MÜHENDİSLİK FAKÜLTESİ

Uluslararası Mühendislik Araştırma ve Geliştirme Dergisi

International Journal of Engineering Research and Development

Cilt/Volume:9 Sayı/Issue:3 Aralık/December 2017 https://doi.org/10.29137/umagd.348744

Araştırma Makalesi / Research Article

The Role of FGATool in Fractional Order System Analysis Education

Bilal Şenol[1]

[1]*İnönü University, Faculty of Engineering, Computer Engineering Department, Malatya, TURKEY*

Başvuru/Received: 08/10/2017 **Kabul/Accepted**: 01/12/2017 **Son Versiyon/Final Version**: 26/12/2017

Abstract

The scenario for engineering education is changing by the effect of theoretical developments in recent years. Appropriate usage of computer technology gives great opportunity to follow these changes and provides student achievement. This paper presents FGATool, a Matlab based interface developed to take role in fractional order system analysis education. Fractional order calculus can be an unfamiliar concept for students, thus it is an indisputable fact that some analysis tools will be useful in the education process. The tool is experienced in classes of master students and the outcomes are recorded. Two main modules of FGATool are presented with their mathematical background all over the paper. Analysis of fractional order systems with fixed and uncertain parameters separately can be realized without complicated technical background. FGATool is also thought to be useful with its user-friendly interface.

Key Words

“Fractional order systems, FGATool, Stability analysis, System uncertainties.”

*** bilal.senol@inonu.edu.tr***

1. INTRODUCTION

A fractional order system can be modeled by a differential equation with orders of arbitrary real numbers. First idea of the concept lays back to late 17th century on a correspondence between Leibniz and L'Hopital. Fractional order calculus is an unfamiliar subject for researchers and especially for students who are new to non-integer world. Thus, some tools will be useful for the education process of fractional order analysis.

Nowadays, fractional order calculus can be used in many areas as an alternative technique for analysis of control systems. It also finds numerous applications in control engineering (Caponetto et al., 2010; Balenau, et al., 2016; Azar et al., 2017; Monje et al., 2010)

Recent years brought intensive usage of computer technology in all areas of education processes. It is a certain fact that computers make human lives easier in so many ways. As the main subject of this paper, control analysis education techniques has also been improved with the help of computer technology (Senol et al., 2012; Senol, 2017)

There can be found some interfaces related to fractional order system analysis in the literature. For example, the CRONE toolbox is famous to be one of the first studies in this direction (Oustaloup, 1991). *ninteger* and PIDLAB are also effective analysis tools (Valerio, 2005). PIDLAB and FOMCON can be shown as a similar tool for fractional order system analysis (Martin&Milos, 2006; Tepljakov, 2017). First version of FGATool named as UFT-FOCS can be shown as a preliminary level analysis tool (Senol, 2012; Senol&Yeroglu, 2015a)

This paper presents FGATool which is developed for easy analysis of fractional order systems. Main difference and proposal of FGATool is its ease of use for researchers who are new to the subject. The tool consists of two main modules for fractional order systems with fixed parameters and for fractional order systems with uncertain parameters. Each module includes six analysis tools. The tool can be freely downloaded from www.fgatool.com.

Organization of this paper is as follows. Section 2 includes a brief introduction to fractional order systems. Section 3 presents the features of FGATool and section 4 shows the usage and effect of FGATool for the control education process. Section 5 has the conclusion.

2. FRACTIONAL ORDER SYSTEMS

A model represented with a differential equation which has orders of non-integer numbers can be classified as a fractional order system. Let us consider the following differential equation which has arbitrary real orders.

$$\begin{aligned} &a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \dots + a_0 D^{\alpha_0} y(t) = \\ &b_m D^{\beta_m} u(t) + b_{m-1} D^{\beta_{m-1}} u(t) + \dots + b_0 D^{\beta_0} u(t) \end{aligned} \tag{1}$$

where, a_i and b_j are the coefficients of the differential equation. α_i and β_j are the real orders. The differential equation in Eq. 1 can be rewritten as a transfer function using the Laplace transform considering zero initial conditions as (Senol et al., 2017)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + \dots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + \dots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}} . \tag{2}$$

Characteristic equation of the transfer function which will be used in some analysis techniques in this paper can be obtained in the following form.

$$\Delta(s) = Y(s) + U(s) \tag{3}$$

We can convert the transfer function in Eq. 2 from s domain to the frequency domain by replacing s to $j\omega$ as follows.

$$G(j\omega) = \frac{Y(j\omega)}{U(j\omega)} = \frac{b_m (j\omega)^{\beta_m} + \dots + b_1 (j\omega)^{\beta_1} + b_0 (j\omega)^{\beta_0}}{a_n (j\omega)^{\alpha_n} + \dots + a_1 (j\omega)^{\alpha_1} + a_0 (j\omega)^{\alpha_0}} \tag{4}$$

Similarly, characteristic equation in Eq. 3 can be written in the frequency domain as,

$$\Delta(j\omega) = Y(j\omega) + U(j\omega) \tag{5}$$

Fractional order of $j\omega$ can be expressed as follows (Yeroglu&Senol, 2013)

$$(j\omega)^{\mu} = \omega^{\mu} (\cos\frac{\pi}{2}\mu + j\sin\frac{\pi}{2}\mu) \tag{6}$$

FGATool is also an effective analysis tool for fractional order systems with uncertain parameters. Coefficients of the numerator and denominator polynomials can vary in an interval in this case. Consider the fractional order plant in Eq. 2 where the coefficients change in an interval.

$$G(s,\mathbf{q})=\frac{Y(s,\mathbf{q})}{U(s,\mathbf{q})}=\frac{b_m(\mathbf{q})s^{\beta_m}+\ldots+b_1(\mathbf{q})s^{\beta_1}+b_0(\mathbf{q})s^{\beta_0}}{a_n(\mathbf{q})s^{\alpha_n}+\ldots+a_1(\mathbf{q})s^{\alpha_1}+a_0(\mathbf{q})s^{\alpha_0}} \tag{7}$$

where, the coefficients $b_j(\mathbf{q})$ and $a_i(\mathbf{q})$ linearly depend on $\mathbf{q}=[q_0,q_1,\cdots,q_q]^T$ which is in the uncertainty box (Senol et al., 2014a),

$$Q=\{\mathbf{q}:q_i\in[\underline{q_i}\ \ \overline{q_i}],\quad i=1,2,\cdots,q\}. \tag{8}$$

$\underline{q_i}$ and $\overline{q_i}$ respectively represent the lower and upper limits of the uncertain parameters q_i. Frequency domain representation of the uncertain plant in Eq. 7 is given in the following way.

$$G(j\omega,\mathbf{q})=\frac{\sum_{k=0}^{m}\left(b_k(\mathbf{q})(r_{bk}\omega^{\beta_k})\right)+\sum_{k=0}^{m}j\left(b_k(\mathbf{q})(i_{bk}\omega^{\beta_k})\right)}{\sum_{l=0}^{n}\left(a_l(\mathbf{q})(r_{al}\omega^{\alpha_k})\right)+\sum_{l=0}^{n}j\left(a_l(\mathbf{q})(i_{al}\omega^{\alpha_k})\right)} \tag{9}$$

where, r_{bk}, r_{al} and i_{bk}, i_{al} are constants of real and imaginary parts respectively. Similar to Eq. 5, an uncertain characteristic equation in the frequency domain can be expressed as,

$$\Delta(j\omega,\mathbf{q})=Y(j\omega,\mathbf{q})+U(j\omega,\mathbf{q}) \tag{10}$$

There can be found numerous studies related to fractional order systems with fixed and uncertain parameters (Senol et al., 2017; Yeroglu&Senol, 2013; Senol et al., 2014a; Senol et al., 2014b; Matusu&Prokop, 2011; Senol&Yeroglu, 2013). Next section of this paper gives brief information about features of FGATool.

3. FGATOOL

FGATool is a MATLAB based interface that is developed for easy analysis of fractional order systems. The tool can be freely downloaded from www.fgatool.com and can be started by typing *fgatool* in Matlab console window. Main window of FGATool is illustrated in Figure 1.

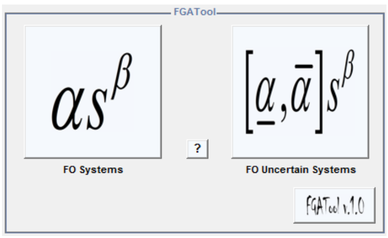

Figure 1: Main window of FGATool

Two main modules of the tool can be listed as,

- Fractional Order Systems
- Fractional Order Uncertain Systems

This section briefly introduces both modules.

3.1. FO Systems

Main appearance of Fractional Order Systems module is given in Figure 2.

Fractional Order Systems module includes the following analysis tools.

- Step Response
- Bode, Nyquist, Nichols
- Hermite-Biehler Analysis
- Roots Region Analysis
- Root Finder
- Frequency Properties

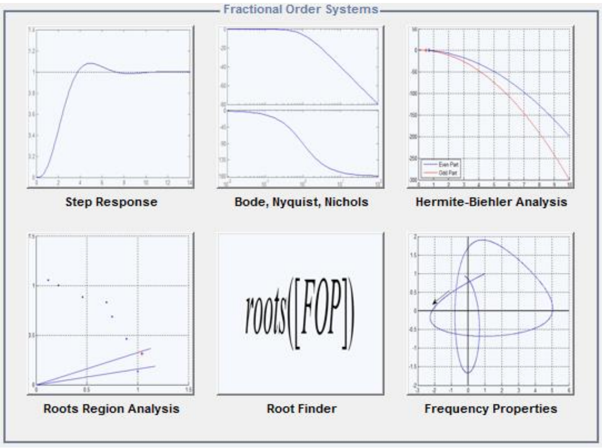

Figure 2: Fractional Order Systems module

Step Response and Bode, Nyquist, Nichols modules are developed using the mathematical background in (Xue et al., 2007). Hermite-Biehler Analysis module is an extension of the interlacing theorem for classical polynomials (Senol et al., 2014b). Roots Region Analysis module works on the roots laying on the first Riemann sheet (Senol et al., 2014a). Root Finder module proposes a method to find the roots of a fractional order polynomial (Senol et al., 2017). Frequency Properties module tests the stability on frequency plot of fractional order polynomials (Şenol&Yeroğlu, 2015b).

3.2. FO Uncertain Systems

Main appearance of Fractional Order Uncertain Systems module is given in Figure 3.

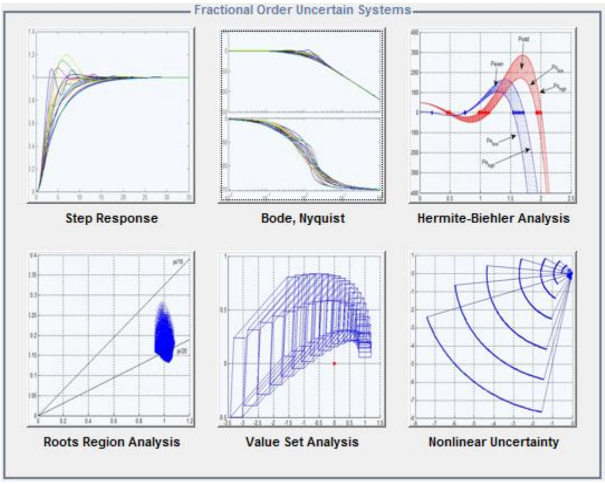

Figure 3: Fractional Order Uncertain Systems module

Fractional Order Uncertain Systems module includes the following analysis tools.

- Step Response
- Bode, Nyquist
- Hermite-Biehler Analysis
- Roots Region Analysis
- Value Set Analysis
- Nonlinear Uncertainty

Value Set Analysis and Nonlinear Uncertainty tools are based on the references (Matusu&Prokop, 2011; Senol&Yeroglu, 2013) respectively. Next section presents usage of FGATool and the easiness that FGATool brought for the control education world.

4. EXAMPLES ON FGATOOL

As known, Matlab has become a world standard with its analysis tools development environment (Senol et al., 2012). In numerous universities and research laboratories, Matlab is used for symbolic and numerical computations. Although Matlab has strong tools for control systems' analysis, fractional order systems cannot be analyzed with base features of Matlab. Thus, some tools will be needed in this case. This section presents the easiness that FGATool brings for the researchers who are interested in fractional order systems.

Let us consider the following plant with orders of fractional numbers.

$$G(s) = \frac{1}{s^{3.1} + 2s^{2.2} + 2s^{1.1} + 1} \qquad (11)$$

It is not possible to build this transfer function with Matlab command *tf()*. However, it is easy to define this plant with FGATool and obtain its step response, Bode, Nyquist and Nichols plots. Figure 4 shows the *Step Response* module of FGATool with the parameter entry for the plant in Eq. 11.

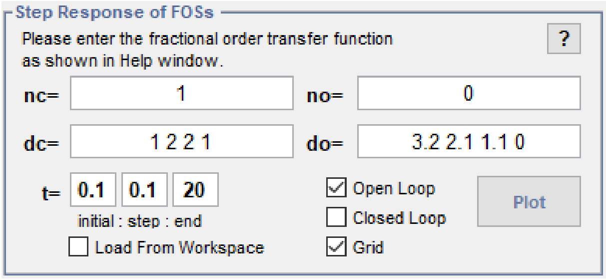

Figure 4: Parameter entry for the plant in Eq. 11.

One can easily obtain the step response of the plant in Eq. 11 by clicking Plot. Figure 5 illustrates the unit step response of Eq. 11 obtained using FGATool.

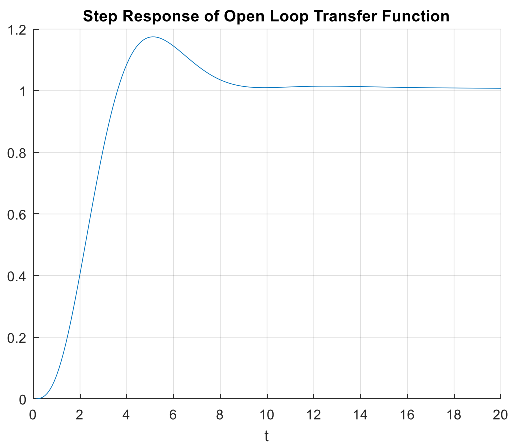

Figure 5: Step response of the plant in Eq. 11.

Characteristic equation of a transfer function should be used in some analysis techniques. Consider the following characteristic equation of the plant in Eq. 11.

$$\Delta(s) = s^{3.1} + 2s^{2.2} + 2s^{1.1} + 2 \qquad (12)$$

The Hermite-Biehler Theorem, also known as the interlacing property of Hurwitz polynomials is an effective analysis method developed for classical systems. Its extension for fractional order case can be found in (Senol et al., 2014b). In order to test the interacing property of any system, even and odd parts of its characteristic equation has to be found. The stability condition is to have the roots of even and odd parts placed in the positive complex plane and interlace one by one.

Considering the characteristic equation form in Eq. 5, even and odd parts can be calculated using following equations.

$$\Delta^e(j\omega) = \sum_{i=0}^{n}\left[a_i \cos\left(\frac{\pi}{2}\alpha_i\right)\right]\omega^{\alpha_i}$$
$$\Delta^o(j\omega) = \frac{\sum_{i=0}^{n} j\left[a_i \sin\left(\frac{\pi}{2}\alpha_i\right)\right]\omega^{\alpha_i}}{j\omega} \qquad (13)$$

With the help of FGATool, plots of the even and odd parts of Eq. 12 can be drawn easily. Figure 6 illustrates the interlacing roots of even and odd parts of the characteristic equation in Eq. 12.

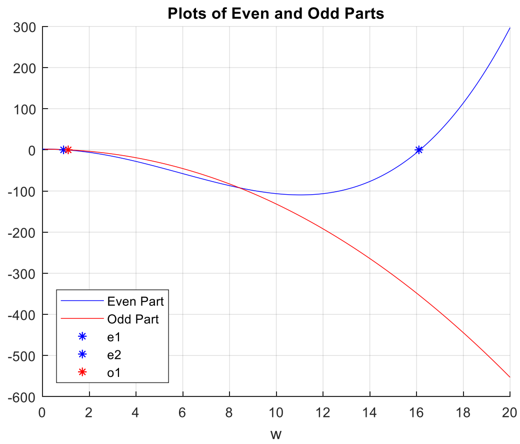

Figure 6: Plots of even and odd parts of Eq. 12.

As the plots of even and odd parts cross the reference line in turn, the fractional order plant in Eq. 11 is stable.

Another analysis method implemented in FGATool investigated the stability by checking the roots placed on the first Riemann sheet (Senol et al., 2014a). The method works on extending the fractional orders of the characteristic equation to integer numbers with a least common multiplier K. For the characteristic equation in Eq. 12, fractional orders can be extended to integer numbers with K=10. Extended integer order form of Eq. 12 is obtained as,

$$\Delta(s^{10}) = s^{31} + 2s^{21} + 2s^{11} + 2 \qquad (14)$$

Now, 31 roots of Eq. 14 can be calculated easily with Matlab *roots()* command. Then, angle values of each root have to be calculated in radians. Stability condition is to have some of the roots in the region $\pi / K < \arg(r_i) < \pi / 2K$ and to have no roots in the region $\pi / 2K < \arg(r_i) < 0$. r_i are the roots calculated from the integer order characteristic equation. This can be checked easily using FGATool. Figure 7 shows the entry of Eq. 14 on FGATool *Roots Region Analysis* module.

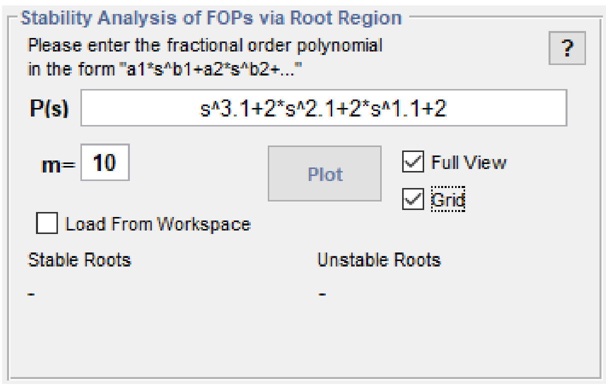

Figure 7: Parameter entry for the characteristic equation in Eq. 12.

Figure 8 illustrates the roots of Eq. 14 placed in the first Riemann sheet. Figure 8shows that there are two roots in the stability region (marked with *) and there is no root in the instability region. Thus, the fractional order plant in Eq. 11 is stable.

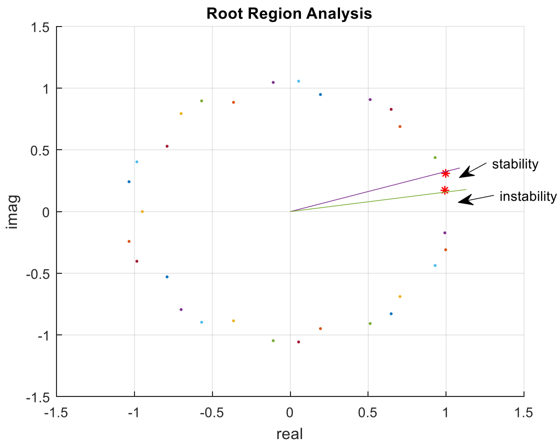

Figure 8: Roots region analysis of the fractional order plant in Eq. 11.

FGATool also includes a module to compute the roots of a fractional order polynomial. Consider the fractional order characteristic equation in Eq. 12. Figure 9 illustrates the parameter entry for *Root Finder* module.

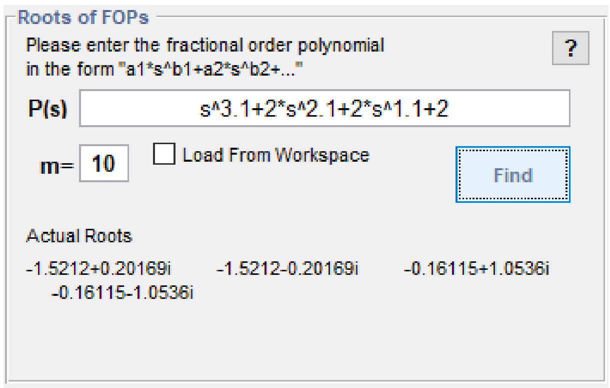

Figure 9: Roots of the characteristic equation in Eq. 12.

As can be seen in Figure 9, roots of Eq. 12 are negative signed. Another feature of FGATool checks the stability of a fractional order polynomial by its *Frequency Properties* module. This module gives the frequency plot of the polynomial. Stability condition is to have the plot start in the positive real axis, move strictly counterclockwise direction and pass $\alpha+1$ regions. α is the order of the polynomial. Frequency plot of the characteristic equation in Eq. 12 is given in Figure 10. It is clear in Figure 10 that the plot provides the stability condition.

Thus, stability of the fractional order plant in Eq. 11 has been analyzed via different methods using FGATool. Researchers and students can freely use FGATool without much knowledge of fractional order calculus and this shows the easiness that FGATool brought to the control world.

Now, let us consider the following fractional order plant with uncertain parameters.

$$G(s) = \frac{1}{s^{3.1} + [1,3]s^{2.2} + [2,3]s^{1.1} + 1} \tag{15}$$

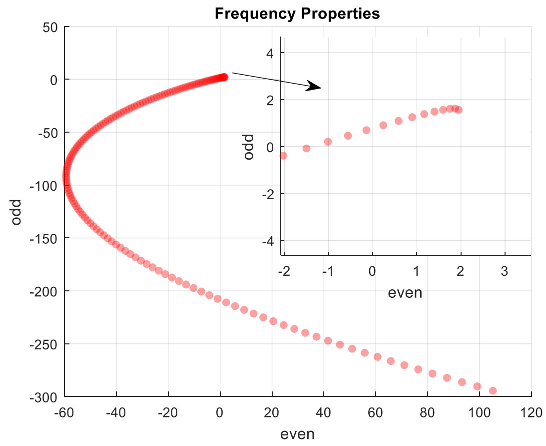

Figure 10: Frequency plot of Eq. 12.

There are two parameters in Eq. 15 that are uncertain. All conditions that the uncertain parameters form have to be investigated. Figure 11 shows the parameter entry for the uncertain plant in Eq. 15.

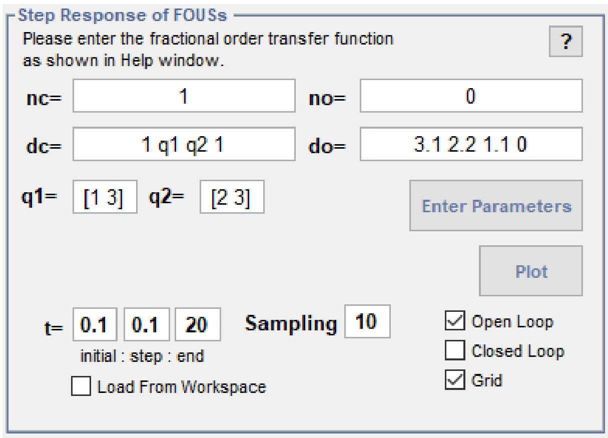

Figure 11: Parameter entry for the uncertain plant in Eq. 15.

Step responses obtained by considering 10 samples on each interval of the uncertain plant in Eq. 15 is illustrated in Figure 12. Roots region analysis considering the roots on the first Riemann sheet can be seen in Figure 13.

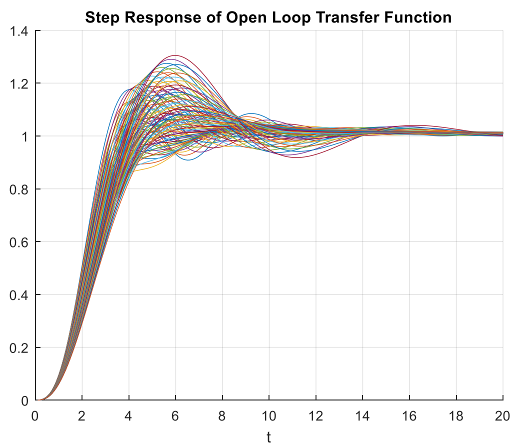

Figure 12: Step responses of the uncertain plant in Eq. 15.

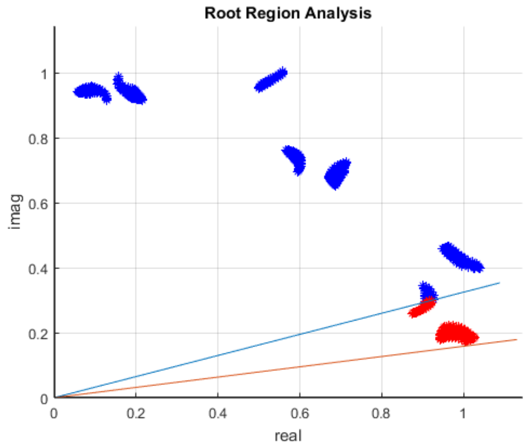

Figure 13: Roots region analysis of the uncertain plant in Eq. 15.

It is clear from Figure 12 and Figure 13 that the fractional order plant in Eq. 15 satisfies the stability criterion with its changing parameters.

5. CONCLUSION

This paper aims on a study to develop a computer interface to help student achievement in fractional order control education. However fractional order calculus describes real world processes better then classical case, its mathematical background may not be easy for student understanding. Education process for fractional order calculus will be improved with the help of easy to use computer interfaces. FGATool, a graphical analysis tool for fractional order systems with fixed and uncertain parameters is brought to the literature in this paper. This tool is thought to be useful for researchers and student who are interested in fractional order world.

REFERENCES

Azar, A. T., Sundarapandian, V. and Adel O. (2017). Fractional order control and synchronization of chaotic systems, vol. 688, Springer.

Baleanu, D. et al., (2016). Fractional calculus: models and numerical methods, vol. 5. World Scientific.

Caponetto, R., Dongola, G., Fortuna L. and Petras I. (2010). Fractional Order Systems, Modeling and Control Applications, World Scientific, Singapore.

Martin, C. And Miloš, S. (2006). PID controller design on Internet: www.PIDlab.com, Department of Cybernetics, University of West Bohemia in Pilsen.

Matusu, R. and Prokop, R. (2011). Graphical analysis of robust stability for systems with parametric uncertainty: an overview, Transactions of the Institute of Measurement and Control, vol. 33 (2), pp. 274–290.

Monje, C. A., Chen, Y. Q., Vinagre, B. M., Dingyü, X. and Feliu, V. (2010). Fractional-order systems and controls, Springer, New York.

Oustaloup, A. (1991). La Commande CRONE: Commande Robuste d’Ordre Non Entier, Hermes.

Senol, B. (2017). FGATool for Time and Frequency Analysis of Systems with Uncertainties, International Conference on Artificial Intelligence and Data Processing (IDAP17).

Senol, B. and Yeroglu, C. (2013). Frequency boundary of fractional order systems with nonlinear uncertainties, Journal of the Franklin Institute, vol. 350 (7), pp. 1908-1925.

Senol, B. and Yeroglu, C. (2015a). Kesir Dereceli Sistemlerin Grafiksel Analiz Arayüzü, Otomatik Kontrol Türk Milli Komitesi Toplantısı 2015 (TOK15).

Senol, B., Ates, A., Alagoz, B. B. and Yeroglu, C. (2014a). A numerical investigation for robust stability of fractional-order uncertain systems, ISA Transactions, vol. 53 (2), pp. 189-198.

Senol, B., Matušů, R. and Gul, E. (2017). Fractional Order Stability of Systems, International Conference on Artificial Intelligence and Data Processing (IDAP17).

Senol, B., Yeroglu, C. and Erturkler M. (2012), Development of a User Friendly Toolbox for Advanced Control Education, 2012 International Conference on Fractional Differentiation and its Applications (ICFDA).

Senol, B., Yeroglu, C. and Tan, N. (2014b). Analysis of Fractional Order Polynomials Using Hermite-Biehler Theorem, 2014 International Conference on Fractional Differentiation and its Applications (ICFDA).

Şenol, B. and Yeroğlu, C. (2015b). Kesir Dereceli Belirsiz Sistemler İçin Dayanıklı Analiz Araçlarının ve Arayüz Programlarının Geliştirilmesi, Ph. D. Thesis, www.bilalsenol.com.

Tepljakov, A. (2017) Fractional Order Modeling and Control (FOMCON), www.fomcon.net.

Valerio, D. (2005) Ninteger v. 2.3 Fractional control toolbox for MatLab, http://web.ist.utl.pt/~duarte.valerio.

Xue, D., Chen, Y. Q. and Atherton, A. (2007). Linear Feedback Control: Analysis and Design with MATLAB, SIAM Press.

Yeroglu, C. and Senol, B. (2013). Investigation of robust stability of fractional order multilinear affine systems: 2q-convex parpolygon approach, Systems & Control Letters, vol. 62 (10), pp. 845-855.