0.2522), w_2{}^{\wedge 2} = (0.2185, 0.2254),$$

$$w_3{}^{\wedge 2} = (0.2554, 0.2568), w_4{}^{\wedge 2} = (0.2503, 0.2656).$$

Compute the subjective and objective scores ${w_k}^{\wedge 1}$ and ${w_k}^{\wedge 2}$ of the expert e_k.

Based on the decision-makers preferences for the objective and subjective weight vectors Equation (3.7) can integrate the subjective weighting vector ${w_1}^{\wedge 1}$, ${w_2}^{\wedge 1}$, ${w_3}^{\wedge 1}$, ${w_4}^{\wedge 1}$ and the objective weighting vector ${w_1}^{\wedge 2}$, ${w_2}^{\wedge 2}$, ${w_3}^{\wedge 2}$, ${w_4}^{\wedge 2}$ into the integrated weighting vector ${w_1}^{\wedge}$, ${w_2}^{\wedge}$, ${w_3}^{\wedge}$, and ${w_4}^{\wedge}$. In Equation (3.7), we assume that $\gamma = 0.5$ and calculate the integrated weighting vector ${w_k}^{\wedge}$, we have

$$w_{1,\mu}{}^{\wedge} = 0.2872\ \textit{and}\ w_{1,\nu}{}^{\wedge} = 0.2518,$$

$$w_{2,\mu}{}^{\wedge} = 0.2025\ \textit{and}\ w_{2,\nu}{}^{\wedge} = 0.2128,$$

$$w_{3,\mu}{}^{\wedge} = 0.2556\ \textit{and}\ w_{3,\nu}{}^{\wedge} = 0.2577,$$

$$w_{4,\mu}{}^{\wedge} = 0.2548\ \textit{and}\ w_{4,\nu}{}^{\wedge} = 0.2777.$$

So, impartial weights are

$$w_1{}^{\wedge} = (0.2872, 0.2518), w_2{}^{\wedge} = (0.2025, 0.2128),$$

$$w_3{}^{\wedge} = (0.2556, 0.2577), w_4{}^{\wedge} = (0.2548, 0.2777).$$

Compute the average intuitionistic fuzzy rough values (IFRVs) ${r_i}^{\wedge(k)}$ of replacement t_i to replacement, using Equation (3.8), we have

From Figure 6 and $B^{\hat{}(1)}$ we get

$$r_1^{\hat{}(1)} = (0.3500, 0.1250), r_2^{\hat{}(1)} = (0.3500, 0.1750),$$

$$r_3^{\hat{}(1)} = (0.2250, 0.3250), r_4^{\hat{}(1)} = (0.2250, 0.2250).$$

From Figure 7 and $B^{\hat{}(2)}$ we get

$$r_1^{\hat{}(2)} = (0.1750, 0.1500), r_2^{\hat{}(2)} = (0.1750, 0.2000),$$

$$r_3^{\hat{}(2)} = (0.2500, 0.1750), r_4^{\hat{}(2)} = (0.1500, 0.1750).$$

From Figure 8 and $B^{\hat{}(3)}$ we get

$$r_1^{\hat{}(3)} = (0.2250, 0.2500), r_2^{\hat{}(3)} = (0.3250, 0.1500),$$

$$r_3^{\hat{}(3)} = (0.3250, 0.2000), r_4^{\hat{}(3)} = (0.2250, 0.2500).$$

From Figure 9 and $B^{\hat{}(4)}$ we get

$$r_1^{\hat{}(4)} = (0.4250, 0.1500), r_2^{\hat{}(4)} = (0.2000, 0.3250),$$

$$r_3^{\hat{}(4)} = (0.1500, 0.4000), r_4^{\hat{}(4)} = (0.2500, 0.2250).$$

By using the Equation (3.9), to find all $r_i{}^{\hat{}(k)}$, k is having $1, 2, \cdots, n$, we have

$$r_{1,\mu}\hat{} = 0.3018, \ and \ r_{1,\nu}\hat{} = 0.1695,$$

$$r_{2,\mu}\hat{} = 0.2700, \ and \ r_{2,\nu}\hat{} = 0.2155,$$

$$r_{3,\mu}\hat{} = 0.2365, \ and \ r_{3,\nu}\hat{} = 0.2817,$$

$$r_{4,\mu}\hat{} = 0.2162, \ and \ r_{4,\nu}\hat{} = 0.2208.$$

Therefore,

$$r_1\hat{} = (0.3018, 0.1695), \ r_2\hat{} = (0.2700, 0.2155),$$

$$r_3\hat{} = (0.2365, 0.2817), \ r_4\hat{} = (0.2162, 0.2208).$$

By Equation (3.10), we have $CC\ (r_i\hat{}) = \mu_i - \nu_i$, we get

$$CC\ (r_1\hat{}) = 0.1323, CC\ (r_2\hat{}) = 0.0545, CC\ (r_3\hat{}) = -0.0452, CC\ (r_4\hat{}) = -0.0046.$$

Therefore

$$CC\ (r_1\hat{}) > CC\ (r_2\hat{}) > CC\ (r_4\hat{}) > CC\ (r_3\hat{}), as\ a\ result\ p_1 > p_2 > p_4 > p_3.$$

Hence, p_1 place the highest position, while p_3 place the last position, finally p_2 and p_4 place the center position orders and which are mentioned in the above tables.

Comparative analysis

TOPSIS technique

The process for selecting the best seed for a cotton crop using an intuitionistic fuzzy rough TOPSIS technique consists of the following phases.

Step 1: Determine the energy of every IFRPR $\forall\ k = 1, 2, \cdots, n$.

$$E\ \left(B^{(k)}\right) = \left|\sum_{i\ =\ 1}^{n} k_i\right| \tag{4.1}$$

Using the following formula, determine the experts' weight based on the IFRGs' energy.

$$w_k^1\ =\ \left((w_\mu)_i\,, (w_\nu)_i\right)\ =\ \left[\frac{E\left((D_\mu)_i\right)}{\sum\limits_{r=1}^{l} E\left((D_\mu)_r\right)},\ \frac{E\left((D_\nu)_i\right)}{\sum\limits_{r=1}^{l} E\left((D_\nu)_r\right)}\right] \forall\ i = 1, 2, \cdots, n. \tag{4.2}$$

Step 2: The following formula may be used to calculate the intuitionistic fuzzy rough weight averaging operator by substituting expert weight values.

$$\left(R_{ij}^{(1)}, R_{ij}^{(2)}, \cdots,\ R_{ij}^{(n)}\right) = \left(1 - \prod_{i=1}^{n}\left(1\ -\ \mu_{jk}^{(i)}\right)^{w_i},\ \prod_{i=1}^{n}\left(\nu_{jk}^{(i)}\right)^{w_i}\right) \tag{4.3}$$

here, w_i is the weight function, μ_{jk} be the membership element, and ν_{jk} be the non-membership element. Using the intuitionistic fuzzy rough weight averaging operator, calculate the IFRPR p_i $(i = 1, 2, \cdots, n)$ of the seed for cotton crop w_i over all other seed for cotton crop models

$$A = (a_{ij})_{n\times n} \tag{4.4}$$

Step 3: Calculate the ranking order of the components of p_b , $b = 1, 2, \cdots, n$ based on the membership degrees of $out - d(p_b)$ and then calculate the ordering. Determine the order in which the factors p_b should be ranked.

Lower approximation of IFRG

The energy of adjacency matrices of IFRG for lower using Equation 4.1 we get,

$$E\ \left(B^{(1)}_{\hat{}}\right) = (1.9224, 2.3129), E\ \left(B^{(2)}_{\hat{}}\right) = (1.3978, 2.4538),$$

$$E\ \left(B^{(3)}_{\hat{}}\right) = (2.0800, 1.6935), E\ \left(B^{(4)}_{\hat{}}\right) = (1.4822, 1.4525).$$

Using Equation 4.2, we get the scores each expert $G_{\hat{}i}$ determined with energy's as follows:

$$w^1_{\hat{}1} = (0.2793,\ 0.2923)\,,\ w^1_{\hat{}2} = (0.2031,\ 0.3101)\,,$$

$$w^1_{\hat{}3} = (0.3022,\ 0.2140)\,,\ w^1_{\hat{}4} = (0.2154,\ 0.1836)\,.$$

From Equation 4.3 and Equation 4.4, we get the collective IFRPR $A\ = (a_{ij})_{n\times n}$ is

$$A\ =\begin{bmatrix} (0.0000, 0.0000) & (0.4212, 0.0000) & (0.2862, 0.3226) & (0.3125, 0.2873) \\ (0.3802, 0.2549) & (0.0000, 0.0000) & (0.3650, 0.0000) & (0.2758, 0.2614) \\ (0.1794, 0.4029) & (0.4065, 0.2125) & (0.0000, 0.0000) & (0.2750, 0.2374) \\ (0.3336, 0.3488) & (0.2508, 0.3535) & (0.2753, 0.3755) & (0.0000, 0.0000) \end{bmatrix}$$

Figure 10. shows collective lower IFRPR

Compute the out degree $out - d(p_{\hat{}b})$ $(b\ =\ 1, 2, 3, 4)$ of all measures in a fractional coordinated system as follows:

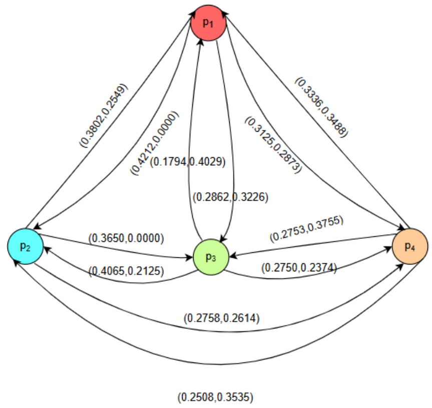

Figure 10. Intuitionistic fuzzy rough digraph for lower

$$out - d(p\hat{}_1) = (1.0199, 0.6099), out - d(p\hat{}_2) = (1.0210, 0.5163),$$

$$out - d(p\hat{}_3) = (0.8609, 0.8528), out - d(p\hat{}_4) = (0.8597, 1.0778).$$

As per to membership degree of $out - d(p\hat{}_b)$ $(b = 1, 2, 3, 4)$, we have the ranking of factors $p\hat{}_b$ $(b = 1, 2, 3, 4)$ as: $p_2 > p_1 > p_3 > p_4$. Thus the best choice is p_2.

Upper approximation of IFRG

The energy of adjacency matrices of IFRG for upper using Equation 4.1 we get,

$$E\left(B^{\hat{}(1)}\right) = (2.3777, 1.7549), E\left(B^{\hat{}(2)}\right) = (1.4848, 1.3983),$$

$$E\left(B^{\hat{}(3)}\right) = (2.0376, 1.8062), E\left(B^{\hat{}(4)}\right) = (2.0659, 2.0238).$$

Using Equation 4.2, we get the score each expert $G_i\hat{}$ determined with energy's as follows:

$$w_1^{\hat{}1} = (0.2985,\ 0.2513),\ w_2^{\hat{}1} = (0.1864,\ 0.2002),$$

$$w_3^{\hat{}1} = (0.2558,\ 0.2586), w_4^{\hat{}1} = (0.2593,\ 0.2898).$$

From Equation 4.3 and Equation 4.4, we get the collective IFRPR is $A = (a_{ij})_{n\times n}$ is

$$A = \begin{bmatrix} (0.0000, 0.0000) & (0.5429, 0.1424) & (0.4072, 0.1958) & (0.3186, 0.2666) \\ (0.4774, 0.2105) & (0.0000, 0.0000) & (0.3781, 0.2514) & (0.2905, 0.3000) \\ (0.2757, 0.2735) & (0.4945, 0.2573) & (0.0000, 0.0000) & (0.2125, 0.4187) \\ (0.2985, 0.3586) & (0.2852, 0.2169) & (0.3993, 0.2434) & (0.0000, 0.0000) \end{bmatrix}$$

Figure 11. shows collective upper IFRPR

Compute the out degree $out - d(p_b\hat{})$ $(b = 1, 2, 3, 4)$ of all measures in a fractional coordinated system as follows:

$$out - d(p_1\hat{}) = (1.2687, 0.6048), out - d(p_2\hat{}) = (1.1460, 0.7619),$$

$$out - d(p_3\hat{}) = (0.9827, 0.9495), out - d(p_4\hat{}) = (0.9830, 0.8189).$$

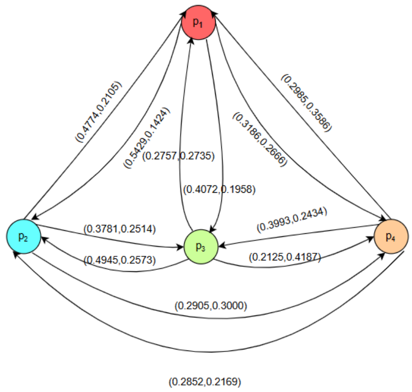

Figure 11. Intuitionistic fuzzy rough digraph for upper

As per to membership degree of $out - d(p_b\hat{})$ $(b = 1, 2, 3, 4)$, we have the ranking of factors p_b $(b = 1, 2, 3, 4)$ as: $p_1 > p_2 > p_4 > p_3$ Thus the best choice p_1.

Hence, p_1 place the highest position, while p_3 place the last position, finally p_2 and p_4 places the centre position orders which are mentioned in the above. The given statement discusses the approximation orders of a sequence, specifically noting that the lower approximation order p_2 is greater than p_1, which is greater than p_3, and p_4 is the least. On the other hand, the upper approximation order is different, with p_1 being greater than p_2, p_4 being greater than p_3. In cases where the lower and upper approximation orders differ, it is asserted that the lower approximation order is considered the best because it provides an exact approximation. Comparing the results, the efficiency of the techniques, their duration, and the ease with which the distinctions between ways, such as the TOPSIS approach and the working method, can be understood. Using these methods, the following are the rankings defined as $p_2 > p_1 > p_3 > p_4$. Furthermore, this strategy produces findings somewhat faster than the TOPSIS method.

5. Conclusion

A new intuitionistic fuzzy rough framework is introduced in this study. It deals with vagueness and makes soft computation better by combining intuitionistic fuzzy sets and rough sets more efficiently. Our approach enables a more thorough comprehension of fuzzy components and their connections by utilising lower and upper approximation spaces. We have shown that our suggested correlation coefficient metric is superior to conventional approaches in evaluating the strength of relationship between intuitionistic fuzzy rough preference relations. Our methodology incorporates this sophisticated correlation coefficient into attribute decision-making procedures, presenting a new technique for expert weight determination based on correlation coefficients and IFRPR uncertainty. This improvement facilitates the decision-making process and enables a more precise evaluation of the choices. Our framework's potential for improving agricultural decision support systems is demonstrated through a cooperative decision-making approach for choosing the best cotton seed kinds.

Additionally, our approach offers significant new directions for future research by extending intuitionistic fuzzy rough sets to intuitionistic fuzzy rough graphs, which are statistical measurement notions. Higher-performing solutions for different kinds of intuitionistic fuzzy rough graphs may be obtained by applying metrics such as association, regression, and variation coefficients. All things considered, our results demonstrate the great potential of incorporating sophisticated statistical techniques into intuitionistic fuzzy rough settings, opening up new avenues for advancements and uses in decision-making and other fields. The work's limitations include its reliance on the correlation coefficient measure within the intuitionistic fuzzy rough framework, which may overlook advanced methods for decision-making, and its application to a specific case, cotton seedling decision-making, which may limit its generalisability.

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APPENDIX I

A list of all abbreviations in this manuscript is summarized in Table 1

Table 1. List of abbreviations

Abbreviation	Full Name
MG	Membership grade
Non-MG	Non-membership grade
FS	Fuzzy set
IF	Intuitionistic fuzzy
IFS	Intuitionistic fuzzy set
RS	Rough set
FRS	Fuzzy rough set
IFRS	Intuitionistic fuzzy rough set
IFRG	Intuitionistic fuzzy rough graph
CC	Correlation coefficient
CCM	Correlation coefficient measure
DM	Decision-making
MCGDM	Multi criteria group decision-making
IFWA	Intuitionistic fuzzy weighted average
IFWG	Intuitionistic fuzzy weighted geometric
IFOWA	Intuitionistic fuzzy ordered weighted average
IFRVs	Intuitionistic fuzzy rough values
IFRPRs	Intuitionistic fuzzy rough preference relations
IFRLM	Intuitionistic fuzzy rough lower matrix
IFRLAM	Intuitionistic fuzzy rough lower adjacency matrix
IFRUM	Intuitionistic fuzzy rough upper matrix
IFRUAM	Intuitionistic fuzzy rough upper adjacency matrix
AO	Aggregation operators