Isı Bilimi ve Tekniği Dergisi, 28, 2, 9-16, 2008
J. of Thermal Science and Technology

ISSN 1300-3615

FORCED CONVECTION FLOW OF VISCOUS DISSIPATIVE POWER-LAW FLUIDS IN A PLANE DUCT
Part 1. Hydrodynamically and Thermally Fully Developed Flow

Mete AVCI* and Orhan AYDIN**
*Karadeniz Technical University, Department of Mechanical Engineering,
61080 Trabzon, mavci@ktu.edu.tr
**Karadeniz Technical University, Department of Mechanical Engineering,
61080 Trabzon, oaydin@ktu.edu.tr

(Geliş Tarihi: 01. 02. 2008, Kabul Tarihi: 12. 05. 2008)

Abstract: In this study, an analysis of laminar forced convection in a plane duct for a power-law fluid with constant thermophysical properties is performed by taking the viscous dissipation into account. This part of the study, Part 1, examines both hydrodynamically and thermally fully developed flow case. Two different thermal boundary conditions are considered: the constant heat flux (H1 boundary condition) and the constant wall temperature (T boundary condition). The combined and interactive influences of the power-law index and the Brinkman number on temperature distribution and the Nusselt numbers are analytically determined both for the wall heating and cooling cases at T and H1 boundary conditions. Singularities are observed in Nu-Br-n behaviors and these are discussed in terms of the energy balance.
Keywords: Non-Newtonian fluid, Power-law fluid, Viscous dissipation, Plane duct, Constant heat flux, Constant wall temperature.

BİR DÜZLEMSEL KANAL İÇERİSİNDEKİ VİSKOZ YAYILIMLI POWER-LAW AKIŞKANLARIN ZORLANMIŞ TAŞINIM AKIŞI
Kısım 1. Hidrodinamik ve Termal Olarak Tam Gelişmiş Akış

Özet: Bu çalışmada, sabit termofiziksel özelliklere sahip power-law akışkanın düzlemsel kanal içerisindeki laminer zorlanmış taşınımı, viskoz yayılım etkileri dahil edilerek, analiz edilmiştir. Çalışmanın bu bölümünde, birinci kısım, hidrodinamik ve termal açıdan tam gelişmiş akış incelenmektedir. Sabit ısı akısı (H1 sınır koşulu) ve sabit yüzey sıcaklığı(T sınır koşulu) olmak üzere iki farklı termal sınır koşulu ele alınmıştır. Power-law indeksi ve Brinkman sayısının sıcaklık dağılımı ve Nusselt sayısı üzerindeki etkisi, T ve H1 sınır koşullarında, sıcak ve soğuk cidar durumları için analitik olarak belirlenmiştir. Nu-Br-n davranışlarında süreksizlikler gözlenmiş ve bu süreksizlikler enerji dengesi açısından tartışılmıştır.
Anahtar kelimeler: Newtonyen olmayan akışkan, Power-law akışkan, Viskoz yayılım, Düzlemsel kanal, Sabit ısı akısı, Sabit yüzey sıcaklığı.

NOMENCLATURE

Br	Brinkman number, Eq. (8)
Br_q	modified Brinkman number, Eq. (11)
c_p	specific heat at constant pressure
k	thermal conductivity [W/mK]
n	power-law index
Nu	Nusselt number
q_w	wall heat flux [W/m^2]
T	temperature [K]
u	velocity [m/s]
y	coordinate in vertical direction [m]
Y	dimensionless vertical coordinate
w	half width of the duct [m]
z	axial direction [m]

Greek symbols

α	thermal diffusivity [m^2/s]
η	consistency factor employed in Eq. (1)
ρ	density [kg/m^3]
υ	kinematic viscosity [m^2/s]
θ	dimensionless temperature, Eq. (7)
θ_q	dimensionless temperature modified, Eq. (13)

Subscripts

c	centerline
m	mean
w	wall

INTRODUCTION

The non-Newtonian fluids exhibit nonlinear relation between the shear stress and the shear rate, as opposed to the linear one of the Newtonian fluids. The flow and heat transfer of non-Newtonian fluids through ducts have wide potential applications in many engineering areas including the chemical, petroleum, polymer, food processing, pharmaceutical and biochemical and biomedical engineering. The most non-Newtonian fluids of practical interest are highly viscous and, therefore, are often processed in the laminar flow regime. Readers are referred to see the excellent reviews by Irvine and Karni (1987) and Hartnett and Choi (1998).

Viscous dissipation changes the temperature distributions by playing a role like an energy source, which, in result, influences heat transfer rates. It is very important when the viscosity is high or for high shear flows. Its effect depends greatly on the thermal boundary conditions applied at the wall. This effect has been studied for the case of Newtonian fluids by several researchers (Brinkman, 1951; Tyagi, 1996; Ou and Cheng, 1973; Lin et al., 1983; Basu and Roy, 1985; Liou and Wang, 1990; Barletta and Zanchini, 1997; Zanchini, 1997; Barletta and Rossi di Schio, 1999) on the thermal boundary conditions applied at the wall. The work of Brinkman (1951) appears to be the first theoretical work dealing with viscous dissipation.

When compared to those available for the Newtonian fluids, there are only a few studies regarding the viscous dissipation effect for non-Newtonian flows. Lawal and Mujumdar (1992) studied viscous dissipation effect on heat transfer for power law fluids in arbitrary cross-sectional ducts. Dang (1983) studied the effect of viscous dissipation in the thermal entrance region in a pipe using the uniform wall temperature. Barletta (1997) studied the asymptotic behavior of the temperature field for the laminar and hydrodynamically developed forced convection of a power-law fluid which flows in a circular duct taking the viscous dissipation into account. The asymptotic Nusselt number and the asymptotic temperature distribution were evaluated analytically in the cases of either uniform wall temperature or convection with an external isothermal fluid. Pinho and Oliveira (2000) investigated the forced convection of Phan-Thien-Tanner fluid in laminar pipe and channel flows including the effects of viscous dissipation. It was shown that the beneficial effects of fluid elasticity were enhanced by viscous dissipation. The effect of viscous dissipation on the laminar forced convection in a circular duct for a Bingham fluid under different thermal boundary conditions was studied by Vradis et al. (1993), Min et al. (1997) and Khatyr et al. (2003). The asymptotic temperature profile and the asymptotic Nusselt number were determined for various axial distributions of wall heat flux which yielded a thermally developed region.

In a recent study, Aydın and Avcı (2006) investigated the effect of viscous dissipation on both hydrodynamically and thermally fully developed laminar forced convective flow in a plane duct. The effect of the Brinkman number on the temperature profile and the Nusselt number was obtained for the constant wall heat flux and the constant wall temperature thermal boundary conditions considering either wall heating (the fluid is heated) case or wall cooling (the fluid is cooled) cases.

Here, our aim is to investigate the effect of viscous dissipation on hydrodynamically and thermally fully developed flow of the power-law type Ostwald-de Waele fluid in a plane duct covering a broad range of the Brinkman number. Here, we will also discuss the possible definitions of the Brinkman number arising from definitions of different dimensionless parameters.

ANALYSIS

We consider steady, laminar hydrodynamically and thermally fully developed flow with constant properties (i.e. The thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature). The axial heat conduction in the fluid and in the wall is neglected. Two different forms of the thermal boundary conditions are applied, which are shown in Fig. 1.

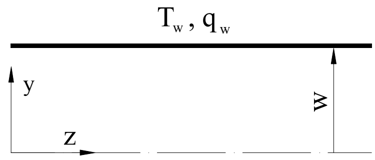


Figure 1. Schematic diagram of the flow domain.

The shear stress and strain relationship for the power-law type Ostwald-de Waele fluid is given as

$$\tau_{yz} = \eta \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy} \tag{1}$$

where n represents the power-law index and the case of n<1, n=1 and n>1 correspond pseudoplastic, Newtonian and dilatant behaviors The velocity profile for fully developed plane duct flow is given as follows:

$$\frac{u}{u_m} = \left(\frac{2n+1}{n+1} \right) \left[1 - \left(\frac{y}{w} \right)^{(n+1)/n} \right] \tag{2}$$

The energy balance equation including the effect of the viscous dissipation is given by

$$\frac{\partial^2 T}{\partial y^2} = \frac{u}{\alpha} \frac{\partial T}{\partial z} - \frac{1}{k} \tau_{yz} \frac{du}{dy} \tag{3}$$

where the second term in the right hand side is the viscous dissipation term. Due to axisymmetry at the center, the thermal boundary condition at $y=0$ can be written as:

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \tag{4}$$

Both the constant wall heat flux (H1-type) and the constant wall temperature (T-type) of the wall thermal boundary condition are separately examined.

For the constant heat flux at wall, the thermal boundary condition can be written as:

$$k\left.\frac{\partial T}{\partial y}\right|_{y=w} = q_w \tag{5}$$

where q_w is positive when its direction is to the fluid (wall heating), otherwise it is negative (wall cooling). For the uniform wall heat flux case, the first term in the right hand side of Eq. (3) is

$$\frac{\partial T}{\partial z} = \frac{d\,T_w}{dz} \tag{6}$$

By introducing the following dimensionless quantities

$$Y = \frac{y}{w}, \quad \theta = \frac{T_w - T}{T_w - T_c} \tag{7}$$

Eq. (3) can be written as

$$\frac{d^2\theta}{dy^2} = a\left(1 - Y^{(n+1)/n}\right) + Br\left(2 + \frac{1}{n}\right)^{n+1} Y^{(n+1)/n} \tag{8}$$

where $a = -\frac{u_m\, w^2}{\alpha\left(T_w - T_c\right)}\left(\frac{2n+1}{n+1}\right)\frac{d\,T_w}{d\,z}$ and Br is the Brinkman number given as:

$$Br = \frac{\eta\, u_m^{n+1}}{k\, w^{n-1}\left(T_w - T_c\right)} \tag{9}$$

For the solution of the dimensionless energy transport equation given in Eq. (8), the dimensionless boundary conditions are given as follows:

$$\theta = 1 \qquad \left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} = 0 \qquad \text{at } Y = 0$$

$$\theta = 0 \qquad \text{at } Y = 1 \tag{10}$$

The solution of Eq. (8) under the thermal boundary conditions given in Eq. (10) is

$$\theta(Y) = \left[-\frac{2n+1}{(4n+1)(n+1)}\left(6n + 2 + Br\left(2 + \frac{1}{n}\right)^n 2n\right)\right] x\left[\frac{Y^2}{2} - \frac{n}{2n+1}\frac{n}{3n+1}Y^{(3n+1)/n}\right] + Br\left(2 + \frac{1}{n}\right)^n\left(\frac{n}{3n+1}\right)Y^{(3n+1)/n} + 1 \tag{11}$$

Since we have developed the above equation for the constant wall heat case, as it is usual in the existing literature, we can also use the modified Brinkman number which is in the following:

$$Br_q = \frac{\eta\, u_m^{n+1}}{w^n\, q_w} \tag{12}$$

In terms of the modified Brinkman number given above, the temperature distribution is obtained as:

$$\theta_q = \frac{T - T_w}{\frac{w\, q_w}{k}} = \left[\left(\frac{2n+1}{n+1}\right)\left(1 + Br_q\left(2 + \frac{1}{n}\right)^n\right)\right] x\left[\frac{Y^2}{2} - \frac{1}{2} - \frac{n}{2n+1}\frac{n}{3n+1}\left(Y^{(3n+1)/n} - 1\right)\right] - Br_q\left(2 + \frac{1}{n}\right)^n\left(\frac{n}{3n+1}\right)\left[Y^{(3n+1)/n} - 1\right] \tag{13}$$

Note that, as seen in Eq. (13), the definition of the dimensionless temperature is different to the earlier one, which in follows, leads to the modified Brinkman number. In fully developed flow, it is usual to use the mean fluid temperature, T_m, rather than the center line temperature when defining the Nusselt number. This mean or bulk temperature is given by:

$$T_m = \frac{\int \rho\, u\, T\, dA}{\int \rho\, u\, dA} \tag{14}$$

The dimensionless mean temperature is obtained as:

$$\theta_m = \frac{T_m - T_w}{T_c - T_w} = \frac{2(2 + 23n + 83n^2 + 96n^3)}{3(1+4n)^2(2+5n)} - Br\left(2 + \frac{1}{n}\right)^n \frac{2n(1+2n)^2}{3(1+4n)^2(2+5n)} \tag{15}$$

In terms of Br_q defined in Eq. (12), the mean temperature is obtained as:

$$\theta_{q,m} = \frac{T_m - T_w}{\frac{q_w w}{k}} = -\frac{2 + 17n + 32n^2}{3(1+4n)(2+5n)} - Br_q\left(2 + \frac{1}{n}\right)^n \frac{1}{3(1+4n)(2+5n)} \tag{16}$$

When the constant temperature is considered at the wall, since $\frac{dT_w}{dz}=0$, the first term in the right hand side of Eq. (3) is

$$\frac{\partial T}{\partial z}=\left(\frac{T_w-T}{T_w-T_c}\right)\frac{dT_c}{dz} \tag{17}$$

Substituting this result into Eq. (3) and introducing the dimensionless quantities given in Eq. (7), we obtain the following dimensionless equation:

$$\frac{d^2\theta}{dY^2}=b\left(1-Y^{(n+1)/n}\right)\theta+Br\left(2+\frac{1}{n}\right)^{n+1}Y^{(n+1)/n} \tag{18}$$

where $b=-\frac{u_m w^2}{\alpha\left(T_w-T_c\right)}\left(\frac{2n+1}{n+1}\right)\frac{dT_c}{dz}$ and Br is the Brinkman number. The boundary conditions given in Eq. (10) is also valid for this case. Actually, no simple closed form solution can be obtained for this equation. However, the variation of θ can be quite easily obtained to any required degree of accuracy by using an iterative procedure (Oosthuizen and Naylor, 1999). The temperature profile for the constant heat flux case is used as the first approximation and Eq. (18) is then integrated to obtain θ. This iterative procedure is repeated until an acceptable convergence is obtained.
From the engineering interest, it is important to determine the forced convective heat transfer coefficient, *h,* from which we can determine the heat transfer rate. *h* can expressed as:

$$h=\frac{k\left(\frac{\partial T}{\partial y}\right)_{y=w}}{T_w-T_m} \tag{19}$$

The Nusselt number is defined in order to obtain *h*:

$$Nu=\frac{q_w\,(2w)}{\left(T_w-T_m\right)k} \tag{20}$$

which is based on the distance between plates. Performing necessary substitutions, we obtain:

$$Nu=\frac{3(1+4n)(2+5n)\left(-2-6n+Br\left(2+\frac{1}{n}\right)^{n}(1+2n)\right)}{-2+n\left(-23+Br\left(2+\frac{1}{n}\right)^{n}(1+2n)^2-n(83+96n)\right)} \tag{21}$$

In terms of the modified Brinkman number, Br_q,

$$Nu=\frac{6(1+4n)(2+5n)}{2+17n+32n^2+Br_q\left(2+\frac{1}{n}\right)^{n}(2+11n+14n^2)} \tag{22}$$

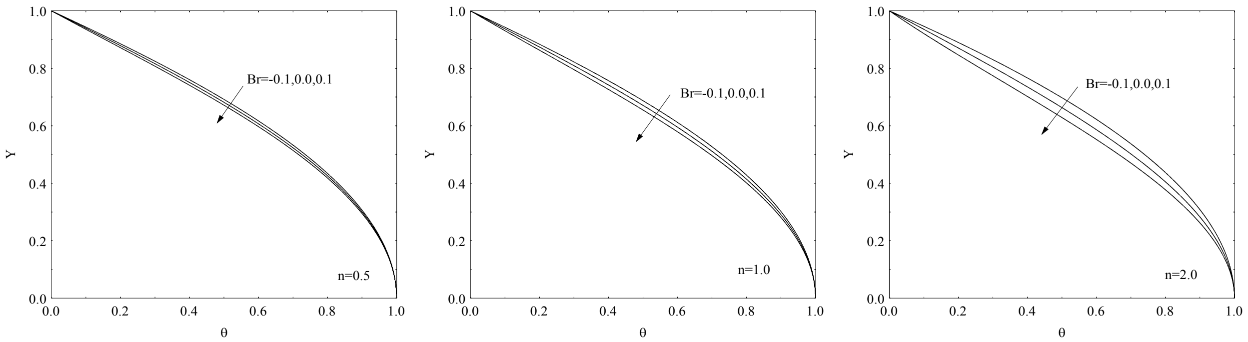
RESULTS AND DISCUSSION

We have studied the problem of hydrodynamically and thermally fully developed flow of a power-law fluid in a duct considering two different thermal boundary conditions at the duct wall, namely: The constant heat flux (H1-type) and the constant wall temperature (T-type). For each boundary condition both wall heating (the hot wall) case or wall cooling (the cold wall) case are examined. In fact, when the viscous dissipation is excluded from the analysis, the solution will be independent of whether there is wall heating or cooling at the wall. However, in the presence of the viscous dissipation, it always contributes to internal heating of the fluid, hence the solution will differ according to the process taking place.

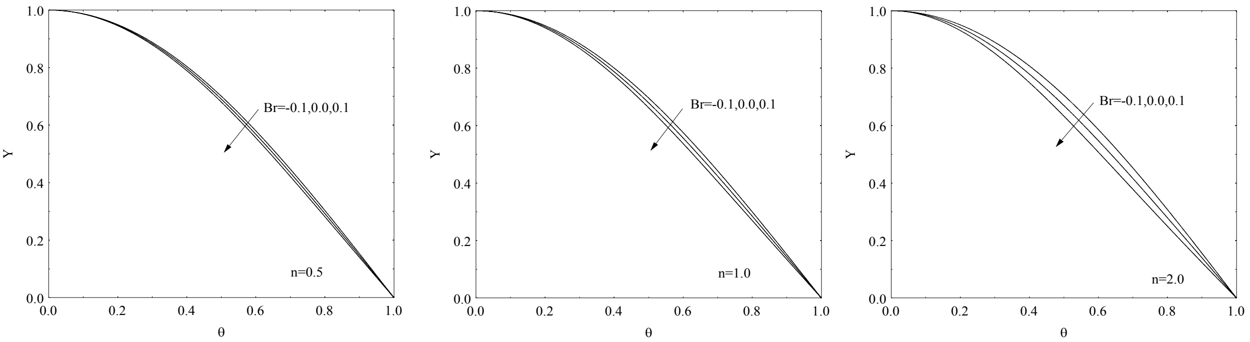
Figure 2a and b depict the dimensionless temperature distributions for different values of Br (Br=-0.1,0,0.1) at n=0.5, 1 and 2 for the wall with the constant heat flux and that with the constant wall temperature, respectively. As seen, in all the cases considered, with an increase at the Brinkman number, dimensionless temperature gradient decreases. Remember positive values of Br correspond to wall heating (heat is being supplied across the walls into the fluid) case (T_w>T_c), while the opposite is true for negative values of Br. Viscous dissipation affects the temperature profile by playing a role like an energy source. It increases the temperature of the bulk fluid. Its effect becomes the most significant near the wall due to the highest shear stress occurring there.

For the constant heat flux cases, as explained earlier, definitions of two different dimensionless temperatures resulted in Brinkman and modified Brinkman numbers. Figure 3a illustrates the variation of the Nusselt number with the power-law index for different Brinkman numbers. As seen for Br=0, the case without viscous dissipation, Nu decreases with increasing n for n>1 (the dilatant behavior), while an enhancement in heat transfer (i.e. Nu increases) with decreasing n for n<1 (the pseudoplastic behavior). As seen, for the lower values of the Brinkman numbers either the wall heating case or the wall cooling case, Br=0.01 and -0.01, Nu versus n is very similar to that at Br=0. Therefore, we can neglect the effect of Br on Nu for its lower values. Increasing dissipation increases the bulk temperature of the fluid due to internal heating of the fluid. For the wall heating case, this increase in the fluid temperature decreases the temperature difference between the wall and the fluid. As a result, at Br=0.1, we obtain lower Nu values. And, as seen, the decreasing effect of the Brinkman number intensifies for the dilatant fluids (n>1). For the cold wall or the cooling wall situation, the viscous dissipation leads to higher temperature differences between the wall and the bulk fluid with the increasing Br. In fact, wall cooling is applied to reduce the bulk temperature of the fluid, while the effect of the viscous dissipation is increasing the bulk temperature of the fluid. As seen for Br=-0.1, either in the pseudoplastic region or in the dilatant region, a shift of n from n=1 increases heat transfer due to the viscous dissipation. Figure 3b shows the variation of the Nusselt number with the power-law index for different modified Brinkman numbers. The behaviors observed in the figure are similar to those in Fig. 3a and can be explained in the same way. For the constant wall temperature case at the wall, Fig. 4 illustrates the variation of the Nusselt number with the power-law index for different Brinkman numbers.

As expected, at n=1, lower values are observed for Nu when compared to the constant wall heat flux case as a result of decreased temperature gradient at the wall. For the H1- and T-types boundary conditions, we compared our results against those by Cotta and Ozisik (1986), Shah and London (1978) respectively, who considered the case without the viscous dissipation. As seen from Table 1, our results present a good agreement with those.



(a)



(b)

Figure 2. The fully developed dimensionless temperature profile for (a) H1-type (b) T-type.

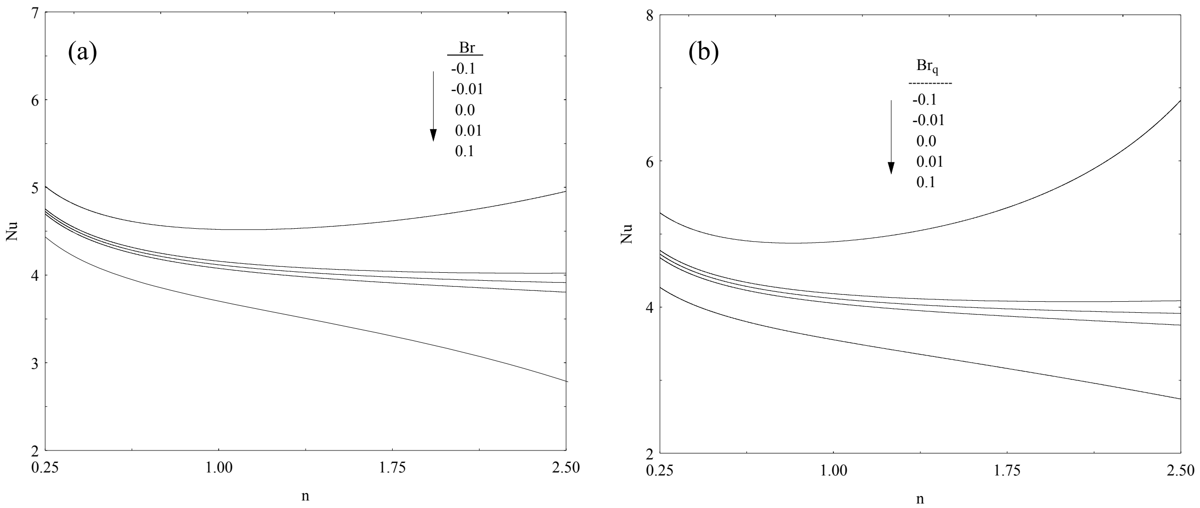


Figure 3. The effect of power law index on fully developed Nusselt number for H1-type at different values of (a) Br (b) Br_q.

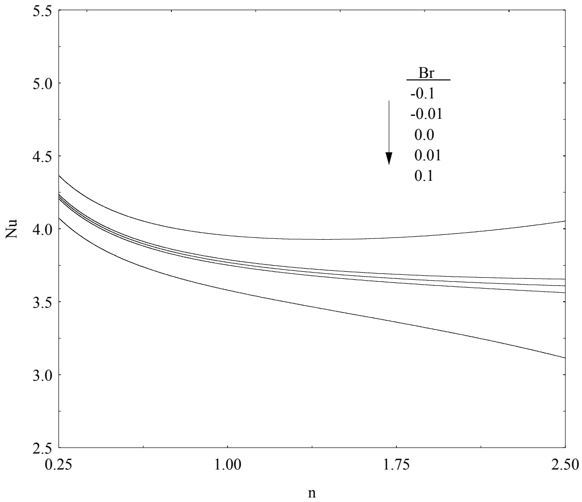
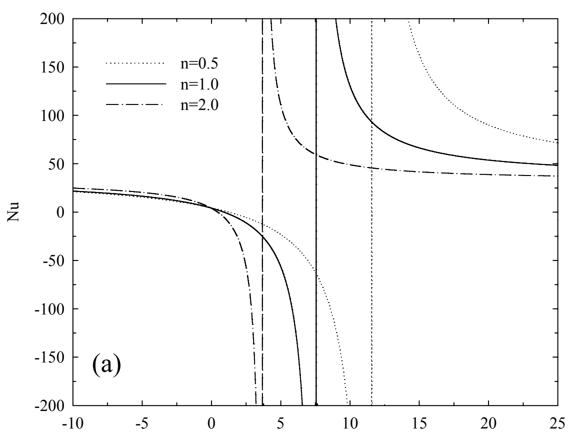


Figure 4. The effect of power law index on fully developed Nusselt number for T-type at different values of Br.

Table 1. Comparison of Nusselt numbers for H1-type and T-type ($Br=Br_q=0$).

n	H1-type Present	H1-type Cotta and Özışık, 1986	T-type Present	T-type Shah and London, 1978
1/3	4.5743	4.5743	4.1140	4.1138
1	4.1177	4.1176	3.7706	3.7704
3	3.8886	3.8886	3.5886	3.5888

In order to see the effect of the Brinkman number in a broader range, the variation of the Nusselt number with the Brinkman number is determined and plotted for different values of the power-law index in Fig. 5a. For n=1, the Newtonian-fluid case, a singularity is observed at Br=64/9, which is explicit when Eq. (21) is closely examined. An increase at Br (i.e. the wall heating case) decreases Nu in the range of 0 < Br < 64/9. This is because the temperature difference which drives the heat transfer decreases. At Br = 64/9, there is a balance between the heat supplied by the wall into the fluid and the internal heat generation by the viscous heating. For Br>64/9, the internally generated heat by the viscous dissipation overcomes the wall heat.



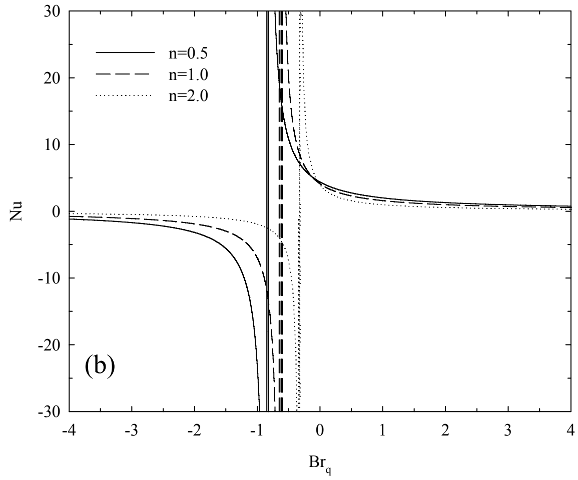


Figure 5. The influence of (a) Br (b) Br_q on Nu at various power law index for the H1-type.

For the cold wall case, Br < 0, the Nusselt number increases with an increase at Br as a result of increasing temperature difference between the wall and the bulk fluid. For n<1, the singularity point arises at higher Br values, while it is reached at lower Br values for n>1. Similarly, again for the hot wall or the wall heating situation, Fig. 5b illustrates the variation of Nu with Br_q. The behavior observed can be explained similarly to that for Br. For Br_q = -17/27, a singularity is observed, which is an expected result from Eq. (22). However, a different shift behavior of the singularity point according to n than Fig. 5a is observed.

For the constant wall temperature condition, Fig. 6 illustrates the variation of Nu with Br for different values of n. The behavior observed can be explained similarly to that for the constant wall heat flux condition. Again, singularities are observed.

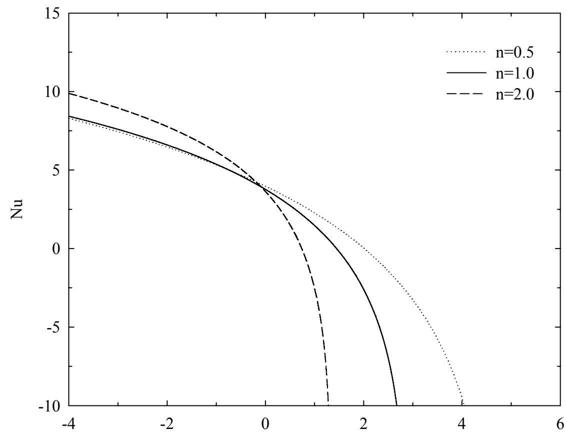


Figure 6. The influence of Br on Nu at various power law index for the T-type.

Tables 2 and 3 summarize some typical results of this study.

Table 2. The Nusselt number values with Br, Br_q for H1-type.

	Br					Br_q				
n	-0.10	-0.01	0.00	0.01	0.10	-0.10	-0.01	0.00	0.01	0.10
0.50	4.6881	4.4096	4.3784	4.3471	4.0633	4.9693	4.4311	4.3784	4.3269	3.9130
1.00	4.5210	4.1585	4.1176	4.0767	3.7034	4.8951	4.1841	4.1176	4.0533	3.5533
2.00	4.7051	4.0285	3.9512	3.8736	3.1551	5.6842	4.0755	3.9512	3.8343	3.0280

Table 3. The Nusselt number values with Br for T-type.

	Br				
n	-0.10	-0.01	0.00	0.01	0.10
0.50	4.1191	3.9846	3.9697	3.9548	3.8180
1.00	3.9545	3.7891	3.7706	3.7512	3.5799
2.00	3.9658	3.6725	3.6391	3.6050	3.2865

CONCLUSION

The problem of hydrodynamically and thermally fully developed forced convection flow of a power-law fluid in a plane duct has been studied by taking the effect of viscous dissipation into account. Two types of wall thermal boundary condition have been considered both axially and peripherally, namely: constant heat flux (H1-type) and constant wall temperature (T-type). Both wall heating and wall cooling case are examined. The Nusselt number has been obtained for different values of the Brinkman number, Br and the, n. The variation of the Nusselt number with the Brinkman number presented some singularities. These singularities have been shown to originate from the thermal energy balance between the wall heat and the viscous dissipation heat during the thermal transport and structure of the related formulations. For the wall heating case, the Brinkman number has been shown to decrease the Nusselt number while the opposite is true for the wall cooling case. The second part of this study analyzes the effect of viscous dissipation for the hydrodynamically fully developed but thermally developing flow.

REFERENCES

Aydın, O. and Avcı, M., Viscous dissipation effects on the heat transfer in a Poiseuille flow, *Applied Energy* 83, 495-512, 2006.

Barletta, A. and Rossi di Schio, E., Effects of viscous dissipation on laminar forced convection with axially periodic wall heat flux, *Heat and Mass Transfer* 35, 9-16, 1999.

Barletta, A. and Zanchini, E., Forced convection in the thermal entrance region of a circular duct with slug flow and viscous dissipation, *Int. J. Heat Mass Transfer* 40, 1181-1190, 1997.

Barletta, A., Fully developed laminar forced convection in circular ducts for power-law fluids with viscous dissipation, *Int. J. Heat Mass Transfer* 40, 15-26, 1997.

Basu, T. and Roy, D. N., Laminar heat transfer in a tube with viscous dissipation, *Int. J. Heat Mass Transfer* 28, 699-701, 1985.

Brinkman, H. C., Heat effects in capillary flow, *Appl. Sci. Res.* A2, 120-124, 1951.

Cotta, R. M. and Özışık, M. N., Laminar forced convection to non-Newtonian fluids in ducts with prescribed wall heat flux, *Int. Comm. Heat Mass Transfer* 13, 325-34, 1986.

Dang, V. D., Heat transfer of a power-law fluid at low Peclet number flow, *J. Heat Transfer* 105, 542-549, 1983.

Hartnett, J. P. and Cho, Y. I., Non-Newtonian Fluids, In *Handbook of Heat Transfer* (edited by W.M. Rohsenow, J.P. Hartnett, Y.I. Cho), McGraw Hill, New York, 1998.

Irvine, T. F. and Karni, J., Non-Newtonian Fluid Flow and Heat Transfer, In *Handbook of Single-Phase Convective Heat Transfer* (Edited by S. Kakaç, R.K. Shah, W. Aung), Wiley, New York, 1987, Chap. 20.

Khatyr, R., Ouldhadda, D. and Il Idrissi, A., Viscous dissipation effects on the asymptotic behavior of laminar forced convection for Bingham plastics in circular ducts, *Int. J. Heat Mass Transfer* 46, 589-598, 2003.

Lawal, A. and Mujumdar, A. S., The effects of viscous dissipation on heat transfer to power law fluids in arbitrary cross-sectional ducts, *Warme Stoffübertragung* 27, 437-446, 1992.

Lin, T. F., Hawks, K. H. and Leidenfrost, W., Analysis of viscous dissipation effect on the thermal entrance heat transfer in laminar pipe flows with convective boundary conditions, *Warme und Stoffübertragung* 17, 97-105, 1983.

Liou, C. T. and Wang, F. S., Solutions to the extended Graetz problem for a power-law fluid with viscous dissipation and different entrance boundary conditions, *Numer. Heat Transfer* A 17, 91-108, 1990.

Min T., Yoo J. Y. and Choi H., Laminar convective heat transfer of a Bingham plastic in a circular pipe-I. Analytical approach – thermally fully developed flow and thermally developing flow (the Graetz problem extended), *Int. J. Heat Mass Transfer* 40, 3025-3037, 1997.

Oosthuizen, P. H. and Naylor, D., *Introduction to Convective Heat Transfer Analysis*, McGraw-Hill, New York, 1999.

Ou, W. J. and Cheng, K.C., Viscous dissipation effects in the entrance region heat transfer in pipes with uniform heat flux *Appl. Sci. Res.* 28, 289-301, 1973.

Pinho, F. T. and Oliveira, P. J., Analysis of forced convection in pipes and channels with the simplified Phan-Thien-Tanner fluid, *Int. J. Heat Mass Transfer* 43, 2273-2287, 2000.

Shah, R. K. and London A. L., *Advances in Heat Transfer* (suppl. 1), Academic Press, New York, 1978.

Tyagi, V.P., Laminar forced convection of a dissipative fluid in a channel, *J. Heat Transfer* 88, 161-169, 1996.

Vradis, G. C., Dougher J. and Kumar, S., Entrance pipe flow and heat transfer for a Bingham plastic, *Int. J. Heat Mass Transfer* 36, 543-552, 1993.

Zanchini, E., Effect of viscous dissipation on the asymptotic behavior of laminar forced convection in circular tubes, *Int. J. Heat Mass Transfer* 40, 169-178, 1997.