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HOW TO DELIVER FREE COAL TO THE POOR FAMILIES? TURKEY CASE¹

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ABSTRACT

Turkish government delivers free coal to poor families via General Directorate of Coal Industry (GDCI) as a part of social and economic policy. Although project has many components, generally the coal is taken from several mines within country, after basic industrial process, the coal is sent to main delivery nodes from mines nodes by highways, seaways or railroads. Finally, the poor families come to main nodes and carry the coal to their houses to use during the winter.

The cost of delivering free coal is reimbursed by Turkish Treasury to GDCI as duty loss under some government regulations. In this study, after giving general information about the problem and describing situation in Turkey, it will be developed transportation and transshipment models by taking into consideration some assumptions. The problem will be solved with alternative methods by giving useful and applicable recommendations for GDCI to reduce cost of this public policy and to enable them to help as many as poor families to have free coal in timely fashion.

Key Words: Optimization, Operations Research, Management Science, Turkey, Free Coal, Poor Families, Transportation Models, Transshipment Models

JEL Classification: C44, C54, C61, G18, H44

YOKSUL AİLELERE ÜCRETSİZ KÖMÜR NASIL DAĞITILIR? TÜRKİYE ÖRNEĞİ

ÖZ

Türk hükümeti, sosyal ve ekonomik politikaların bir parçası olarak Türkiye Kömür İşletmeleri Genel Müdürlüğü (GDCI) vasıtasıyla yoksul ailelere ücretsiz kömür sağlamaktadır. Projenin birçok bileşeni olmasına rağmen, genel olarak kömür ülkedeki birkaç madenden alınmakta; temel sanayi

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işlemlerinden sonra da karayolu, deniz ya da demiryolları ile ana dağıtım noktalarına

gönderilmektedir. Projenin son aşamasında ise, yoksul aileler ana dağıtım noktalarına gelerek

kömürlerini almakta ve kış boyunca kullanmak için evlerine taşımaktadır.

Ücretsiz kömür dağıtma projesinin maliyeti, yapılan düzenlemeler uyarınca, Hazine

Müsteşarlığı tarafından GDCI'ye görev zararı olarak geri ödenmektedir. Bu çalışmada, proje hakkında

genel bilgi verildikten ve Türkiye'deki mevcut durum ortaya konduktan sonra, bazı varsayımlarla,

ulaştırma ve aktarma modelleri geliştirilecektir. Problem alternatif yöntemlerle çözülerek, proje

çervesinde uygulanacak kamu politikasının maliyetini düşürmek ve mümkün olduğu kadar çok sayıda

yoksul ailenin projeden faydalanarak zamanında ücretsiz kömür ihtiyaçlarını gidermelerini sağlamak

için GDCI'na faydalı ve uygulanabilir öneriler sunulacaktır.

Anahtar Kelimeler: Optimizasyon, Yöneylem Araştırması, Yönetim Bilimi, Türkiye, Ücretsiz Kömür,

Yoksul Aileler, Ulaştırma Modelleri, Aktarma Modelleri

Jel Sınıflandırması: C44, C54, C61, G18, H44

1. INTRODUCTION

Turkish government delivers free coal to poor families via General Directorate of Coal Industry

(GDCI) as a part of social and economic policy. Although project has many components, generally the

coal is taken from several mines within country, after basic industrial process, the coal is sent to main

delivery nodes from mines nodes by highways, seaways or railroads. Finally, the poor families come to

main nodes and carry the coal to their houses to use during the winter.

The cost of delivering free coal is reimbursed by Turkish Treasury to GDCI as duty loss under some

government regulations. The strategic objective of the project, by providing free coal, is to help poor

families to sustain harsh and cold winters. The fundamental objectives are to close the gap of life quality

between poor and rich people and to reduce child sickness because of cold weather. The means

objectives are to reduce imported oil consumption for heating houses and to stimulate transportation

industry.

The 3E's (Economy, Efficiency and Effectiveness) objectives are to decrease the cost of delivering

coal via shortest path, to produce coal in timely fashion and using best way of transportation method.

Those objectives are main areas to use Operations Research and Management Science tools concerned

with scientifically deciding to design and operate systems optimally by using of mathematical models

41

<u>Yönetim ve Ekonomi Araştırmaları Derqisi / Journal of Management and Economics Research</u>
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to investigate complex problems under conditions requiring the best allocation of scarce resources (Winston and Venkataraman, 2003:1-10; Taha, 2003; Ragsdale, 2004; Johnson, 2006).

According to project of "Delivering Free Coal to Poor Families", the coal is taken from several mines within Turkey. Those mine nodes have certain amount of production capacity and they also have previously engaged supply commitment (GDCI, 2006). Therefore they will use only some part of their production capacity for this project. For example; despite the fact that the Manisa node has total capacity of 15.000 tons/year, it has 9.000 ton/year of previously engaged production capacity. Therefore Manisa mine node has just 6.000 ton/year maximum capacity devoted to this project. This operations restriction is illustrated by Figure 1.

At the very beginning of the project GDCI determines the total coal supply coming from several mines. The second stage of the project is to determine the total demand of coal. The demand is determined by the Turkish Treasury with consulting the Social Solidarity Fund (SSF), Ministry of Environmental Affairs (MOEA), Ministry of Finance (MOF), Ministry of Development, State Meteorology Institute (SMI), Ministry of Transportation (MOT), Ministry of Energy and Natural Resources (MOER), Local Municipalities (LM) and other responsible governmental agencies.

The relevant Department of the Turkish Treasury gets following information necessary to determine the needs of each demand nodes (Republic of Turkey Ministry of Transportation, 2006; Republic of Turkey Ministry of Energy and Natural Resources, 2006):

- The number of poor families from SSF and STO,
- Income level of those families from MOF and LMs,
- Pollution rate of local area MOEA.
- Weather conditions of demand node SMI,
- Transportation facility options from MOT,
- Available/alternative natural resources in demand node from MOER,

In our model, to give an insight about determining the demand in cities, we used the weather conditions, pollution rates, number of people in cities and their indexes with respect to total level these variables. After collecting these data by Turkish Treasury, it assigns the GDCI to deliver the free coal to the families in need in across the Turkey as the cost of delivery is reimbursed by the Turkish Treasury.

2. METHOD, MODELS AND ASSUMPTIONS

2.1) Transportation Methods

The all transportation methods, except airway, are generally used by the GDCI to delivery. GDCI is given the distances between all the supply nodes and demand nodes. The distances can be railroad

(km), seaway (mile) and highway (km) by the MOT. Based on past experiences, GDCI also have the information about delivery ton/km cost of coal. The delivery cost structure is given at the Table 1:

Table 1: The Delivery Cost Structure

Type of Transportation	Unit Cost (ton/km or mile)
Highway	\$1.00
Railroad	\$0.10
Seaway	\$0.15

Source: Republic of Turkey Ministry of Transportation & GDCI

As seen from the Table 1 above, the unit cost for delivery with railroad is the cheapest (\$0.10 ton/km) among the others. Although there is a highway between all the supply nodes and the demand nodes, the restricted number of demand nodes has railroad and seaway transshipment nodes. In those cases, the coal is first sent to transshipment nodes by GDCI and then to final demand nodes, if the cost is cheaper than highway. We want to mention that all transshipment nodes have 0 (zero) demand and supply.

2.2) Type of Problem

Our project is related with network flow problem. These problems are used for a number of practical decisions in business. Based on the information above; we have determined that our model is a typical transshipment problem.

Our model is a transshipment problem. Transshipment problem deals with the distribution of goods from several points of supply to a number of points of demand (Greenberg, 2006). It may also contain transshipment points through which coal can be shipped on their way from a supply point (shipments from sources) to demand point (to destinations) so that total production and transportation costs of project of "Delivering Free Coal to Poor Families" are minimized (Winston and Venkataraman, 2003:360-406; Albright, 2001; Hillier, and Lieberman, 1995; Daskin, 1995; Johnson, 2006).

2.3) Model and Assumptions

GDCI has 21 supply nodes and it is generally assign to deliver the free coal to 1738 demand nodes of which are cities and townships. To simplify the solution, in our model, we will only use 6 supply nodes including Bolu, Canakkale, Corum, Kutahya, Manisa and Sirnak whereas we will use 8 demand nodes including Adana, transshipment nodes some of which are seaway and the others are

railroad centers. Izmir, Mersin and Samsun are the transshipment nodes which seaway and highway are used while Ankara and Erzurum are the transshipment nodes that railroad and highway are used.

As it is seen in Figure 1 below, the coal can be transported just by one transportation method or as long as there are alternative transportation methods combination of two or three transportation method can be applied. For instance from sample mine to a sample node coal can be transported by highway and this route is illustrated by dashed black line with truck icon. The same transportation can be realized by either highway and truck combination or first railroad second seaway and seaway to final destination.

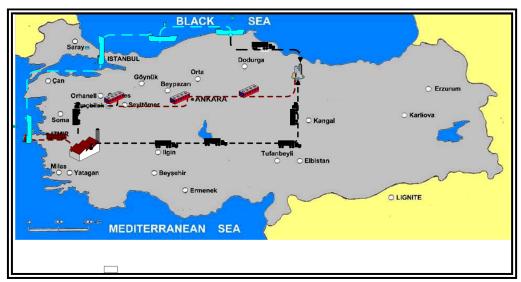


Figure 1: Illustration of a Sample Transshipment Structure of the Project

Source: Republic of Turkey Ministry of Transportation

GDCI has very complicated production process. Therefore we have to make some other assumptions (Cohon, 1978; Fourer, Gay and Kernighan, 2003; Johnson, 2006):

- a- The capacity of mines are fixed and does not change because of any interruption of production,
- **b-** Production function is linear.
- **c-** Loading and dumping costs are ignored
- **d-** It is assumed that shortest path between nodes are always available. The weather conditions of seaway or maintenance of railroads or other possible interruptions are ignored

2.4) Purposes and Goals of the Project

The project has the following objectives:

1- Minimizing the total cost of coal transportation by choosing best (cheapest) combination of transportation methods with respect to fulfilling demand on poor families in demand nodes. (Transshipment Problem)



2- Minimizing the number of facility location that is required to cover all the coal demands for poor family in demand nodes. (Facility Location Set-Covering Problem)

3. ALTERNATIVE SOLUTION METHODS & RESULTS

We think that we can solve our problem by using either Facility Location Set-Covering Problem or Transshipment Method. Despite we have decided to solve our model by using transshipment method we presented both models' objective functions' and constraints' notations.

3.1) Facility Location Set-Covering Problem

Decision Variable: $x_{i=1}$ if Transshipment node (facility) is located in city j, 0 otherwise.

Objective function:

Minimize:
$$\sum_{i=1}^{7} x_j$$

So we can write out LP formula as

```
(1)
Min x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12 + x13
s.t.
       x1 + x4 + x9 + x11 >= 1
       x2 + x5 + x7 + x8 + x13 >= 1
       x3 + x4 + x11 + x12 + x13 >= 1
       x1 + x3 + x4 + x7 + x9 + x11 + x12 >= 1
       x2 + x5 + x7 + x13 >= 1
       x6 >= 1
       x2 + x5 + x7 + x13 >=1
       x2 + x8 >= 1
       x1 + x4 + x9 + x11 >= 1
       x9 + x10 + x12 >= 1
       x1 + x3 + x4 + x9 + x11 + x12 >= 1
       x3 + x4 + x11 + x12 >=1
       x2 + x3 + x5 + x7 + x13 >=1
```

xj binary

- Build the minimal number of facility location such that at least one location is within 500 km of each city. (demand nodes)
- 13 cities in our model
- The distance (km) between the cities is in the table below.

Table 2: The Distance (km) Between the Cities

CITY NAME	ADANA	AĞRI	AMASYA	ANKARA	ARTVİN	EDIRNE	ERZURUM	HAKKARİ	MERSIN	izmir	KIRŞEHİR	SAMSUN	TUNCELİ
ADANA	1	966	612	490	1046	1168	809	899	69	901	375	729	622
AĞRI	966	1	734	1057	391	1634	183	434	1035	1636	945	754	423
AMASYA	612	734	1	336	708	900	553	1145	639	915	308	131	495
ANKARA	490	1057	336	1	999	682	115	1366	483	579	186	419	818
ARTVİN	1046	391	708	999	1	1546	237	770	1115	1578	966	580	461
EDİRNE	1168	1634	900	682	1546	1	1453	2045	1161	534	868	966	1395
ERZURUM	809	183	553	876	237	1453	1	617	878	1455	764	573	242
HAKKARİ	899	434	1145	1366	770	2045	617	1	968	1800	1180	1188	650
MERSIN	69	1035	639	483	1115	1161	878	968	1	893	368	743	691
İZMİR	901	1636	915	579	1578	534	1455	1800	75	1	754	120	1397
KIRŞEHİR	375	945	308	186	966	868	764	1180	368	754	1	391	706
SAMSUN	729	754	131	419	580	966	573	1188	743	998	391	1	575
TUNCELİ	622	423	495	818	461	1395	242	650	691	1397	706	575	1

Source: Republic of Turkey Ministry of Transportation

- Coverage distance is 500 km driving distance
- Coverage matrix is as below.

Table 3: Coverage Matrix

CITY NAME	ADANA	AĞRI	AMASYA	ANKARA	ARTVİN	EDIRNE	ERZURUM	НАККАВІ	MERSIN	izmir	KIRŞEHİR	SAMSUN	TUNCELİ
ADANA	1	0	0	1	0	0	0	0	1	0	1	0	0
AĞRI	0	1	0	0	1	0	1	1	0	0	0	0	1
AMASYA	0	0	1	1	0	0	0	0	0	0	1	1	1
ANKARA	1	0	1	1	0	0	1	0	1	0	1	1	0
ARTVİN	0	1	0	0	1	0	1	0	0	0	0	0	1
EDİRNE	0	0	0	0	0	1	0	0	0	0	0	0	0
ERZURUM	0	1	0	0	1	0	1	0	0	0	0	0	1
HAKKARİ	0	1	0	0	0	0	0	1	0	0	0	0	0
MERSIN	1	0	0	1	0	0	0	0	1	0	1	0	0
İZMİR	0	0	0	0	0	0	0	0	1	1	0	1	0
KIRŞEHİR	1	0	1	1	0	0	0	0	1	0	1	1	0
SAMSUN	0	0	1	1	0	0	0	0	0	0	1	1	0
TUNCELİ	0	1	1	0	1	0	1	0	0	0	0	0	1

As seen in the LINDO output of Appendix (Annex 1), if we locate only four locations (x2 (Agri), x3 (Amasya), x6 (Edirne) x9 (Edirne) then our objective value will be 4 and all the demand nodes will be within 500 distance to the at least one of the facility location.

3.2) Transshipment Problem

a) Model

The main objective of this project is to minimize the total cost of transportation (shipments from sources to destinations) free coal with respect to fulfilling demand on poor families in demand nodes.

b) List and description of all sets or indexes

Set k= transportation methods where k=1,2,3 (highway, railroad, seaway)

Set i = supply nodes where i=1,2,...,6 (n)

Set j= demand nodes where j=1,2.....8 (m)

c) Description of all data elements and decision variables

 X_{ijk} = number of tons coal transported from node i to node j by method k

 $C_k = cost of transportation method k per km(mile)/ton$

D_{ij}= distance from node i to node j

 S_i = supply capacity in node i

 P_i = demand in node i

d) Defining the Objective Function

The objective of the project is to minimize the total cost of transportation for free coal with respect to meeting demand of poor families in demand nodes.

Then objective function for the project is expressed as:

MIN:
$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{n} D_{ij} X_{ijk} C_k$$
 (2)

e) Defining the Constraints



$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} \le S_i \quad \forall \quad i \in$$
 (Supply constraint)

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} \ge P_j \ \forall \ j \in$$
 (Demand constraint) (4)

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} - \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} = 0 \quad \forall i \in \text{(Transshipment constraint)}$$
 (5)

X_{ij}, S_i, P_j non negativity

f) Implementing the Model

MIN:
$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{n} D_{ij} X_{ijk} C_{k}$$
 (6)

S.T.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} \le S_i \quad \forall i \in \text{(Supply constraint)}$$
 (7)

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} \ge P_j \ \forall \ j \in$$
 (Demand constraint) (8)

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} - \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} = 0 \quad \forall \quad i \in \text{ (Transshipment constraint)}$$
 (9)

X_{ij}, S_i, P_j non negativity

An appropriate way of implementing this model for the project is shown in Excel sheets (Figure 2: Excel Sheet Showing Appropriate way of implementing Model, Figure 3: Excel Sheet Showing Key Formulas in the Model & Figure 4: Excel Sheet Showing Solutions of the Model) of Appendix Section of the study (See Annex 2 for the Excel solution of the Model).

g) Results of the Model

As seen from the Figure 4 in Appendix section, the minimum total cost (optimal solution) for this problem would be TC=\$ 6,723,310 where all the constraints are meet.

The solutions of the model will also provide us best transportation type and quantity from the related sources to destinations shown in Table 4 below.

Table 4: Transportation Type and Quantity from Sources to Destinations

X _{ij} will be as		
From- to	Quantity	Type of Transportation
X _{Manisa-Edirne:}	630 ton	Highway
X _{Manisa-Izmir:}	5370 ton	Highway
X _{Canakkale-Edirne:}	1150 ton	Highway
X Sirnak-Hakkari:	600 ton	Highway
X Corum-Kirsehir:	750ton	Highway
X _{Kutahya-Ankara:}	3200 ton	Highway
X _{Bolu-Ankara:}	850 ton	Highway
X _{Izmir-Samsun:}	2590 ton	Seaway
X Izmir-Mersin:	2780 ton	Seaway
X _{Samsun-Amasya:}	670 ton	Highway
X _{Samsun-Tunceli:}	1920 ton	Highway
X _{Mersin-Adana:}	570 ton	Highway
X _{Mersin-Hakkari} :	810 ton	Highway
X Mersin-Kirsehir:	1400 ton	Highway
X _{Ankara-Erzurum:}	4050 ton	Railway
X Erzurum-Artvin:	1040 ton	Highway
X Erzurum-Tunceli:	600 ton	Highway
X Erzurum-Agri:	2410 ton	Highway

h) Sensitivity Analysis of the Model

According to Sensitivity Analysis of the Model (Frontline Systems, 2006), we will have the following results by changing either "Right Hand Side Values" or "Objective Function Coefficient" (See Annex 3 for the Sensitivity Analysis of the Model):

Changing Right Hand Side Values: After solving an LP problem, it should be determined how much better or worse the solution would be if we have more or less resources for the project. As seen from the Lindo Sensitivity Output, the shadow (dual) price of the constraints indicates the amount by which the objective function value changes given a unit increase in the right hand side (RHS) value of the constraint, assuming all other coefficients remain constant (ceteris paribus). If a shadow price is positive that means unit increase in the RHS value of the associated constraint results in increase in the optimal objective function of the value. For example as shadow price of Amasya is 167, therefore, if the amount of coal delivered to Amasya changes between the allowable increase and decrease limit

<u>Yönetim ve Ekonomi Araştırmaları Derqisi / Journal of Management and Economics Research</u>

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which are 0 and 670 respectively, our objective function values change by \$167 for each additional unit. Also note that some variables that have non-binding constraints have 0 shadow price.

Changing Objective Function Coefficient: If the one of the coefficient of the variable change in an allowable decrease or increase limit, the optimal solution of the objective function will not change. For example, if X 111 (from Manisa to Edirne) change between allowable increase (167) and decrease (365), the optimal solution of the objective function will not change.

4. CONCLUSIONS AND POLICY RECOMENDATIONS

Turkish government delivers free coal to poor families via General Directorate of Coal Industry (GDCI) as a part of social and economic policy. Although project has many components, generally the coal is taken from several mines within country, after basic industrial process, the coal is sent to main delivery nodes from mines nodes by highways, seaways or railroads. Finally, the poor families come to main nodes and carry the coal to their houses to use during the winter.

The cost of delivering free coal is reimbursed by Turkish Treasury to GDCI as duty loss under some government regulations. After giving general information about the problem and describing situation in Turkey, transportation and transshipment models were developed by taking into consideration some assumptions. One of the fundamental objectives should also be closing the gap of life quality between poor and rich people and reducing child sickness because of cold weather.

The project should also aim 3E's (Economy, Efficiency and Effectiveness). In other words, to decrease the cost of delivering coal via shortest path, to produce coal in timely fashion and using best transportation method for the project of "Delivering Free Coal to Poor Families". The problem was solved with alternative methods by giving also following useful and applicable recommendations for GDCI to reduce cost of this public policy and to enable them to help as many as poor families to have free coal in timely fashion:

- Our model shows better solution for current operation,
- The model can be applied to current operations with real distance data among all nodes,
- For full implementation of model given assumptions should be eliminated in timely fashion,
- Management should provide professional consultant service to implement other operations research tools and techniques.

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APPENDIX

Annex 1: Lindo Solution of the Model/1

LP OPTIMUM FOUND AT STEP 10
OBJECTIVE VALUE = 3.50000000

NEW INTEGER SOLUTION OF 4.00000000 AT BRANCH 0 PIVOT 10 RE-INSTALLING BEST SOLUTION...

OBJECTIVE FUNCTION VALUE

OB	OLCIIVL	1011011011	V1111011	
1)	4 .	.000000		
VARIABLE		VALUE		REDUCED COST
X1		0.000000	1	1.000000
X2		1.000000	1	1.000000
х3		1.000000	1	1.000000
X4		0.000000	1	1.000000
X5		0.000000	1	1.000000
Х6		1.000000	1	1.000000
X7		0.000000	1	1.000000
X8		0.000000	1	1.000000
Х9		1.000000	1	1.000000
X10		0.000000	1	1.000000
X11		0.000000	1	1.000000
X12		0.000000	1	1.000000
X13		0.000000	1	1.000000
ROW	SLACE	OR SURPL	US	DUAL PRICES
2)		0.000000	١	0.000000
3)		0.000000	ı	0.00000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.000000
3)	0.00000	0.000000
4)	0.00000	0.000000
5)	1.000000	0.000000
6)	0.00000	0.000000
7)	0.00000	0.000000
8)	0.00000	0.000000
9)	0.00000	0.000000
10)	0.00000	0.000000
11)	0.00000	0.000000
12)	1.000000	0.000000
13)	0.00000	0.000000
14)	1.000000	0.000000

NO. ITERATIONS= 10

BRANCHES= 0 DETERM.= 1.000E 0

Annex 2: Excel Solution of the Model

Figure 2: Excel Sheet Showing Appropriate Way of Implementing Model

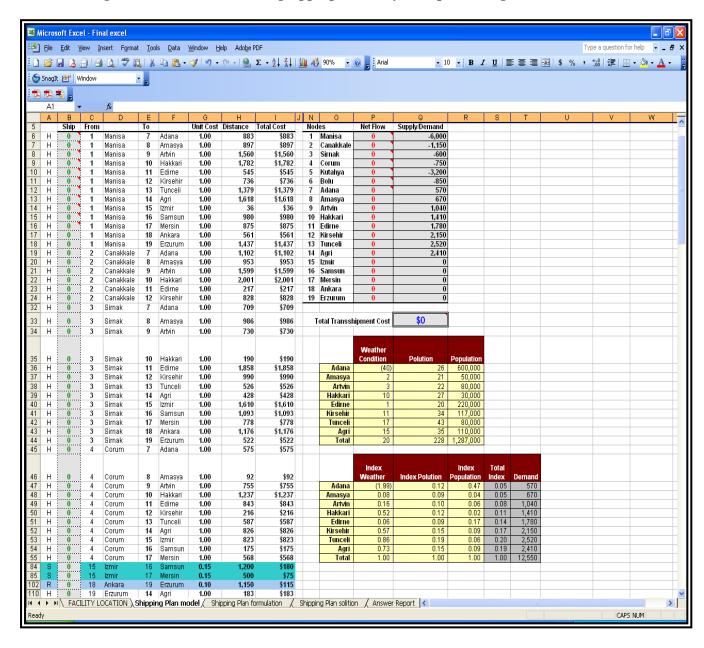


Figure 3: Excel Sheet Showing Key Formulas in the Model

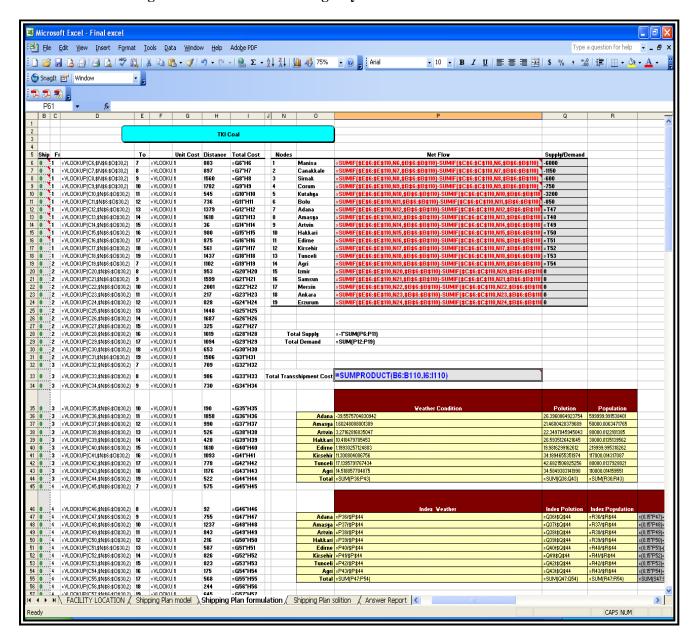
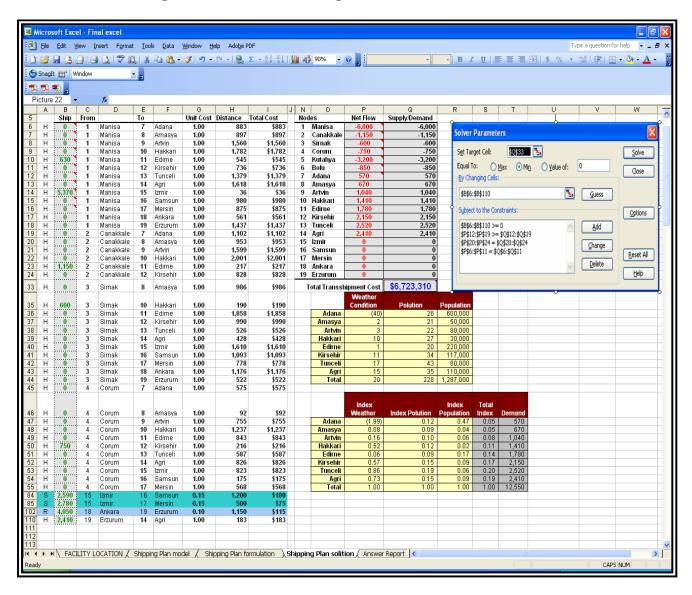


Figure 4: Excel Sheet Showing Solutions of the Model



Annex 3: Sensitivity Analysis of the Model

RANGES IN WHICH THE BASIS IS UNCHANGED:

	CHERTHE	OBJ COEFFICIENT	
VARIABLE	CURRENT	ALLOWABLE	ALLOWABLE
X17	COEF 883.000000	INCREASE INFINITY	DECREASE 703.000000
X17 X18	897.000000	INFINITY	550.000000
X19	1560.000000	INFINITY	774.000000
X110	1782.000000	INFINITY	703.000000
X111	545.000000	167.000000	365.000000
X112	736.000000	INFINITY	257.000000
X113	1379.000000	INFINITY	588.000000
X114	1618.000000	INFINITY	886.000000
X115	36.000000	127.000000	167.000000
X116 X117	980.000000 875.000000	INFINITY INFINITY	764.000000 764.000000
X117 X118	561.000000	INFINITY	127.000000
X119	1437.000000	INFINITY	1257.000000
X27	1102.000000	INFINITY	1250.000000
X28	953.000000	INFINITY	934.000000
X29	1599.000000	INFINITY	1141.000000
X210	2001.000000	INFINITY	1250.000000
X211	217.000000	547.000000	INFINITY
X212	828.000000	INFINITY	677.000000
X213	1488.000000 1687.000000	INFINITY	1025.000000 1283.000000
X214 X215	325.000000	INFINITY INFINITY	617.000000
X215 X216	1019.000000	INFINITY	1131.000000
X217	1094.000000	INFINITY	1311.000000
X218	653.000000	INFINITY	547.000000
X219	1506.000000	INFINITY	1654.000000
X37	709.000000	INFINITY	1418.000000
X38	986.000000	INFINITY	1528.000000
X39	730.000000	INFINITY	833.000000
X310	190.000000 1858.000000	585.000000	INFINITY 2202.000000
X311 X312	990.000000	INFINITY INFINITY	1400.000000
X313	526.000000	INFINITY	624.000000
X314	428.000000	INFINITY	585.000000
X315	1610.000000	INFINITY	2463.000000
X316	1093.000000	INFINITY	1766.000000
X317	778.000000	INFINITY	1556.000000
X318	1176.000000	INFINITY	1631.000000
X319	522.000000 575.000000	INFINITY INFINITY	1231.000000 658.000000
X47 X48	92.000000	INFINITY	8.000000
X49	755.000000	INFINITY	232.000000
X410	1237.000000	INFINITY	421.000000
X411	843.000000	INFINITY	561.000000
X412	216.000000	8.000000	INFINITY
X413	587.000000	INFINITY	59.000000
X414	826.000000	INFINITY	357.000000
X415	823.000000	INFINITY	1050.000000 222.000000
X416 X417	175.000000 568.000000	INFINITY INFINITY	720.000000
X418	244.000000	INFINITY	73.000000
X419	645.000000	INFINITY	728.000000
X57	674.000000	INFINITY	617.000000
X58	647.000000	INFINITY	423.000000
X59	1310.000000	INFINITY	647.000000
X510	1573.000000	INFINITY	617.000000
X511	589.000000	INFINITY	167.000000
X512 X513	486.000000 1129.000000	INFINITY INFINITY	130.000000 461.000000
X514	1368.000000	INFINITY	759.000000
X514 X515	334.000000	INFINITY	421.000000
X516	730.000000	INFINITY	637.000000
X517	666.000000	INFINITY	678.000000
X518	311.000000	130.000000	INFINITY



X519	1187.000000	INFINITY	1130.000000
X67	677.000000	INFINITY	740.000000
X68	409.000000	INFINITY	305.000000
X69	1055.000000	INFINITY	512.000000
X610	1554.000000	INFINITY	718.000000
X611	491.000000	INFINITY	189.000000
X612	377.000000	INFINITY	141.000000
X613	904.000000	INFINITY	356.000000
X614	1143.000000	INFINITY	654.000000
X615	595.000000	INFINITY	802.000000
X616	475.000000	INFINITY	502.000000
X617	670.000000	INFINITY	802.000000
X618	191.000000	141.000000	INFINITY
X619	962.000000	INFINITY	1025.000000
X1516	180.000000	8.000000	87.000000
X1517	75.000000	87.000000	8.000000
X167	729.000000	INFINITY	765.000000
X168	131.000000	8.000000	167.000000
X169	580.000000	INFINITY	10.000000
X1610	1188.000000	INFINITY	325.000000
X1611	966.000000	INFINITY	637.000000
X1612	391.000000	INFINITY	128.000000
X1613	575.000000	10.000000	87.000000
X1614	754.000000	INFINITY	238.000000
X177	69.000000	167.000000	INFINITY
X177	639.000000	INFINITY	403.000000
X178	1115.000000	INFINITY	440.000000
X1710	968.000000	87.000000	585.000000
X1710 X1711	1161.000000	INFINITY	727.000000
X1711 X1712	368.000000	128.000000	8.000000
X1712 X1713	691.000000	INFINITY	11.000000
X1713 X1714	1035.000000	INFINITY	414.000000
X1714 X1819	115.000000	130.000000	73.000000
X1019	809.000000	INFINITY	1178.000000
X197	553.000000	INFINITY	755.000000
X199	237.000000	10.000000	606.000000
X1910	617.000000	INFINITY	87.000000
X1910 X1911	1453.000000		1457.000000
X1911 X1912	764.000000	INFINITY	834.000000
X1912 X1913	242.000000	INFINITY 87.000000	10.000000
			552.000000
X1914	183.000000	238.000000	552.000000
		RIGHTHAND SIDE I	RANGES
ROW	CURRENT	ALLOWABLE	ALLOWABLE
1000	RHS	INCREASE	DECREASE
2	6000.000000	INFINITY	0.000000
3	1150.000000	630.000000	0.000000
4	600.000000	810.000000	0.000000
5	750.000000	1400.000000	0.000000
6	3200.000000	1920.000000	0.000000
7	850.000000	1920.000000	0.000000
8	0.000000	0.000000	INFINITY
9	0.000000	0.000000	2590.000000
	0.000000	0.000000	INFINITY
10 11	0.000000	0.000000	1920.000000
12	0.000000	0.000000	1920.000000
13			
	570.000000	0.000000	INFINITY
14 15	670.000000 1040.000000	0.000000	670.000000 1040.000000
16	1410.000000	0.000000	810.000000
17	1780.000000	0.00000	630.000000
18	2150.000000	0.000000	1400.000000
19	2520.000000	0.000000	1920.000000
20	2410.000000	0.000000	1920.000000
∠∪	7410.000000	0.00000	1920.000000