An Improved LQR-based Controller for PEMFC Interleaved DC-DC Converter

M. Habib and F. Khoucha

***Abstract*—Proton Exchange Membrane Fuel Cell (PEMFC) is one of the promising technologies for the distributed power generation, especially electric vehicles. For designing high efficiency fuel cell power systems, a suitable DC-DC converter is required. Among the various topologies, interleaved boost converter (IBC) is considered as a better solution for fuel cell systems, due to improved electrical performance; minimize inductor size, current ripples and harmonic content. In this paper, the electrochemical modelling of a Proton Exchange Membrane Fuel Cell (PEMFC) has been carried out. A two phase interleaved boost converter is suggested for fuel cells. In additional, an LQR control is developed and compared with classical PI control when designing an interleaved boost regulator. Simulation study for the IBC interfaced with fuel cells has been presented.**

***Index Terms*—PEMFC, IBC, LOR control.**

I. Introduction

THE Fuel Cells are electrochemical energy converters. They can be seen as black-boxes turning chemical energy contained in a fuel directly into electrical energy while generating heat and water as byproducts [1]. The mechanism involved in this conversion is the same as the one for batteries. The primary difference is that the battery contains the reactants that generate electricity whereas those reactants need to be supplied externally to the fuel cell i.e. a battery needs to be fearful away or recharged once those reactants are depleted while the fuel cell can be refueled by refilling the tank with hydrogen. In this respect they are comparable to internal combustion engines which generate mechanical power with heat and exhaust gases as byproducts [2].

The DC-DC Converter is an integral part of fuel cell power conditioning unit, therefore this paper intends to present modeling of fuel cell as well as of DC/DC Converter [3]. The design of DC/DC converter and their controller play an important role to control power regulation particularly for a common DC bus. The boost converter offers higher efficiency and less component counts compared to other DC/DC converters topologies like push pull, half bride and full bridge etc. which could possibly be used to interface fuel cell system to the load.

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The remaining sections are organized as follows. First, in Section II, we present the chosen PEMFC model. In Section III, we introduce the uncertain state space model of an IBC. In Section IV, we give a basic LQR control theory background, with the formulation needed to solve the LQR problem. Finally, in section V, numeric simulation for the IBC associated to an LQR controller has been realized and discussed.

TABLE I
PEMFC PARAMETERS

PARAMETER	VALUE
Maximum power	7 Kw
Cell open circuit voltage	1.22 V
Nominal voltage	47 V
Nominal current	30 A
Voltage at 1A	44 V
Membrane resistance	0.079 Ω
Hydrogen partial pressure H_2	2.61 bar
Oxygen partial pressure O_2	0.3 bar

II. PEM Fuel Cell Model

A PEMFC consists of an electrolyte sandwich between two electrodes. The electrolyte has a special property that allows positive ions (protons) to pass through while blocking electrons. Hydrogen gas passes over one electrode, called an anode, and with the help of a catalyst, separates into electrons and hydrogen protons, as shown in Fig. 1. The chemical reaction that described this process is:

$$2H_2 \rightarrow 4H^+ + 4e^- \quad (1)$$

The protons pass through the electrolyte towards the cathode, and the electrons close the circuit through the electric load, performing electric work. In the cathode, the protons and electrons combine with oxygen produce water, this reaction is described by:

$$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O \quad (2)$$

The overall reaction:

$$2H_2 + 2H_2 \quad (3)$$

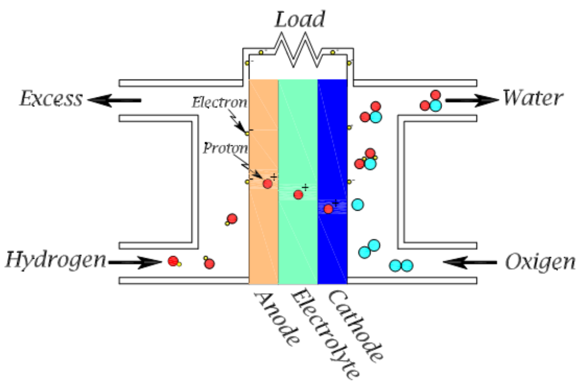

Fig. 1 Scheme of one cell internal process

In this section, by means of energy balance, the fuel cell terminal voltage is calculated. In this respect, three types of losses are shown. The electrical equivalence circuit is also briefly explained [4].

A. Fuel Cell Open Circuit Voltage:

$$\mathrm{E} = 1.229 - 0.85 \times 10^{-3}(\mathrm{T_{FC}} - 298.15) \\ +4.3085 \times 10^{-5} T_{FC}[\ln(P_{H_2}) + 0.5 \times ln(P_{O_2})] \quad (4)$$

T_{FC}: Temperature of the cells (Kelvin).
P_{H_2}et P_{O_2} : partial pressure of oxygen and hydrogen respectively (bar).
A_{FC} : Surface of the cells.
N : Numberof cells.

B. Activation Losses:

$$V_{act} = v_0 + v_a(1 - e^{-c_1 i}) \quad (5)$$

Wherev_0 (volts) is the voltage loss (in open circuit), andv_a (volts) and c_1 are constants. The values of v_0, v_a and c_1 can be experimentally determined.

C. Ohmic Losses:

$$V_{ohm} = i \cdot R_{ohm} \quad (6)$$

Ohmic loss is a function of membrane conductivity $(\Omega \cdot cm)^{-1}\sigma_m$ with this form:

$$R_{ohm} = \frac{t_m}{\sigma_m} \quad (7)$$

Where R_{ohm} is the internal resistance expressed in $\Omega. cm^2$. t_m is the thickness of the membrane, and σ_m is the membrane conductivity. There is a dependency between the resistance and the membrane humidity and internal temperature. The ohmic resistance is a function of the membrane conductivity σ_m, $(\Omega. cm^2)^{-1}$, which is also a function of water content of the membrane (λ_m) and the FC temperature.

D. Concentration Losses:

$$V_{con} = i\left(c_2 \frac{i}{i_{max}}\right)^{c_3} \quad (8)$$

Where the constantsc_2, c_3 and I_{max} are obtained empirically and depends on the temperature and the reactant partial pressure. In (8) I_{max} is the current density that generates abrupt voltage drop.

By adding the different voltage drops and the voltage given by (4), the operation terminal voltage of the FC is given by:

$$V_{FC-cell} = E - V_{act-cell} - V_{ohm-cell} - V_{con-cell} \quad (9)$$

$$\begin{cases} V_{FC} = N \times V_{FC-cell} \\ \quad I \ = \dfrac{I_{FC}}{A_{FC}} \end{cases} \quad (10)$$

Witch I is the current density $(\Omega. cm^2)$.

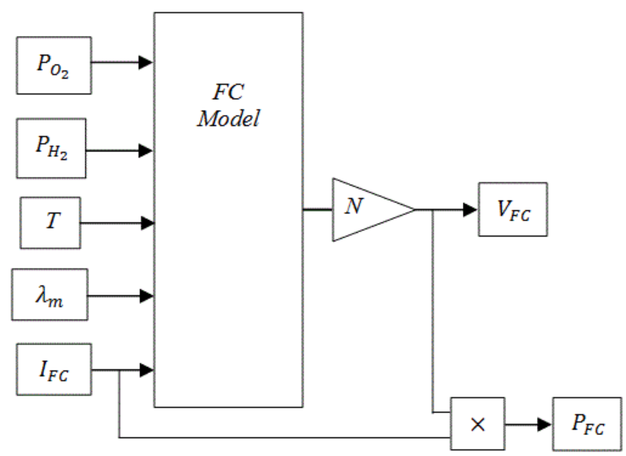

Fig. 2 PEMFC Simulink model

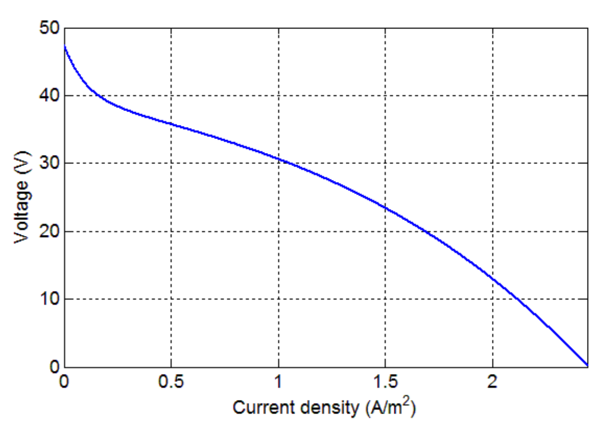

Fig. 3 PEMFC Voltage - Current density characteristic
$(\mathrm{P_{O_2}} = 0.3\ \mathrm{bar}, \mathrm{P_{H_2}} = 2.6\ \mathrm{bar}, \mathrm{T} = 80°\ \mathrm{C}, \lambda_\mathrm{m} = 14, \mathrm{N} = 50, \mathrm{A} = 200\ \mathrm{cm}^2)$

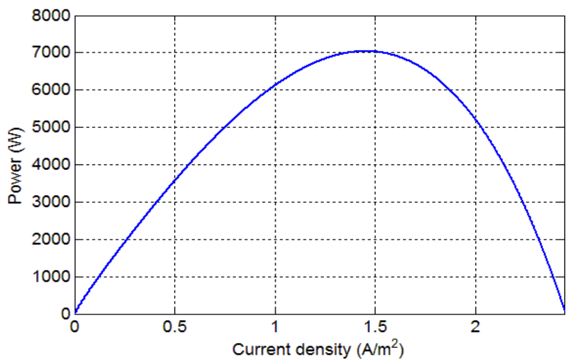

Fig. 4 PEMFC Power-Current density characteristic
($P_{O_2} = 0.3$ bar, $P_{H_2} = 2.6$ bar, $T = 80°$ C, $\lambda_m = 14$, $N = 50$, $A = 200$ cm^2)

III. INTERLEAVED BOOST CONVERTER MODEL

By adding another power stage, connecting inputs and outputs in parallel and shifting drive signals by 180° a two phase interleaved boost converter is created. Principle schematics is shown in Fig. 5:

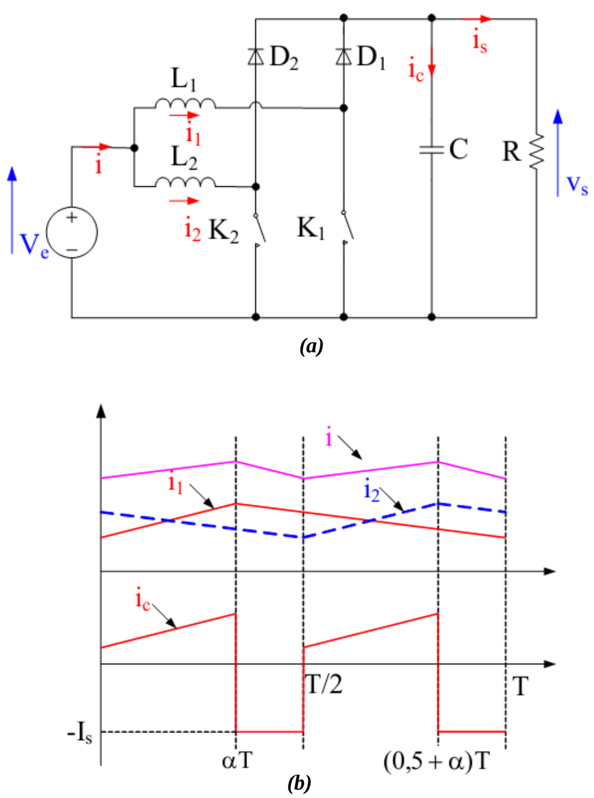

Fig. 5 (a) typical structure of IBC with two phases. (b) Inductances current and source current for α<0,5.

For a two-phase interleaved boost converter, two distinct modes of operation can be analyzed. One is for duty cycles lower than 0.5 (50%) and the other one for duty cycles above 0.5. At exactly 0.5 the converter benefits from ripple cancellation and input current ripple is zero, while output capacitor ripple is at minimum [5].

TABLE II
IBC PARAMETERS

PARAMETER	VALUE
Resistance R	50 Ω
Inductance $L_1 = L_2 = L$	0.005 H
Capacitor C	0.001 F
Fuel cell voltage (at nominal current) V_g	46 V
Desired voltage Vs	100 V
Switching frequency f	10 KHz

A. Average model of IBC converter:

The average model of the interleaved boost converter is the same as classical boost with this time two inductance currents are presented:

$$\begin{cases} L_1 \dfrac{dI_{L1}}{dt} = V_{FC} - V_C(1 - D_1) \\ L_2 \dfrac{dI_{L2}}{dt} = V_{FC} - V_C(1 - D_2) \\ I_{FC} = I_{L1} + I_{L2} \\ C\dfrac{dV_c}{dt} = I_{L1}(1 - D_1) + I_{L2}(1 - D_2) - I_{ch} \end{cases} \quad (11)$$

With V_{FC} is the fuel cell voltage, V_c is the capacitor voltage. L_1, L_2 ($L_1 = L_2 = L$) and $C$are respectively the phases inductances and the capacitor. D_1 and D_2 ($D_1 = D_2 = D$) are the duty cycles of each phase (which one is shifted by 180° from the other).

When only one phase works complimentary with the other phase, the state-space matrices of IBC is the same as a boost converter. Its are written as follows:

$$A_{ON} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix}, A_{OFF} = \begin{bmatrix} 0 & 0 \\ 0 & -1/RC \end{bmatrix}$$

$$B_{ON} = B_{OFF} = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \quad (12)$$

The state vector :

$$X = \begin{bmatrix} I_{FC} \\ V_c \end{bmatrix} \quad (13)$$

The control vector :

$$U = D \quad (14)$$

The model is linearized around the following equilibrium points:

$$X_0 = \begin{bmatrix} \dfrac{V_g}{RD_d'^2} \\ \dfrac{V_g}{D_d'^2} \end{bmatrix} \quad (15)$$

$$U_0 = D_d \quad (16)$$

WhereD_d' is the complementary operating point duty-cycle:

$D'_d = 1 - D_d$.

This model is then augmented with an additional state variable $x_3(t)$ which stands for the integral of the output voltage error, i.e., $x_3 = -\int v_c$, such that the steady-state error is zero.

Since we consider the control of the boost converter around the equilibrium point, we can neglect the nonlinear term of the converter model and obtain a linearized augmented model as in (18), where the system matrices are as follows:

$$x = X - X_0,\ u = U - U_0 \tag{17}$$

$$\dot{x} = Ax + Bu \tag{18}$$

With:

$$A = \begin{bmatrix} 0 & -\frac{D'_d}{L} & 0 \\ \frac{D'_d}{C} & -\frac{1}{RC} & 0 \\ 0 & -1 & 0 \end{bmatrix} \tag{19}$$

$$B = \begin{bmatrix} \frac{V_g}{D'_d L} \\ -\frac{V_g}{RCD'_d} \\ 0 \end{bmatrix} \tag{20}$$

Some elements involved in the system matrices may be uncertain or time varying. Then, matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ depend on such uncertain or time-varying terms ($p = [V_{FC} \quad R \quad D']^T$), and we can express (18) as a function of these parameters [6].

$$\dot{x}(t) = A(p)x(t) + B(p)u(t) \tag{21}$$

TABLE III
UNCERTAIN PARAMETERS DOMAINS

PARAMETER	**VALUE**
V_{FC}	[45 50] V
R	[45 55] Ω
D'	[0.5 0.55]

IV. ROBUST LQR CONTROL

This section presents the key concepts of the robust LQR control method proposed in this paper. First of all, we introduce the basic result of quadratic stability for dynamic systems. Second, we will formulate the uncertain LQR problem. These concepts will be applied to derive an LQR controller for an interleaved boost converter.

A. Quadratic stability for a dynamic system:

Given a linear time-invariant system:

$$\dot{x} = Ax \tag{22}$$

It is well known that Lyapunov theory states that the existence of a positive definite matrix $\boldsymbol{P}$ such that the quadratic function:

$$V(x) = x'\boldsymbol{P}x > 0 \tag{23}$$

This satisfies:

$$V(\dot{x}) = x'(A'\boldsymbol{P} + \boldsymbol{P}A)x < 0 \tag{24}$$

Is a necessary and sufficient condition to assure that the system is quadratically stable (i.e., all trajectories converge to zero) [7].

In the inequality (24), $\boldsymbol{P}$ is the matrix variable (noted in bold notation) that must be found to assure quadratic stability but also to solve the Riccati equation that arise in the LQR control of a switched-mode DC-DC converter.

B. LQR controller designing:

Given the system presented in (22), the optimal LQR controller is obtained by using the state-feedback gain K ($u = \boldsymbol{K}x$) that minimizes a performance index:

$$\boldsymbol{J} = \int_0^\infty (x'\boldsymbol{Q}x + u'\boldsymbol{R}u)dt \tag{25}$$

Where $\boldsymbol{Q}$ is a symmetric and semi definite positive matrix. $\boldsymbol{R}$ is a symmetric and definite positive matrix. The pair ($\boldsymbol{A} \quad \boldsymbol{B}$) must be controllable.

The matrix $\boldsymbol{P}$ is the solution of Riccati equation:

$$P\boldsymbol{A} + \boldsymbol{A}P + Q - P\boldsymbol{B}R^{-1}\boldsymbol{B}'P = 0 \tag{26}$$

Then, the optimal feed-back gain is obtained [8]:

$$\boldsymbol{K} = R^{-1}B'\boldsymbol{P} \tag{27}$$

The LQR problem can be viewed as the weighted minimization of a linear combination of the states $\boldsymbol{x}$ and the control input $\boldsymbol{u}$. The weighting matrix $\boldsymbol{Q}$ establishes which states are to be controlled more tightly than others. $\boldsymbol{R}$ weights the amount of control action to be applied depending on how large is the deviation of the state $\boldsymbol{x_i}$ [9]. This optimization cost weight constraints the magnitude of the control signal [10].

V. NUMERIC SIMULATION

In this section, we present a comparative simulation with MATLAB/Simulink between the classical PI controller and LQR controller response. The converter parameters are given in table II. The parametric variation fields are specified in table III. The chosen PEMFC model is that studied in section II, their main parameters are given in Table I.

The simulation has been realized for two situations: FC voltage and load changes. The classical PI controller has been optimized choosing the following coefficients: PI voltage controller with $K_p = 0.2$, $K_i = 10$. P current controller with $K_p = 1$.

For LQR controller, two parameters have been tested:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10^{-3} & 0 \\ 0 & 0 & 10^{4} \end{bmatrix}, \; R = 10$$

(a)

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{4} \end{bmatrix}, \; R = 10^{2}$$

(b)

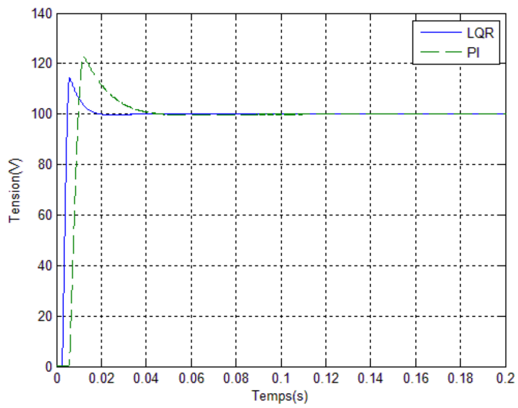

Fig. 6 100 V echelon response for two controllers with LQR parameters (*a*)

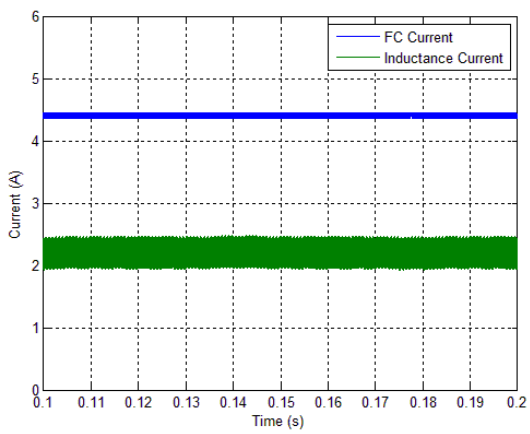

Fig. 7 FC current and inductance current undulation

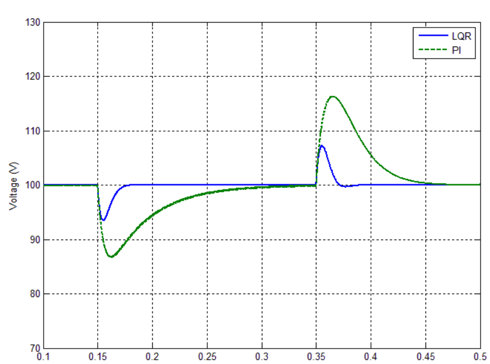

Fig. 8 Response of two controllers for ±3 *A* load changes with LQR parameters (a)

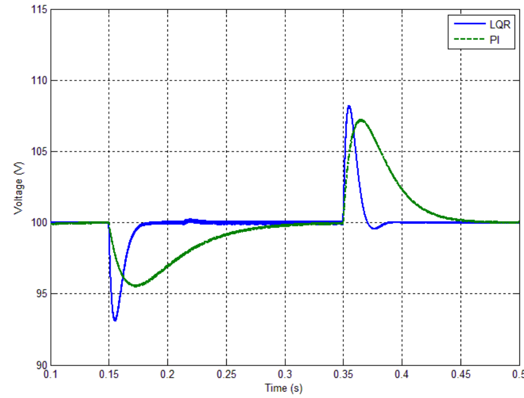

Fig. 9 Response of two controllers for ±3 *A* current changes with LQR parameters (b)

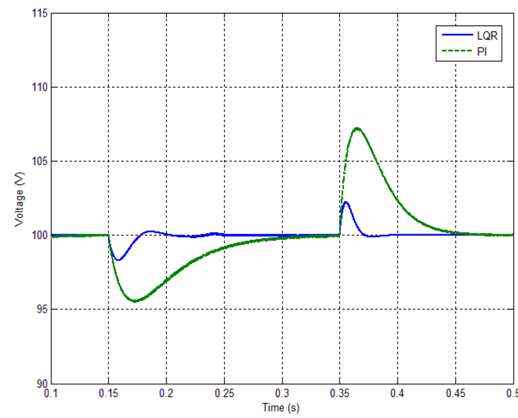

Fig. 10 Response for two controllers for ±5 *V* PEMFC voltage changes with LQR parameters (*a*)

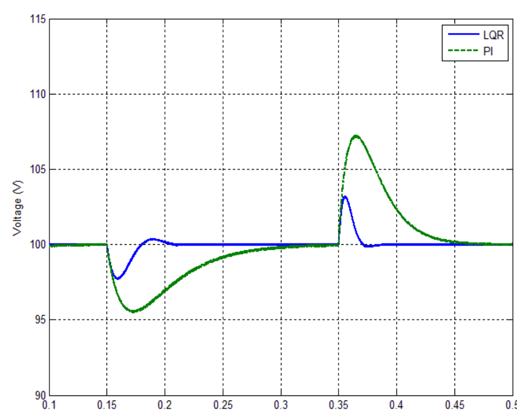

Fig. 11 Response of two controllers for ±5 *V* FEMFC voltage changes with LQR parameters (*b*)

It is clear after the simulation results, with this new converter topology, the current ripple are considerably minimized. These results prove also that with good chosen parameters, robust LQR controller can easily make fast corrections without any undulations than the classical PI controller. On the other hand, this controller is robust facing external parameters changes like PEMFC voltage and load current.

VI. CONCLUSION

This paper has proposed a new power conditioning strategy for PEM fuel cell applications. The DC-DC converter structure is a two phase interleaved boost converter, this structure reduces the FC current ripples increasing the membrane life time. A robust control technique based in quadratic stability with optimal LQR method has tested for our PWM switching DC-DC converter. The proposed model allows one to consider parametric uncertainty. The proposed controller exhibits a more predictable response than a classical PI controller when operating conditions changes.

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BIOGRAPHY

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