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ANALYSIS OF HEAT AND FLUID FLOW IN CONCENTRIC ANNULAR SQUARE DUCTS

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Abstract: A numerical study is conducted on the heat and fluid flow characteristics in an annulus between two concentric square ducts. Inner and outer walls are assumed to be isothermal, but at different temperatures. The flow through the annular duct is assumed to be laminar, steady, and both hydrodynamically and thermally fully developed with constant physical properties. For the Cartesian coordinate system, the governing equations are discretized by using the control volume method and are solved by the ADI method. The upwind scheme and the central difference scheme were employed to represent the convection and diffusion terms, respectively. The Stone's method was employed to solve the pressure-correction equation based on the SIMPLE Algorithm. Solutions were obtained for air *(Pr=0.7)*. The velocity and temperature fields, the friction coefficients and Nusselt numbers are presented depending on the dimension ratio, a/b. With the increasing dimension ratio, it has been shown that the convective heat transfer is remarkably enhanced at the inner wall, while it becoming worse at the outer wall. The present results are compared with those for an annulus between two concentric cylinders and, finally, it is disclosed that the present or former geometry suggest lower heat transfer rates and friction factors

Keywords: Laminar flow, heat transfer, annular duct flow, concentric, square ducts, constant wall temperature

EŞMERKEZLİ HALKA KESİTLİ KARE KANALLARDA ISI VE AKIŞKAN AKIŞININ ANALİZİ

Özet: Bu çalışmada, eşmerkezli halka kesite sahip kare kanallarda ısı ve akış karakteristikleri sayısal olarak incelenmiştir. İç ve dış duvarlarda birbirinden farklı olmak koşuluyla sabit yüzey sıcaklığı öngörülmüştür. Akışın laminer, daimi, hidrodinamik ve ısıl olarak tam gelişmiş ve akışkanın sabit fiziksel özelliklerde olduğu kabul edilmiştir. Kartezyen koordinatlarda ifade edilmiş temel korunum denklemleri kontrol hacim yöntemi kullanılarak ayrıklaştırılmış ve ADI yöntemi ile çözülmüştür. Taşınım ve yayınım terimleri sırasıyla yukarı fark ve merkezi fark yöntemiyle ayrıklaştırılmıştır. SIMPLE algoritmasına dayalı basınç-doğrultman denklemini çözmek için Stone metodu kullanılmıştır. Çözümler hava için elde edilmiştir (Pr=0.7). Hız ve sıcaklık alanları, sürtünme faktörü ve Nusselt sayıları boyut oranına (a/b) bağlı olarak gösterilmiştir. Boyut oranının artışıyla, ısı transferinin iç duvarda önemli düzeyde arttığı, dış duvarda ise azaldığı belirlenmiştir. Elde edilen sonuçlar eşmerkezli halka kesitli dairesel kanal akışı ile karşılaştırılmış ve sonuç olarak mevcut geometri akışında daha düşük ısı transferi ve sürtünme faktörünün oluştuğu ortaya konmuştur.
Anahtar kelimeler: Laminer akış, ısı transferi, halka kanal akışı, eşmerkezli, kare kanallar, sabit yüzey sıcaklığı

NOMENCLATURE

a	width or height of the outer wall [m]
A	cross-sectional area of the annular duct [m^2]
b	width or height of the inner wall [m]
c_p	specific heat [kJ/kg K]
dP/dz	axial pressure gradient [Pa/m]
dT/dz	axial temperature gradient [K/m]
D_h	hydraulic diameter [m]
f	friction factor
k	thermal conductivity [W/m K]
n	the normal of the wall surface
Nu	Nusselt number
P	pressure [Pa]
Pr	Prandtl number

r^*	annulus dimension ratio [$=r_i/r_o$ for the annulus between two concentric cylinders and, =b/a for the annulus between two concentric square ducts]	Γ	diffusion coefficient
Re	Reynolds number	ϕ	dependent (generic) variable
S	source term	μ	dynamic viscosity [kg/s m]
T	temperature [K]	ν	kinematic viscosity [m^2/s]
u, v, w	velocity components in x-, y- and z-directions [m/s]	ρ	density [kg/m^3]
x, y, z	Cartesian coordinates [m]	θ	dimensionless temperature
x^*	dimensionless x-direction coordinate		

Greek symbols

α thermal diffusivity [m^2/s]

Subscripts

f	fluid
i	inner
L	local
m	mean value
o	outer
w	wall

INTRODUCTION

Convective heat transfer and fluid flow in rectangular ducts is frequently encountered in many industrial applications such as the cooling of electronic components, cooling channels in gas turbine blades, ventilation and air conditioning systems, turbomachinery, nuclear reactors and various compact heat exchangers.
For a given cross-sectional area, rectangular ducts have greater surface area than other ducts. From the point of increasing heat transfer, this characteristic is favorable for the design of heat exchangers. However, the increased surface area may also increase the friction between the fluid flowing inside and wall of the duct, which is undesirable. A compromise must, therefore, be made in using rectangular ducts as the heat transfer elements in heat exchangers (Su and Lin, 1997).

The rectangular shape provides an enhancement of heat transfer surface with no variation of the duct section. However, the analysis of the hydrodynamics and heat transfer of rectangular duct flows (a 2-D analysis), is generally more complicated than in the case of circular pipe flow (the usual 1-D analysis).

Laminar flow through rectangular ducts has been extensively investigated for different boundary conditions in the last decades. An extensive literature review on the subject is presented by Shah and London (1978), Shah and Bhatti (1987) and Harnett and Kostic (1989). Shah and London (1974) proposed a systematic exposition of the types of main thermal boundary conditions. The pressure drop characteristics of the flow in a constant, rectangular duct has been studied by Han (1960) and Wiginton and Dalton (1970) and the results were compared with experimental measurements by Beaverset et al. (1970)

Cheng (1991) studied the problem of heat transfer in fully developed laminar flow in a rectangular duct using a symbolic finite element method. The Nusselt number is obtained as a power series of the aspect ratio of the duct.

Xie and Harnett (1992) reported the heat transfer enhancement data for mineral oil in a 2:1 rectangular duct. By the numerical studies of Shin et al. (1993) and Chou and Tung (1995), the mechanism of heat transfer enhancement for mineral oil in a 2:1 rectangular duct has been investigated. They found that for the case of top wall heated, the heat transfer enhancement is caused mainly by the axial velocity distortion due to temperature dependence of viscosity. Spiga and Morini (1996) obtained analytical solutions for the H2 boundary condition. They predicted the temperatures and Nusselt numbers as a function of the aspect ratio of the rectangular duct. Chang et al. (1998) performed a numerical study on the heat transfer mechanism of Newtonian and non-Newtonian fluids in 2:1 horizontal rectangular ducts. Morini (2000) analytically obtained a rigorous solution for the temperature and the Nusselt numbers in the fully developed thermal region of rectangular ducts, wherein a laminar fully developed velocity profile occurred. Du Toit (2002) described a finite-element technique to determine the wall shear stresses and heat fluxes for fully developed laminar flow at the wall of a rectangular duct. For the flow in convergent and divergent ducts of rectangular cross section, the pressure drop and heat transfer characteristics have been investigated numerically and experimentally by Su and Lin (1997).

The objective of the present study is to predict the temperature and velocity distributions, and in the following, the friction coefficient and the Nusselt numbers as a function of the dimension ratio under the assumption of a steady-state laminar flow, fully developed both hydrodynamically and thermally. To the best knowledge of the authors, this is the first study on this geometry in the existing literature.

ANALYSIS

The physical configuration and coordinate system of the problem under consideration are depicted in Fig. 1. All thermophysical properties of the fluid are assumed to be constant. The axial conduction, the buoyancy and the viscous dissipation effects are neglected.

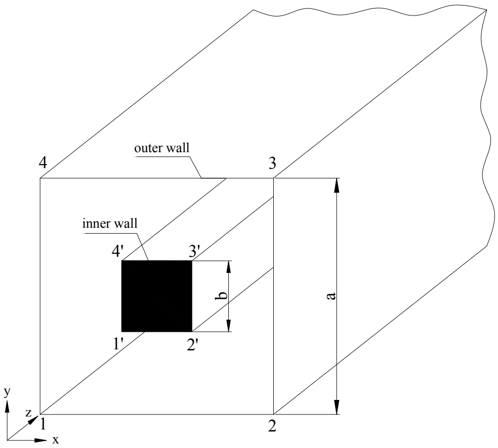


Figure 1. Problem geometry and coordinate system.

The fully developed flow and the constant axial temperature gradient assumptions result in the following conditions for velocity and temperature profiles are:

$$\frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=\frac{\partial w}{\partial z}=0\ ;\ \frac{\partial T}{\partial z}=const.$$

The governing equations for steady, hydrodynamically and thermally fully developed, incompressible laminar flow can be written as:

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equations

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{4}$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{5}$$

In the fully developed flow, the pressure gradient varies only in the cross-section of the annular duct. Therefore, the pressure gradient in the axial direction, $\partial P/\partial z$, remains constant. No-slip conditions are applied at all solid surfaces. The equations are subjected to the following boundary conditions at the inner and outer walls:

$$u=v=w=0\ ,\quad T_o=const.\ \text{and}\quad T_i=const. \tag{6}$$

The equations are approximated with finite difference equations by the control volume-based finite difference method for the dependent variables, u, v, w and T. The convection and diffusion terms are discretized by using the upwind scheme and the central difference scheme, respectively. The finite difference equations for the dependent variable of interest are solved by ADI (Alternating-Direction Implicit) method. This method uses the Tri-Diagonal Matrix Algorithm, TDMA, making successive sweeps over the computational field. Because the pressure-correction equation is a Poisson equation, Alternating-Direction Implicit solution of the difference equations is replaced by the Stone's method solution based on the SIMPLE Algorithm (Stone, 1968). A staggered grid system is employed in this study and the solutions are obtained by an iterative scheme. Iteration is repeated until the residuals in each equation are enough small. The relaxation factor is optimized 0.5, 0.5, 1, 0.7 and 0.45 for u, v, w, T and P, respectively. A convergence study with respect to the mesh size was performed and a mesh size of 101×101 was chosen. To check the adequacy of the numerical code developed in the present study, the forced convection flow in a square duct is simulated. For this geometry, the value of the mean Nusselt number is found to be equal to 2.985, which is in an excellent agreement with its literature value, 2.976 (Shah and London, 1978).

After numerically determining the axial velocity, the average velocity is calculated as

$$w_m=\frac{1}{A}\iint w\,dx\,dy \tag{7}$$

The Reynolds number is given as

$$\mathrm{Re}=\frac{\rho w_m D_h}{\mu} \tag{8}$$

The major parameters of practical interest for a forced convection study are the Nusselt number and friction coefficient, which lead to the heat transfer rate and pressure loss, respectively. The local Nusselt numbers can be obtained by an energy balance at the channel walls and its expression is given in following:

$$Nu=-\left.\frac{\partial\theta}{\partial n}\right|_{wall} \tag{9}$$

where n represents the normal of the wall surface and the dimensionless temperature, θ is defined as:

$$\theta=\frac{T_i-T}{T_i-T_o} \tag{10}$$

The average Nusselt number can be expressed as

$$\overline{Nu}=\int_s Nu\,ds \tag{11}$$

The friction factor is given by

$$f\,\mathrm{Re} = \left(\frac{-2D_h \left({dP}/{dz} \right)}{\rho w_m^2} \right) \mathrm{Re} \tag{12}$$

RESULTS AND DISCUSSION

This study is aimed at determining heat and fluid flow characteristics for steady, laminar and both hydrodynamically and thermally fully developed flow in an annulus between two concentric square ducts. Computations are carried out for air as a working fluid with a Prandtl number of 0.7. In order to study, the effect of the dimension ratio, a/b on the heat and fluid flow, its four different values are considered: 2.36, 2.90, 3.80 and 5.50.

Fig. 2 and 3 show dimensionless velocity and temperature fields, respectively. Since the peripherally the same hydrodynamic and thermal boundary conditions are used, the above mentioned fields are found to be symmetrical in both x- and y-directions, as expected. As seen in Fig. 2, the form of the axial velocity field is not affected by the increasing value of the annulus dimension ratio. However, the temperature profiles are affected by the annulus dimension ratio (Fig. 3). Decreasing the annulus dimension ratio intensifies the temperature profiles near the outer wall. The thermal boundary layers near the outer wall become thinner with the decreasing annulus dimension ratio, while those near the inner wall becoming thicker. Accordingly, as will be shown later, this will affect the heat transfer rates.

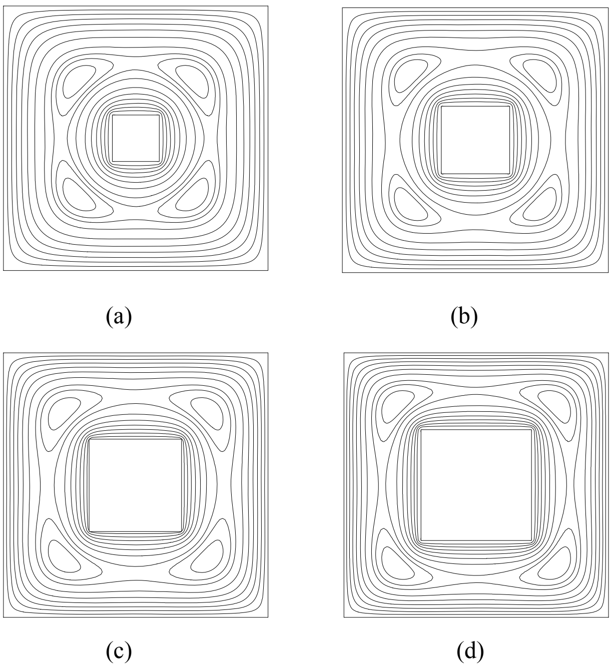


Figure 2. Axial velocity contours with different annulus dimension ratio a/b= (a) 5.5 (b) 3.8 (c) 2.9 (d) 2.36.

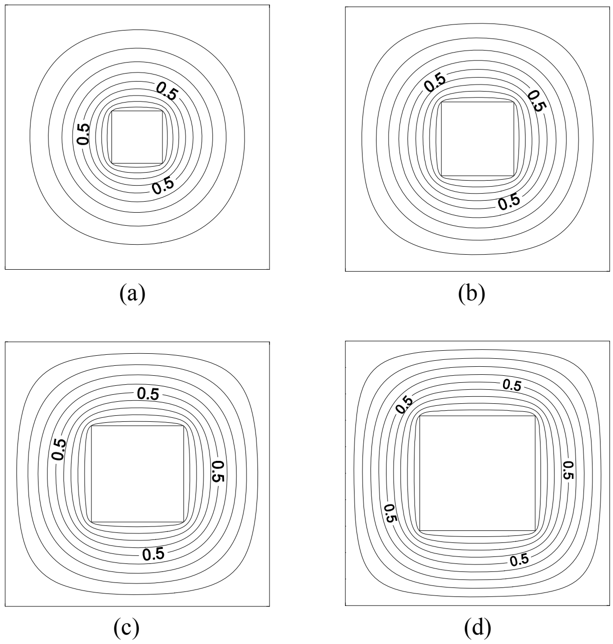


Figure 3. Temperature contours with different annulus dimension ratio a/b= (a) 5.5 (b) 3.8 (c) 2.9 (d) 2.36.

Fig. 4 represents the dimensionless axial velocity profile in the midplane of the annulus. In the annulus, the velocity profiles are not axisymmetrical, as it is in the cylindrical annular geometry, suggesting a maximum close to the inner wall. As shown in figure, a decrease at the dimension ratio will result in an increasing at w/w_m.

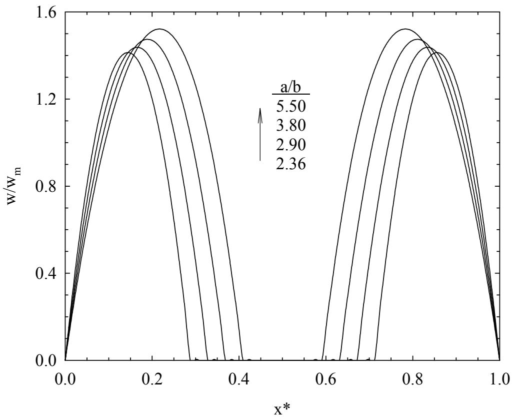


Figure 4. The variations of dimensionless axial velocity distribution at horizontal symmetry lines.

Fig. 5a and b show the peripheral distribution of the local Nusselt number at the outer and inner walls, respectively. Increasing the dimension ratio, a/b increases the average Nusselt number at the inner wall, while decreasing the average Nusselt number at the outer wall. This can be explained in terms of the temperature profiles obtained before. Fig. 6 shows the average Nusselt number for the inner wall (a) and for the outer wall (b) as a function of a/b. The friction factor of a/b is shown in Fig. 7. As shown, an increase at a/b leads to decrease of *f Re*.

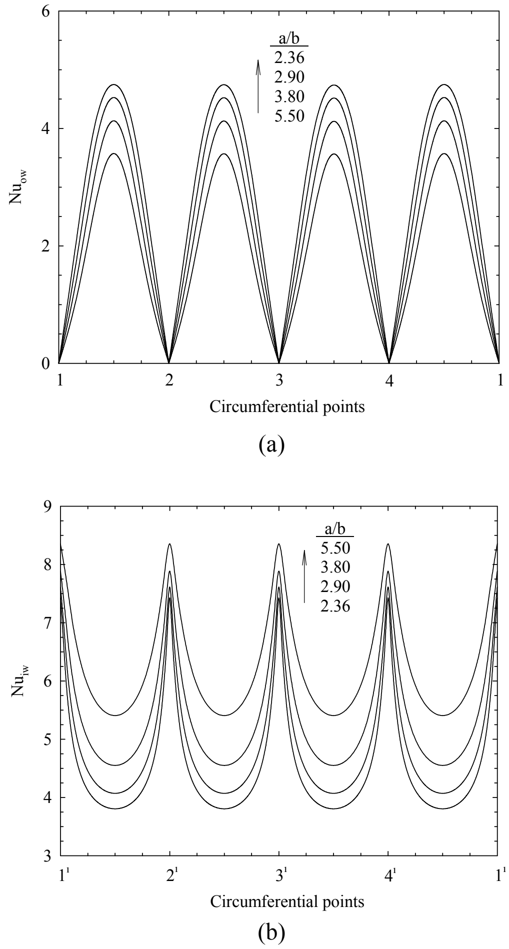


Figure 5. The variation of local Nusselt number at the outer wall (a) at the inner wall (b).

The hydraulic diameter for an annulus between two concentric cylinders is given as

$D_h = D_o - D_i$

while that for an annulus between two concentric square *ducts is given by*

$D_h = a - b$

The present results are compared with those for an annulus between two concentric cylinders (Shah and London, 1978) and, finally, it is disclosed that the present or former geometry suggest lower Nusselt numbers and friction factors (Fig. 8). Note that r^* is the annulus dimension ratio which is the inverse of the dimension ratio. Its value is defined as $r^*=b/a$ for the present geometry while it is $r^*=r_i/r_o$ for the latter geometry.

In fact, the above two geometries can be well compared in terms of the area goodness factor, j/f, which is defined as the ratio between the Colbourn number and the friction factor for the duct considered and defined as follows (Shah and London, 1978):

$$j/f. = Nu \, \mathrm{Pr}^{-1/3} / f \, \mathrm{Re} \tag{13}$$

Since the dimensionless quantities j and f are independent of the scale geometry (D_h), the area goodness factor j/f for different channels represents the influence of the geometric factors on the pressure losses and the heat transfer (Su and Lin, 1997). Fig. 9 compares the two geometries examined in Fig. 8 in terms of the area goodness factor, j/f, and a similar behavior to that in Fig. 8 is observed.

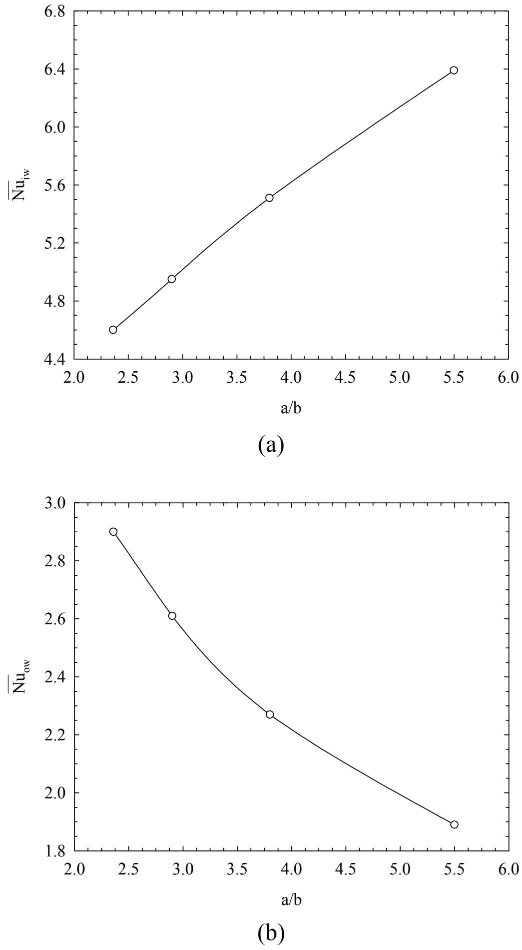


Figure 6. The variation of average Nusselt number at the outer wall (a) at the inner wall (b).

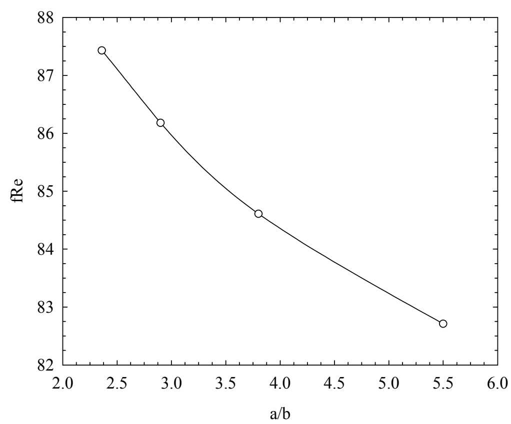
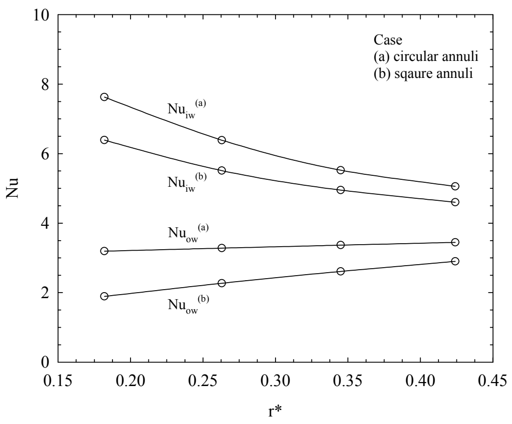
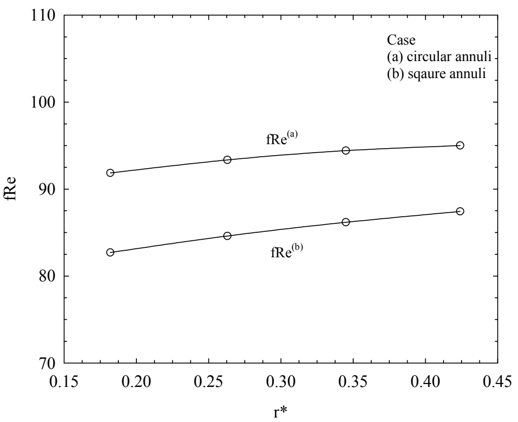


Figure 7. The variation of fRe with a/b.



(a)



(b)

Figure 8. Comparison of the mean Nusselt number (a) and friction factor (b) between the square annular duct and circular annular duct.

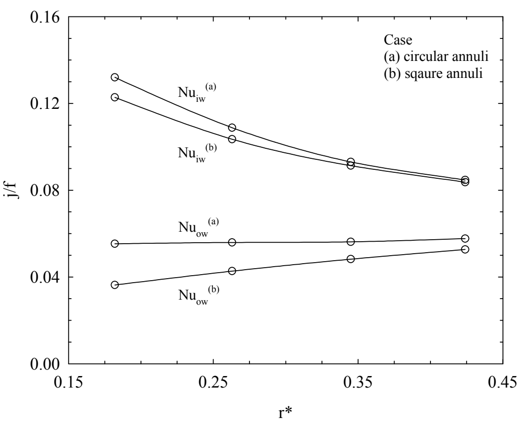


Figure 9. The variation of the area goodness factor with r^*.

CONCLUSION

A numerical study has been done to investigate the laminar forced convective flow and heat transfer of air in an annulus between two concentric square ducts by means of the control volume method. The annular flow is assumed to be both hydrodynamically and thermally fully developed. The validity of the code developed here is checked for the square channel geometry and the agreement is found to be excellent. The friction factors and the Nusselt numbers are determined. Numerical results reveal that an increase at the dimension ratio, a/b will result in an increase at the average Nusselt number at the inner wall, while resulting in an decrease at the average Nusselt number at the outer wall.

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