

# Zaman Gecikmeli Diferansiyel Denklem Tabanlı Kaotik Sistemlerde Çevrimiçi Zaman Gecikmesi Kestirimi

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## ÖZET

Bu çalışmada, gecikmeli fark denklemleri tabanlı kaotik sistemlerde zaman gecikmesi kestirimi ele alınmıştır. Zaman gecikmesi, sistemin doğrusallığını bozan bir parametre olarak düşünülmüştür. Bu düşünce doğrultusunda, doğrusal olmayan bir kestirim yönteminden faydalanılmıştır. Bu yöntem, Lyapunov kararlılık analizlerine dayanmaktadır ve tüm sinyallerin küresel olarak sınırlı kalmasını ve kestirim hatasının sıfıra yakın bir noktaya yakınsamasını garanti etmektedir. Zaman gecikmesi kestirimi yönteminin etkinliğini göstermek için, birbirinden farklı, gecikmeli fark denklemleri tabanlı kaotik sistem modelleri kullanılarak birden fazla sayısal benzetim çalışmaları yapılmıştır. Sayısal benzetim çalışmaları sonucunda, yöntemin etkili bir şekilde çalıştığı görülmüştür.

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# Online Time Delay Estimation in Delay Differential Equation Based Chaotic Systems

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## ABSTRACT

In this work, time delay estimation in delay differential equation based chaotic systems is handled. The time delay is handled as a parameter which effects the system nonlinearly. Under the light of this consideration, a nonlinear parameter estimator is utilized. The aforementioned method is based on Lyapunov stability analysis and assures global boundedness of all the signals and the convergence of the estimation error to the vicinity of zero. Several simulations are given to demonstrate the efficiency of the proposed time delay estimator for various delay differential equation based chaotic systems. From the numerical simulation studies, it is observed that the method works efficiently.

## 1. Introduction (Giris)

At the beginning of 1990s, the study of Carroll and Pecora [1] on chaos synchronization has initiated research on chaos-based communications. After [1], several aspects of communication applications of chaos were researched [2, 3, 4]. Since time delay increases the complexity of any system [5], it was observed that utilizing time delay in chaotic systems can grow better security in chaotic communication systems [6, 7, 8, 9, 10]. In [7], the delayed nonlinear feedback chaotic systems were considered and it was shown that signal encoding with such systems can be broken. In [9], the time-delay is assumed as a parameter to be estimated and an optimization method called chaotic ant swarm was used to estimate the time delay in the chaotic system and also other system parameters. In [10], synchronization between two different time-delayed systems is considered to construct a robust cryptosystem. This observation brought delay differential equation (DDE) based chaotic systems into secure communication where several rather simple chaos models were obtained [11, 12, 13, 14, 15, 16]. Especially, in [11], Yalçın and Özoğuz presented a delay differential equation of which nonlinearity is based on a hard limiter function, to generate an n-scroll chaotic attractor. One special DDE is the Mackey–Glass delay differential model which was come across when modeling physiological systems [17, 18, 19, 20].

Several research problems associated with DDE based chaotic systems were investigated. In [21], Tian and Gao designed a model reference adaptive controller for Mackey–Glass delay differential model where the delay was considered to be known. Wang et al. studied on designing a linear controller for time delay Lorenz systems [22]. In [23, 24, 25], researchers designed methods to extract messages masked in DDE based chaotic communication systems. Synchronization of chaotic systems with delay was addressed in [6, 26, 27, 28, 29, 30].

The focus of some of the previous research was devoted to cracking DDE based chaotic systems by estimating time delay. In [23, 24, 25], time series analysis methods were utilized to reconstruct DDE based chaotic system models. In [29], cross-correlation function based methods were fused with sliding mode observers to estimate time delay. In [9] and [31], Tang et al. investigated time delay estimation problem in DDE based chaotic systems and proposed a solution by converting it to an optimization problem where chaotic ant swarm and a

differential evolution algorithm were used. In [7] and [8], Udaltsov et al. utilized time series analysis fused with different methods including auto-correlation function to estimate time delay. In [32], time delay and some other system parameters in time delayed chaotic systems were estimated by using optimization techniques. While satisfactory performance was obtained in these past works, all of these methods were offline where they were applied to a previously saved data.

Motivated by the currently available time delay estimation methods applied to DDE based chaotic systems being offline, in this work, we applied the method in [33] and [34] to estimate uncertain time delay online. This method considers the time delay as parameter which affects the system nonlinearly. The aforementioned is initiated via the design of a tuning function based observer signal for the state. The observer signal includes a sensitivity function based time delay estimation law. The tuning function in the state observer and the sensitivity function in the time delay estimator are the derived from a min–max optimization problem. The stability of the estimator is investigated in two sub-parts. In the first part, Lyapunov–type analysis is utilized to ensure the boundedness of all system signals under closed-loop operation. In the second part, convergency of time delay estimator is ensured providing that satisfaction of a nonlinear persistent excitation (PE) condition. The proof is based on showing that the time delay estimation error decreases by a finite number over every interval of time till it reaches to the vicinity of zero. Extensive simulation results are given to validate the estimation technique. Specifically, DDE based chaotic system models from [7, 9, 10, 11] are borrowed and the performance of the time delay estimator is demonstrated.

## 2. Delay Differential Equation based Chaotic System Model (Gecikmeli Türev Eşitliği Tabanlı Kaotik Sistem Modeli)

In this study, we consider the following general model of scalar DDE based chaotic systems:

$$\frac{dx}{dt} = f(x, x_\tau) \quad (1)$$

where  $x(t) \in \mathfrak{R}$  is state,  $x_\tau = x(t - \tau) \in \mathfrak{R}$  is the delayed state with  $\tau \in \mathfrak{R}$  denoting the time delay, and  $f \in \mathfrak{R}$  is a nonlinear function including state and delayed state. In the subsequent

development, we will assume that state  $x(t)$  and its past values are available, the structure of the nonlinear function  $f$  is known but the time delay  $\tau$  is uncertain.

There are several models that fit the general description in (1) [7, 31, 10, 11]. While, in this work, we focus on the general model and base our findings on it, we also present numerical studies on the following models which may be given as examples of (1). The model 1 is the time delayed logistic chaotic system which is given as [31]:

$$\dot{x} = -26x + 104x_\tau(1 - x_\tau) \tag{2}$$

which shows chaotic behavior for  $\tau = 0.5 \text{ sec}$ . The second model is given as [11]:

$$\dot{x} = -0.2x_\tau + 0.2 \tanh(10x_\tau) \tag{3}$$

for  $\tau = 10 \text{ sec}$  a chaotic double-scroll attractor is observed in  $x - x_\tau$  plane. The third model is given as [7]:

$$\dot{x} = \frac{1}{9 \times 10^{-6}}(-x + 3.5 \sin^2(x_\tau - 2.43)) \tag{4}$$

where chaotic behavior is observed for  $\tau = 514 \times 10^{-6} \text{ sec}$ . The fourth model is given as [10]:

$$\dot{x} = 1.7x_\tau - x_\tau^3 \tag{5}$$

where chaotic behavior is observed for  $\tau = 1.0 \text{ sec}$ . Following model assumptions are required by the subsequently designed time delay estimation method.

*Assumption 1:* The uncertain time delay  $\tau$  is bounded with respect to  $\tau \in R_\tau$  with the region which is stated as  $R_\tau = [\tau_{\min}, \tau_{\max}]$  where  $\tau_{\min}$  is known lower and  $\tau_{\max}$  is known upper bounds, respectively. The nonlinear function  $f$  is either convex or concave in a region  $R_f = [\underline{\tau}, \tau]$  of  $\tau$  that includes  $R_\tau$  (i.e.  $0 < \underline{\tau} \leq \tau_{\min} \leq \tau \leq \tau_{\max} \leq \tau$ ).

*Assumption 2:*  $x(t)$  is bounded and Lipschitz in time as follows

$$|x(t_1) - x(t_2)| \leq L_1 |t_1 - t_2| \quad \forall t_1, t_2 \in \mathbb{R}^+$$

where  $L_1 \in \mathbb{R}^+$  is a Lipschitz constant. The function  $f$  is assumed to be Lipschitz wrt its arguments as

$$|f(t_1) - f(t_2)| \leq L_2 (|x(t_1) - x(t_2)| + |\tau_0(t_1) - \tau_0(t_2)|) \tag{7}$$

for some time-varying function  $\tau_0$  where  $L_2 \in \mathbb{R}^+$  is a positive Lipschitz constant.

### 3. Time Delay Estimator (Zaman Gecikmesi Kestirimcisi)

The time delay estimator is based on design of an observer signal. This auxiliary observer signal will be introduced for developing error system and design of the time delay estimator will follow.

The observer signal is shown as  $\hat{x}(t) \in \mathfrak{R}$  and updated as given below

$$\dot{\hat{x}} = -\alpha(\tilde{x} - \text{sat}(\frac{1}{\varepsilon}\tilde{x})) + \hat{f} - a^* \text{sat}(\frac{1}{\varepsilon}\tilde{x}) \tag{8}$$

where  $\alpha \in \mathbb{R}^+$  is a constant gain,  $a^*(t) \in \mathfrak{R}$  is the tuning function that is yet to be obtained,  $\varepsilon \in \mathfrak{R}$  is the desired precision,  $\hat{f} \doteq f(x, x_\tau)$  where  $\hat{\tau}(t)$  is the subsequently designed estimate of uncertain time delay  $\tau$ , and  $\tilde{x}(t) \in \mathfrak{R}$  is the observer error stated as:

$$\tilde{x} \doteq \hat{x} - x \tag{9}$$

and  $\text{sat}(\cdot)$  denotes saturation function as:

$$\text{sat}(z) = \begin{cases} 1 & , \quad 1 \leq z \\ z & , \quad |z| < 1 \\ -1 & , \quad z \leq -1 \end{cases} \tag{10}$$

The observer error signal  $\tilde{x}$  is analyzed by taking the time derivative of observer error in (9) (10).

$$\dot{\tilde{x}} = -\alpha\tilde{x}_\varepsilon + \hat{f} - f - a^* \text{sat}(\frac{1}{\varepsilon}\tilde{x}) \tag{11}$$

where (1) and (8) were utilized and  $\tilde{x}_\varepsilon(t) \in \mathfrak{R}$  is defined as

$$\tilde{x}_\varepsilon \doteq \tilde{x} - \varepsilon \text{sat}(\frac{1}{\varepsilon}\tilde{x}). \tag{12}$$

(6)

The following update law is proposed:

$$\dot{\hat{\tau}} = \text{Proj}\{-\Gamma \tilde{x}_\varepsilon \phi^*\} \quad (13)$$

where  $\phi^*(t) \in \mathfrak{R}$  is sensitivity function that will subsequently be obtained,  $\Gamma$  is a constant positive adaptation gain, and the projection strategy  $\text{Proj}\{\cdot\} \in \mathfrak{R}$  is introduced to ensure that the estimated time delay  $\hat{\tau}$  always resides in the known bounds in Assumption 1 (i.e.,  $\hat{\tau} \in R_\tau$ ) and has the following general form

$$\hat{\tau} = \begin{cases} \tau_{\max} & \text{if } \tau_{\max} < \hat{\tau} \\ \hat{\tau} & \text{if } \tau_{\min} \leq \hat{\tau} \leq \tau_{\max} \\ \tau_{\min} & \text{if } \hat{\tau} < \tau_{\min} \end{cases} \quad (14)$$

The tuning function,  $a^*(t)$  in (8) and the sensitivity function,  $\phi^*(t)$  in (13) are the solutions of min-max optimization problem which is given as [34]

$$a^* = \min_{\phi} \max_{\tau \in R_f} J(\phi, \tau) \quad (15)$$

$$\phi^* = \arg \min_{\phi} \max_{\tau \in R_f} J(\phi, \tau) \quad (16)$$

where the performance index  $J$  is given as

$$J = \text{sat}\left(\frac{1}{\varepsilon} \tilde{x}\right) [\hat{f} - f - \Gamma \tilde{\tau} \phi] \quad (17)$$

where  $\tilde{\tau}(t) \in \mathbf{R}$  is estimation error as follows

$$\tilde{\tau} \doteq \hat{\tau} - \tau. \quad (18)$$

The solutions of the min-max optimization problem in (15) and (16) for  $\phi^*(t)$  and  $a^*(t)$  are obtained as [34], [35]

when  $\tilde{x}(t) < 0$

$$a^* = \begin{cases} 0 & \text{f is concave on } R_f \\ a_1 & \text{f is convex on } R_f \end{cases} \quad (19)$$

$$\phi^* = \begin{cases} \frac{\partial f}{\partial \tau} |_{\tau=\hat{\tau}} & \text{f is concave on } R_f \\ \phi_1 & \text{f is convex on } R_f \end{cases} \quad (20)$$

when  $0 \leq \tilde{x}(t)$

$$a^* = \begin{cases} a_1 & \text{f is concave on } R_f \\ 0 & \text{f is convex on } R_f \end{cases} \quad (21)$$

$$\phi^* = \begin{cases} \phi_1 & \text{f is concave on } R_f \\ \frac{\partial f}{\partial \tau} |_{\tau=\hat{\tau}} & \text{f is convex on } R_f \end{cases} \quad (22)$$

In (19)–(22),  $a_1$  and  $\phi_1$  are scalar time-varying functions obtained from:

$$a_1 = \frac{(\hat{f} - \underline{f})(\hat{\tau} - \bar{\tau}) - (\bar{f} - \underline{f})(\hat{\tau} - \underline{\tau})}{\beta(\bar{\tau} - \underline{\tau})} \quad (23)$$

$$\phi_1 = \frac{\bar{f} - \underline{f}}{\Gamma(\bar{\tau} - \underline{\tau})^T} \quad (24)$$

where  $\underline{f} \doteq f(x, x_\tau)$ ,  $\bar{f} \doteq f(x, x_{\bar{\tau}})$ , and  $\beta \in \mathfrak{R}$  is as follows:

$$\beta = \begin{cases} 1 & \text{f is convex on } R_f \\ -1 & \text{f is concave on } R_f \end{cases} \quad (25)$$

#### 4. Stability Analysis (Kararlılık Analizi)

In this study, Lyapunov based methods are utilized in stability analysis. Lyapunov based methods are used to determine the behaviour of systems. And, especially in control theory, it is used to investigate the stability of the system. Reader is referred to [36] and [37] for deep information about Lyapunov based methods.

The stability analysis is conducted in two sub-parts which are merged in the following theorem. The first part focuses on ensuring global boundedness of signals. In the second part, convergence of time delay estimator is analyzed.

*Theorem 4.1:* Adaptive update law in (13) guarantees that  $\tilde{x}_\varepsilon \in L_2 \cap L_\infty$  therefore, global boundedness of overall adaptive system and stability of the estimator. Furthermore, the estimator ensures the convergence of time delay estimation error  $\tilde{\tau}(t)$  to the vicinity of the origin in the sense that:

$$|\tilde{\tau}(t)| \leq \sqrt{\gamma} a s t \rightarrow \infty \quad (26)$$

provided the following nonlinear PE condition holds:

$$\beta[f(\hat{\tau}(t_1), x(t_2)) - f(\tau, x(t_2))] \geq \varepsilon_u |\hat{\tau}(t_1) - \tau| \quad (27)$$

where  $t_2 \in [t_1, t_1 + T_0]$  for some positive constant  $T_0$ ,  $t_0 < t_1$ , and  $\varepsilon_u$  is positive constant. In (26),  $\gamma$  is a positive constant defined as:

$$\gamma = \frac{8\varepsilon}{\varepsilon_u^2} (4L_1L_2 + 2\Gamma L_2L_\phi + \Gamma L_\phi^2) \quad (28)$$

where  $L_\phi$  is the known upper bound of  $\phi^*(t)$ .

*Proof:* The proof is performed in two sub-parts. Firstly, global boundedness of signals under closed-loop operation are given. Due to simplicity reasons, only a sketch of the proof is given. For the boundedness proof, Lyapunov based stability analysis methods are utilized. Specifically, a Lyapunov function, denoted by  $V(\tilde{x}_\varepsilon, \tilde{\tau})$ , is proposed

$$V \doteq \frac{1}{2} \tilde{x}_\varepsilon^2 + \frac{1}{2} \tilde{\tau}^2 \quad (29)$$

which is nonnegative. After taking time derivative of (29) and then substituting necessary terms yielded

$$\dot{V} \leq -\alpha \tilde{x}_\varepsilon^2 \quad (30)$$

from which the boundedness of the signals could be demonstrated. After ensuring the boundedness of the closed-loop signals, convergence of time delay estimator is examined with detailed analysis. The reader is referred to [34] for rest of the proof.

It is to be highlighted that, from the definition of  $\gamma$  in (28), it is clear that  $\gamma$  can be made smaller by choosing a smaller  $\varepsilon$ . Specifically, as the desired precision  $\varepsilon \rightarrow 0$ , then  $\gamma \rightarrow 0$ ; thus, the time delay estimation error is driven to the origin.

**5. Numerical Simulation Results (Sayısal Benzetim Sonuçları)**

The efficiency of the proposed time delay estimator was tested by conducting simulations using Matlab/Simulink for the models in (2)–(5). Since the chaotic behavior of these models were previously shown in the references, we only presented the performance of the proposed time delay estimation method in subsequent subsections.

*Results for the system model in (2)*

During the simulation, the desired precision and the gains for time delay estimator in (13) were selected as  $\varepsilon = 0.2$ ,  $\alpha = 400$  and  $\Gamma = 2$ . The bounds of the time delay  $\tau$  were taken as 0.2sec and 1.1sec. The initial values of state ( $x$ ) and estimated state ( $\hat{x}$ ) were chosen as 1.1 and the initial value of  $\hat{\tau}$  was chosen as lower bound of the time delay. The chaotic behavior of the system in (2) and estimated time delay  $\hat{\tau}$  are presented in Figure 1 and Figure 2, respectively. From Figure 2, it is obvious that estimation of time delay is obtained.

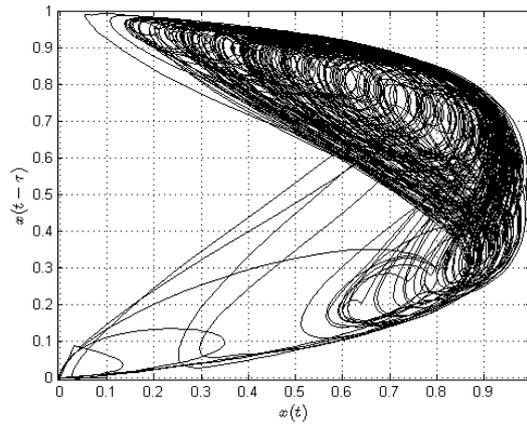


Figure 1: Chaotic behavior in phase plane of model in (2)

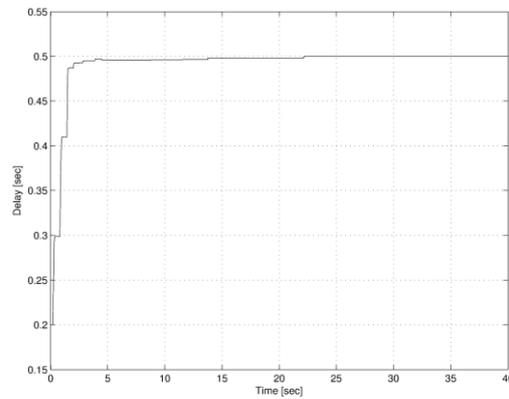


Figure 2: The time delay estimate  $\hat{\tau}$  for the model in (2)

### Results for the system model in (3)

During the simulation, the desired precision and the gains for time delay estimator in (13) were selected as  $\varepsilon = 0.00001$ ,  $\alpha = 12$  and  $\Gamma = 4500$ . The bounds of the time delay  $\tau$  were taken as 8sec and 15sec. The initial values of state ( $x$ ) and estimated state ( $\hat{x}$ ) were chosen as 0.1 and the initial value of  $\hat{\tau}$  was chosen as its lower bound. The chaotic behavior of the system in (3) and estimated time delay  $\hat{\tau}$  are presented in Figure 3 and Figure 4, respectively. From Figure 4, it is obvious that estimation of time delay is obtained.

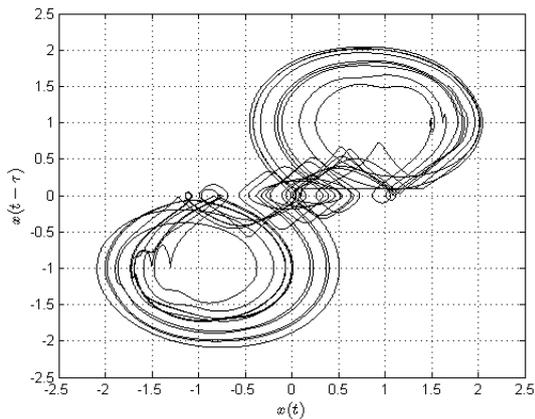


Figure 3: Chaotic behavior in phase plane of model in (3)

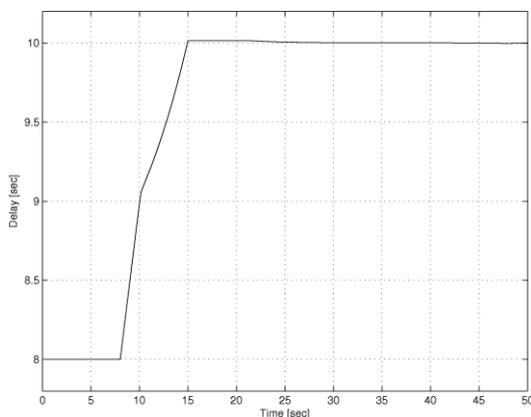


Figure 4: The time delay estimate  $\hat{\tau}$  for the model in (3)

### Results for the system model in (4)

For the system model in (4), since the time delay is very small (i.e.,  $514 \times 10^{-6}$  sec), time scaling was utilized. Specifically, the time of the dynamic model in (4) was scaled with a factor of  $10^{-6}$ , then the estimation method was applied, and the result is presented back in its original time scale. During the simulation, the desired precision and the gains for time delay estimator in (13) were selected as

$\varepsilon = 0.00001$ ,  $\alpha = 100$  and  $\Gamma = 90$ . The bounds of the time delay  $\tau$  were taken as 0.0004 sec and 0.0006 sec. The initial values of state ( $x$ ) and estimated state ( $\hat{x}$ ) were chosen as 0 and the initial value of  $\hat{\tau}$  was chosen as its lower bound. The chaotic behavior of the system in (2) and estimated time delay  $\hat{\tau}$  are presented in Figure 5 and Figure 6, respectively. From Figure 6, it is obvious that estimation of time delay is obtained.

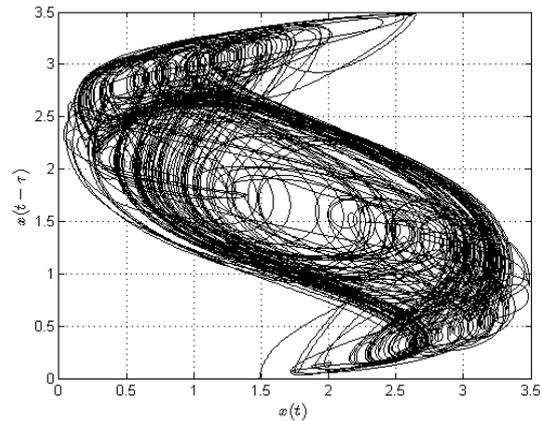


Figure 5: Chaotic behavior in phase plane of model in (4)

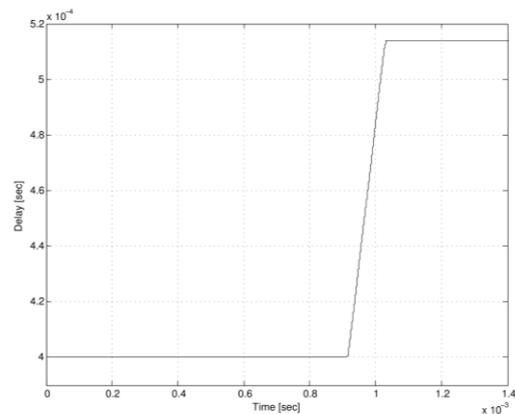


Figure 6: The time delay estimate  $\hat{\tau}$  for the model in (4)

### Results for the system model in (5)

During the simulation, the desired precision and the gains for time delay estimator in (13) were selected as  $\varepsilon = 0.0001$ ,  $\alpha = 28$  and  $\Gamma = 1$ . The bounds of the time delay  $\tau$  were taken as 0.2sec and 1.22sec. The initial values of state ( $x$ ) and estimated state ( $\hat{x}$ ) were chosen as 0.1 and the initial value of  $\hat{\tau}$  was chosen as 0.5sec. The chaotic behavior of the system in (2) and estimated time delay  $\hat{\tau}$  are presented in Figure 7 and Figure 8, respectively. From Figure 8, it is obvious that estimation of time delay is obtained.

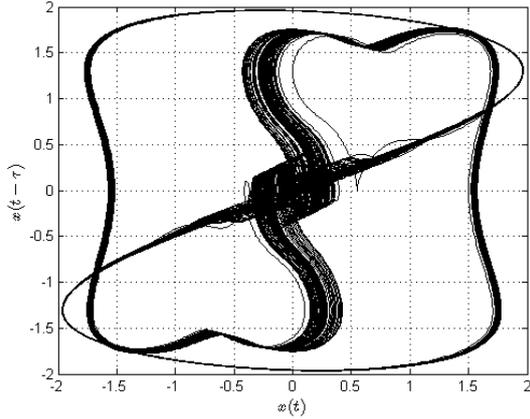


Figure 7: Chaotic behavior in phase plane of model in (5)

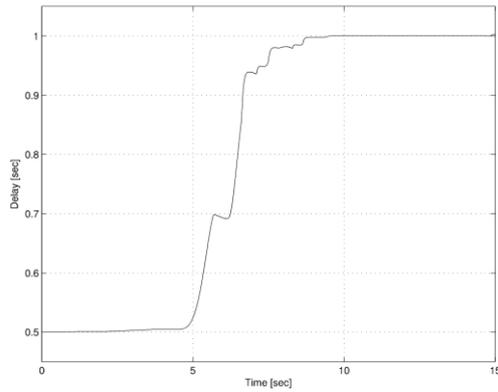


Figure 8: The time delay estimate  $\hat{\tau}$  for the model in (5)

## 6. Conclusions (Sonuçlar)

In this work, the time delay was considered as a parameter which affects the system nonlinearly and an online nonlinear parameter estimator was utilized to identify time delay in DDE based chaotic systems. The aforementioned estimation technique was based on an auxiliary state observer which was utilized to minimize the time delay estimation error. Lyapunov-based techniques were utilized to ensure the boundedness of the state observation error and time delay estimation error. The sketch of the proof of the convergence of the time delay estimation was given where a full proof can be found in [34]. The convergence proof depended on a nonlinear persistent excitation condition which may be seem as restrictive, however, as a direct consequence of the nature of the chaotic systems, it seems to be satisfied all the time. Several numerical results were presented that demonstrated the effectiveness of the proposed technique.

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