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RESEARCH ARTICLE

Operational Implications of Time Window Relaxation in Vehicle Routing Problems

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ABSTRACT
The Vehicle Routing Problem with Time Windows (VRPTW) poses a significant challenge in logistics, requiring vehicles to meet the objective of minimising costs—such as distance travelled and total travel time—while adhering to specified delivery time constraints and vehicle capacities. This study investigates the implications of relaxing time window constraints by transitioning from VRPTW instances to standard Vehicle Routing Problem (VRP) instances. Our findings highlight notable differences between VRP and VRPTW configurations, particularly in total route length and consistency of route metrics. Removal of time window constraints generally resulted in shorter and more uniform route lengths, indicating operational benefits under certain conditions. However, our comparisons also revealed substantial variability in route structures across datasets, emphasising the cost implications of adhering to strict time windows. This study underscores the critical balance logistics firms must strike between operational efficiency and customer satisfaction when navigating the complexities of VRPTW. This research provides a foundation for future investigations into optimizing route planning under varying logistical constraints, with potential implications for enhanced flexibility and reduced operational costs despite dynamic delivery requirements. We used a state-of-the-art heuristic solver to solve instances from standard benchmark datasets heavily used for VRPTW literature.

Keywords: logistics, minimizing costs, delivery time constraints, operational efficiency, route planning

Introduction

The Vehicle Routing Problem (VRP) is a complex combinatorial optimisation problem that seeks to design optimal routes for multiple vehicles delivering goods or services to various destinations under specified constraints. Originating from the Travelling Salesman Problem (TSP), where the goal is to find the shortest possible route for a salesperson to visit each city and return to the origin, VRP extends this goal by incorporating multiple vehicles with capacities operating from one or more depots.

VRP aims to reduce costs like distance, time, and fuel while respecting limits on vehicle capacity, delivery times, customer needs, and driver shifts. The problem's complexity increases with the number of vehicles, geographic spread of delivery points, and quality and quantity of constraints, making it challenging to solve the problem optimally.

VRP is pivotal in logistics and has significant applications in transportation, distribution, and supply chain management, impacting overall efficiency and operational costs. It has spurred diverse variants with multiple attributes and solution methodologies, from exact algorithms to heuristic and metaheuristic approaches, to address the different forms and industry-specific requirements.

The Vehicle Routing Problem with Time Windows (VRPTW) is a generalisation of the VRP, where each delivery location has a specific delivery time window. The objective is to minimise the total route cost while respecting these time constraints. VRPTW involves determining optimal routes for vehicles delivering to multiple locations. Each customer must be served within a specified time period, and vehicles must arrive on time. The solution aims to reduce costs, such as distance travelled or total travel time, while adhering to all time window constraints and vehicle capacity limits.

The primary focus of this study is to explore the effects of time window constraints on route structuring in VRP. Specifically, we aim to demonstrate, compare, and analyse the modifications in route configurations when these constraints are altered or removed.

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Traditionally, time window constraints are integral to ensuring customer satisfaction; they dictate when deliveries must be made. However, firms may be unable to adhere to these constraints due to various unforeseen circumstances.

This research investigates the implications of transitioning from a standard VRP with strict time windows to a Capacitated Vehicle Routing Problem (CVRP) in which these constraints are either absent or significantly relaxed. By removing or loosening time window constraints, an alternative route structure emerges, which we examine using numerous metrics to elucidate different aspects of route formation.

The fundamental question we address is as follows: What potential benefits can logistics firms accrue from not satisfying time window constraints under certain conditions? This relates to operational flexibility and potential gains in efficiency and customer service under varying logistical constraints.

Our paper is structured in the following manner. The literature review section provides a succinct yet comprehensive overview of existing VRPTW studies. Subsequently, the research problem section articulates our specific investigative focus within this field, setting the stage for further discussion. The methods section delineates the methodologies and analytical techniques used to address the research problem. The results section then presents the findings of our study. Finally, the conclusions section summarises the implications of our findings, discusses the limitations of our study, and suggests avenues for future research.

Literature Review

While the scope of our article necessarily limits the expansiveness of our literature review, we have endeavoured to include a substantial number of citations. These references are intended to guide interested readers towards a more in-depth exploration of topics related to VRPTW. By doing so, we ensure that although our review may not be exhaustive, it still serves as a valuable starting point for readers seeking extensive knowledge on current research and methodologies in this area.

In particular, we have included essential review papers and their brief descriptions to address the scope limitations and ensure comprehensive topic coverage. This approach provides readers with foundational insights and context while directing them to more detailed reviews of the cited works. Each selected review has been chosen for its relevance and contribution to the field, ensuring that despite the brevity of our review section, the quality and depth of the information remain uncompromised.

Vehicle Routing Problem with Time Windows

VRPTW addresses the challenge of designing optimal routes from a depot to a set of geographically scattered points, where each delivery or pickup location is constrained within specific time periods (Bräysy & Gendreau, 2002). This problem variant not only aims to minimise the total routing cost, including the distance and travel time, and it also adds the complexity of scheduling within tight time frames, which significantly impacts solution feasibility and optimisation strategies.

In the context of the problem, time constraints can be categorised into two types: soft and hard. Soft time windows allow for some flexibility in arrival times, often incurring penalties for early or late arrivals. In contrast, hard time windows strictly enforce arrival within designated periods, with violations typically rendering a solution infeasible. Studies using hard time windows are more common than those using soft windows (Figliozzi, 2010).

We can also differentiate between tight and loose time windows and between narrow and wide time windows. A tight time window significantly affects the solution by acting as an active constraint. Conversely, a time window is deemed narrow if it is small compared to the planning horizon, for example, 10 min within 12 h. However, a narrow time window does not automatically imply tightness. When time windows are predominantly broad, the VRPTW resembles a Capacitated Vehicle Routing Problem (CVRP) (Desaulniers et al., 2014, pp. 119–120).

Researchers have developed a range of exact and heuristic solution methodologies to address the VRPTW. Exact algorithms (Pecin et al., 2017), including branch-and-cut (Bard et al., 2002) and dynamic programming techniques, have been designed to guarantee optimality. Although these methods are powerful for solving smaller instances or instances in which the solution's optimality is critical, their computational expense often becomes prohibitive as the problem size increases.

On the other hand, heuristic approaches, such as genetic algorithms (Nazif & Lee, 2010), simulated annealing (Wang et al., 2013), and tabu search (Cordeau et al., 2001), do not offer the same optimality guarantees but are notably fast and effective for larger problem instances. These methods have been engineered to produce high-quality solutions within acceptable computational times, making them more suitable for practical applications in which near-optimal solutions are sufficient and computational resources or time are limited (Laporte et al., 2014). Each of these heuristic techniques leverages different mechanisms to explore the solution space and avoid local optima, thereby increasing the likelihood of identifying feasible and reasonable solutions, (see Prodhon & Prins, 2016; Labadie et al., 2016).

The following are critical works on vehicle routing problem with time windows that are highly beneficial for researchers. We provide summaries of each work to elucidate their contributions to the field.

We would like to begin with a book chapter. For a rigorous introductory exploration of the VRPTW, Desaulniers et al. (2014) is indispensable. This comprehensive study delves into various mathematical formulations of VRPTW and critiques both exact and heuristic solution methodologies. This study examines exact techniques, such as branch-and-cut-and-price, branch-and-cut, and set partitioning, alongside heuristic and metaheuristic approaches, including local search techniques and evolutionary algorithms.

Bräysy and Gendreau, (2005a) examine conventional heuristic methods and modern local search algorithms, analysing their effectiveness using Solomon's (Solomon, 1987) benchmark problems. Their paper highlights the importance of evaluating heuristics through Pareto optimality to effectively compare different methodologies. The paper details each method's basic features and their experimental outcomes.

In the second part of their study, Bräysy and Gendreau (2005b) comprehensively reviewed metaheuristics applied to VRPTW, focusing on strategies to design cost-efficient routes from depots to various locations. The paper details these methods' structures and benchmarks their performance on Solomon's test problems (see Solomon, 1987), demonstrating how they effectively navigate and optimise complex routing challenges.

X. Liu et al. (2023) examined the evolution of solving methods for the VRPTW, emphasising its relevance in real-life logistics challenges. Following the PRISMA guidelines (see Page et al., 2021) for methodical research and analysis, their paper reviewed literature from 2018-2022, revealing a dominance of approximate methods (86%), with a significant inclination towards meta-heuristics over simple heuristics. Notably, the authors reported that approximately 40% of the studies integrate hybrid approaches, blending multiple algorithms to tackle the multi-constrained, multi-objective nature of VRPTW.

Research Problem

Background

The increasing service expectations of customers present a formidable challenge in the logistics sector, particularly for parcel delivery companies. For a comprehensive meta-analysis on logistics customer service research, (see Leuschner et al., 2013). Customers increasingly demand high-quality services and expect these services to be delivered within specific time frames that align with their availability and immediate needs (Salari et al., 2022). In response, logistics firms have begun to offer time-window services designed to meet these precise temporal requirements. Although this approach ostensibly enhances customer satisfaction and allows companies to charge premiums for tailored delivery options, it also introduces significant operational complexities and costs (Köhler et al., 2023).

One critical issue in adhering to time window constraints is the potential disruption to route optimisation, which directly affects efficiency (Schaumann et al., 2023). Traditional routing algorithms aim to minimise the total route length while reducing travel time and fuel consumption. However, when specific time windows are imposed, these algorithms must be adjusted or completely redesigned to accommodate the constraints, often at the expense of route efficiency. This adjustment can lead to suboptimal routes that are longer and more convoluted than those generated under a more flexible system.

While time window services can provide a competitive edge and cater to individual customer preferences, they require careful consideration of their associated costs and logistical challenges. Balancing these factors is essential for logistics companies that aim to maintain efficiency while satisfying evolving service expectations (Deflorio et al., 2012).

Problem Statement

This investigation discerns the immediate benefits and potential costs and efficiency trade-offs engendered by time window constraints. These constraints typically arise from the need to align pick-up and delivery schedules with customer availability, thereby purportedly enhancing service quality and customer satisfaction. However, the consequences of these constraints on operational efficiency and resource utilisation still need to be adequately understood.

To systematically assess the impacts, we consider a series of critical metrics reflective of route optimisation and demand fulfilment, segmented into three primary categories: route lengths, route demands, and route customers. Members of these metrics will be explained in the methods section, detailing how each metric is calculated and utilised in the analysis.

Purpose of the Study

The study methodically investigated the effects of time window constraints on logistical operations, specifically, in the context of vehicle routing problems. This research aims to present a comprehensive analysis of how these constraints influence key performance metrics, such as route length, demand fulfilment, and service efficiency. By using empirical data and graphical representations, this study seeks to elucidate the operational trade-offs and costs associated with the enforcement of time window constraints, thereby contributing valuable insights to the optimisation strategies employed by logistics firms.

Significance

This study quantifies the additional operational complexities introduced by time window constraints in vehicle routing problems. Through numerical analysis, this paper systematically compares various performance metrics of routing processes with and without the imposition of time window constraints. The comparative analysis aims to elucidate the impact of these constraints on logistical efficiency, cost, and service quality, providing insights that can guide the optimisation of transportation and delivery systems.

Methods

Benchmark Set

We employed a widely recognised benchmark dataset in this study, i.e., Homberger and Gehring, (1999) dataset. Our analysis was limited to a subset of the entire dataset following the selection employed by Vidal et al. (2013), where only instances involving 200 customers were analysed. However, distinct from their approach, our study encompassed all instances within this subset.

The Homberger and Gehring (1999) and Gehring (1999) dataset has been extensively utilised across various studies in diverse settings, highlighting its relevance and robustness for academic investigation. Representative VRP studies that have deployed this dataset include, but are not limited to, Alfredo Tang Montané and Galvão (2006) and Pisinger and Ropke (2007), R. Liu et al. (2013) and Vidal et al. (2014). More recent research examples include Y. Zhang et al., (2020); Beling et al. (2022); Kool et al. (2022), Y. Liu et al. (2023).

We derive our benchmark set from a subset of Homberger and Gehring (1999). We selected instances with 200 customers representing an expanded edition of the initial framework outlined by Solomon (1987). Solomon’s original research investigated vehicle routing problem instances with 100 customers and various vehicle capacities of 200, 700, and 1000. Homberger and Gehring extended Solomon’s dataset (Meira et al., 2020) and maintained vehicle capacities while increasing the number of customers across different datasets, namely 200, 400, 600, 800, and 1000.

We selected the 200-customer dataset because the optimum total route lengths are known. In addition, the solver that we use in this study is capable of solving these instances well. It can provide solutions that either achieve the optimum or approximate it very closely. The efficacy of the solver is noteworthy because it can solve 80% of the instances optimally. The following table illustrates instances in which our solver could not resolve optimally, including the percentage gap between the solver’s solution and the known optimal solution.

Table 1. Solver Performance Gaps (%) of best known solutions.

Instance	Gap (%)	Instance	Gap (%)	Instance	Gap (%)
R1_2.18	0.12	R1_7.18	0.07	R1_4.18	0.14
R1_6.18	0.02	R1_9.18	0.15	R1_10.18	0.27
R2_7.4	0.56	RC1_2.18	0.02	RC1_5.18	0.18
RC1_7.18	0.03	RC1_9.18	0.69	RC1_10.18	0.11

To elucidate the characteristics and specifics of the dataset, we included comprehensive details in graphical representations and tabular format in the subsequent sections.

Following the methodology established by Solomon (1987), Homberger and Gehring (1999) organised their dataset into six distinct subsets: C1, C2, R1, R2, RC1, and RC2. The notation used denotes ‘C’ for clustered configurations, ‘R’ for random configurations, and ‘RC’ for random clustered or semi-clustered configurations.

Variability exists in the number of vehicles; for instance, the subset C1 exhibits variability with 18, 19, or 20 vehicles, R1 exhibits variability with 18 or 20 vehicles, and RC2 ranges from 4 to 6 vehicles. Conversely, subsets C2, R2, and RC1 maintain a consistent number of vehicles, with C2 employing 6, R2 4, and RC1 18 vehicles. The following table provides information about the number of vehicles and vehicle capacities of the instance sets.

Table 2. Summary of vehicle numbers and capacities by instance

Instance Set	Vehicle Number	Vehicle Capacity
C1	18 (5), 19 (1), 20 (1)	200
C2	6	700
R1	18 (9), 20 (1)	200
R2	4	1000
RC1	18	200
RC2	4 (8), 5 (1), 6 (1)	1000

Customer locations were placed within a 140 × 140 coordinate system, with the depot consistently at the centre (70, 70). There are differences in customer locations between subsets C1 and C2, whereas R1 and R2, and RC1 and RC2, share identical customer locations. The following graphics show the customer locations of the datasets.

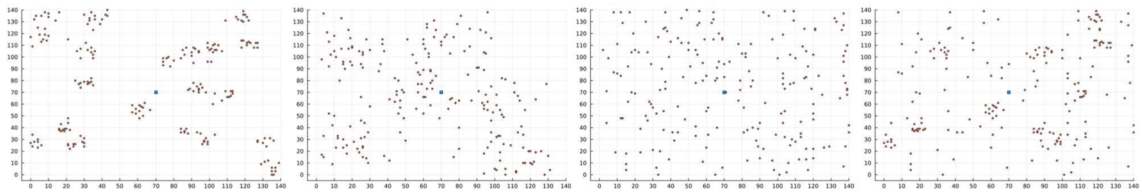

Figure 1. Customer Locations of the Datasets (Left to Right) C1, C2, R1-R2, RC1-RC2

C1 and C2 have different customer demand distributions, while R1 and R2, RC1 and RC2 have the same. Here, we provide histograms representing the demand data.

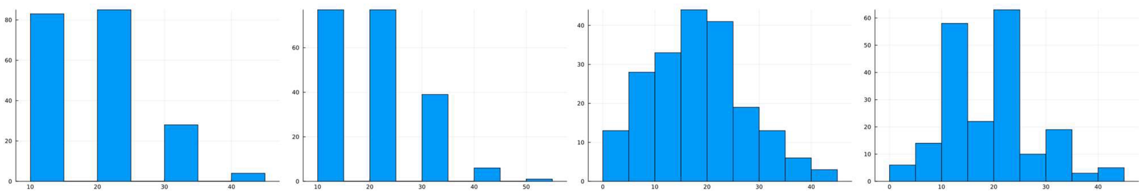

Figure 2. Customer Demands of Datasets (Left to Right) C1, C2, R1-R2, RC1-RC2

Figures showing the distribution of time windows for C1, C2, R1, R2, RC1, and RC2.

Datasets are provided in the appendix.

Solver

We utilised LKH-3 (Helsgaun, 2017), an enhanced Lin-Kernighan-Helsgaun (Helsgaun, 2000) TSP solver designed for various constrained routing problems, such as the travelling salesman and vehicle routing problems with specific limitations like capacity and time windows. The extension effectively converts these complex issues into standard symmetric TSP problems using penalty functions to manage constraints.

Helsgaun (2017) reported that extensive testing demonstrated LKH-3's efficacy, frequently matching or surpassing the best-known solutions. The solver uses a hybrid iterated local search that blends an iterative local search heuristic and a set partitioning formulation, mixing a metaheuristic with mixed integer programming to solve various VRP variants (Muniasamy et al., 2023). LKH-3 and its resources are freely available for academic use.

LKH-3 is a well-recognised solver extensively used across diverse research contexts. We provide direct quotations from several recent scholarly articles in which LKH-3 was used to substantiate the solver's versatility and efficacy. These excerpts were carefully chosen to reflect the breadth of application and diverse analytical assessments that researchers make in their respective studies. This approach underscores the prevailing acceptance and reliability of the solver within the academic community and enriches our methodology by situating it within a broader scholarly dialog.

- “powerful heuristic solver” (Li et al., 2021, p. 1)
- “well-known heuristic solver” (Hottung et al., 2022, p. 1)
- “widely used by the ML community for solving VRPs” (Accorsi, 2022, p. 103)
- “known as one of the best mTSP heuristics” (Kim et al., 2022, p. 7)
- “one of the best heuristic solvers for the TSP” (Löwens et al., 2022, p. 2)
- “able to tackle many VRP variants. Although less efficient with respect to other state-of-the-art CVRP heuristics” (Cavaliere et al., 2022, p. 751)
- “strong heuristic solver” (Sun & Yang, 2023, p. 24)
- “powerful extension of LKH that can solve many TSP variants” (Zheng et al., 2023, p. 1)
- “one of the leading heuristics algorithms” (Pan et al., 2023, p. 9345)
- “a highly optimised solver for routing problems” (Ye et al., 2023, p. 43707)
- “allows an efficient exploration of the search space” (Osorio-Mora et al., 2023, p. 3)
- “state-of-the-art heuristic solver which efficiently produces solutions with a very small optimality gap and has good scalability” (Yang & Fan, 2024, p. 4)

Study Steps

We outline our study methodology in a detailed, step-by-step format, presented as a numbered sequence. We describe each study stage to ensure clarity and reproducibility and facilitate fellow researchers conducting similar or replication studies. Additionally, we highlight that the data generated during our investigation are available for sharing with researchers upon request. We hope that this transparency will help foster a collaborative environment within the academic community.

1. We derive our benchmark dataset from Homberger and Gehring (1999). The Capacitated Vehicle Routing Problem Library (CVRPLIB) (see Uchoa et al., 2017) hosts this benchmark on its website. To maintain consistency in the data complexity, the selection of benchmark instances was limited to those involving 200 customers.
2. To solve these instances, we utilised the LKH-3 (Helsgaun, 2017). The LKH-3 solver is renowned for its efficacy in handling various Vehicle Routing Problem (VRP) variants, such as VRP with Time Windows (VRPTW). Notably, LKH-3 combines ease of use with robust performance, rapidly achieving high-quality results although optimal outcomes are only occasionally achieved and are not guaranteed.
3. We downloaded the source code for LKH-3 and implemented it according to the guidelines provided on the official website. The solver includes comprehensive documentation; we followed the author’s recommendations for parameter settings. Since it is open source, it is always possible to investigate the source code and follow annotations if certain aspects are unclear. For example, we successfully refactored certain functions without any issues.
4. The computational experiments were executed on a GNU/Linux operating system with an Intel G5500 CPU complemented by 8 GB of RAM. The choice of this modest hardware setup and operating system was deliberate and aimed at assessing the performance and replicability of the study on systems that are not high-end, thereby reflecting a more typical user environment.
5. Our programming tool was Julia (Bezanson et al., 2017), a programming language renowned for its high performance, exceptional modularity, and superior composability, which it achieves through the implementation of multiple dispatch and just-in-time compilation techniques (Christ et al., 2023, p. 2). This study was conducted entirely within the Julia environment, utilising its multi-threading capabilities to deploy LKH-3 across all available CPU cores, resulting in enhanced computational efficiency. We also used DataFrames.jl (Bouchet-Valat & Kamiński, 2023), Plots.jl (Christ et al., 2023), StatsBase.jl, and Statistics.jl.
6. Moreover, we performed our statistical analyses and comparisons using Jamovi (The Jamovi Project, 2024), a versatile statistical spreadsheet (see Şahin & Aybek, 2020) built on top of the R statistical language (see R Core Team, 2024). Consistent with our commitment to transparency and accessibility in research, we prioritised using non-proprietary, open-source software whenever feasible.
7. For data storage and accessibility, we employed durable ordinary text-based files that can be easily opened and modified with any standard text editor, thus avoiding the need for specialised software. This approach ensures compatibility across different platforms and enhances the longevity and reproducibility of our research outputs.
8. We used plain text-based instance files in CVRPLIB rather than XML files in VRP-REP (see Mendoza et al., 2014), as they are generally easier to read and manipulate. We investigated the VRPTW instance files and determined how to convert them into VRP instance files. We made the necessary changes to text files using only Julia’s string manipulation functions.
9. After conversion, we solved the original instances and then all modified versions. We compared the results obtained from these different instances to identify any significant discrepancies. To facilitate the comprehension of our findings, we constructed tables and graphs, which served as effective visual aids.
10. Several graphs did not yield significant insights and were therefore excluded from the analysis.

Results

When time window constraints are removed from VRPTW, the problem undergoes a transformation to CVRP. This shift induces notable changes in the structure of the solution routes and other related aspects. A diverse array of metrics was employed in our analysis to elucidate these transformations. In this section, we provide a concise description of the metrics used, followed by a detailed presentation of the findings.

Homberger and Gehring's dataset comprises six subsets: C1, C2, R1, R2, RC1, and RC2. To facilitate the comparison, we organised our graphs as follows: C1 at the top left and C2 at the bottom left; R1 at the top centre and R2 at the bottom centre; RC1 at the top right and RC2 at the bottom right. In these graphs, the blue and red bars denote the values associated with the modified versions of the problem set (VRP) and the original versions (VRPTW). Detailed data tables for these instances are presented in the Appendix.

We note that in the datasets, C denotes clustered R denotes randomized, and RC denotes random clustered or semi-clustered.

Performance Metrics

Total Route Length

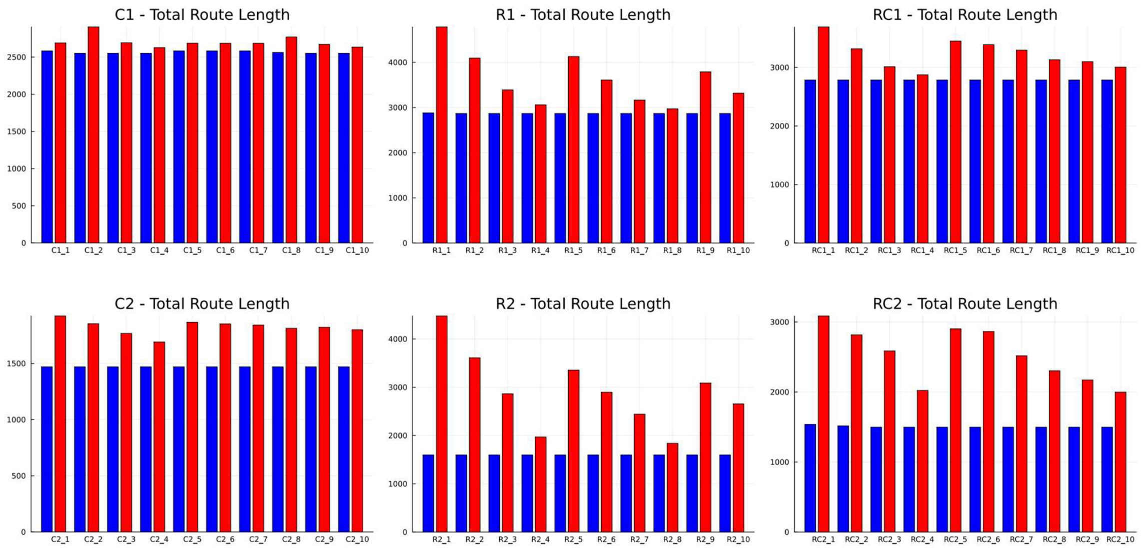

Figure 3. Comparison of total route lengths

Total route length refers to the cumulative distance or travel time traversed by a fleet of vehicles while delivering goods to customers, starting and ending at a depot. Minimising total route length is a critical objective in VRP because it directly affects the operational efficiency, fuel consumption, and overall logistical costs.

In accordance with established methodologies aimed at ensuring compatibility of comparative analyses, the calculated route lengths were rounded. In contrast, no rounding adjustments were applied to any other metrics while generating figures. For convenience, we rounded values to two decimal places in the data tables.

In addition, we observed no significant changes in the total route for the C1 instance set. Although the VRPTW versions consistently exhibited slightly higher values than the VRP versions, the differences were negligible. The C2 and RC1 instances showed similar values, suggesting a resemblance between these subsets. In both cases, the presence of time window constraints tended to increase the total route length. The differences were more pronounced on the R1, R2, and RC2 datasets. Certain instances in these datasets almost doubled the total route length in one version compared to the other.

Generally, the imposed time window constraints increased the total route length, although the magnitude of this effect was not universally consistent. Factors such as customer demand, location, and the specific structure of the time window constraints may influence these variations.

Minimum Route Length

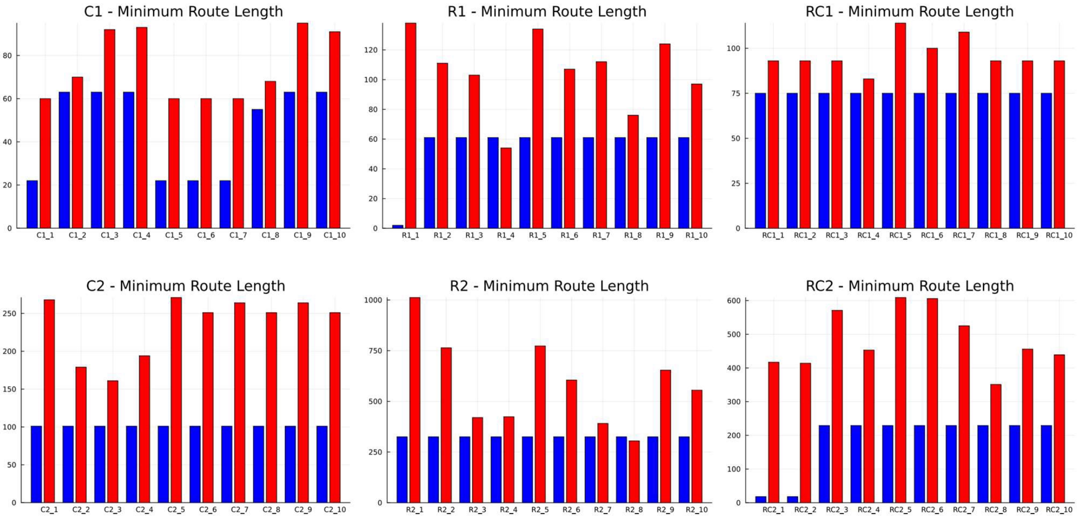

Figure 4. Comparison of minimum route lengths

The minimum route length identifies the shortest distance a vehicle in a fleet travels to complete its assigned deliveries and return to the depot. It helps determine the vehicle that completes its route within the least distance, often also resulting in it being the earliest to return to the depot.

Our observations indicate that the minimum route length is one of the most variable metrics under investigation. The VRP versions demonstrate greater stability and consistency across all datasets. Conversely, VRPTW ones exhibited significant variability. In nearly all instances, the VRPTW versions registered higher minimum route lengths, with exceptions noted in datasets R1_4, R2_8, RC_1, and RC_2. We can confidently assert that including the time window constraints significantly affected the minimum route length. In other words, removing the time window constraints resulted in more stable, consistent, and almost identical or similar minimum route length values. Route Length identifies the shortest distance a vehicle in a fleet travels to complete its assigned deliveries and return to the depot. It helps determine the vehicle that completes its route within the least distance, often also resulting in it being the earliest to return to the depot.

R1_1, RC2_1, and RC2_2 demonstrated notably small values due to the formation of routes that served only a single customer in these instances. In our observations, the highest degree of volatility in the VRP versions was noted for the C1 instance, whereas the other instances exhibited significantly lower levels of volatility. Conversely, the VRPTW versions demonstrated the minimum volatility for the RC1 instance, and the remaining instances demonstrated much higher volatility.

Maximum Route Length

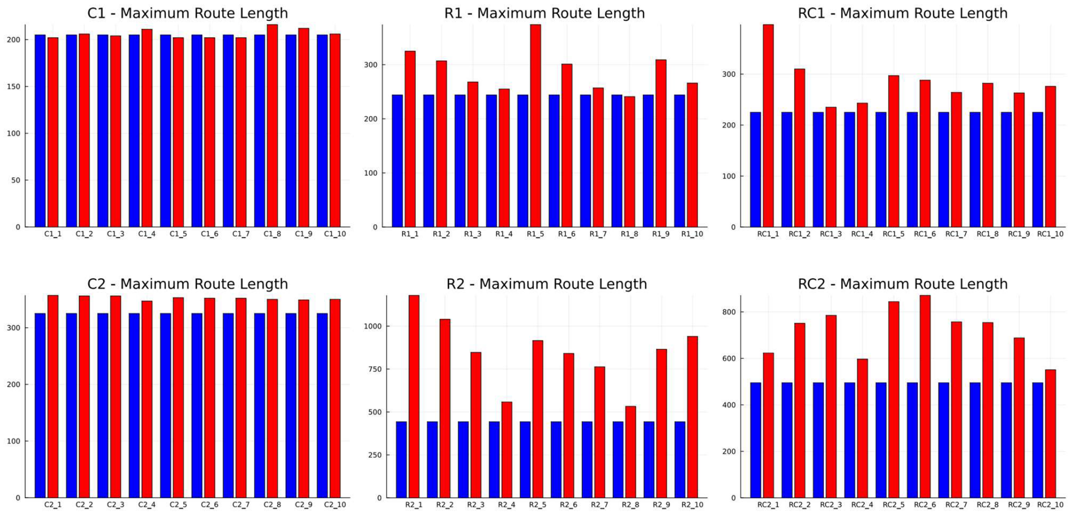

Figure 5. Comparison of maximum route lengths

Maximum Route Length measures the longest distance a vehicle in a fleet travels to complete its deliveries and return to the depot. This metric is essential for identifying the most extended and potentially resource-intensive routes, which may impact fuel consumption, driver fatigue, and vehicle maintenance. By analysing the maximum route length, logistics planners can ensure that every vehicle is equally burdened, thus promoting balanced workloads and improving overall fleet efficiency.

In addition, the maximum route length signifies that the vehicle assigned the longest route will be the last to return to the depot after all other vehicles have already completed their routes. Consequently, the duration of the vehicle's route determines the overall duration of the entire operation. Thus, this metric provides an opportunity to compare the total operation duration.

For each dataset, we observed that the versions associated with the VRPTW generally exhibited higher maximum route length values. The most substantial differences are observed in datasets RC2, R2, R1, and RC1, listed in descending order of observed variations. It is essential to note that no differences exist between the VRP versions regarding the maximum route length. This consistency provides a valuable opportunity to compare values across different instances. Instances C1 and C2 exhibited the highest values. They were followed by instance R1, with RC1 and RC2 showing slightly lower values. Finally, instance R2 obtained the lowest values among the observed instances.

Moreover, in the R1, R2, RC1, and RC2 datasets, variations in the time constraints significantly affected the maximum route length. The datasets C1 and C2 did not significantly contribute to our insights because the values across these datasets were nearly identical. The results indicate that time window constraints do not significantly affect the metric in these two instances.

Mean Route Length

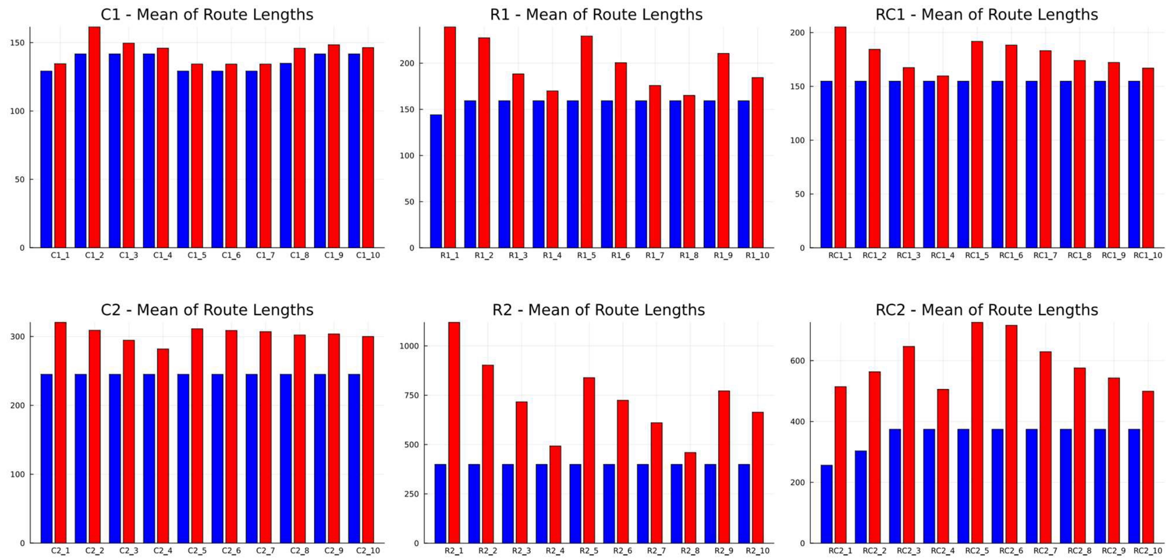

Figure 6. CComparison of mean route lengths

The mean route length calculates the average distance travelled by all vehicles in a fleet to complete their respective routes and return to the depot. This metric provides a comprehensive overview of the overall routing plan efficiency and balance. By analysing the mean of the route lengths, logistics managers can identify trends, detect inconsistencies, and make informed decisions optimise route planning.

A lower mean route length generally indicates more efficient routing, reduced operational costs, improved service levels, and better resource allocation across the fleet. A higher mean route length indicates that, on average, the vehicles in the fleet travel longer distances to complete their deliveries and return to the depot. This metric can indicate potential inefficiencies in the routing plan, such as suboptimal route assignments, imbalanced workloads, or delivery point clustering issues. Higher mean route lengths often mean increased operational costs, fuel consumption, driver fatigue, and increased vehicle wear and tear.

Our observations indicate that mean route lengths are generally higher in the VRPTW than in the VRP. We found the most significant variability in mean route lengths in R1, R2, and RC2. Removing the time window constraints in all datasets resulted in lower mean route lengths. Specifically, the clustered datasets (C1 and C2) exhibited minimal differences between the VRP and VRPTW values, whereas the randomised datasets (R1 and R2) exhibited the highest discrepancies. The semi-clustered datasets (RC1 and RC2) had values that fell between those of the purely clustered and purely randomised datasets.

In C1, we observed a strong correlation between the values derived from the VRP and VRPTW. The VRPTW versions demonstrated marginally higher values than the VRP versions.

Median Route Length

Our analysis did not reveal any significant differences in the median route lengths across the datasets except for RC2_1. The median route length metric did not yield any insights into the impact of removing the time window constraints. Consequently, we could not make meaningful comparisons within or between the instances. Given the lack of informative value, we opted not to include graphical representations.

Variance of Route Lengths

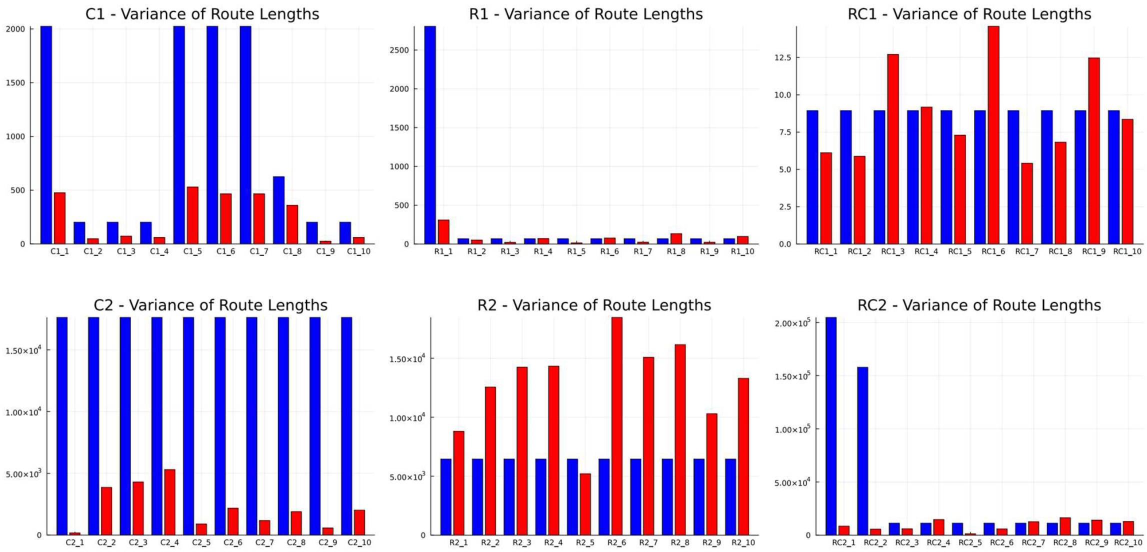

Figure 7. Comparison of route length variance

The variance of route lengths can provide insights into the consistency and balance of the routes assigned to vehicles. High variance suggests significant discrepancies in the lengths of routes, potentially leading to inefficiencies, such as unequal workload distribution among drivers and inefficient utilisation of resources. Conversely, low variance indicates more uniform route lengths, which contributes to balanced operations, predictable delivery times, and optimised fleet performance. Analysing route length variance helps identify areas for improvement in route planning and enhances the overall logistics efficiency.

In the C2, R2, and RC1 instances, removing the time window constraints significantly altered the route length variance, which was uniform across the datasets. R1 and RC2 datasets exhibited minimal variance with only one and two exceptions, respectively. In contrast, eliminating the time window constraints in C2 and RC2 led to notably high variances. Upon closer examination of the C2 datasets, we found consistently high variance values across all VRP versions. However, this pattern was less pronounced on the RC2 datasets (only two versions exhibiting similarly high variance. In the R2 dataset, except for the R2_5 instance, removing the time window constraints generally reduced the variance in the route lengths. The RC1 dataset presented a mixture of characteristics from both clustered and randomised datasets. The effect of removing the time window constraints in RC1 was inconsistent; in certain instances, it led to an increase in variance, while in other instances, a decrease was observed. Notably, on this dataset, the RC1_4 and RC1_10 instances exhibited minimal variance changes upon removing the time window constraints, albeit in opposite directions.

Standard Deviation of Route Lengths

The standard deviation of the route lengths is a metric used to assess the variability or dispersion of the lengths of different routes. It quantifies how much the route lengths deviate from the average route length, providing insight into the consistency and balance of the routes. A lower standard deviation indicates that the routes are relatively uniform in length, indicating that their efficiency and equitable distribution, whereas a higher standard deviation indicates more significant disparities, which could imply inefficiencies or issues in route planning. This metric helps evaluate and optimise the overall routing performance.

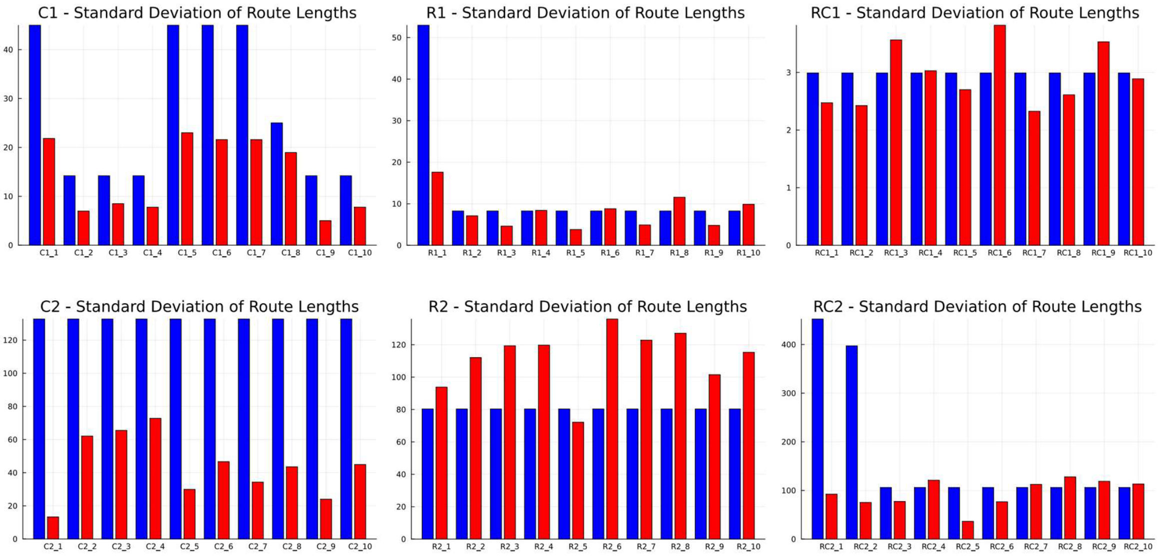

Figure 8. Comparison of the route length standard deviations

As anticipated, the standard deviation of the route lengths in the presented graphs closely mirrors the variance of the route lengths. For the VRP versions, all instances (except C1) exhibited consistent standard deviations of route lengths. In the VRPTW versions, particularly with clustered datasets, removing the time window constraints increased the standard deviation of the route lengths. Conversely, for the other datasets, we observed fluctuations in both directions; some instances exhibited increased standard deviations, while others demonstrated decreased values. The lowest standard deviation values were recorded for RC1.

Furthermore, the values were comparable between C1 and R1 and C2 and R2. However, in RC1, the standard deviation values were markedly low, whereas in RC2, they were significantly higher. For R1 and RC2, the observed values were generally similar, with exceptions noted in instances R1_1, RC2_1, and RC2_2.

Minimum Route Demand

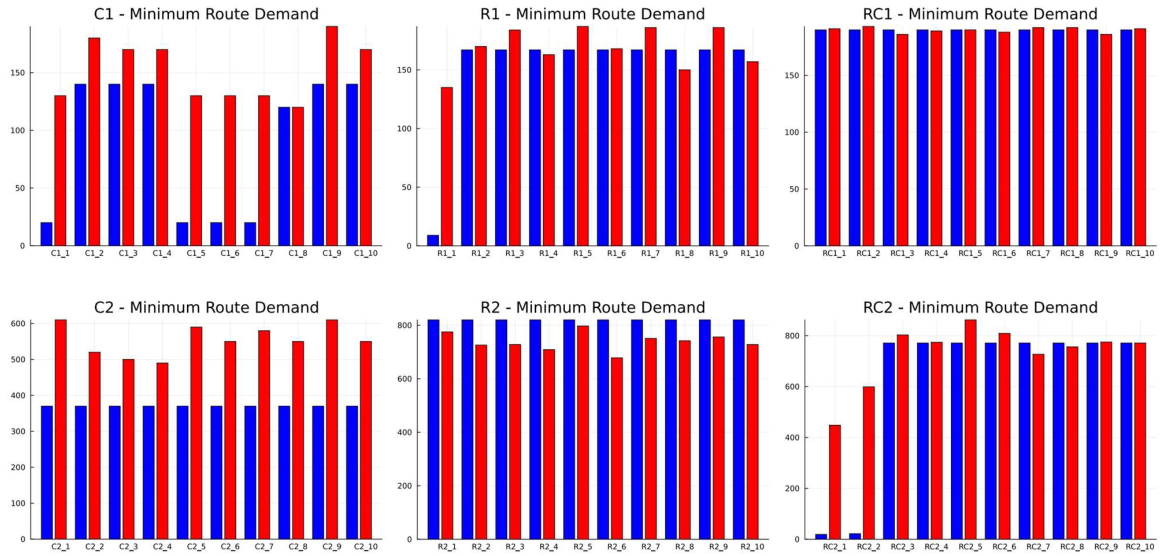

Figure 9. Comparison of minimum route demand

Minimum route demand (MRD) represents the minimum load or demand for a single route or vehicle. It captures the smallest amount of goods, passengers, or services that must be delivered or attended to on any given route. MRD is essential for optimising resource allocation, ensuring that no route is underutilised, and balancing logistics efficiency with operational cost-effectiveness. MRD helps craft efficient, sustainable, and cost-effective routing plans by minimising unproductive distances and reducing fuel consumption and labour expenses.

Randomised and semi-clustered datasets did not provide insights into minimum route demand. Instances R1, R2, RC1, and RC2 exhibit no significant disparities, with the notable exceptions of R1_1, RC2_1, and RC2_2 demonstrating substantial decreases in minimum route demand.

In the clustered datasets, for example in C2, the minimum route demands remained consistent after removing the time window constraints and displayed values lower than those in the VRPTW versions. We observed noteworthy decreases in C1_1, C1_5, C1_6, and C1_7, with C1_8 showing no change and the rest of the dataset experiencing moderate variations. The lowest variance in minimum route demands was observed in the RC1 category, whereas the highest variance was observed in C1.

Maximum Route Demand

We did not observe any significant differences in any instances between the VRP and VRPTW versions or between them. The maximum route demand metric yielded no meaningful insights. Therefore, we did not include a graphical representation of this metric.

Median Route Demand

We did not observe any significant differences between the VRP and VRPTW versions. The median route demand metric did not provide meaningful insights. Therefore, we did not include a graphical representation of this metric.

Variance of Route Demands

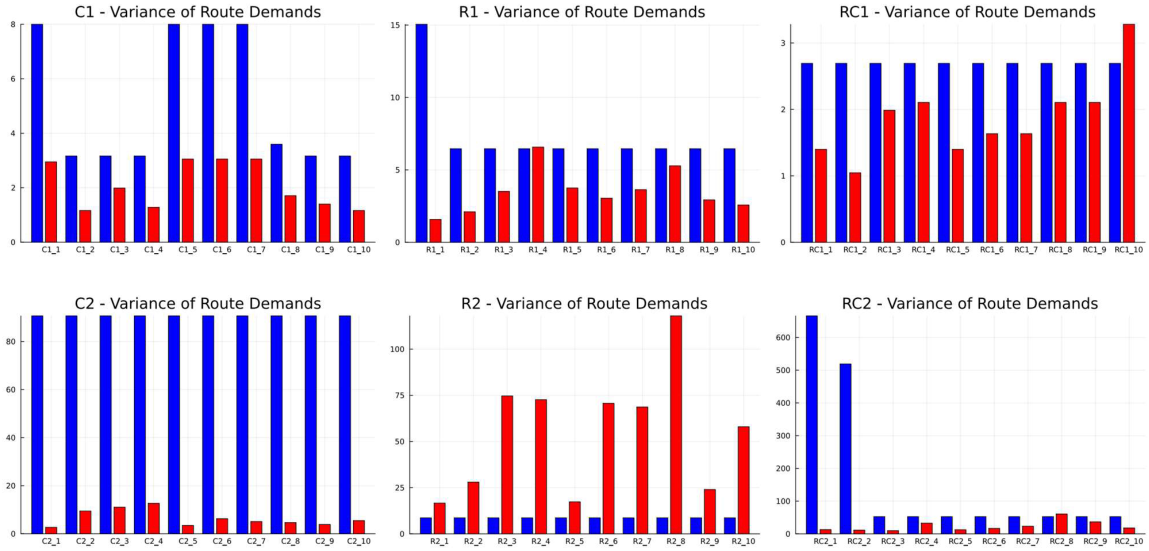

Figure 10. Comparison of the Route Demand Variance

The variance of route demands is a metric for evaluating the efficiency and balance of solutions. It measures the degree to which demands are spread across different routes, indicating how evenly workloads are distributed among vehicles. A lower variance suggests a more balanced allocation, where each vehicle has a similar load; this leads to improved operational efficiency and customer satisfaction. Conversely, a high variance may indicate an imbalance, potentially resulting in increased travel times, higher fuel consumption, and uneven service levels.

After removing the time window constraints, we observed significant variations in the route demands, although in different directions between datasets C2 and R2. In particular, on the C2 dataset, variances were uniformly higher than the original values. Conversely, on the R2 dataset, variances decreased and equalised. Notably, all instances exhibit substantial increases and decreases. For in most instances, the variance in the VRP versions equalised. We detected no deviations in the C2, R2, and RC1 datasets. However, an outlier was found for dataset R1, specifically R1_1. We note four exceptions to dataset C1, specifically in instances C1_1, C1_5, C1_6, and C1_7. The variance values of C1 and RC1 were notably low.

In contrast, the RC2 dataset exhibited the highest variance values. Interestingly, on the RC1 dataset, only the RC1_10 instance demonstrated a decrease in variance after removing the time window constraints. A similar trend was observed for datasets RC2 and R1, with only the instances RC2_8 and R1_4, respectively, showing decreased variance. Dataset C2 uniformly exhibited increased variance, whereas dataset R2 uniformly exhibited decreased variance upon removing the time window constraints. The remaining datasets exhibited a mix of increased and decreased variances.

Standard Deviation of Route Demands

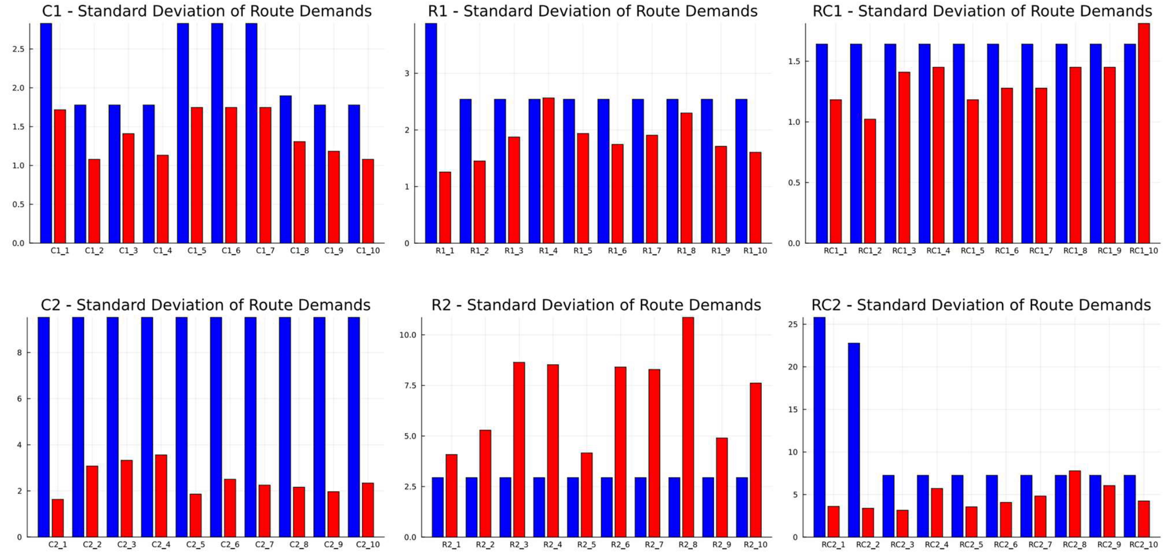

Figure 11. Comparison of the Route Demand Standard Deviations

The standard deviation of the route demands is a critical metric for assessing the variability in demand across different routes. It quantifies the extent to which individual route demands deviate from the average demand, providing insights into the consistency and efficiency of route planning. Higher standard deviations indicate more significant fluctuations, suggesting potential inefficiencies and imbalances among the routes. Conversely, a lower standard deviation indicates a more uniform demand distribution, contributing to optimised vehicle use and a balanced workload among drivers.

The observed standard deviations of the route demands in various instances exhibit an increasing trend in the following sequence: RC1, C1, R1, C2, R2, and RC2. Notably, the R2 dataset demonstrated the highest variability in the standard deviations of the route demands, indicating significant fluctuations. Conversely, the R1 and RC1 datasets displayed low variability, suggesting more consistent demand patterns. Furthermore, we observed variations in the standard deviations for both small and large magnitudes across different instances. Notably, the standard deviation values are comparable among the datasets categorised as C1, R1, and RC1, as well as among C2, R2, and RC2. This pattern may imply underlying differences in demand characteristics within these categorizations, warranting further examination to understand the factors contributing to such variability.

Minimum Number of Customers per Route

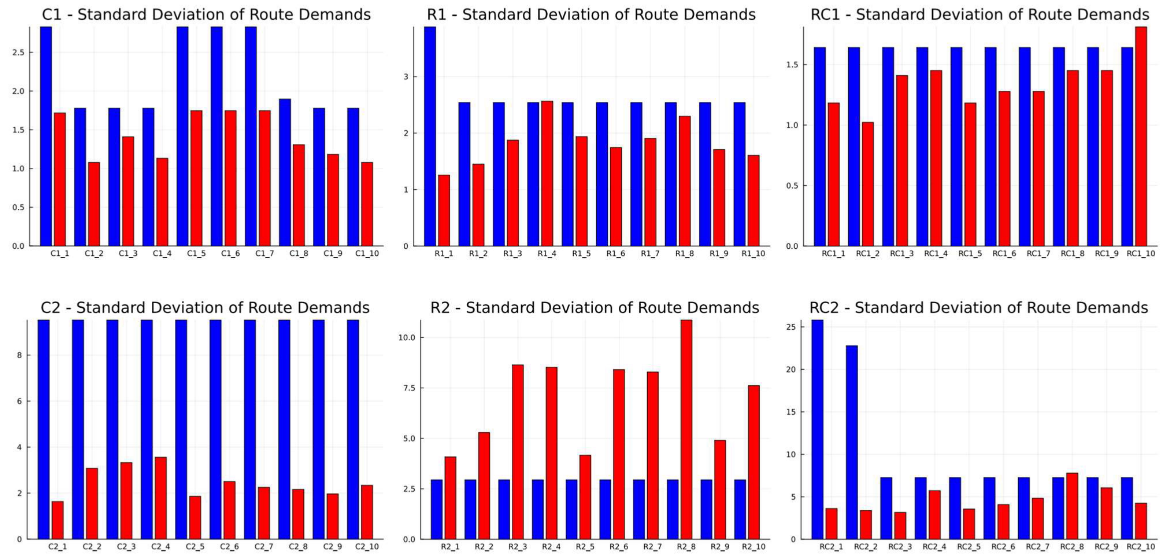

Figure 12. Comparison of minimum number of customers per route.

The minimum number of customers per route represents the smallest number of customers served on any given route in the solution. This metric helps evaluate route efficiency and distribution balance by ensuring that every route is utilised. It is particularly useful for identifying potential route planning and resource allocation improvements to balance workloads and enhance service delivery. By understanding this metric, decision makers can make informed adjustments to achieve more optimised and equitable vehicle routing solutions.

Our analysis revealed no significant differences in R1, R2, RC1, and RC2. However, exceptions were noted in instances R1_1, RC2_1, and RC2_2, which parallel the trends in minimal route demand. In R2, we observed increased values after removing the time window constraints, which deviated from the trend observed across the other datasets. Conversely, C1, R1, and RC2 consistently recorded the lowest values.

Maximum Number of Customers per Route

We observed no significant variations in the maximum number of customers per route. Although minor differences were observed, they were deemed negligible and did not warrant further discussion. Therefore, we do not include graphical representations for this metric.

Median Number of Customers per Route

The results revealed that the median number of customers per route remained consistent across all datasets after removing the time window constraints. Consequently, we determined that graphical representations would not add value to the interpretation of the data, which led to their omission.

Variance of the Number of Route Customers

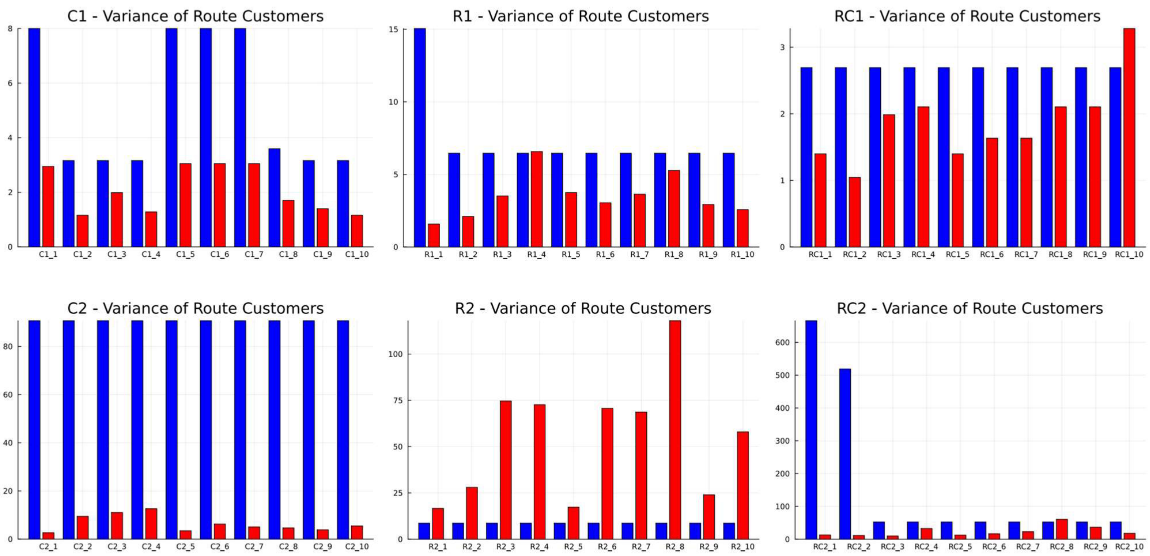

Figure 13. Comparison of variances of the number of customers per route.

The variance of the number of route customers is a metric used to assess the distribution uniformity of customer assignments among different routes in a vehicle routing problem. It quantifies how evenly customers are allocated to various routes by measuring the spread or dispersion of the average number of customers per route. A high variance indicates significant imbalances, which suggests that some routes may be heavily loaded while others are underutilised. Conversely, a low variance indicates a more balanced distribution, which is often desirable for efficiency and workload equity in logistics and transportation planning.

The comparison of the minimum number of customers per route in R1, R2, and RC1 revealed no significant differences between the VRP and the VRPTW versions. Notably, three specific instances, R1_1, RC2_1, and RC2_2, exhibited substantial reductions in this metric. This phenomenon can be attributed to the inclusion of a single-customer route in both VRP versions.

The observed variance among VRPTW versions was notably higher than that of the VRP versions. We observed that the R2 category exhibited the highest variance for VRPTW, whereas the RC2 category demonstrated the highest variance for VRP. Conversely, the lowest variance for VRPTW was observed in both the C1 and RC1 categories, whereas for VRP, it was most minimal in the RC1 category.

All datasets consistently exhibited stable metrics for the VRP and VRPTW versions, with the exception of R2. After removing the time window constraints, all instances in this dataset displayed decreased and equalised metrics. Dramatic reductions were observed in datasets C1 and R1, particularly in instances C_1, C1_5, C1_6, C1_7, and R1_1. Conversely, we observed increased results for the R2 and RC2 datasets.

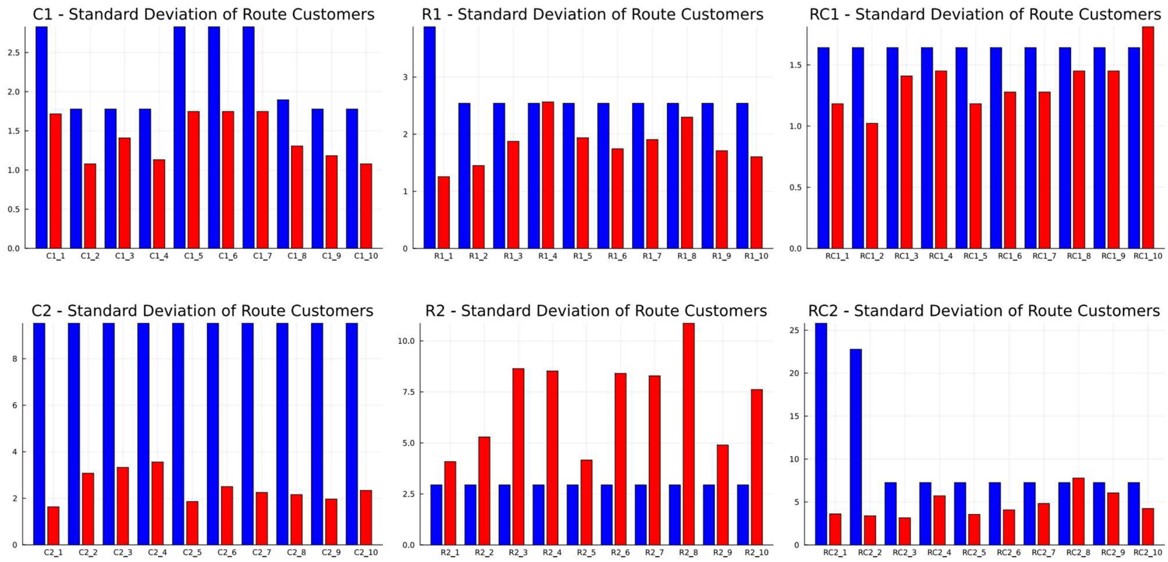

Figure 14. Comparison of standard deviation of number of customers per route.

The standard deviation of the number of route customers measures the average deviation from the mean number of customers per route, providing insight into the variability and balance of the distribution. A high standard deviation indicates considerable differences between routes, with some having significantly more or fewer customers than others, potentially leading to inefficiencies and uneven workloads. In contrast, a low standard deviation suggests that customer assignments are distributed more evenly, which contributes to a more balanced and efficient routing solution.

After removing the time window constraints, the C1 dataset showed a consistent increase in standard deviation values. The C2 dataset exhibited a significant increase in all measured values. For the R1 dataset, an increase was observed in all instances except for R1_4. In the R2 dataset, a decrease in the standard deviation values was noted consistently. These values became more equalised, with the smallest difference observed in R2_5 and the largest difference observed in R2_8. The RC1 dataset primarily showed increases in standard deviation values, with the exception of RC1_10. All VRP versions in the RC1 dataset tended to equalise in terms of standard deviation. In the RC2 dataset, primarily moderate changes were observed, with notable exceptions to RC2_1 and RC2_2. RC2_8 was the sole instance exhibiting a decreased value in this dataset.

Route Formations

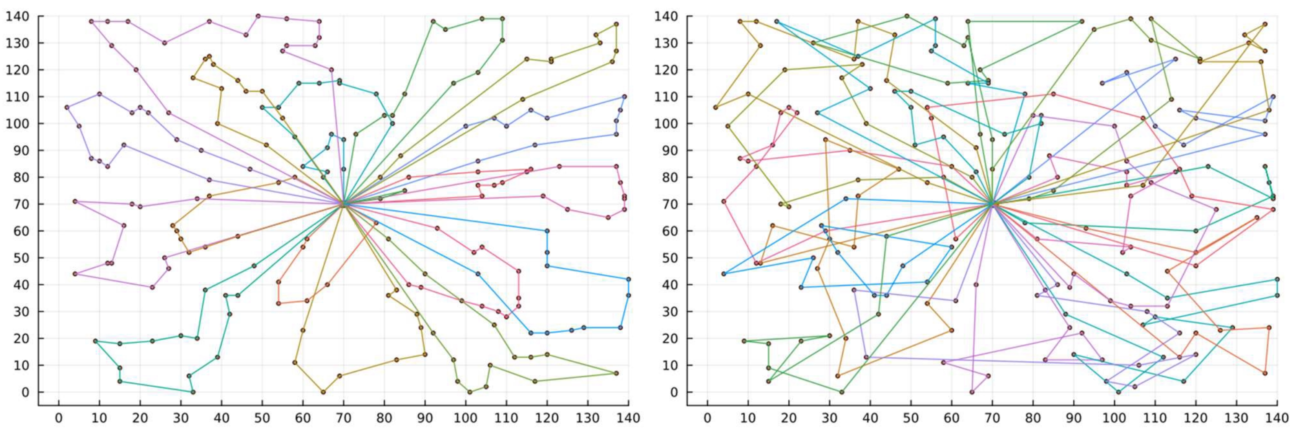

Figure 15. Route formations for R1_1 instance. Without Time-Windows Constraints (Left) and with Time-Windows Constraints (Right) Versions

We observed the most significant total and mean route length differences in instance R2_1. The most significant disparity in minimum route length was observed in instance R1_1. Furthermore, we observed the most substantial differences in the variance and standard deviation of the number of route customers in instance RC2_1. Here, we provide route graphs for these instances to illustrate them. These representations include the VRP and VRPTW versions to demonstrate how including or excluding time window constraints can affect route structure. The selected figures highlight significant variations between the VRP and VRPTW versions of the instances. By comparing and contrasting these versions, valuable insights into their structural and operational differences can be gained.

In figure 15, left graph illustrates relatively sparse routing patterns with minimal overlap, which is characteristic of a typical VRP structure. On the other hand, the right graph presents a denser network with significant intersections and overlaps, which is typical for VRPTW scenarios where the synchronisation of service times is critical.

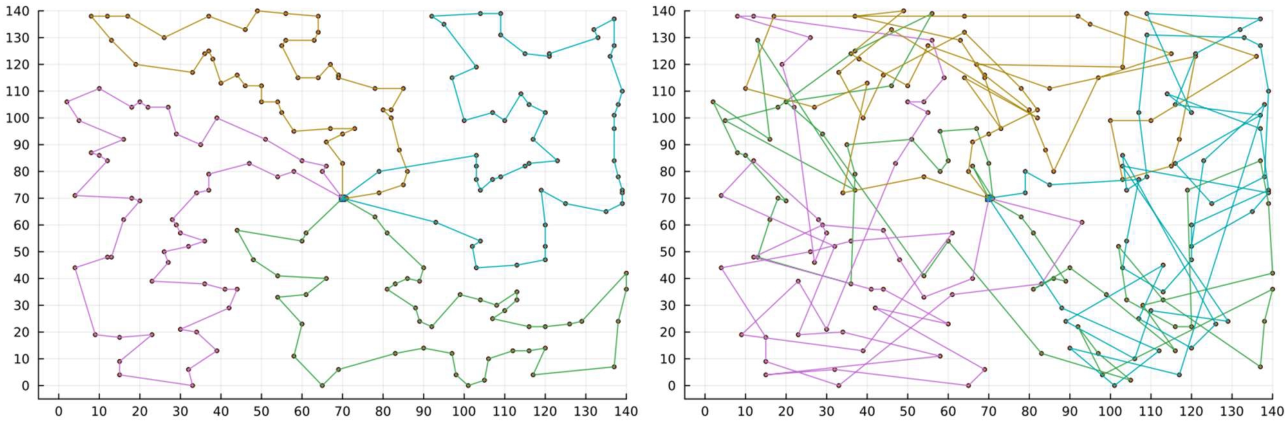

Figure 16. Route formations for R2_1 instance. Without Time-Windows Constraints (Left) and with Time-Windows Constraints (Right) Versions

In figure 16 on the left side, the routes are distinct and separated, indicating fewer constraints on the routing process. Conversely, the right side demonstrates tightly packed and interwoven routes, highlighting the complexities introduced by the time window constraints.

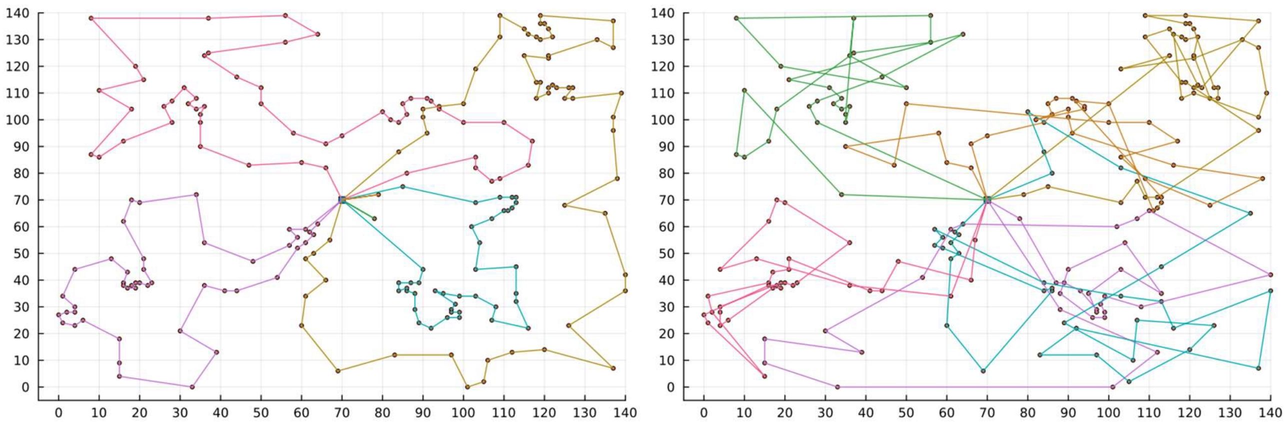

Figure 17. Route formations of RC2_1 instance. Without Time-Windows Constraints (Left) and with Time-Windows Constraints (Right) Versions

In figure 17 on the left, we see routes that appear more spaced out and cover a broader area, while the right side depicts a more compact and tangled routing network, which is indicative of the VRPTW versions' operational characteristics.

Structural Comparisons

Route Density

The VRP figures (figures on the left) depict relatively less dense routes than their VRPTW counterparts (figures on the right). The routes displayed in the VRPTW versions are markedly denser, indicating a higher frequency of stops within closer proximity. This density is attributed to the time window constraints that necessitate more localised route planning to meet stringent scheduling requirements.

Route Complexity

The VRPTW figures demonstrate increased route complexity, characterised by spaghetti-like patterns and overlapping paths. This complexity arises due to the adherence to specified time windows, which introduces the need for more detailed planning to optimise service times and travel durations. In contrast, VRP figures exhibit more straightforward and less intertwined routes, reflecting the absence of temporal constraints and the resulting more linear route optimisation.

Operational Differences

Service Efficiency

The increased density and complexity of the VRPTW versions underscore a focus on maximising service efficiency within the restricted time windows. Clustering routes within smaller geographical segments enhances the ability to fulfil multiple service requests within limited time frames. This is in contrast with the VRP models, where the primary objective centres on minimising travel distance without the constraint of specific service times, as evidenced by the more spatially distributed routes.

Geographical Coverage

VRP routes tend to exhibit wider geographical coverage, with routes sprawling across wider areas. This approach is suitable for scenarios in which the service time is flexible, emphasising covering a larger service area with minimal travel expenditure. Conversely, VRPTW routes are more concentrated within confined areas, effectively using available time slots for service delivery.

Summary

The comparison between the VRP and VRPTW route graphs shows pronounced differences in the route density, complexity, operational efficiency, and geographical coverage. The VRP models prioritise covering larger areas with minimal travel, whereas VRPTW models emphasise route optimisation within specified time windows, leading to denser and more complex routing patterns. These distinctions underscore the need for tailored approaches to address various routing challenges in logistics and service delivery contexts.

Conclusion

In this study, we removed the time window constraints from Homberger and Gehring (1999) VRPTW instances and solved them as VRP instances to observe the resulting changes. We observed significant differences between the VRP and VRPTW versions for almost all metrics, with a few notable exceptions. Various figures, graphs, and tables thoroughly illustrate these phenomena.

Our analysis revealed significant variability in total route length. Based on the obtained data, we can confidently assert that time window constraints typically affect the total route length. When the time constraints are removed, the mean route lengths tend to be smaller and more consistent across all datasets. Furthermore, removing the time window constraints led to more stable measures regarding the route lengths, route demands, and number of customers per route. Conversely, this study did not derive any substantive insights from metrics such as median route length, maximum route demand, median route demand, maximum number of customers, and median number of customers.

For logistics firms aiming to meet their customers' temporal requests and requirements, it is critical to recognise the substantial variations observed in specific datasets. The imposition of time window constraints incurs significant changes in the route structures, which may result in high costs.

Limitations and Directions for Future Research

This study utilised a dataset comprising 200 customer-size instances. Relatively small dataset size is a notable limitation. Future research can benefit significantly from the use of larger datasets. The selection of dataset size was influenced by the capability of our solver to handle these instances and achieve near-optimal solutions. However, the scalability of our approach may not hold for larger datasets, which poses a challenge for future work in this field.

Conversely, future researchers may also explore the use of smaller datasets. Utilising smaller datasets facilitates the application of exact solvers, which guarantees optimal solutions. This approach can yield more definitive insights but may be less representative of larger, real-world scenarios.

Our study aimed at achieving near-optimal solutions; however, in practical applications, attaining near-optimality may only occasionally be feasible. Comparative studies focusing on VRP and VRPTW solutions that are significantly far from optimal can offer valuable insights into the performance and applicability of various methodologies.

This research was constrained to a single dataset. Future research could explore combining multiple datasets to enhance the generalizability of the findings. Additionally, while our study employed specific performance metrics for comparison, future researchers' development and utilisation of alternative performance metrics can provide a broader understanding of solver performance.

One limitation of our dataset is that it only allows for one-on-one comparisons. If larger datasets or combinations of datasets are employed in future studies, various statistical tests and advanced data analysis methods can be applied. This would enhance the robustness and applicability of the findings.

Future research could also extend to different VRP variants. For example, standard VRP instances can be examined as Open VRP instances. Comparative analyses of multi-depot versus single-depot VRP using identical customer locations and demands could yield insights into the effect of depot quantity on solution quality. Such studies could involve varying the number of depots to evaluate how the depot quantity affects the solutions.

Although our study provides significant insights, the above-mentioned limitations and avenues for future research highlight the potential for further advancements in this field.

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Appendix A

Appendix 1. C1 Data

Instance Name	RLT	RLM_1	RLM_2	RLM_3	RLM_4	RLV	RLS	RDM_1	RDM_2	RDM_3	RDV	RDS	RCM_1	RCM_2	RCM_3	RCV	RCS
C1_1.20.vrp	2583	22	205	129.15	200	2023.95	44.99	20	200	200	8	2.83	1	13	11	8	2.83
C1_2.18.vrp	2551	63	205	141.72	200	201.63	14.2	140	200	200	3.16	1.78	6	13	12	3.16	1.78
C1_3.18.vrp	2551	63	205	141.72	200	201.63	14.2	140	200	200	3.16	1.78	6	13	12	3.16	1.78
C1_4.18.vrp	2551	63	205	141.72	200	201.63	14.2	140	200	200	3.16	1.78	6	13	12	3.16	1.78
C1_5.20.vrp	2583	22	205	129.15	200	2023.95	44.99	20	200	200	8	2.83	1	13	11	8	2.83
C1_6.20.vrp	2583	22	205	129.15	200	2023.95	44.99	20	200	200	8	2.83	1	13	11	8	2.83
C1_7.20.vrp	2583	22	205	129.15	200	2023.95	44.99	20	200	200	8	2.83	1	13	11	8	2.83
C1_8.19.vrp	2562	55	205	134.84	200	625.73	25.01	120	200	200	3.6	1.9	6	13	11	3.6	1.9
C1_9.18.vrp	2551	63	205	141.72	200	201.63	14.2	140	200	200	3.16	1.78	6	13	12	3.16	1.78
C1_10.18.vrp	2551	63	205	141.72	200	201.63	14.2	140	200	200	3.16	1.78	6	13	12	3.16	1.78
C1_1.20.vrptw	2690	60	202	134.5	180	476.58	21.83	130	200	180	2.95	1.72	7	13	11	2.95	1.72
C1_2.18.vrptw	2906	70	206	161.44	200	48.69	6.98	180	200	200	1.16	1.08	9	13	12	1.16	1.08
C1_3.18.vrptw	2693	92	204	149.61	200	72.22	8.5	170	200	200	1.99	1.41	8	13	12	1.99	1.41
C1_4.18.vrptw	2627	93	211	145.94	200	60.46	7.78	170	200	200	1.28	1.13	9	13	12	1.28	1.13
C1_5.20.vrptw	2687	60	202	134.35	180	529.21	23	130	200	180	3.05	1.75	7	13	11	3.05	1.75
C1_6.20.vrptw	2686	60	202	134.3	180	466.05	21.59	130	200	180	3.05	1.75	7	13	11	3.05	1.75
C1_7.20.vrptw	2686	60	202	134.3	180	466.05	21.59	130	200	180	3.05	1.75	7	13	11	3.05	1.75
C1_8.19.vrptw	2771	68	216	145.84	190	359.06	18.95	120	200	190	1.71	1.31	8	13	11	1.71	1.31
C1_9.18.vrptw	2671	95	212	148.39	200	25.16	5.02	190	200	200	1.4	1.18	9	13	12	1.4	1.18
C1_10.18.vrptw	2634	91	206	146.33	200	60.46	7.78	170	200	200	1.16	1.08	9	13	12	1.16	1.08

RLT Route Length Total
RLM1 Route Length Minimum
RLM2 Route Length Maximum
RLM3 Route Length Mean
RLM4 Route Length Median
RLV Route Length Variance
RLS Route Length Standard Deviation
RDM1 Route Demand Minimum
RDM2 Route Demand Maximum
RDM3 Route Demand Median
RDV Route Demand Variance
RDS Route Demand Standard Deviation
RCM1 Route Customer Minimum
RCM2 Route Customer Maximum
RCM3 Route Customer Median
RCV Route Customer Variance
RCS Route Customer Standard Deviation

Appendix B

Appendix 2. C2 Data

Instance Name	RLT	RLM_1	RLM_2	RLM_3	RLM_4	RLV	RLS	RDM_1	RDM_2	RDM_3	RDV	RDS	RCM_1	RCM_2	RCM_3	RCV	RCS
C2_1.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_2.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_3.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_4.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_5.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_6.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_7.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_8.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_9.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_10.6.vrp	1470	101	325	245	700	17616.67	132.73	370	700	700	90.67	9.52	17	41	37	90.67	9.52
C2_1.6.vrptw	1923	268	357	320.50	630	176.67	13.29	610	640	630	2.67	1.63	31	36	37	2.67	1.63
C2_2.6.vrptw	1854	179	356	309	640	3856.67	62.10	520	690	640	9.47	3.08	28	36	37	9.47	3.08
C2_3.6.vrptw	1767	161	356	294.50	645	4296.67	65.55	500	690	645	11.07	3.33	27	36	37	11.07	3.33
C2_4.6.vrptw	1691	194	347	281.83	640	5296.67	72.78	490	700	640	12.67	3.56	27	36	37	12.67	3.56
C2_5.6.vrptw	1867	271	353	311.17	625	896.67	29.94	590	670	625	3.47	1.86	31	36	37	3.47	1.86
C2_6.6.vrptw	1852	251	352	308.67	635	2176.67	46.65	550	690	635	6.27	2.50	30	36	37	6.27	2.50
C2_7.6.vrptw	1842	264	352	307.00	625	1176.67	34.30	580	670	625	5.07	2.25	31	36	37	5.07	2.25
C2_8.6.vrptw	1813	251	350	302	640	1896.67	43.55	550	670	640	4.67	2.16	30	36	37	4.67	2.16
C2_9.6.vrptw	1822	264	349	303.67	620	576.67	24.01	610	670	620	3.87	1.97	31	36	37	3.87	1.97
C2_10.6.vrptw	1800	251	350	300	635	2016.67	44.91	550	670	635	5.47	2.34	30	36	37	5.47	2.34

RLT Route Length Total
RLM1 Route Length Minimum
RLM2 Route Length Maximum
RLM3 Route Length Mean
RLM4 Route Length Median
RLV Route Length Variance
RLS Route Length Standard Deviation
RDM1 Route Demand Minimum
RDM2 Route Demand Maximum
RDM3 Route Demand Median
RDV Route Demand Variance
RDS Route Demand Standard Deviation
RCM1 Route Customer Minimum
RCM2 Route Customer Maximum
RCM3 Route Customer Median
RCV Route Customer Variance
RCS Route Customer Standard Deviation

Appendix C

Appendix 3. R1 Data

Instance Name	RLT	RLM_1	RLM_2	RLM_3	RLM_4	RLV	RLS	RDM_1	RDM_2	RDM_3	RDV	RDS	RCM_1	RCM_2	RCM_3	RCV	RCS
R1_1.20.vrp	2881	2	244	144.05	197	2804.98	52.96	9	200	197	15.05	3.88	1	17	11	15.05	3.88
R1_2.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_3.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_4.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_5.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_6.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_7.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_8.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_9.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_10.18.vrp	2866	61	244	159.22	197.5	68.38	8.27	167	200	197.5	6.46	2.54	7	17	11	6.46	2.54
R1_1.20.vrptw	4784	138	325	239.20	180	309.40	17.59	135	199	180	1.58	1.26	8	12	11	1.58	1.26
R1_2.18.vrptw	4093	111	307	227.39	197	50.50	7.11	170	200	197	2.10	1.45	9	14	11	2.10	1.45
R1_3.18.vrptw	3390	103	268	188.33	196.5	21.32	4.62	184	200	196.5	3.52	1.88	8	15	11	3.52	1.88
R1_4.18.vrptw	3059	54	255	169.94	198	70.62	8.40	163	200	198	6.58	2.56	8	18	11	6.58	2.56
R1_5.18.vrptw	4128	134	374	229.33	196	14.50	3.81	187	200	196	3.75	1.94	8	15	11	3.75	1.94
R1_6.18.vrptw	3609	107	301	200.50	199	77.44	8.80	168	200	199	3.05	1.75	8	14	11	3.05	1.75
R1_7.18.vrptw	3165	112	257	175.83	197	24.03	4.90	186	200	197	3.63	1.91	8	15	11	3.63	1.91
R1_8.18.vrptw	2971	76	241	165.06	199	133.79	11.57	150	200	199	5.28	2.30	7	17	11	5.28	2.30
R1_9.18.vrptw	3790	124	309	210.56	197	23.09	4.81	186	200	197	2.93	1.71	9	14	11	2.93	1.71
R1_10.18.vrptw	3318	97	266	184.33	198	97.44	9.87	157	200	198	2.58	1.60	8	14	11	2.58	1.60

RLT Route Length Total
RLM1 Route Length Minimum
RLM2 Route Length Maximum
RLM3 Route Length Mean
RLM4 Route Length Median
RLV Route Length Variance
RLS Route Length Standard Deviation
RDM1 Route Demand Minimum
RDM2 Route Demand Maximum
RDM3 Route Demand Median
RDV Route Demand Variance
RDS Route Demand Standard Deviation
RCM1 Route Customer Minimum
RCM2 Route Customer Maximum
RCM3 Route Customer Median
RCV Route Customer Variance
RCS Route Customer Standard Deviation

Appendix D

Appendix 4. R2 Data

Instance Name	RLT	RLM$_1$	RLM$_2$	RLM$_3$	RLM$_4$	RLV	RLS	RDM$_1$	RDM$_2$	RDM$_3$	RDV	RDS	RCM$_1$	RCM$_2$	RCM$_3$	RCV	RCS
R2_1.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_2.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_3.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_4.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_5.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_6.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_7.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_8.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_9.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_10.4.vrp	1598	325	443	399.5	848	6452.25	80.33	820	997	848	8.67	2.94	47	53	50	8.67	2.94
R2_1.4.vrptw	4477	1012	1179	1119.25	872.5	8804.92	93.83	775	993	872.5	16.67	4.08	47	56	50	16.67	4.08
R2_2.4.vrptw	3610	764	1040	902.5	896.5	12554.92	112.05	726	994	896.5	28.00	5.29	43	55	50	28.00	5.29
R2_3.4.vrptw	2865	420	847	716.25	897	14239.58	119.33	728	991	897	74.67	8.64	38	58	50	74.67	8.64
R2_4.4.vrptw	1970	424	558	492.5	907	14328.92	119.70	709	990	907	72.67	8.52	43	62	50	72.67	8.52
R2_5.4.vrptw	3355	773	916	838.75	887.5	5203.58	72.14	797	941	887.5	17.33	4.16	45	55	50	17.33	4.16
R2_6.4.vrptw	2897	605	841	724.25	931	18454.92	135.85	678	973	931	70.67	8.41	39	59	50	70.67	8.41
R2_7.4.vrptw	2442	391	763	610.5	883	15080.25	122.80	751	996	883	68.67	8.29	41	61	50	68.67	8.29
R2_8.4.vrptw	1838	305	533	459.5	891.5	16148.25	127.08	742	988	891.5	118.00	10.86	36	60	50	118.00	10.86
R2_9.4.vrptw	3087	654	865	771.75	888	10298.25	101.48	756	981	888	24.00	4.90	44	56	50	24.00	4.90
R2_10.4.vrptw	2655	555	940	663.75	902	13292.92	115.29	728	981	902	58.00	7.62	44	61	50	58.00	7.62

RLT Route Length Total
RLM1 Route Length Minimum
RLM2 Route Length Maximum
RLM3 Route Length Mean
RLM4 Route Length Median
RLV Route Length Variance
RLS Route Length Standard Deviation
RDM1 Route Demand Minimum
RDM2 Route Demand Maximum
RDM3 Route Demand Median
RDV Route Demand Variance
RDS Route Demand Standard Deviation
RCM1 Route Customer Minimum
RCM2 Route Customer Maximum
RCM3 Route Customer Median
RCV Route Customer Variance
RCS Route Customer Standard Deviation

Appendix E

Appendix 5. RC1 Data

Instance Name	RLT	RLM_1	RLM_2	RLM_3	RLM_4	RLV	RLS	RDM_1	RDM_2	RDM_3	RDV	RDS	RCM_1	RCM_2	RCM_3	RCV	RCS
RC1_1.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_2.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_3.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_4.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_5.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_6.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_7.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_8.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_9.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_10.18.vrp	2786	75	225	154.78	199	8.94	2.99	190	200	199	2.69	1.64	8	15	11	2.69	1.64
RC1_1.18.vrptw	3693	93	397	205.17	198.5	6.12	2.47	191	200	198.5	1.40	1.18	9	13	11	1.40	1.18
RC1_2.18.vrptw	3319	93	310	184.39	198	5.88	2.43	193	200	198	1.05	1.02	9	13	11	1.05	1.02
RC1_3.18.vrptw	3013	93	235	167.39	199	12.71	3.56	186	200	199	1.99	1.41	8	14	11	1.99	1.41
RC1_4.18.vrptw	2875	83	243	159.72	199	9.18	3.03	189	200	199	2.10	1.45	8	14	11	2.10	1.45
RC1_5.18.vrptw	3451	114	297	191.72	198	7.29	2.70	190	200	198	1.40	1.18	8	13	11	1.40	1.18
RC1_6.18.vrptw	3390	100	288	188.33	199	14.59	3.82	188	200	199	1.63	1.28	9	13	11	1.63	1.28
RC1_7.18.vrptw	3296	109	264	183.11	198	5.41	2.33	192	200	198	1.63	1.28	9	14	11	1.63	1.28
RC1_8.18.vrptw	3132	93	282	174.00	199	6.82	2.61	192	200	199	2.10	1.45	9	14	11	2.10	1.45
RC1_9.18.vrptw	3099	93	263	172.17	199	12.47	3.53	186	200	199	2.10	1.45	8	13	11	2.10	1.45
RC1_10.18.vrptw	3006	93	276	167.00	199	8.35	2.89	191	200	199	3.28	1.81	8	15	11	3.28	1.81

RLT Route Length Total
RLM1 Route Length Minimum
RLM2 Route Length Maximum
RLM3 Route Length Mean
RLM4 Route Length Median
RLV Route Length Variance
RLS Route Length Standard Deviation
RDM1 Route Demand Minimum
RDM2 Route Demand Maximum
RDM3 Route Demand Median
RDV Route Demand Variance
RDS Route Demand Standard Deviation
RCM1 Route Customer Minimum
RCM2 Route Customer Maximum
RCM3 Route Customer Median
RCV Route Customer Variance
RCS Route Customer Standard Deviation

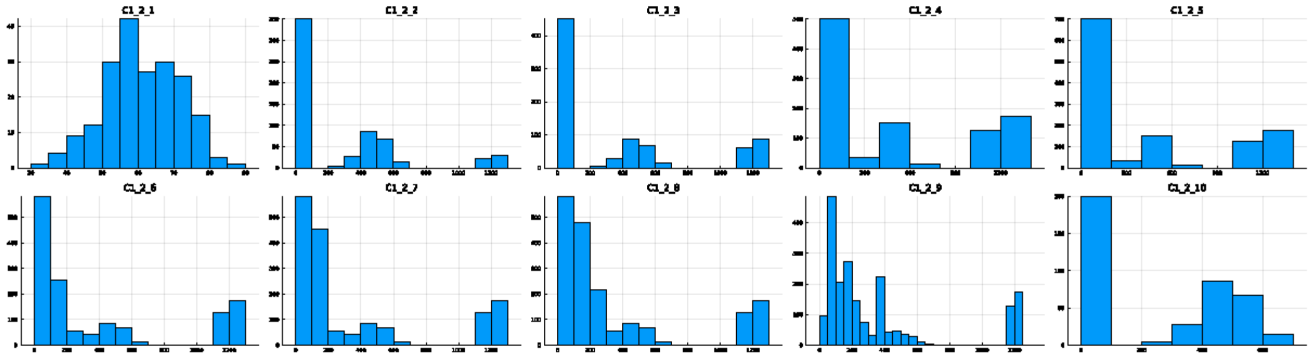
Appendix F

Appendix 6. RC2 Data

Instance Name	RLT	RLM_1	RLM_2	RLM_3	RLM_4	RLV	RLS	RDM_1	RDM_2	RDM_3	RDV	RDS	RCM_1	RCM_2	RCM_3	RCV	RCS
RC2_1.6.vrp	1538	18	495	256.33	779.5	204654	452.39	19	984	779.5	665.87	25.80	1	59	43.5	665.87	25.80
RC2_2.5.vrp	1516	18	495	303.2	807	157808	397.25	22	984	807	519	22.78	1	59	47	519	22.78
RC2_3.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_4.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_5.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_6.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_7.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_8.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_9.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_10.4.vrp	1498	229	495	374.5	901.5	11257.67	106.10	771	984	901.5	52.67	7.26	42	59	49.5	52.67	7.26
RC2_1.6.vrptw	3086	417	623	514.33	584	8519	92.30	448	728	584	13.07	3.61	27	38	43.5	13.07	3.61
RC2_2.5.vrptw	2816	414	751	563.2	743	5680	75.37	599	792	743	11.50	3.39	36	44	47	11.5	3.39
RC2_3.4.vrptw	2586	571	785	646.5	883.5	5980.33	77.33	803	988	883.5	10	3.16	46	53	49.5	10	3.16
RC2_4.4.vrptw	2022	453	597	505.5	894	14643.67	121.01	774	996	894	32.67	5.72	44	57	49.5	32.67	5.72
RC2_5.4.vrptw	2903	609	844	725.75	877	1329	36.46	862	942	877	12.67	3.56	47	54	49.5	12.67	3.56
RC2_6.4.vrptw	2864	606	871	716.0	878	5885.67	76.72	809	993	878	16.67	4.08	45	55	49.5	16.67	4.08
RC2_7.4.vrptw	2517	525	757	629.25	926	12654.33	112.49	727	979	926	23.33	4.83	46	57	49.5	23.33	4.83
RC2_8.4.vrptw	2303	351	754	575.75	901.5	16371	127.95	756	999	901.5	60.67	7.79	41	57	49.5	60.67	7.79
RC2_9.4.vrptw	2172	456	688	543.0	893	14121	118.83	775	997	893	36.67	6.06	44	58	49.5	36.67	6.06
RC2_10.4.vrptw	1998	439	551	499.5	899.5	12855	113.38	771	988	899.5	18	4.24	47	56	49.5	18	4.24

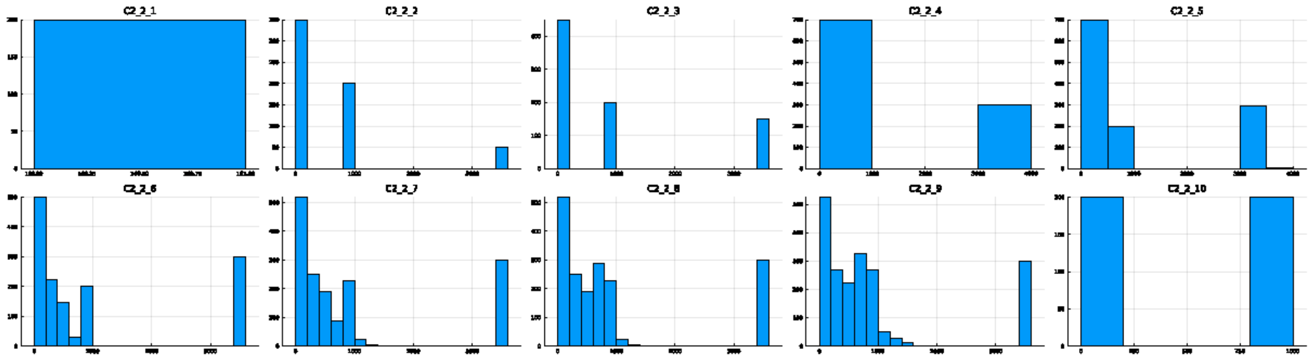
RLT Route Length Total
RLM1 Route Length Minimum
RLM2 Route Length Maximum
RLM3 Route Length Mean
RLM4 Route Length Median
RLV Route Length Variance
RLS Route Length Standard Deviation
RDM1 Route Demand Minimum
RDM2 Route Demand Maximum
RDM3 Route Demand Median
RDV Route Demand Variance
RDS Route Demand Standard Deviation
RCM1 Route Customer Minimum
RCM2 Route Customer Maximum
RCM3 Route Customer Median
RCV Route Customer Variance
RCS Route Customer Standard Deviation

Appendix G



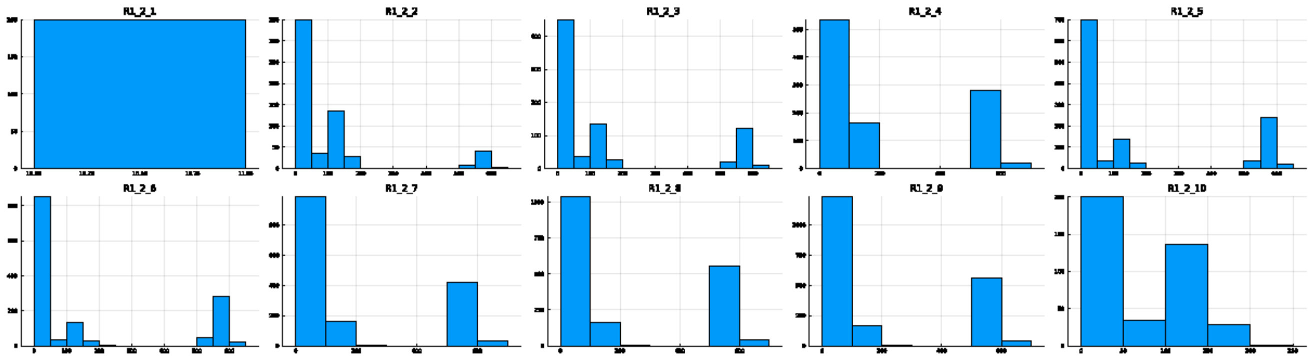
Appendix 7. C1 Time Windows Distributions

Appendix H



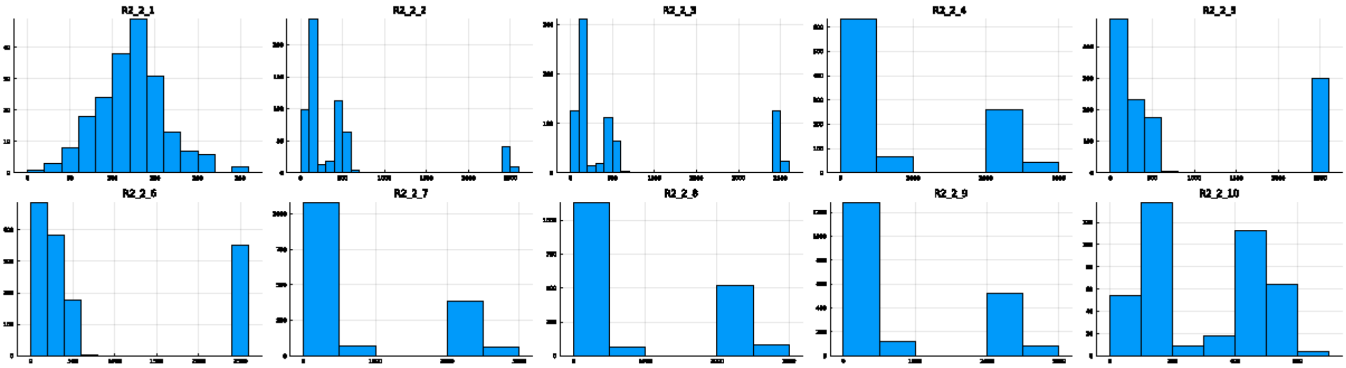
Appendix 8. C2 Time Windows Distributions

Appendix I



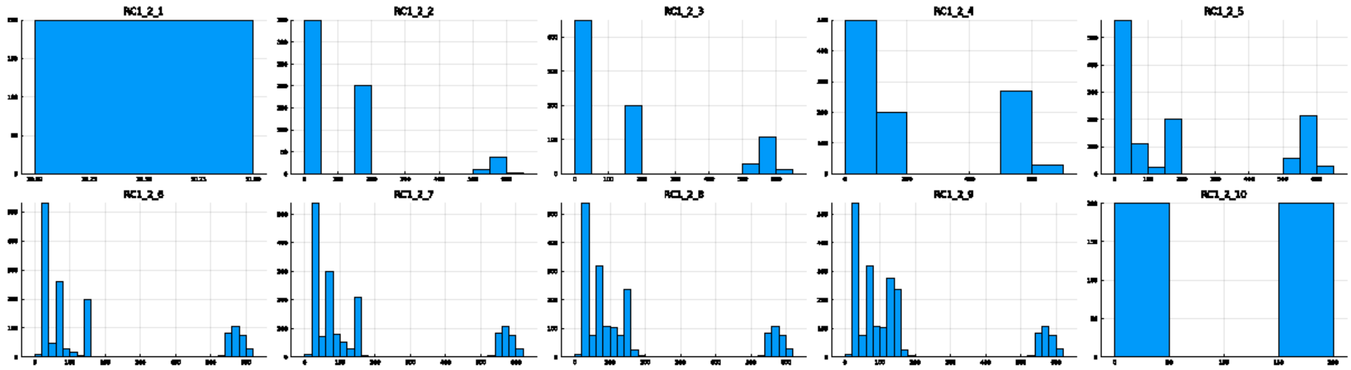
Appendix 9. R1 Time Windows Distributions

Appendix J



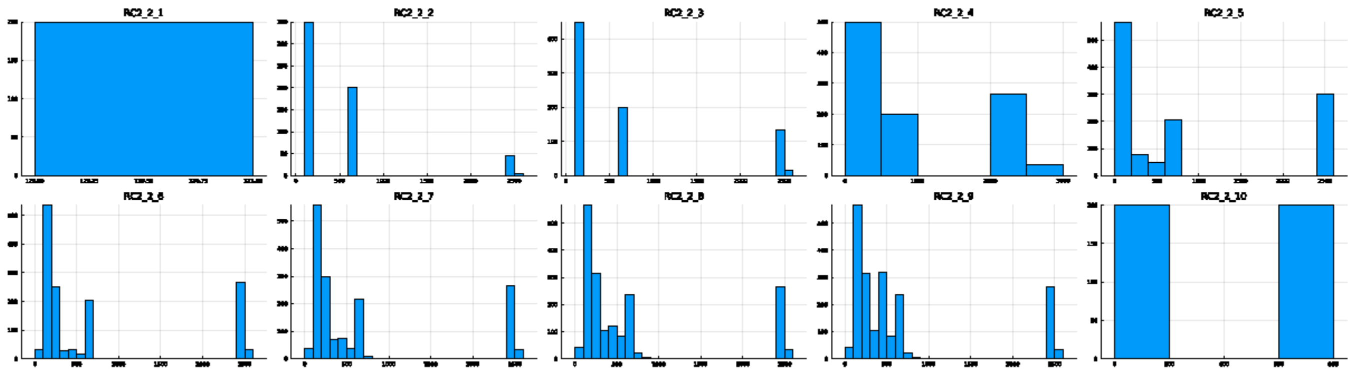
Appendix 10. R2 Time Windows Distributions

Appendix K



Appendix 11. RC1 Time Windows Distributions

Appendix L



Appendix 12. RC2 Time Windows Distributions