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A Contribution To Solve A Linear Equation With One Variable And Reverse A Matrix In 3-Cyclic Refined Neutrosophic Ring

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Abstract

n-cyclic refined neutrosophic rings have become the focus of scientific research and various applications in recent years. Due to the complexity and versatile uses of these structures, much research and development is still needed. There are many open problems waiting to be solved in this field. This article focuses on solving some of these problems. In particular, 3-cyclic refined neutrosophic rings are emphasized and the solution of some basic problems in this context is discussed. First, a linear equation is solved and then the inverse of 2x2 matrices is found. In order to do all these, the inverse of an element taken from this ring is used. There is a need for convenience for this. Therefore, the algorithm developed to easily find the inverse of an element in 3-cycle refined neutrosophic rings is presented.

Keywords: Neutrosophic Ring, n-Cyclic Refined Neutrosophic Ring, 3-Cyclic Refined Neutrosophic Ring, 3-Cyclic Refined Neutrosophic equation, Inverse of 3-Cyclic Refined Neutrosophic Real Number.

1. Introduction

Neutrosophy is an important concept in philosophy and has formed the basis of the widely used neutrosophic set theory. Thanks to the contributions of this theory to classical set theory, new algebraic structures and topologies such as neutrosophic groups, rings, spaces and modules have been defined and applied in various fields. The wide use of neutrosophic clusters is clearly evident in various studies. These clusters are used as an important tool, especially in the analysis of situations such as uncertainty, inconsistency and incompleteness [1,2,3,4,6,7,9,10,11]. Recent advances in this field include the introduction of improved neutrosophic clusters, as in [8]. The results of these developments are notable in the study of refined neutrosophic rings as well as refined neutrosophic modules and spaces [5, 12, 13].Florentin Smarandache [21] refined the concept of literal indeterminacy I *as* $I_1, I_2, ..., I_n$ *and defined a different multiplication on them called n-cyclic refined neutrosophic ring. The introduction of n-refined neutrosophic clusters reflects an effort to capture and analyze the complexities of uncertainty in more detail. The elements in any n-refined neutrosophic ring and an n-cyclic refined neutrosophic ring are equivalent. Despite this equality in elements, the multiplication of elements differs significantly based on the defined operation between sub-indeterminacies. This differentiation leads to the emergence of a new class of refined neutrosophic rings [14]. n-cyclic refined neutrosophic rings'' has become a widely studied in [15-23]. Due to the complexity and versatile uses of these structures, much research and development is still needed. There are many open problems waiting to be solved in this field. This article focuses on solving some of these problems. Particularly, 3-cyclic refined neutrosophic rings are focused on and the solution of some basic problems in this context is discussed. First, a linear equation is solved, then the inverse of the 2x2 matrices is found. To do all this, the inverse of an element taken from this ring is used. To facilitate this process, an algorithm is presented to easily find the inverse of an element in 3-cyclic refined neutrosophic rings.*

2. Materials and Methods

Definition 2.1: [4] Let R be a ring and $I_1, I_2, ..., I_n$ be n sub-indeterminacies. $R_n(I) = \{a_0 + a_1 I_1 + ... + a_n I_n : a_i \in R,\ i=1,2,...,n\}$ is called n-cyclic refined neutrosophic ring with the following operations:

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$$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i ,$$

$$\sum_{i=0}^{n} x_i I_i . \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i . y_j) I_i I_j = \sum_{i,j=0}^{n} (x_i . y_j) I_{(i+j \text{ mod} n)} .$$

Definition 2.2: Let R be a ring. $R_3(I)$ is called 3-cyclic refined neutrosophic ring defined by $R_3(I) = \{m_0 + m_1 I_1 + m_2 I_2 + m_3 I_3 : m_i \in R, i = 1,2,3\}$ .It is commutative if $\forall m,n \in R_3(I)$, $mn = nm$. If there is $1 \in R_3(I)$ and $1.m = m.1 = m$, then it is called an 3-cyclic refined neutrosophic ring with unity.

Theorem 2.3: [12]
$R_2(I) = \{m_0 + m_1 I_1 + m_2 I_2 : m_i \in R\}$ be 2-cyclic refined neutrosophic ring of real numbers. Let $A = m_0 + m_1 I_1 + m_2 I_2 \in R_2(I)$. Then, the algorithm of invers of A is given the following:

$$A^{-1} = \frac{1}{m_0} + I_1 \left[\frac{-m_1}{(m_0 - m_1 + m_2)(m_0 + m_1 + m_2)} \right]$$

$$+ I_2 \left[\frac{-m_1 - m_2}{m_0 (m_0 + m_1 + m_2)} + \frac{m_1}{(m_0 + m_1 + m_2)(m_0 - m_1 + m_2)} \right]$$

where $m_0 \neq 0$, $m_2 \neq 0$, $m_0 - m_1 + m_2 \neq 0$.

Remark 2.4: A linear 2-cyclic refined neutrosophic equation with one variable $AX + B = 0$ can be solved easily by using above algorithm of the inverse of a 2-cyclic refined neutrosophic real number A.

Example 2.5: By getting 2-cyclic refined neutrosophic linear equation:

$$(2 - 3I_1 + 2I_2) X + (1 - 4I_1 + 2I_2) = 0$$

Let us solve this linear equation.

Solution: Let $AX + B = 0$ be a linear 2-cyclic refined neutrosophic equation with one variable, where

$A = 2 - 3I_1 + 2I_2$, $B = 1 - 4I_1 + 2I_2$,

$X = x_0 + x_1 I_1 + x_2 I_2$. We compute A^{-1} with the algorithm given in theorem 2.3.

For $a_0 = 2 \neq 0$, $a_2 = 2 \neq 0$, $a_0 - a_1 + a_2 = 1 \neq 0$,

$$A^{-1} = \frac{1}{2} + I_1 \left[\frac{3}{1.1} \right] + I_2 \left[\frac{-(-3) - 2}{2.1} + \frac{-3}{1.1} \right] \Rightarrow$$

$$A^{-1} = \frac{1}{2} + 3I_1 - \frac{5}{2} I_2$$

Now, solve one variable equation given in example 2.5

$$AX + B = 0 \Rightarrow X = A^{-1}.B$$

$$\Rightarrow X = \left(\frac{1}{2} + 3I_1 - \frac{5}{2} I_2 \right) . (-1 + 4I_1 - 2I_2)$$

$$\Rightarrow X = -\frac{1}{2} + I_1 [2 - 3 - 6 - 10] + I_2 \left[-1 + 12 + \frac{5}{2} + 5 \right]$$

So, the solution is $X = -\frac{1}{2} - 17I_1 + \frac{37}{2} I_2$.

3.Results and Discussion

Definition 3.1: Let R be any ring with unity and $R_3(I) = \{a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3 : a_i \in R, i = 1,2,3\}$ be the 3-cyclic refined neutrosophic ring. Then $A = a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3 \in R_3(I)$ is called invertible (unit) if and only if there exists $A^{-1} = K = k_0 + k_1 I_1 + k_2 I_2 + k_3 I_3 \in R_3(I)$ such that $A.A^{-1} = A.K = 1$.

Theorem 3.2: Let R be any ring of real numbers with unit and $R_3(I) = \{a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3 : a_i \in R\}$ be the 3-cyclic refined neutrosophic ring of real numbers. Then, $A = a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3$ is invertible if and only if the following Diophantine equation is true:

$$(a_0 + a_3)^3 + (a_1)^3 + (a_2)^3 - 3a_1 a_2 (a_0 + a_3) \neq 0$$

Proof: The invertibility of A means that compute the 3-cyclic refined neutrosophic equation $A.A^{-1} = 1$, where

$A = a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3 \in R_3(I)$ and
$A^{-1} = K = k_0 + k_1 I_1 + k_2 I_2 + k_3 I_3 \in R_3(I)$.

$$A.K = a_0 k_0 + I_1 [a_0 k_1 + a_1 k_0 + a_1 k_3 + a_3 k_1 + a_2 k_2]$$
$$+ I_2 [a_0 k_2 + a_2 k_0 + a_2 k_3 + a_3 k_2 + a_1 k_1]$$
$$+ I_3 [a_0 k_3 + a_3 k_0 + a_1 k_2 + a_2 k_1 + a_3 k_3] = 1$$

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This implies that;

$$a_0k_0=1,\quad (a_0+a_3)k_1+a_2k_2+a_1k_3=-a_1k_0$$
$$a_1k_1+(a_0+a_3)k_2+a_2k_3=-a_2k_0$$
$$a_2k_1+a_1k_2+(a_0+a_3)k_3=-a_3k_0$$

Get three linear equations with three variables k_1,k_2,k_3 and write the following system:

$$\begin{bmatrix}(a_0+a_3) & a_2 & a_1\\ a_1 & (a_0+a_3) & a_2\\ a_2 & a_1 & (a_0+a_3)\end{bmatrix}\begin{bmatrix}k_1\\k_2\\k_3\end{bmatrix}=\begin{bmatrix}-a_1/a_0\\ -a_2/a_0\\ -a_3/a_0\end{bmatrix}$$

Where the coefficients matrix is called M. The system is solvable uniquely if and only if the determinant of the coefficients matrix M is invertible, that is,

$$\det M=\begin{vmatrix}a_0+a_3 & a_2 & a_1\\ a_1 & a_0+a_3 & a_2\\ a_2 & a_1 & a_0+a_3\end{vmatrix}\neq 0$$

By easy computing the determinant of the coefficients matrix, we get the equations

$$(a_0+a_3)^3+(a_1)^3+(a_2)^3-3a_1a_2(a_0+a_3)\neq 0$$

Theorem 3.3: Let R be any ring of real numbers with unity and

$$R_3(I)=\{a_0+a_1I_1+a_2I_2+a_3I_3 : a_i\in R\}$$

be the 3-cyclic refined neutrosophic ring of real numbers. Let $A=a_0+a_1I_1+a_2I_2+a_3I_3\in R_3(I)$ and $A^{-1}=K=k_0+k_1I_1+k_2I_2+k_3I_3\in R_3(I)$

Then, find the following algorithm of $A^{-1}=K$ is true.

$$A^{-1}=K=\frac{1}{a_0}+\left(\frac{a_2^2-a_1(a_0+a_3)}{N}\right)I_1+\left(\frac{a_1^2+a_2(a_0+a_3)}{N}\right)I_2$$
$$+\left(\frac{-(a_1^3+a_2^3+a_3^3)+a_1a_2(2a_0+3a_3)-a_0a_3(a_0+2a_3)}{a_0N}\right)I_3$$

Where $N=\det M\neq 0$ and $a_0\neq 0$.

Proof: Let $A=a_0+a_1I_1+a_2I_2+a_3I_3\in R_3(I)$ and $A^{-1}=K=k_0+k_1I_1+k_2I_2+k_3I_3\in R_3(I)$.

Compute $A.K=1$.

$$A.K=a_0k_0+I_1[a_0k_1+a_1k_0+a_1k_3+a_3k_1+a_2k_2]$$
$$+I_2[a_0k_2+a_2k_0+a_2k_3+a_3k_2+a_1k_1]$$
$$+I_3[a_0k_3+a_3k_0+a_1k_2+a_2k_1+a_3k_3]=1$$

$$\Rightarrow a_0k_0=1,$$
$$a_0k_1+a_1k_0+a_1k_3+a_3k_1+a_2k_2=0$$
$$a_0k_2+a_2k_0+a_2k_3+a_3k_2+a_1k_1=0$$
$$a_0k_3+a_3k_0+a_1k_2+a_2k_1+a_3k_3=0$$

$$\Rightarrow (a_0+a_3)k_1+a_2k_2+a_1k_3=-\frac{a_1}{a_0}$$
$$a_1k_1+(a_0+a_3)k_2+a_2k_3=-\frac{a_2}{a_0}$$
$$a_2k_1+a_1k_2+(a_0+a_3)k_3=-\frac{a_3}{a_0}$$

Then write the following system;

$$\begin{bmatrix}(a_0+a_3) & a_2 & a_1\\ a_1 & (a_0+a_3) & a_2\\ a_2 & a_1 & (a_0+a_3)\end{bmatrix}\begin{bmatrix}k_1\\k_2\\k_3\end{bmatrix}=\begin{bmatrix}-a_1/a_0\\ -a_2/a_0\\ -a_3/a_0\end{bmatrix}$$

(1)

Where the coefficients matrix is called M. By multiplying above system from the right and left by the inverse of M, obtain the coefficients of K, that is, find an algorithm of $A^{-1}=K$.

Now, compute $M^{-1}=\dfrac{1}{\det M}AdjM$.

Where

$$\det M=(a_0+a_3)^3+(a_1)^3+(a_2)^3-3a_1a_2(a_0+a_3)\neq 0$$

$$AdjM=\begin{bmatrix}\begin{vmatrix}(a_0+a_3) & a_2\\ a_1 & (a_0+a_3)\end{vmatrix} & -\begin{vmatrix}a_1 & a_2\\ a_2 & (a_0+a_3)\end{vmatrix} & \begin{vmatrix}a_1 & (a_0+a_3)\\ a_2 & a_1\end{vmatrix}\\ -\begin{vmatrix}a_2 & a_1\\ a_1 & (a_0+a_3)\end{vmatrix} & \begin{vmatrix}(a_0+a_3) & a_1\\ a_2 & (a_0+a_3)\end{vmatrix} & -\begin{vmatrix}(a_0+a_3) & a_2\\ a_2 & a_1\end{vmatrix}\\ \begin{vmatrix}a_2 & a_1\\ (a_0+a_3) & a_2\end{vmatrix} & -\begin{vmatrix}(a_0+a_3) & a_1\\ a_1 & a_2\end{vmatrix} & \begin{vmatrix}(a_0+a_3) & a_2\\ a_1 & (a_0+a_3)\end{vmatrix}\end{bmatrix}^T$$

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$$AdjM=\begin{bmatrix}(a_0+a_3)^2-a_1a_2 & -a_1(a_0+a_3)+a_2^2 & a_1^2-a_2(a_0+a_3)\\ -a_2(a_0+a_3)+a_1^2 & (a_0+a_3)^2-a_1a_2 & -a_1(a_0+a_3)+a_2^2\\ a_2^2-a_1(a_0+a_3) & -a_2(a_0+a_3)+a_1^2 & (a_0+a_3)^2-a_1a_2\end{bmatrix}^T$$

$$AdjM=\begin{bmatrix}(a_0+a_3)^2-a_1a_2 & -a_2(a_0+a_3)+a_1^2 & a_2^2-a_1(a_0+a_3)\\ -a_1(a_0+a_3)+a_2^2 & (a_0+a_3)^2-a_1a_2 & -a_2(a_0+a_3)+a_1^2\\ a_1^2-a_2(a_0+a_3) & -a_1(a_0+a_3)+a_2^2 & (a_0+a_3)^2-a_1a_2\end{bmatrix}$$

By easy computing , write the following matrix is inverse of M;

$$M^{-1}=\begin{bmatrix}\frac{(a_0+a_3)^2-a_1a_2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)} & \frac{-a_2(a_0+a_3)+a_1^2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)} & \frac{a_2^2-a_1(a_0+a_3)}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)}\\ \frac{-a_1(a_0+a_3)+a_2^2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)} & \frac{(a_0+a_3)^2-a_1a_2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)} & \frac{-a_2(a_0+a_3)+a_1^2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)}\\ \frac{a_1^2-a_2(a_0+a_3)}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)} & \frac{-a_1(a_0+a_3)+a_2^2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)} & \frac{(a_0+a_3)^2-a_1a_2}{(a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)}\end{bmatrix}$$

By multiplying the system (I) from the right and left by M^{-1}, obtain the coefficients of K. So,

$$k_0=\frac{1}{a_0},\quad k_1=\frac{a_2^2-a_1(a_0+a_3)}{N},\quad k_2=\frac{a_1^2-a_2(a_0+a_3)}{N},\quad k_3=\frac{-(a_1^3+a_2^3+a_3^3)+a_1a_2(2a_0+3a_3)-a_0a_3(a_0+2a_3)}{a_0N}$$

Where $N=\det M\neq 0$ and $a_0\neq 0$.

Finally, the algorithm of $A^{-1}=K$;

$$K=\frac{1}{a_0}+\left(\frac{a_2^2-a_1(a_0+a_3)}{N}\right)I_1+\left(\frac{a_1^2-a_2(a_0+a_3)}{N}\right)I_2+\left(\frac{-(a_1^3+a_2^3+a_3^3)+a_1a_2(2a_0+3a_3)-a_0a_3(a_0+2a_3)}{a_0N}\right)I_3$$

4. Numerical Application

Problem 4.1: Take the following 3-cyclic refined neutrosophic linear equation:
$(2+I_1-3I_2+I_3)X+I_1-I_2-I_3=0$. Let us solve this linear equation.

Solution: This problem is a linear 3-cyclic refined neutrosophic equation with one variable $AX+B=0$, where
$A=2+I_1-3I_2+I_3$, $X=x_0+x_1I_1+x_2I_2+x_3I_3$ and $B=I_1-I_2-I_3$. Firstly, Let us check whether A is invertible or not. So that, use the algorithm given in theorem 3.2.

$$\Rightarrow (a_0+a_3)^3+a_1^3+a_2^3-3a_1a_2(a_0+a_3)\neq 0$$

$$\Rightarrow (2+1)^3+1^3+(-3)^3-3.1.(-3).(2+1)=28\neq 0$$

So, A is invertible.

Now, compute A^{-1} with the algorithm given in theorem 3.3.

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$$A^{-1}=\frac{1}{2}+\left(\frac{(-3)^2-1(2+1)}{28}\right)I_1+\left(\frac{1^2-(-3)(2+1)}{28}\right)I_2$$

$$+\left(\frac{-\left(1^3+(-3)^3+1^3\right)+1(-3)(2.2+3.1)-2.1(2+2.1)}{2.28}\right)I_3$$

$$\mathrm{A}^{-1}=\frac{1}{2}+\frac{3}{14}I_1+\frac{5}{14}I_2-\frac{1}{14}I_3$$

Finally, solve one variable equation given in problem:

$$AX+B=0\Rightarrow X=A^{-1}B$$

$$\Rightarrow X=\left(\frac{1}{2}+\frac{3}{14}I_1+\frac{5}{14}I_2-\frac{1}{14}I_3\right).\left(-I_1+I_2+I_3\right)$$

$$\Rightarrow X=\left(-\frac{1}{2}+\frac{3}{14}+\frac{5}{14}+\frac{1}{14}\right)I_1$$

$$+\left(\frac{1}{2}-\frac{3}{14}+\frac{5}{14}-\frac{1}{14}\right)I_2+\left(\frac{1}{3}+\frac{3}{14}-\frac{5}{14}-\frac{1}{14}\right)I_3$$

$$\Rightarrow X=\frac{1}{7}I_1+\frac{4}{7}I_2+\frac{2}{7}I_3$$

Problem 4.2. Let $A=\begin{bmatrix}1 & -2I_2\\ I_1 & 1-I_3\end{bmatrix}\in R_3(I)$. Is ıt invertible?

Solution: Let us solve the problem using the found algorithms. Compute A^{-1}.

$$A^{-1}=\frac{1}{\det A}AdjA\Rightarrow \det A=1+I_3$$

Let $\frac{1}{\det A}=B$. Using the algorithm of an element in $R_3(I)$ in theorem 3.3, get $B=1-\frac{1}{2}I_3$.

$$AdjA=\begin{bmatrix}1-I_3 & 2I_2\\ -I_1 & 1\end{bmatrix}$$

$$\Rightarrow A^{-1}=\left(1-\frac{1}{2}I_3\right)\begin{bmatrix}1-I_3 & 2I_2\\ -I_1 & 1\end{bmatrix}$$

$$\Rightarrow A^{-1}=\begin{bmatrix}1-I_3 & I_2\\ -\frac{1}{2}I_1 & 1-\frac{1}{2}I_3\end{bmatrix}$$

So, the inverse of a given matrix in $R_3(I)$ can be found easily using algorithms.

5. Conclusion

In this article, methods for solving basic mathematical problems in 3-cycle refined neutrosophic rings are developed and applied. Operations such as finding the inverse of elements, solving linear equations and calculating the inverse of 2x2 matrices are examined in detail. These solutions provide a basis for more complex problems in this field and open new avenues for researchers.

Author’s Contributions

Hamiyet Merkepçi: Drafted and wrote the manuscript, performed the experiment and result analysis.

Ethics

There are no ethical issues after the publication of this manuscript.

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