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A CORRELATION FOR OPTIMUM FIN SPACING OF VERTICALLY-BASED RECTANGULAR FIN ARRAYS SUBJECTED TO NATURAL CONVECTION HEAT TRANSFER

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Abstract: A new expression has been developed for prediction of the optimal fin spacing for vertical rectangular fins protruding from a vertical rectangular base. This expression was determined using the results of experimental investigation available in the literature over a wide range of test variables. Data collated from literature covered the range of fin spacing from 2.85 mm to 85.5 mm. The range of base-to-ambient temperature difference was quite extensive, from 14 ºC to 162 ºC. The fin length range was from 100 mm to 500 mm, the fin height from 5 mm to 90 mm, the fin thickness from 1 mm to 19 mm, the width of rectangular base plate from 180 mm to 250 mm. Its dimensionless parameters were formulated by the intersection of asymptotes method proposed in (Yildiz, and Yüncü, 2004). These dimensionless parameters correlate the experimental data of seven investigators. The expression resulting from this correlation predicted the experimental data within an average overall error of less then 24 percent.
Keywords: Optimal fin spacing, Extended surfaces, Natural convection.

DÜŞEY YÜZEYLER ÜZERİNDEKİ, DÜŞEY KANATÇIKLAR ARASINDAKİ OPTİMUM ARALIĞI HESAPLAMAK İÇİN YENİ BİR EŞ-İLİŞKİ

Özet: Bu çalışmada düşey yüzeyler üzerindeki, düşey kanatçıklar arasındaki optimum aralığı hesaplamak için yeni bir eş-ilişki geliştirilmiştir. Eş-ilişki literatürdeki deneysel sonuçlardan elde edilmiştir. Eş-ilişkiyi elde etmekte kullanılan deneysel sonuçların elde edildiği kanatçıkların aralığı 2,85 mm ile 85,5 mm arasında, kalınlığı 1 mm ile 19 mm arasında, uzunluğu 100 mm ile 500 mm arasında ve yüksekliği de 5 mm ile 90 mm arasında değişmektedir. Kanatçıkların taban sıcaklığı ile ortam sıcaklığı arasındaki fark 14 ºC ile 162 ºC, kanatçıkların oturduğu yüzeyin eni de 80 mm ile 250 arasında değişmektedir. Eş -ilişkiyi türetmede kullanılan boyutsuz parametreler asimptotların kesiştirilmesi olarak adlandırılan ve Yıldız ve Yüncü, 2004 den alınan yöntemle oluşturulmuştur. Bu boyutsuz parametreler ve literatürdeki 7 araştırmacının deneysel sonuçları kullanılarak geliştirilen eş-ilişki yardımı ile düşey yüzeyler üzerindeki, düşey kanatçıklar arasındaki optimum aralık ortalama %24 hata ile hesaplanabilmektedir.
Anahtar Kelimler: Optimum kanatçık aralığı, Kanatçık, doğal konveksiyon.

NOMENCLATURE

$\bar{h}$	Average Convection heat transfer coefficient, $\left[W/(m^2 \cdot K)\right]$
H	Fin height, $[m]$
k	Thermal conductivity, $[W/(m \cdot K)]$
L	Fin length, $[m]$
n	Number of fins
$\overline{Nu}_L$	Average Nusselt number $\left[=\frac{\bar{h}L}{k}\right]$
$\dot{Q}_{c_{max}}$	Maximum heat transfer rate, $[W]$
$\dot{Q}_c$	Convection heat transfer rate, $[W]$
$(\dot{Q}_o)_c$	Total heat transfer rate from vertical plate, $[W]$

Ra_L Rayleigh number $\left[= \dfrac{g\beta L^3 \Delta T}{\nu\alpha} \right]$

s Fin spacing, $[m]$

T_a Ambient Temperature, $[K]$

T_f Film Temperature, $[K]$

T_w Base plate temperature, $[K]$

W Fin base width, $[m]$

α Thermal diffusivity , $\lfloor m^2 / s \rfloor$

β Volumetric thermal expansion coefficient $\lfloor K^{-1} \rfloor$

ΔT Base-to-ambient temperature difference, $[K]$

INTRODUCTION

The operation of many engineering systems results in generation of heat. If the heat which is generated within a system is not dissipated rapidly to its surroundings the temperatures of the components may rise and the system may not function effectively and safely. The use of natural convection air-cooling extended surfaces provides a reliable, cheap and widely used method of cooling for dissipating unwanted heat. Besides, their design is simple, economic and without any acoustic noise. However the designs of these need to be optimized so the rate of convective heat dissipation through them are maximized. Without exceeding a maximum temperature and by keeping the power input fixed, the convective heat transfer from an extended surface can be increased either by increasing heat transfer coefficient or the surface area or both of these quantities. Increasing the heat transfer area is preferred as the simplest method to enhance heat dissipation rate, because the use of better fluid to increase the heat transfer coefficient is not an economical and practical solution. The only the controllable variable to enhance the convective heat transfer rate from an extended surface is the geometry of the fins.

The designer must optimize the size and spacing of the fin arrays. Otherwise; using fins can bring more disadvantages than its advantages to the design. Attaching fins increases the surface area but it also increases the resistance to the flow of air. The heat transfer coefficients based on base area of the fin arrays may be less than that of the base plate. If the decrease in the heat transfer coefficient is more than the increase in the surface area, the heat transfer rate will decrease.

Available literature has shown that the most common configurations used in applications involve horizontal or vertical surfaces to which plate or pin fin arrays are attached. In this study, the results of experimental investigation available in literature over a wide range of test variables are collated and analyzed to determine the effect of geometric parameters on heat transfer performance of rectangular fins protruding from a vertical rectangular base. The optimum parameters that will maximize the convective heat transfer rate from fin arrays are determined and correlations for prediction of the optimal fin spacing and maximum convective heat transfer rate are developed.

The experimental results for the optimal fin spacing of rectangular fins protruding from a vertical rectangular base are collated from (Leung et al., 1985 and 1986), (Leung and Probert, 1987) , (KO et al., 1989). (Güvenç and Yüncü 2001) and (Yazicioglu, and Yüncü, 2007) are analyzed in such a way that the separate roles of fin height, fin spacing and fin base to ambient temperature difference on convective heat transfer rate from the fin arrays are demonstrated. Then, data are rearranged to be presented in terms of the non-dimensional parameters as suggested by the intersection of asymptotes method proposed in (Bejan, 1984). The equations of the curves which are provided to fit non-dimensional optimal fin spacing and maximum convective heat transfer rate are obtained by least square regression.

FIN CONFIGURATION GEOMETRY AND EXPERIMENTAL SET-UP

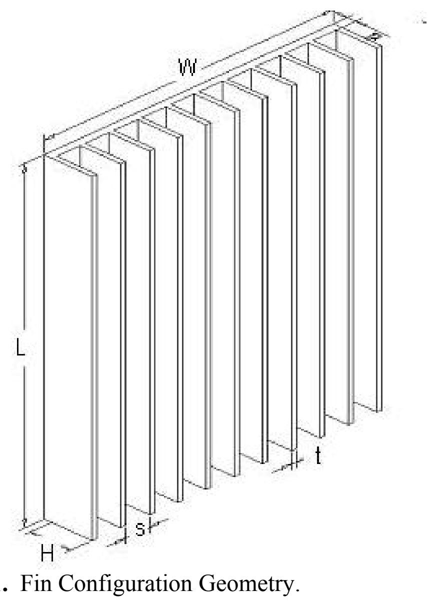

Figure 1. Fin Configuration Geometry.

Figure 1. illustrates the geometry of the fin-configurations and defines the significant dimensions. The dimensions of the fin configurations considered in this study are listed in Table 1. The range of fin thickness t was from 1mm to 19 mm, the fin spacing s, from 3 mm to 85.5mm, the fin length L, from 100 mm to 500 mm, the fin height H, from 5 mm to 90 mm, the width of base plate W from 180 mm to 250 mm.

Table 1. Geometrical Parameters.

Reference	Number of Fin Arrays Tested	Fin Length L(mm)	Fin Width W(mm)	Fin Thickness, t(mm)	Fin Height, H(mm)	Fin Spacing s(mm)	Base-to-Ambient Temperature Difference ΔT(°C)	Optimum Fin Spacing s_{opt}(mm)	Fin Material
1	16	150	190	3	10-17	3-45	20-40	9-9.5	Duralumin
2	11	250	190	3	60	2.85-33.2	20-80	10	Light Aluminum Alloy
3	36	250	190	3	32-90	3-77	20-60	10.5	Duralumin
4	22	250-375	190	3	60	5-77	40	10-11	Duralumin
5	40	500	190	1-19	65	3-54	20-40	11-19	Duralumin
6	14	500	190	3	60	5-77	20-40	12	Stainless Steel
8	15	100	250	3	5-25	4.5-58.75	14-106	7	Aluminum
9	30	250-340	180	3	5-25	5.85-85.5	21-162	10-11	Aluminum Alloy

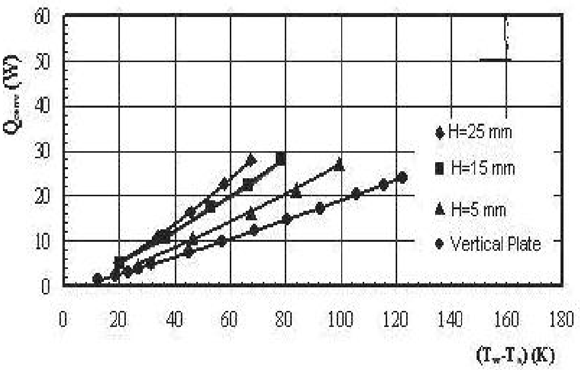

Figure 2. Variation of Convection Heat transfer Rate with Base-to-Ambient Temperature Difference (s=32.4mm, L=100mm) (Güvenç and Yüncü 2001).

The fin arrays tested in (Leung et.al., 1985, 1985 and 1986) and (KO et al., 1989) were composed of various combinations of fins and spacer bars. Nuts and bolts were used to connect the fins and spacer bars to form a rigid body. In (Güvenç and Yüncü, 2001) and (Yazicioglu and Yüncü, 2007) the fins were integral with the base plate. The fin-base plate assembly was fabricated by milling longitudinal grooves (i.e. inter fin gaps) into one of the face of a rectangular bar. The experimental set-ups and methods used in literature were basically similar. The experimental set-up in (Leung et al., 1985 and 1986) and (Leung and Probert, 1987) consists of externally insulated wooden case which covers the base of fin array, the heater plate and the thermal insulation. A plate heater was placed at the rear surface of the rectangular fin array. The experimental set-up in (Leung and Probert, 1989) and (KO et al., 1989) is similar to that of (Leunget et al., 1985, 1985 and 1986) and (Leung and Probert, 1987) but a guard heater was installed behind the main heater and fibrous insulation was applied to back surface of the heater plate. In works of (Güvenc and Yüncü, 2001) and (Yazicioglu and Yüncü, 2007) fiberboard case or aerated concrete was used to enclose the base of fin array. Behind the base plate of fin array, the copper plate and the heater were placed and the external surface of the case was insulated with fiber glass wool .To determine the rate of heat transferred from heaters through the fin-arrays, the experimental set-up was calibrated. For further details of experimental set-ups and experimental procedures see references.

ANALYSIS OF EXPERIMENTAL RESULTS

As a point of departure for the presentation of convective heat transfer rates Q_c, from fin arrays are plotted as a function of base-to-ambient temperature difference $(T_w - T_a)$. Four samples of these plots are given in Figs. 2, 3, .4 and 5. In Figs. 2 and 3 the results for the fin heights of H = 5 mm, H=15 mm and H = 25 mm and for vertical flat plate are shown. In Fig.2 and Fig.3 the fin spacing is s= 32.3 mm and the fin lengths are L=100mm and L=340 mm respectively. These figures reveal that convection heat transfer rate from fin

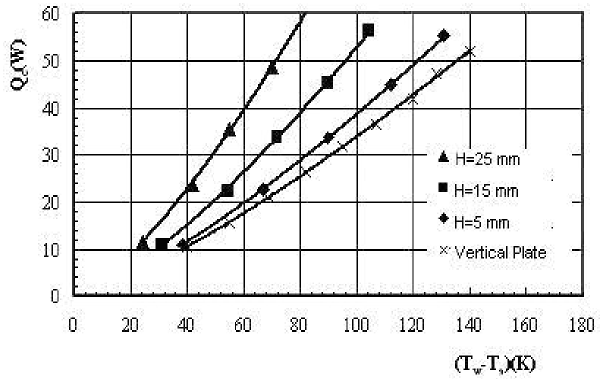

Figure 3. Variation of Convection Heat transfer Rate with Base-to-Ambient Temperature Difference (s=32.4mm, L=340mm) (Yazicioglu and Yüncü, 2007).

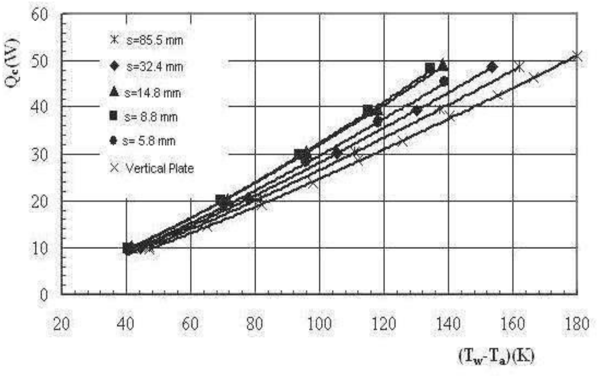

Figure 4. Variation of Convection Heat transfer Rate with Base-to-Ambient Temperature Difference (s=32.4mm, L=250mm) (Yazicioglu and Yüncü, 2007).

arrays is dependent on fin height, fin length and base-to-ambient temperature difference. Essentially, fin heat transfer rate increases with fin height, fin length and base-to -ambient temperature difference. At low temperature differences, heat transfer rates are closer to each other and tend to diverge at higher temperature differences. Moreover, with larger fin heights, the increase of convection heat transfer rate with temperature difference is sharper. For all fin arrays, the convection heat transfer rate is higher than that of the vertical flat plate.

In Figs. 4 and 5 the results for the fin spacing of s = 85.5 mm, s= 32.4 mm, s = 14.8 mm, s = 8.8 mm, s=5.8 mm and for vertical flat plate are given. In these figures the fin length is 250 mm and the fin heights are 5mm and 25mm respectively. These figures show that as fin spacing is increased, fin heat transfer rates approach each other and the vertical flat plate, and fin height does not play a significant role. The effect of fin height can be seen more clearly on fins with smaller fin spacings. For all fin arrays, the ratio of fin convection heat transfer rate to that of the vertical flat plate at the same base-to-ambient temperature difference is greater than unity; for some fin arrays, it may be as high as 4. This reveals that good enhancement is achieved by attaching fins to a vertical flat plate.

To see the effect of base-to-ambient temperature difference more clearly. In Figs. 6, 7 and 8, convection heat transfer rates are plotted as a function of fin spacing.

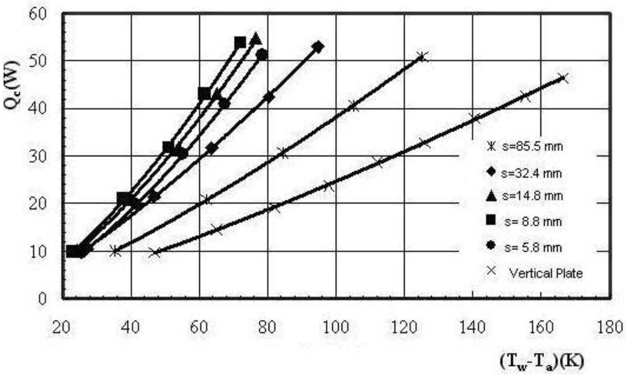

Figure 5. Variation of Convection Heat Transfer Rate with Base-to-Ambient Temperature Difference (H=25 mm, L=250mm) (Yazicioglu and Yüncü, 2007).

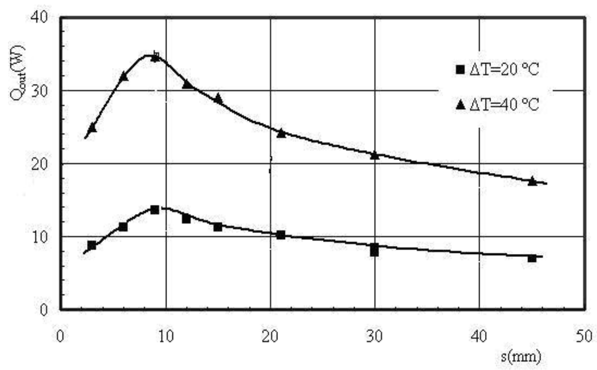

Figure 6. Variation of Convection Heat Transfer Rate with Fin Spacing (H = 17 mm, L=150 mm) (Leung et al., 1985).

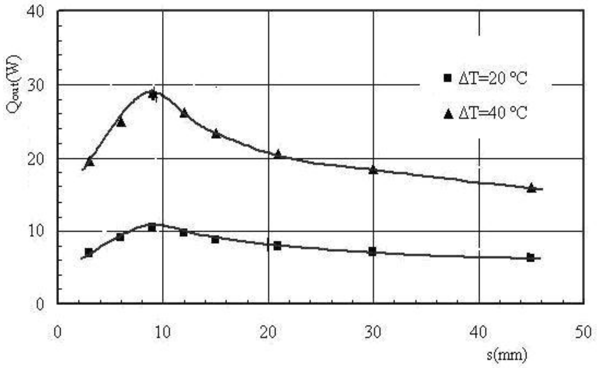

Figure7. Variation of Convection Heat Transfer Rate with Fin Spacing (H = 10 mm, L=150 mm) (Leung et al., 1985).

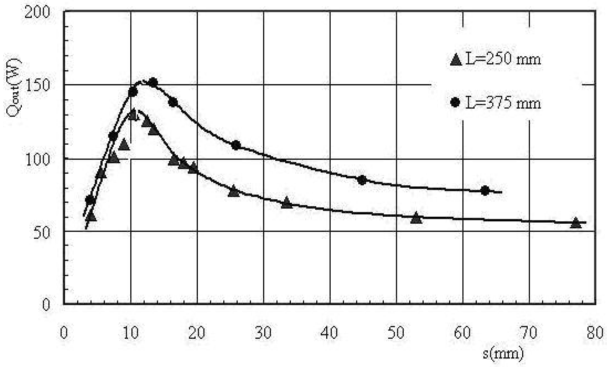

Figure 8. Variation of Convection Heat Transfer Rate with Fin Spacing (ΔT=40 ºC, H=60mm) (Leung and Probert, 1987).

The figures are for fin heights of H =17 mm, H =10 mm and H=60mm for base-to-ambient temperature difference of ΔT= 20^0 C and ΔT= 40^0 C and fin lengths of L= 150 mm, L= 250 mm and L=375 mm. Closer inspection of these figures reveals that, at a given fin height and base-to-ambient temperature difference, convection heat transfer rate from a fin array first increases, reaches a maximum and then starts decreasing. The value for the fin spacing at which the convection heat transfer rate is maximized is called optimum fin spacing s_{opt} . The optimum fin spacing depends on fin height, fin length and base-to-ambient temperature difference, but the dependence on fin length is not very significant.

CORRELATION FOR OPTIMUM FIN SPACING

The experimental data for optimum fin spacing and maximum convective heat transfer rate are correlated in terms of non-dimensional parameters obtained from scale analysis. For the scale analysis, a procedure applied for annular fins on horizontal cylinder in Ref. [9] was adapted for the current geometry. For this reason, the vertical base plate of length L is modeled as isothermal at temperature T_w . The temperature of ambient air is taken as constant at T_a . The thickness of the fins is considered as negligible. The flow is assumed to be laminar. The fin surfaces are assumed to be sufficiently smooth to justify the use of heat transfer results for natural convection over vertical smooth wall.

The base-to-ambient temperature difference, $\Delta T = T_w - T_a$ in the order of magnitude sense, is assumed to be representative of the temperature difference in the flow field. The order of magnitude of optimum fin spacing s_{opt} for maximum heat transfer rate is obtained as

$$\frac{s_{opt}}{L} \sim Ra_L^{-1/4} \tag{1}$$

For dimensionless presentation of the order of magnitude of optimum fin spacing, the Rayleigh number is employed according to its definition.

$$Ra_L = \left(\frac{g\beta L^3 \Delta T}{\nu\alpha} \right) \tag{2}$$

In Figure 9 the variation of experimentally estimated optimum fin spacing values s_{opt}/L is plotted as a function of Ra_L. All the physical properties necessary to evaluate Rayleigh number were taken at film temperature $T_f = (T_a + T_w)/2$. A curve is also provided to fit the data in Fig. 9 to demonstrate the trend of the data points. The equation of this curve is obtained by least square regression as.

$$\frac{s_{opt}}{L} = 3.15 Ra_L^{-1/4} \tag{3}$$

As observed from Fig. 9 the experimental points follow the trend predicted by the range correlation given in Eq. (3). Bearing in mind the number of data sources and wide range of experimental parameters, the degree of correlation is encouraging. It is interesting to note that the correlation includes measurement made on fins integral with the base plate or connected to spacer by bolts and nuts.

An order-of-magnitude estimate for the maximum convection heat transfer rate from fins can also be obtained as: $\dot{Q}_{c_{\max}}$

$$\left[\dot{Q}_{c_{\max}} - \left(\dot{Q}_o\right)_c\right] \sim Ra_L^{1/2} kH\Delta T \frac{W}{L} \tag{4}$$

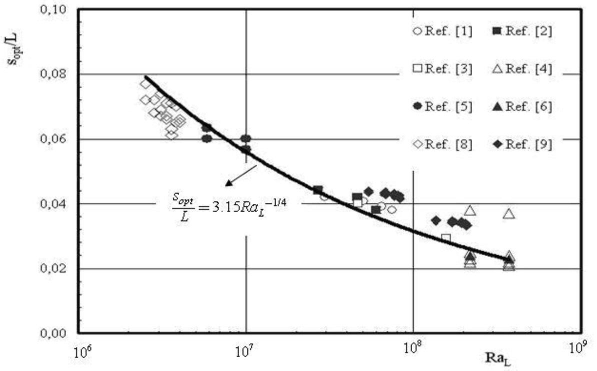

Figure. 9. Comparison of Eq. (3) with Experimentally Estimated Optimum Fin Spacing .

The right hand side of Eq. (4) represents the order of magnitude of maximum heat transfer rate. A non-dimensional form of Eq. (4) may be obtained as

$$\frac{\left[\dot{Q}_{c_{\max}} - \left(\dot{Q}_o\right)_c\right]}{kH\Delta T (W/L)} \sim Ra_L^{1/2} \tag{5}$$

The variation of $\left[\dot{Q}_{c_{\max}} - \left(\dot{Q}_o\right)_c\right] / kH\Delta T (W/L)$ is plotted as a function Ra_L for all experimental results in Fig.10. Several functional forms are suitable to represent these data but the following form was selected as the best, which was obtained by least square regression

$$\left[\dot{Q}_{c_{\max}} - \left(\dot{Q}_o\right)_c\right] = 0.2116 Ra_L^{1/2} kH\Delta T \frac{W}{L} \tag{6}$$

In Fig. 10, it is seen that some experimental data points are slightly above the curve of Eq. (6). This is presumably due to the maximum convection heat transfer rates readings taken from plots of variation of convection heat transfer rate with fin spacing

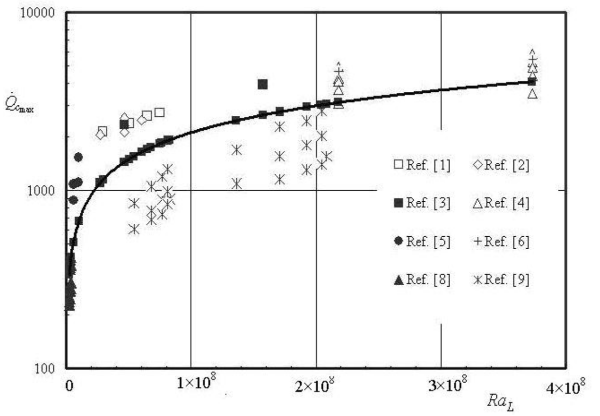

Figure. 10. Comparison of Eq. (6) With Experimentally Estimated Experimentally Maximum Convection Heat Transfer Rate.

The correlations numbered as Eqs. (3) and (6) were obtained from experimental results of eight authors. This fact indicates the dimensional analysis is incomplete and missing parameter or parameters are present. The convection heat transfer from the base-plate $\left(\dot{Q}_o\right)_c$, can be calculated by the Newton's law of cooling

$$\dot{Q}_c = \bar{h} WL\Delta T \tag{7}$$

where the average heat transfer coefficient $\bar{h}$ may be obtained from the knowledge of Rayleigh number and a suitable correlation for this flow. The correlation suitable for the laminar flow is of the form

$$\overline{Nu}_L = 0.59 Ra_L^{1/4} \tag{8}$$

Hence, the average heat transfer coefficient is

$$\bar{h} = 0.59 \frac{k}{L} Ra_L^{1/4} \tag{9}$$

The convection heat transfer from the base-plate can be obtained by substituting Eq. (8), and Eq. (9). The result is

$$\left(\dot{Q}_o\right)_c = 0.59 Ra_L^{1/4} k W \Delta T \tag{10}$$

To quantize the effect of fins on augmentation technique,

the variation of the ratio of maximum convective heat transfer rate to that of vertical plate at the same base to-ambient temperature difference is calculated as a function of Ra_L and H/L .

$$\frac{\dot{Q}_{c_{\max}}}{\left(Q_o\right)_c} = 1 + 0.36 Ra_L^{1/4} \frac{H}{L} \tag{11}$$

The ratio H/L is dimensionless parameter to represent the geometry of fin array for maximum heat transfer rate. The zero value of this parameter corresponds to the case of vertical plate. For the range of fin arrays for which the data are collated from literature, the ratio of heat transfer rates is greater than unity, for some fins it maybe as high as 4. This reveals that good enhancement is achieved by attaching fins to a vertical flat plate

CONCLUSIONS

This work presents experimental results of heat transfer by free convection from rectangular fins on a vertical base. From the experimental results, it can be concluded that the geometric parameters of fin array, fin height, fin length and fin spacing and base-to-ambient temperature difference affects the rate of convection heat transfer primarily. The separate roles of these parameters and base-to-ambient temperature difference are given in Figs. 2 to 8.

Experimental results showed that the convective heat transfer rate from all fin arrays is greater than that for vertical plate. In some cases the ratio of convective heat transfer rates may be as high as 4.This shows that good enhancement can be achieved by attaching fin to vertical surface.

From the results plotted in the figures, it can be seen that the larger fin height results in higher convection heat transfer rate from the fin arrays. Although, at low base-to-ambient temperature differences, the increase in convective heat transfer rate with fin height is not very significant, at higher temperature differences the convective heat transfer rate increases significantly with fin height. For a given fin spacing, heat transfer rate increases monotonously with fin height and base-to-ambient temperature difference. For a given base-to-ambient temperature difference, the increase in heat transfer rate with fin height is steeper for smaller fin spacing. Since, the fin array with smaller fin spacing has higher fin number; the increase in fin height causes larger heat transfer area and higher convection heat transfer rate.

The experimental results showed that, the change of fin length from 250 mm to 340 mm causes an increase in convection heat transfer rates for each fin configuration. The average relative improvements in convection heat transfer rates from identically spaced fin arrays for fin heights of 5, 15 and 25 mm are 37.44 %, 39.01 % and 41.28 %, respectively.

From Figs. 2 to 8, for a given fin height , fin length and base-to-ambient temperature difference the convection heat transfer rate from fins increases as fin spacing decreases, attains a maximum and then starts to decrease with the further decrease in the fin spacing. The value of fin spacing at which the maximum convection heat transfer occurs is called the optimum fin spacing. As the fin spacing is decreased, the heat transfer area of the fin array increases. As a result of increases in heat transfer area the convective heat transfer rate from the fin array increases. On the other hand, decreasing the fin spacing below a certain value produces a resistance to flow of air. Further decreases in fin spacing, even though the heat transfer area is increased, decreases heat transfer rate.

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