

MODELING OF TWO-PHASE FLOW INSTABILITIES IN CONVECTIVE IN-TUBE BOILING HORIZONTAL SYSTEMS

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Abstract: This work aims to predict the low frequency oscillations in a horizontal in-tube boiling system using a refrigerant-11 as the working fluid. The steady state system pressure-drop characteristics are determined by a numerical solution of the governing equations as derived from the Drift-flux models. Oscillations were obtained for various heat inputs, flow rates and exit restriction diameters. Both numerical solutions of the steady-state and transient solutions obtained are compared by experimental findings and a satisfactory agreement between the theory and experiments is obtained.

Keywords: Instabilities, Two-phase flow, In-tube boiling

YATAY BORU IÇERISINDE KONVEKSIYONLA KAYNAMALI SISTEMLERDE IKI-FAZLI AKIM KARASIZLIKLARININ MODELLENMESI

Özet: Bu çalışmanın amacı, düşük frekanslı osilasyonları, bir soğutucu akışkan kullanarak, yatay bir boruda soğutucu akışkanın konveksiyonla kaynamasında elde etmektir. Drift-flux ve homojen modeller kullanılarak, süreklilik, momentum ve enerji denklemlerini nümerik olarak çözerek, sistemin sürekli rejimde basinç düşümü karakteristikleri tayin edilmektedir. Iki-fazlı akım kararsızlıkları, çesitli ısı akıları, akışkan debileri ve çıkışda orifis çaplarının bir fonksiyonu olarak elde edilmektedir. Sayısal metotla elde edilen sürekli ve zamana bağlı rejim sonuçları, deneysel sonuçlar ile karşılaştırılmış ve teori ile deneyler arasında kabul edilebilir sonuçlar elde edilmiştir. **Anahtar Kelimler:** Kararsızlıklar, Osilasyonlar, İki-fazlı akım, Kaynama

NOMENCLATURE

- A Tube inner surface area, m^2
- A_P Surge tank cross-section area $[m^2]$
- d Inner diameter of the heater tube [m]
- f Friction factor, dimensionless
- F_M Two-phase flow friction multiplier, dimensionless
- g Gravitational acceleration, 9.806 [m/s²]
- G Fluid mass velocity (= ρu) [kg/ (m²/s)]
- G_i Inlet mass velocity to surge tank [kg/ (m²/s)]
- G_o Outlet mass velocity from surge tank [kg/ (m²/s)]
- h Specific enthalpy of the fluid [J/kg]
- h_{lv} Latent heat of vaporization [J/kg]
- j Volumetric flux [m/s]
- K_I Inlet restriction coefficient, dimensionless
- L_H Heated length [m]
- P Pressure [Pa]
- P_o System inlet pressure during density-wave type oscillations [Pa]

- P_s Pressure in the surge tank [Pa]
- P_{so} Steady-state pressure in surge tank [Pa]
- q_f Steady-state heat input into the fluid [W]
- q' Heat input into the fluid during oscillations [W]
- t Time [s]
- T Fluid inlet temperature [K]
- u Fluid velocity [m/s]
- V_o Volume of noncondensable gas in surge tank $[m^3]$
- x Quality of the liquid-vapor mixture, dimensionless
- z Axial distance along the flow path [m]

Greek Symbols

- α Heat transfer during pressure-drop oscillations $[W/m^2 K]$
- β Exit restriction diameter ratio, d_i/d
- μ Dynamic viscosity of the fluid [Pa s]

- ρ Density [kg/m³]
- σ Surface tension [N/m]
- Ψ Void Fraction, dimensionless
- Δ Incremental length along tube

Subscripts

~~~~~	-P+5
e	Exit
f	Fluid
i	Inlet
1	Liquid
0	Heater exit to restriction
s	Surge tank
SP	Single-phase
ТР	Two-phase
ν	Vapor
	XX 7 11

w Wall

# INTRODUCTION

Two phase flows have been observed to occur in many industrial situations like refrigeration systems, turbomachinery, and power plant heat exchangers. Prediction of flow parameters such as steady-state pressure-drop characteristics, stability boundaries during boiling, and oscillations is very crucial in the design of two-phase flow equipments. Thermal oscillations are undesirable as they can cause problems of system control, and can lead to tube failure due to wall temperature increase. They can also lead to fatigue failure of the tube caused by a continuous cycling of the wall temperature. It is clear from these problems that plant equipments should possess an adequate margin of safety against oscillations.

Stenning (1964) investigated and identified three types of dynamic instabilities, namely, density-wave type, pressure-drop type, and thermal oscillations in a single channel upflow boiling system. Maulbetsch and Griffith (1965) studied Ledinegg and pressure-drop instabilities in single horizontal tubes, the working fluid being water. The onset of instability was deduced from CHF in the test section. Daleas and Bergles (1965) investigated the effect of the compressibility of upstream volumes of subcooled water on system instability. They investigated pressure-drop instabilities in a single horizontal tube system. Mayinger and Kastner (1968) also identified thermal oscillations. Among other investigators, Kakaç et al. (1977) identified dynamic oscillations and presented (Kakaç et al. 1990, Gurgenci et al. 1982) investigations of thermal instabilities in forced convection boiling in a single vertical system. Experiments with various heat inputs, inlet temperatures and different size heater tubes were conducted. Various reviews of two-phase flow dynamic instabilities and some of theoretical studies for in-tube boiling vertical systems are cited in the references (Bergles 1977, Boure et al. 1973, Kakaç and Bon 2008).

The objective of this work is to study theoretically and experimentally the steady-state and oscillatory characteristics of flow boiling in a single horizontal channel system. The Drift-Flux and homogeneous models are used for prediction of amplitude and frequencies of oscillations and its results are compared with experimental findings to validate the frequencies of oscillations and its results are compared with experimental findings to validate the model.

# EXPERIMENTAL STUDY

As part of the experimental studies, two-phase flow of R-11 is studied and different flow parameters are measured. The steady-state characteristics are obtained via pressure-drop vs. mass flow rate measurements. Different heat inputs and exit restriction diameters are considered. As part of the oscillations (unsteady flow problem) study, a surge-tank was used to simulate system compressible volume. The change in surge-tank level coincides with the occurrence of oscillations. A pressure transducer is used to measure oscillatory pressure and tube-wall temperatures.

Figure 1 is a schematic diagram of a multi-channel intube boiling horizontal system experimental apparatus which is used in this study. In the present study, only single-channel flow is studied.

*Fluid Supply*: This part of the system set-up consists of a main tank, accessories, pressurizing vessel, filter, rotameter, and main control valve.

*Test Section*: The components from the surge tank to the exit restriction form the test section, which is the most important part of the system. The surge tank is a four inch diameter, ten inch high copper cylinder with caps on both ends. Pressure gage and sight glass are mounted on it to visualize the pressure and liquid level inside. The main function of the surge tank is to provide a compressible volume upstream of the system. The test section tube is thermally insulated and electrically heated and the heat input can be regulated by a DC rectifier. The rectifier has a capacity of 20 kW and is equipped with continuous current settings. Electrical insulation of the heating system to the test chamber is ensured.

*Fluid Recovery*: First the two-phase mixture is sent to a chiller where it condenses and the condensed liquid is stored in the recovery tank. When the experiment is finished the fluid collected in the recovery tank is sent back to the main tank.

*Temperature Measurement*: The temperature measurement at various locations on the line will be made by using E-type thermocouple wires of 0.01 inch diameter. The temperature signals are sent to two measuring devices, i.e., data logger and recorder. Signals from the data logger will be used for computer

analysis, e.g., Fourier transforms; signals are sent to the recorder and printed on the chart.

*Pressure Measurement*: Pressure signals are measured at the following locations: onlet and outlet plenum, before inlet plenum, after restriction valve, before and after the heater. Pressures before and after the heater are sensed by both pressure gages and transducers, while the others are measured by gages only. The pressure gages have arrange from zero to 150 psi with subdivisions of 0.2 psi. The pressure transducers also have the same range, but a 16V DC power supply is needed to exit them.

*Flow Measurement*: The mass flow rate in the system was regulated by the control valve. It is an important parameter, thus of its accurate measurement is required. Two measuring devices, therefore, are used to monitor and control the flow rate, i.e., turbine flowmeter and rotameter. The turbine flowmeter has a linear DC voltage output from zero to 5V within its range.

The rotameter is used to monitor the flow variation visually. These two devices are connected in series before the main control valve.

*Experimental Procedure.* With a typical setting of 7.6 bars in the main supply tank, experimental data of dynamic instabilities are obtained following the procedure outlined below. The main control valve which admits fluid in the test section is open. The initial setting of the mass flow is usually 70-80%. The recirculating cooler is turned on and the temperature control dial is set to the desired fluid inlet temperature. The surge tank is filled with nitrogen and its liquid level is set to a desired location. The rectifier is turned on and

the current output is set to the desired valve. The heat input to the test section was applied gradually. The condenser is turned on. Next, the system is allowed some time to settle down and reach steady-state. Then, all parameters are recorded. The next step was to reduce the flow rate by a small amount and repeat the measurements. Different sets of experiments are preformed for different heat input values. Details of the experimental study on vertical channels can be found in Ding (1993), Patki et al (1991), Kakaç et al (1995), Kakaç, Bon (2008), Karslı et al (2002), Widman et al (1995) and Çomaklı et al (2001).

## THEORETICAL STUDY

In the most general formulation of the two-phase flow problem, the conservation equations are written separately for each of the phases, hence it is called as a "separated flow model". Various forms of the conservation equations have been used in the literature (Kakaç et al. 1977), (Cao et al. 2000), (Padki et al. 1991). In most of the practical problems, however, one dimensional time-dependent equations are used. To close the set of six conservation equations (three for each phase), seven constitutive laws are required friction and heat transfer for the two phases at the tube wall, and three conservation equations for shear force, mass balance, and energy balance at the interface of the two phases. The requirement of seven constitutive laws makes the use of this model very difficult. In order to reduce the complexity involved in the formulation of the problem in its most general form, as noted above, several models have been suggested, which attempt at correlating different parameters of the two-phases, e.g. drift velocity, void fraction, slip ratio, etc.



Figure 1. Schematic diagram of the experimental system.

In the solution of two-phase problems by one these correlations, the six phase equations are written separately and then combined for the mixture (Yadigaroglu and Lahey 1976). For a number of twophase flow regimes, such as annular or slug flow, the homogeneous model does not reflect the physics of the phenomena, since the assumption of equality of the phase velocities is not justifiable. The drift-flux formulation which has gained much acclaim in the last decade, takes the relative velocity between the phases into account, while assuming thermodynamic equilibrium (Zuber and Findlay 1965). In the boiling region, the fluid flow is treated as a mixture of saturated liquid and vapor phases travelling at different velocities. The relative velocity between the phases is represented by the so-called drift velocity, UV, of the vapor phase with respect to the center-of-volume of the mixture.

#### **Mathematical Formulation of the Drift-Flux Model**

The following equations are written for a horizontal, single channel upflow boiling system.

*Conservation Equations*. The continuity, energy, and momentum equations are as follows:

Continuity equation:

$$\frac{\partial}{\partial t} [\rho_l (1 - \Psi) + \rho_v \Psi] + \frac{\partial}{\partial t} [\rho_l u_l (1 - \Psi) + \rho_v u_v \Psi] = 0$$
(1)

Here  $\Psi$  is the local void fraction or the volumetric concentration. The above equation is actually a sum of two equations, one each for the liquid and vapor phases, as can be seen from the weighing parameter  $\Psi$ .

Energy equation:

$$\frac{\partial}{\partial t} [\rho_l (h_l - Pv_l)(1 - \Psi) + \rho_v (h_v - Pv_v)\Psi] + \frac{\partial}{\partial z} [\rho_l u_l h_l (1 - \Psi) + \rho_v u_v h_v \Psi] = \frac{q'}{AL_h}$$
(2)

Note here that the energy equation is basically a thermal balance over a part of the heated section.

Momentum equation:

$$-\frac{\partial P}{\partial z} = \frac{\partial}{\partial t} \left[ \rho_l u_l (1 - \Psi) + \rho_v u_v \Psi \right] + \left( \frac{\partial P}{\partial z} \right)_{\Psi fric} + \frac{\partial}{\partial z} \left[ G^2 \left[ \frac{(1 - x)^2}{\rho_l (1 - \Psi)} + \frac{x^2}{\rho_v \Psi} \right] \right] + g \left[ \rho_l (1 - \Psi) + \rho_v \Psi \right]$$
(3)

The first and second terms on the right side of the equation denote the momentum buildup due to fluid flow and the two-phase frictional drop respectively. The pressure drop is a function of the wall roughness and flow regime and is generally an empirical correlation in terms of the Reynolds number. The last term on the right hand side of the equation is the net efflux of momentum due to vapor and liquid flows. The mass flux, G, in the momentum equation is defined as

$$G = \rho_l u_l (1 - \Psi) + \rho_v u_v \Psi \tag{4}$$

The phase velocities,  $u_1$  and  $u_v$ , will be related to each other using the drift-flux kinematic constitutive relation (Stenning 1964).

*Auxiliary Terms.* The various auxiliary terms in the governing equations are defined below. Mass velocity, G,

$$G = \rho_l u_l (1 - \Psi) + \rho_v u_v \Psi \tag{5}$$

Quality (dryness fraction), x,

$$x = \frac{\rho_v u_v A_v}{\rho_v u_v A_v + \rho_l u_l A_l} \tag{6}$$

Void Fraction, Ψ,

$$\Psi = \frac{A_v}{A} \tag{7}$$

Total enthalpy at a given section,

$$h = \rho_l h_l (1 - \Psi) + \rho_v h_v \Psi \tag{8}$$

*Equations of State.* The thermal-physical properties of Refrigerant-11 have been correlated in a polynomial form from data available in (Irvine and Hartnett 1976). The saturated properties are in terms of pressure and are valid in the range of 0.6 to 6.2 bar.

Saturated Liquid Enthalpy:

$$h_f = 20.11198 + 42.14375 p - 10.8984 p^2 + 1.659071 p^3 - 0.09865 p^4$$
(9)

Saturated Vapor Enthalpy:

$$h_{v} = 215.8316 + 24.78123 p - 6.6406 p^{2} + 1.009815 p^{3} - 0.06008 p^{4}$$
(10)

Saturated Liquid Density:

$$\rho_l(p) = 1489.7 - 0.11 \, p + 0.0473 \, p^2 \tag{11}$$

Saturated Vapor Density:

$$\rho_{\nu}(p) = 0.233297 + 5.798266 \ p - \ 0.23475 \ p^{2} + 0.036673 \ p^{3} - 0.00204 \ p^{4}$$
(12)

*Two-phase Frictional Pressure Drop.* To estimate the magnitude of the two-phase frictional pressure gradient, a two-phase friction multiplier  $F_M$  is used along with an expression for the single-phase pressure drop.  $F_M$  is defined as

$$F_M = \frac{\Delta P_{TP}}{\Delta P_{SP}} \tag{13}$$

The single-phase pressure gradient can be estimated by using the definition of the Fanning friction coefficient as

$$\left(\frac{dP}{dz}\right)_{sp\ fric} = 4\frac{f_0}{d}\frac{G^2}{2\rho_l} \tag{14}$$

where  $f_0$  is the single phase friction factor. It can be estimated by using Blasius' formula as

$$f_0 = 0.079 \left(\frac{\mathrm{Gd}}{\mu_\mathrm{l}}\right)^{-n} \tag{15}$$

where n is taken as 0.25. The above relation is valid for smooth pipes with flow Reynolds numbers in the range 3000 < Re < 10000. Thus the two phase pressure frictional pressure gradient takes the form

$$\left(\frac{dP}{dz}\right)_{TP} = 4\frac{f_0}{d}\frac{G^2}{2\rho_l}F_M \tag{16}$$

The Kinematic Correlation for Void Fraction. To solve the three conservation equations presented in the previous sections, we need to find relationships between the phase velocities in terms of the volumetric flow rates and void fraction. A volumetric flux for the twophase mixture is defined as

$$j = u_l(1 - \Psi) + u_v \Psi \tag{17}$$

Here,  $\Psi$  is the local void fraction. Note that the volumetric flux is the weighted sum of the liquid and vapor contributions. Equating the mass flux of the liquid at any section to the liquid fraction of the total mass velocity, we get,

$$\rho_l u_l (1 - \Psi) = G(1 - x)$$
(18)

Similarly for the vapor mass flux, we can write,

$$\rho_n u_n \Psi = G x \tag{19}$$

Thus the average volumetric flux, j, can be expressed in terms of the mass velocity, G, as

$$j = \frac{G(1-x)}{\rho_l} + \frac{Gx}{\rho_v}$$
(20)

According to the Zuber-Findlay (1965) model, the vapor velocity,  $u_v$ , may be related to the volumetric flux as,

$$u_v = C_0 j + u_{vj} \tag{21}$$

Where  $C_0$  is the distribution parameter and  $u_{vj}$  is the drift velocity of the vapor phase with respect to the center of mass of the mixture. In the literature, there are

various correlations for  $C_0$  and  $u_{vj}$  depending primarily on the regime of the two-phase. For well mixed flows, the distribution parameter  $C_0$  is near unity. For stratified flow conditions, like annular flow it is near 1.5. In this study,  $C_0$  is chosen to be 1.2. The following expressions used in the present study are reported to give good results for steam-water flows irrespective of the flow pattern (Zuber and Findlay 1965).

$$C_0 = 1.2$$
 (22)

$$u_{\nu j} = 1.2 \left[ \frac{\sigma g(\rho_l - \rho_{\nu})}{\rho_l^2} \right]^{1/4}$$
(23)

# TWO-PHASE FLOW CHARACTERISTICS - SOLUTION PROCEDURE

The study of two-phase dynamic instabilities, in general, requires the knowledge of the steady-state pressure-drop versus mass flow rate characteristics, over the range of interest. The stability boundaries for pressure-drop and density-wave type oscillations are usually shown on the plot of these relationships. These relationships, which are the steady-state solutions to the conservation equations, are also used to determine the initial conditions for both types of oscillations. Therefore, initially solutions are obtained for various heat inputs and/or inlet subcoolings under steady-state conditions. The heater inlet temperature is taken to be 24°C.

### **Steady-State Characteristics**

Under steady-state conditions, the time-dependent terms in the governing Eqs. (1), (2), and (3) drop out and the following equations are obtained:

Continuity equation:

$$\frac{\partial}{\partial z} [\rho_l u_l (1 - \Psi) + \rho_v u_v \Psi] = 0$$
⁽²⁴⁾

Energy equation:

$$\frac{q'}{A} = \frac{\partial}{\partial z} \left[ \rho_l u_l h_l (1 - \Psi) + \rho_v u_v h_v \Psi \right]$$
(25)

Momentum equation:

$$-\frac{\partial P}{\partial z} = \frac{\partial}{\partial z} \left[ G^2 \left[ \frac{(1-x)^2}{\rho_l (1-\Psi)} + \frac{x^2}{\rho_v \Psi} \right] \right] + \left( \frac{\partial P}{\partial z} \right)_{fric} + g [\rho_l (1-\Psi) + \rho_v \Psi]$$
(26)

Finite-Difference Equations. Region-wise The momentum and energy equations are to be integrated over the system. The experimental system is described in figure 1. Due to the pressure dependence of the properties, it is necessary to numerically perform the integrations. All fluid properties are expressed in terms of pressure. In writing the finite-difference equations, five distinct regions are identified along the experimental system, having different each

characteristics (see Figure 2). For each of these regions, finite-difference formulations of the flow are made.

*Exit restriction:* Two-phase flow is observed at this section. The exit restriction is a sharp-edged orifice of diameter 2.64 mm. An empirical correlation, based on experimental data, is used to calculate the two-phase pressure-drop across the restriction.

$$\Delta P_e = \Delta P_{SP} F_M \tag{27}$$

where  $\Delta P_{SP}$  is the single-phase pressure drop across the orifice plate which is experimentally determined as

$$\Delta P_{SP} = 175 \frac{G^2}{\rho_l} \tag{28}$$

The two-phase multiplier,  $F_M$ , has been experimentally determined as

$$F_M = 1 + 28.73 x_e - 6.68 x_e^2 + 25.518 x_e^3 \quad (29)$$

*Boundary Conditions.* The conservation equations, together with the equations of state and the constitutive relations, are to be solved for the following boundary conditions:

- Constant inlet temperature,  $T_i = constant$ ,
- Constant heat input,  $\phi = \text{constant}$ , and
- Constant exit pressure,  $P_e = constant$

Scheme of Solution. Calculations start with given values of mass velocity, G, inlet temperature,  $T_i$ , heat input,  $\phi$ , and exit pressure, Pe. Assuming an inlet (surge tank) pressure, P_s, the flow parameters and properties are calculated from the exit of the surge tank to the inlet of the heater. The enthalpy, pressure, and density are calculated at each successive node in the heater. At each step, the enthalpy is checked against the saturated enthalpy at that pressure. Boiling is assumed to start when the fluid enthalpy exceeds the saturation liquid enthalpy. Appropriate state and constitutive equations are chosen according to the state of the fluid. The calculation is continued along the rest of the experimental system. The calculated exit pressure is checked against the given value of Pe. If the difference is found to be within an acceptable margin, the assumed value of surge tank pressure Ps is substituted for the surge tank pressure; otherwise the whole procedure is repeated with a different value of Ps, until convergence is obtained.



Figure 2. Schematic diagram of the model for finite-difference analysis; Horizontal flow channel.

### TIME-DEPENDENT SOLUTIONS – PRESSURE-DROP TYPE OSCILLATIONS

Using the method and results developed in the previous sections, the time dependent behavior of the system under consideration is predicted for pressure-drop type oscillations.

Model for Pressure-Drop Type Oscillations. Pressuredrop type oscillations that occur in two-phase flow systems are triggered by a small instability in the negative slope region of the steady state characteristics curve. The surge tank is an important dynamic component of the system that serves as an "external compressible volume". These oscillations have relatively low frequencies, and their periods are usually much larger than the residence time of single fluid particle in the system. For the surge tank, the continuity equation can be obtained as

$$\frac{\mathrm{d}P_s}{\mathrm{dt}} = P_s^2 \frac{(G_l - G_o)A_p}{P_{so}V_o\rho_l} \tag{30}$$

The momentum equation for the mass velocity,  $G_i$ , between the main tank and the surge tank is written as:

$$G_{i} = \left[\frac{(P_{I} - P_{S})\rho_{l}}{\kappa_{l}}\right]^{1/2}$$
(31)

where  $K_I$  is the inlet restriction coefficient of the valve between the main and the surge tanks and  $P_I$  is the pressure in the main tank. The above expression has been experimentally determined as

$$G_{i} = \left[\frac{(P_{I} - P_{s})\rho_{l}}{328}\right]^{1/2}$$
(32)

The steady state solution, which is assumed to be valid for the section of the system after the surge tank, can be expressed as:

$$G_{o} = G_{o}(P_{s}, q')$$
(33)

The above equation is incorporated in a computer program as an "inverse procedure" numerical program in the solution. Thus, these equations given above form the basis of predicting the pressure-drop type oscillations. It is assumed that thermodynamic equilibrium exist between the vapor and liquid phases of the working fluid, the working fluid vapor and air in the surge tank constitute an ideal gas mixture, and the surge tank pressure stays constant.

The derivatives of the variables with respect to time are approximated by forward differences. The numerical scheme is stable for (Padki 1990 and Kakaç et al. 1990)

$$\Delta t^{j+1} \le \frac{\Delta z}{Max \left| u_i^j \right|} \tag{34}$$

*Step of the solution.* The calculations start with given fluid temperature and pressure. The initial flow parameters and properties, corresponding to the given inlet mass velocity and heat input, are calculated using the steady-state program. These results are saved as the initial conditions at the stable operating point.

The system is perturbed by increasing the pressure  $P_s$  in the surge tank. The inlet mass velocity for the surge tank is calculated using Eq. (30). Calling the steadystate program again, the mass velocity and other flow parameters and properties along the system are computed using Eq. (31), corresponding to the increased surge tank pressure. Since the pressure in the surge tank is increased, the flow rate through the heater decreases. The heat transfer coefficient and the heat input into the fluid also change. Using Eq. (30), the pressure  $P_s$  in the surge tank is calculated. As the exit mass velocity G₀ from the surge tank decreases more than the mass velocity G_i into the surge tank, P_s increases further during the time step  $\Delta t$ . This procedure is repeated in the successive time steps. In the successive time steps, the pressure difference between the surge tank and the system exit increases along the negative slope, until the top of one of the steady-state mass flow rate versus

pressure-drop curves is reached, depending on the instantaneous heat input into the fluid. Then the calculation point leaps from the top of the curve to the liquid region automatically. The flow rate through the heater increases. The surge tank pressure decreases with increasing flow rate through the heater, since the exit flow rate is larger than the inlet flow rate. The pressure in the surge tank decreases till the bottom of one of the steady-state mass flow rate versus pressure-drop curves. Then another flow excursion takes place from the liquid region to two-phase or vapor region. Theses limit cycles are then repeated. In the following chapter, the results obtained from the modeling are presented along with the obtained experimental data. A comparison of the results of the drift-flux and homogeneous flow models is also presented.

# TIME-DEPENDENT SOLUTIONS – THERMAL OSCILLATIONS

During the pressure-drop oscillations, the mass flow rate, heat transfer coefficient, and heat input into the fluid keep changing. However the heat generated in the heater wall is constant. Therefore, when the limit cycle enters the liquid region, the wall temperature decreasing as the liquid heat transfer coefficient is usually high; whereas when the limit cycle enters the vapor region, the wall temperature increases. Thus the wall temperature fluctuates during the limit cycle. These are called thermal oscillations (Kakaç et al. 1990), (Kakaç and Bon 2007). Note that the thermal oscillations are a result of the pressure drop oscillations.

*Model for Thermal Oscillations.* The rate of heat transfer into the fluid is given by:

$$q_f = \int_{A_h} \alpha \big( T_w - T_f \big) dA_h \tag{35}$$

The heat transfer coefficient,  $\alpha$ , in this equation is a local, instantaneous value. There are no studies available in the literature which can predict the local, instantaneous heat transfer coefficients under oscillatory conditions. Therefore, a correlation which was developed in one of the earlier studies is utilized in this work. This correlation is as follow (Kakaç and Bon 2007):

$$Nu = 190C_s \left(\frac{P}{P_{crit}}\right)^{0.25} \left(\frac{q'd_e}{h_{lv}\mu_l}\right)^{0.7} e^{-0.125x}$$
(36)

where  $Nu(=\alpha d/k_l)$  is the Nusselt number.

The heater wall temperature change can be calculated from the energy balance for the heater, yielding,

$$\frac{\mathrm{d}(T_{w})}{\mathrm{dt}} = \frac{(q'-q_f)}{m_h c_h} \tag{37}$$

The solutions of these equations yield the thermal oscillation at any node of the heater.

Method of Solution. In the pressure-drop oscillation model, fluid parameters and properties are calculated along the system during the oscillations. The fluid temperature inside the heater at any node is known. The heat transfer coefficient is calculated using Eq. (36). The heat input into the fluid is assumed to start with as the rate of electrical heat generation in the tube. Then the heater wall temperature can be calculated. During the oscillations, the heat transfer coefficient and the heat input change, and the heater wall temperature changes accordingly. The governing equations are written in forward finite difference form. The solution of these yield the thermal oscillations at any node of the heater. Details of the experimental investigation and the numerical relation can be found in (Ishii 1977, Kakaç et al. 1977).

# COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Two different models were used to simplify the governing equations of two-phase flow. Results from the homogeneous and drift-flux models were each obtained for two sets of parameters. The first set of results were obtained for heat inputs ranging from 0 - 2500 W and exit restriction 2.64 mm. The second set of results were for the same range of heat inputs but with exit restriction diameter of 3.175 mm. The fluid inlet temperature was assumed to be 24°C in all the cases. We shall now examine the results from the theoretical study and experimental work performed on horizontal flow systems.

### **Time-Dependent Results**



**Figure 3.** Pressure-drop oscillations in horizontal two-phase flow; experimental results. The exit restriction diameter is 3.175 mm and the tube diameter is 10.90 mm. The heat input to the fluid is 2500 W. Working Fluid: R-11, Mass Flow Rate: 0.0717 kg/s.

Figures 3, 4, and 5 and Table 1 and 2 show the results of the model predictions and experimental findings of the pressure-drop and thermal oscillations for a horizontal flow system. The rising portion of the pressure-drop oscillations corresponds to an increasing vapor flow that carries away more heat, thus lowering the wall temperature. The decreasing portion corresponds to a decreasing liquid mass flow predicted by the theory. Table 3 summarizes the comparison of the amplitude of the pressure-drop type oscillations between the experimental and theoretical results of the Drift-Flux model. Under the present experimental conditions, high frequency density-wave oscillations also occur, which are superimposed on the pressure-drop oscillations (Fig.3). The present model can predict the pressure-drop oscillations quite well. However, density-wave oscillations cannot be predicted by this model, because it does not take into account explicitly the propagation of continuity waves that generate these oscillations.



**Figure 4.** Thermal oscillations in horizontal two-phase flow; experimental results. The exit restriction diameter is 2.6 mm and the tube diameter is 10.90 mm. The heat input to the fluid is 2000 W. Mass Flow Rate: 0.0717 kg/s.



**Figure 5.** Horizontal two-phase flow steady-state characteristics; Comparison of drift-flux model and experimental results. The exit restriction diameter is 3.175 mm and the tube diameter is 10.90 mm.

### **Steady-State Results**

Figure 5 and Table 1 show the pressure-drop versus mass flow rate results for a constant inlet fluid temperature ( $T_i = 20^{\circ}C$ ), with various electrical heat input rates ( $Q_0 = 0 - 2500$  W). The theoretical predictions obtained by drift-flux model for horizontal two-phase flow system can be seen to be in very close

agreement with the experimental results over the entire range of parameters involved.

**Table 1.** Comparison of results from the homogeneous-flow model and experimental studies; Horizontal single-channel flow; Steady-state results; exit restriction = 2.64 mm; working fluid: R-11: tube diameter ID = 8.34 mm.

Mass	Heat Input	Model	Experimental
Flow	(W)	Pressure-	Pressure-Drop
Rate		Drop	(Bar)
		(Bar)	
0.02	0	0.08	0.1
0.02	1500	1.8	2
0.02	2000	2.8	3
0.02	2500	5.8	6
0.04	0	0.31	0.34
0.04	1500	2.0	2.2
0.04	2000	3.9	4.1
0.04	2500	7.0	7.2
0.06	0	0.73	0.75
0.06	1500	1.8	2
0.06	2000	1.8	2

**Table 2.** Comparison of experimental and theoretical results; Pressure drop type oscillations in a horizontal single-channel flow; tube diameter ID = 8.34 mm; mass flow rate = 0.0717 kg/s.

Exit Restrictio n (mm)	Heat	Expe	rimental Theoretical		oretical
	Inpu	Perio	Amplitud	Perio	Amplitu
	t	d	e	d	de
	(W)	(s)	(Bar)	(s)	(Bar)
2.64	200 0	18	1.5	24	1.9
2.64	250 0	16	2.2	18	2.0
3.175	200 0	13	1.1	12	1.25
3.175	250 0	14	1.2	12	1.4

We observed that at higher mass flow rates the slope of the pressure-drop versus mass flow rate curve is positive. This signifies an all liquid flow for which the obtained curves appear intuitively correct.

**Table 3.** Comparison of results from the Drift-Flux model and experimental studies. Steady-state characteristics horizontal single-channel, tube diameter ID = 8.34 mm; OD = 10.6 mm; exit restriction = 3.175 mm.

Mass Flow Rate (kg/s)	Heat Input (W)	Experimental Amplitude Pressure-Drop (Bar)	Theoretical Amplitude Pressure-Drop (Bar)
0.02	0	0.12	0.14
0.02	2000	1.70	1.80
0.02	2500	1.70	1.80
0.06	0	0.90	1.10
0.06	2000	4.30	4.50
0.08	0	1.30	1.40
0.08	2000	3.60	3.70
0.08	2500	4.80	5.00

At a certain mass flow rate the curve gets a negative slope, indicating the start of two-phase flows. Since a lower mass flow rate signifies a greater residence time in the test section for fluid particle, the flow absorbs greater heat. Thus, some of it gets vaporized. The pressure-drop multiplier for two-phase flow which is function of dryness fraction causes the curve to take a negative slope. This is simply because vapor undergoes more pressure-drop in flow than a corresponding mass flow rate of liquid. Notice that the onset of boiling is shifted to the left as heat input decreases. This is to be expected as it takes more time for a lower heat input rate to cause boiling at a given flow rate. The obtained results agree quite well with the model, especially at lower heat input rates. In conclusion we observe that pressure-drop values were higher for the lesser diameter This observation is supported restriction. by observations in fluid mechanics of single-phase flows, greater the restriction, greater the frictional pressure loss. The details of the modeling can be found in Venkataraman (1993).

### CONCLUSIONS

Experiments have been performed at a constant inlet temperature for different heat inputs. The following conclusions can be reached based on the experimental and theoretical studies:

- Both the pressure-drop type and thermal oscillations occur at all heat inputs. At a given inlet subcooling, the amplitudes and periods of the oscillations increase with increasing heat input rate.
- Both the pressure-drop type and thermal oscillations occur at all inlet subcoolings. At a given heat input rate, the amplitudes and periods of the oscillations increase with increasing inlet subcooling.
- Thermal oscillations accompany the pressuredrop type oscillations. Oscillations of pressure and temperature are in phase; but the maximum of temperature oscillations always lag the maximum of pressure oscillations.
- The periods and amplitudes of the pressuredrop type oscillations increase with decreasing mass flow rate at the initial operating point on the negative slope.
- The steady-state characteristics and the oscillations predicted with the use of the drift-flux model (Fig. 5) are in reasonably good agreement with experimental results.

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