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ANALYTICAL CALCULATION OF WET COOLING TOWER PERFORMANCE WITH LARGE COOLING RANGES

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Abstract: Cooling tower performance calculations are usually performed numerically. In this paper, a simple differential equation for counter flow wet cooling tower is solved analytically taking into consideration the non-linear dependency of the saturated air enthalpy on temperature. The method allows analytical calculation of cooling tower performance with large cooling ranges. The analytically obtained values are compared with the well-known logarithmic mean enthalpy method (LMED) and corrected LMED method. It is seen that analytically obtained values are much more accurate than the values obtained using these two methods. The analytical results are also compared with experimental ones and it is seen that there is a good agreement between them.
Keywords: Cooling tower, Cooling range, Analytical solution, Numerical calculation.

YÜKSEK SICAKLIK FARKLARINDA ISLAK SOĞUTMA KULELERİ ETKİNLİĞİNİN ANALİTİK OLARAK HESABI

Özet: Soğutma kulesi hesaplamaları genellikle sayısal olarak yapılmaktadır. Bu makalede, karşıt akışlı ıslak soğutma kulesi için basit bir diferansiyel denklem doymuş hava antalpisinin sıcaklığa lineer olmayan bağımlılığı göz önüne alınarak analitik olarak çözülmüştür. Bu metod soğutma kulesi etkinliğinin büyük sıcaklık farklarında analitik olarak hesaplanmasına olanak vermektedir. Analitik olarak hesaplanan değerler çok iyi bilinen logaritmik ortalama antalpi farkı (LMED) ve düzeltilmiş-LMED metodlarıyla karşılaştırılmıştır. Bu çalışmada analitik olarak elde edilen değerlerin bu iki metodla elde edilen değerlere göre çok daha iyi neticeler verdiği görülmüştür. Analitik olarak hesaplanan değerler ayrıca deneysel değerlerle karşılaştırılmış ve neticelerin birbirleriyle uyum içinde olduğu görülmüştür.
Anahtar kelimeler: Soğutma kulesi, Sıcaklık farkı, Analitik çözüm, Sayısal hesaplama.

Nomenclature

A	cross-sectional area of cooling tower [m^2]
a	constant in eq. (26)
c_p	specific heat [J/kgK]
D	constant in eqs. (30) and (33)
DS	determinant given by eq. (28)
d	diameter [m]
F	wetted area [m^2]
H	height of the packing in cooling tower [m]
h	enthalpy [kJ/kg]
I	integral given by eq. (19)
$\dot{M}$	mass flow rate [kg/s]
Me	Merkel number defined in eq. (62)
N	number of transfer unit, defined by eq. (9)
$\dot{Q}$	heat transferred, [kW]
T	temperature [K]
x	absolute humidity [kg H_2O/kg dry air]
z	coordinate in vertical direction (bottom : $z=0$)

Greek letters

α	convective heat transfer coefficient [W/m^{2}K]
β	convective mass transfer coefficient [m/s]
ε	efficiency of cooling tower, defined by eq. (24)
ρ	density [kg/m^3]
θ	dimensionless temperature, defined by eq. (8)

Subscripts

a	dry air
a,w	air at air-water surface
e	equivalent
i	inlet
l	latent
m	mean
max	maximum
min	minimum
o	outlet, reference, initial
s	sensible
T	total
w	water, at air-water surface
$1/2$	at $\theta = 0.5$

Superscripts

*	dimensionless

INTRODUCTION

Cooling towers are commonly used devices for heat rejection into ambient air in many industrial applications such as condensers of refrigeration machines, power generation plants and the textile industry.

Cooling tower theory was first given by Merkel, (1925). Effectiveness-NTU and logarithmic mean enthalpy methods were described by Jaber and Webb, (1989). They took the nonlinearity of the saturated air enthalpy versus temperature into consideration with a correction factor. Kloppers and Kröger, (2005) analyzed the derivation of heat and mass transfer equations in counter flow wet cooling towers in detail. They described Merkel, NTU and Poppe methods and concluded that Poppe method yields higher Merkel numbers. El-Dessouky et al. (1997) concluded that the effect of water evaporation on the cooling tower performance is not conservative but it can be as low as 1.3 % so that the assumption of constant water flow rate is justified.

Mohiuddin and Kant, (1996) explained different numerical methods for the analysis of wet cooling towers. Khan et al. (2003) showed through numerical analysis that most of the heat transfer occurs by evaporation. The ratio of heat transfer by evaporation to total heat transfer was 90% at the top and 62.5% at the bottom of cooling tower packing.

Assuming linear dependency of saturation enthalpy of air on temperature, Halasz (1999) showed that cooling tower efficiency depends on two dimensionless numbers. However, the results were corrected with a coefficient of linearization. It can be concluded from his results that his method is applicable only if the cooling range is less than 10 ^{o}C. Bedekar et al. (1998) presented experimentally-obtained results on cooling tower performance for different inlet water temperatures and different water flow rates. They showed that tower characteristics and tower efficiencies are influenced by water inlet temperature. Experimental and numerical results for different filling materials in pilot-scale and industrial cooling towers were given by Milosavljevic and Heikkilae (2001). Cooling towers were extensively described by Berliner (1975) and Kröger (2004).

In this work, the Merkel equation for counter flow wet cooling tower is solved analytically. The non-linearity of the saturated air enthalpy on the temperature is taken into consideration. The analytically derived equations allow for the description of cooling towers with large cooling ranges. With the presented method, water temperature and air enthalpy along the cooling tower packing can also be calculated analytically. The analytically and numerically obtained results are compared with each other. The analytical results are also compared with the experimental results given by Milosavljevic and Heikkilae (2001).

DERIVATION OF GOVERNING EQUATIONS

In the present analysis, countercurrent flow of air and water to be cooled in vertical direction in a cooling tower is considered. The wetted packing surface area is F and the wetted surface area in the height of dz is dF. The total amount of heat transferred in dz is $d\dot{Q}$.

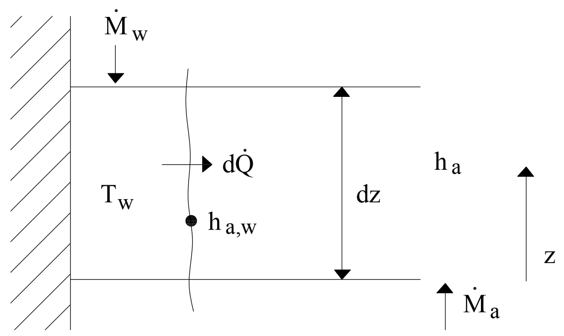

Figure 1. Water-air counter flow in cooling tower

The amount of heat transferred in the height of dz (figure 1) can be written for water and air, respectively, as:

$$d\dot{Q} = \dot{M}_w \, c_{p_w} \, dT_w \tag{1}$$

$$d\dot{Q} = \dot{M}_a \, dh_a \tag{2}$$

$\dot{M}_w$ and $\dot{M}_a$ are mass flow rates of water and dry air, respectively. Water mass flow rate ($\dot{M}_w$) is assumed to be constant, that means evaporated water mass flow rate is very low compared to the water mass flow rate. T_w is the water temperature and h_a is the enthalpy of air. $\dot{Q}$ consists of two parts, namely sensible ($\dot{Q}_s$) and latent ($\dot{Q}_l$) heats. For the length dz, one can write;

$$d\dot{Q} = d\dot{Q}_s + d\dot{Q}_l \tag{3}$$

where $d\dot{Q}_s$ and $d\dot{Q}_l$ can be calculated by considering heat and mass transfer between water and air:

$$d\dot{Q}_s = \alpha_a \left(T_w - T_a\right) dF \tag{4}$$

$$d\dot{Q}_l = \beta_a \left(\rho_{w,w} - \rho_{w,a}\right) h_{gl} \, dF \tag{5}$$

$\rho_{w,a}$ and $\rho_{w,w}$ are partial densities of water vapor in the air and at the water-air interface, respectively. h_{gl} is latent heat of evaporation. α_a and β_a are convective heat and mass transfer coefficients between the water-air interface and air (air side), respectively.
If one assumes further that Lewis number is unity, one obtains the following well-known Merkel equation (Berliner 1975 and Kröger 2004):

$$\frac{\alpha_a}{c_{p_a}} \frac{F}{\dot{M}_w \, c_{pw}} = \int_{T_{wo}}^{T_w} \frac{dT_w}{h_{a,w} - h_a} \tag{6}$$

The amount of heat transferred from water is given as;

$$\dot{Q} = \dot{M}_w \, c_{p_w} \left(T_{wi} - T_{wo}\right) \tag{7}$$

Dimensionless temperature θ, number of transfer units N and equivalent temperature difference $\Delta T_{aw,e}$ are defined as follows:

$$\theta = \frac{T_w - T_{wo}}{T_{wi} - T_{wo}} \tag{8}$$

$$N = \frac{\alpha_a \, F_T \, \Delta T_{aw,e}}{\dot{Q}} \tag{9}$$

$$\Delta T_{aw,e} = \frac{h_{a,wi} - h_{a,wo}}{c_{p,a}} \tag{10}$$

The following equation can be obtained from eq. (6) using the dimensionless quantities:

$$N \frac{F}{F_T} = \int_0^{\theta} \frac{\left(h_{a,wi} - h_{a,wo}\right) d\theta}{h_{a,w} - h_a} \tag{11}$$

Here F_T and F are total wetted surface and wetted surface between 0 and z in the cooling tower, respectively. With the definitions of,

$$h^*_{a,w} = \frac{h_{a,w} - h_{a,wo}}{h_{a,wi} - h_{a,wo}} \tag{12}$$

$$h^*_a = \frac{h_a - h_{a,i}}{h_{a,wi} - h_{a,i}} \tag{13}$$

$$h^*_{a,wo} = \frac{h_{a,wo} - h_{a,i}}{h_{a,wi} - h_{a,wo}} \tag{14}$$

$$z^* = \frac{F}{F_T} \tag{15}$$

the following equation is obtained from eq. (11):

$$N \, z^* = \int_0^{\theta} \frac{d\theta}{h^*_{a,w} - \left(1 + h^*_{a,wo}\right) h^*_a + h^*_{a,wo}} \tag{16}$$

Defining I_θ as

$$I_\theta = \int_0^{\theta} \frac{d\theta}{h^*_{a,w} - \left(1 + h^*_{a,wo}\right) h^*_a + h^*_{a,wo}} \tag{17}$$

it follows from eq. (16)

$$I_\theta = N \, z^* \tag{18}$$

For the total cooling tower, one gets from eqs. (17) and (18)

$$I = \int_0^{1} \frac{d\theta}{h^*_{a,w} - \left(1 + h^*_{a,wo}\right) h^*_a + h^*_{a,wo}} \tag{19}$$

$$I = N \tag{20}$$

In the above equations water inlet and outlet are denoted by indices wi and wo, respectively. $h_{a,i}$ is the enthalpy of air entering the cooling tower.

Description of enthalpies dependent on temperature

$h^*_{a,w}$ and h^*_a should be defined as a function of θ for the determination of the I_θ values from eq. (17).

From eqs. (1) and (2), one gets

$$\frac{dh_a}{dT_w} = \frac{\dot{M}_w \, c_{pw}}{\dot{M}_a} \tag{21}$$

After integration, it follows

$$h_a = h_{a,i} + \frac{\dot{M}_w \, c_{pw}}{\dot{M}_a} \left(T_w - T_{w,o}\right) \tag{22}$$

This equation can be rewritten in the dimensionless form as:

$$h^*_a = \varepsilon \, \theta \tag{23}$$

where

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} \tag{24}$$

$$\dot{Q}_{\max} = \dot{M}_a \left(h_{a,wi} - h_{a,i}\right) \tag{25}$$

Here ε and $\dot{Q}_{\max}$ are the efficiency of the cooling tower and the maximum possible heat flow, respectively.

For different inlet and outlet temperatures, $h^*_{a,w}$ can be determined as a function of θ as follows:

$$h^*_{a,w} = (1-a)\theta^2 + a\,\theta \tag{26}$$

This function fulfills the boundary conditions $h^*_{a,w} = 0$ at $\theta = 0$ and $h^*_{a,w} = 1$ at $\theta = 1$. Eq. (26) is plotted in figure 2. For *a*=1, the relation is linear. For *a*=0, a parabolic (quadratic) relation is obtained. Normally, *a* will have values between these limiting values.

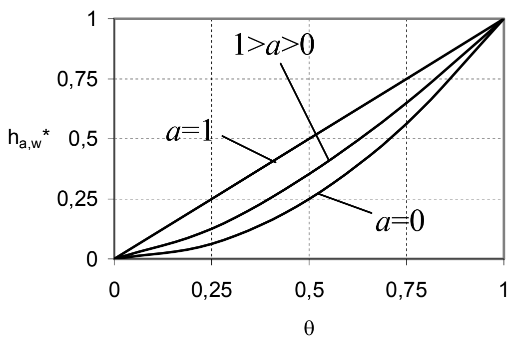

Figure 2. Demonstration of $h^*_{a,w}$ as a function of θ.

ANALYTICAL SOLUTION

Using eqs. (23) and (26), it follows from eq. (17) that

$$I_\theta = \int_0^\theta \frac{d\theta}{(1-a)\theta^2 - \left[\varepsilon\left(1+h^*_{a,wo}\right)-a\right]\theta + h^*_{a,wo}} \quad (27)$$

This integral can be solved analytically.
Defining DS as follows,

$$DS = 4(1-a)h^*_{a,wo} - \left[\varepsilon\left(1+h^*_{a,wo}\right)-a\right]^2 \quad (28)$$

the following solution of eq. (27) is obtained for DS>0;

$$I_\theta = \frac{2}{D} arctg\left(\frac{D\theta}{2h^*_{a,wo} - \left[\varepsilon\left(1+h^*_{a,wo}\right)-a\right]\theta}\right) \quad (29)$$

Where

$$D = \sqrt{DS} \quad (30)$$

The value of *I* follows from eq. (29) with $\theta = 1$:

$$I = \frac{2}{D} arctg\left(\frac{D}{2h^*_{a,wo} - \left[\varepsilon\left(1+h^*_{a,wo}\right)-a\right]}\right) \quad (31)$$

From the above equations, one can see that I and therefore N are dependent on $h^*_{a,wo}$ and ε. The maximum allowable values of $\varepsilon_{\max}$ can be calculated if D=0. For this case, one gets then from eq. (28)

$$\varepsilon_{\max} = \frac{a + 2\sqrt{(1-a)h^*_{a,wo}}}{1+h^*_{a,wo}} \quad (32)$$

If DS<0, it would mean that $\varepsilon > \varepsilon_{\max}$ and this is not realistic.

The minimum possible air mass flow rate $\dot{M}_{a,\min}$ can be determined from eqs. (24) and (25).

$$\dot{M}_{a,\min} = \frac{\dot{Q}}{\varepsilon_{\max}\left(h_{a,wi} - h_{a,i}\right)} \quad (33)$$

From eqs. (18) and (20), the dependency of θ on z^* can be written:

$$z^* = \frac{I_\theta}{I} \quad (34)$$

Determination of a in equation (26)

One can use the values of $h^*_{a,w,1/2}$ at $\theta = 0.5$ from eq. (26)

$$a = 4h^*_{a,w,1/2} - 1 \quad (35)$$

Another way of determination of a is that both integration values of the enthalpy $h_{a,w}$ between $h_{a,wi}$ and $h_{a,wo}$ should be equal to the value obtained using eq. (26). Then the following equation can be written for a:

$$\int_0^1 h^*_{a,w}\, d\theta = \int_0^1 \left[(1-a)\theta^2 + a\theta\right] d\theta \quad (36)$$

From this equation, one obtains then

$$a = 6\int_0^1 h^*_{a,w}\, d\theta - 2 \quad (37)$$

Determination of a for an exponential function

The enthalpy of saturated air between 10-90 °C taken from ASHRAE [12] is given in figure 3. $h_{a,w}$ can be described with the function

$$h_{a,w} = b\exp(cT) \quad (38)$$

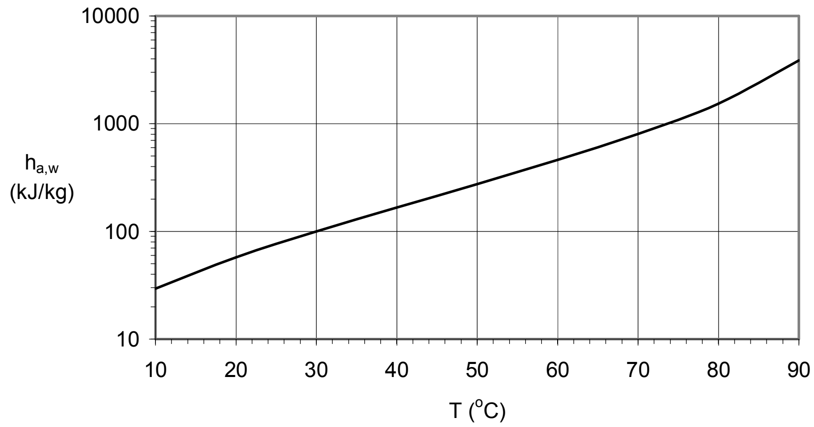

Figure 3. Saturated air enthalpy dependent on temperature.

b and c values are given in table 1. It can be seen from this table that, one can determine b and c with R^2>0.9994 between 20-70 °C.

Therefore, eq. (39) can be written:

$$h_{a,w} = h_{a,wo}\left(\frac{h_{a,wi}}{h_{a,wo}}\right)^{\theta} \quad (39)$$

Logarithms of the functions (38) and (39) are linear functions of T and θ, respectively. Therefore, they are similar functions. Besides, eq. (39) fulfills the limiting conditions for $\theta = 0$ and $\theta = 1$.
This equation can be rewritten in dimensionless form using eq. (12) as

$$h^*_{a,w} = \frac{1}{h^*_{io} - 1}\left(h^{*\,\theta}_{io} - 1\right) \quad (40)$$

where

$$h^*_{io} = \frac{h_{a,wi}}{h_{a,wo}} \quad (41)$$

Table 1. Determination of b and c in eq. (38) for temperatures between 10-90 °C.

T_o-T_i (°C)	b	c	R^2
10-70	18.678	0.0539	0.9981
10-90	16.564	0.0576	0.9932
20-80	19.366	0.0537	0.9985
30-90	15.168	0.0589	0.9876
20-70	20.501	0.0522	0.9997
30-50	21.923	0.0506	1.0000
30-60	21.750	0.0509	1.0000
30-70	20.907	0.0518	0.9996
40-50	22.385	0.0502	1.0000
40-60	21.756	0.0509	0.9999
40-70	20.299	0.0523	0.9994

If $h^*_{a,w,1/2}$ is calculated from eq. (40) and this value is inserted in eq. (35), the following equation for a is obtained:

$$a = 4\frac{h_{aw,g} - h_{a,wo}}{h_{a,wi} - h_{a,wo}} - 1 \quad (42)$$

where $h_{aw,g}$ is the geometric mean enthalpy

$$h_{aw,g} = \sqrt{h_{a,wi}\, h_{a,wo}} \quad (43)$$

Inserting eq. (40) in eq. (37), the following equation for a is yielded:

$$a = 6\frac{h_{a,wl} - h_{a,wo}}{h_{a,w} - h_{a,wo}} - 2 \quad (44)$$

Here, $h_{a,wl}$ is the logarithmic mean enthalpy of saturated air between inlet and outlet saturated air enthalpies

$$h_{a,wl} = \frac{h_{a,wi} - h_{a,wo}}{\ln\left(\frac{h_{a,wi}}{h_{a,wo}}\right)} \quad (45)$$

It is clear that a must have no negative values for the validity of the calculations. For a parabolic function, if the Simpson integral method is applied to eq. (37), eq. (35) would be obtained again. Therefore, eq. (35) will be applied for the calculations if saturation enthalpy of air ($h_{a,w}$) can not be described by eq. (38) in the given cooling range.

RESULTS AND DISCUSSION

Comparison between analytical and numerical results

Numerical values of I obtained according to eq. (19) are calculated using Simpson Integral Method. Comparison between analytical and numerical methods are presented in tables 2 and 3 for the water inlet and outlet temperatures between 10-90 °C and for different cooling ranges between 4 and 16 °C. For each water inlet temperature and cooling range two different air mass flow rates are chosen. High and low ε values are selected for comparison in tables 2 and 3, respectively.
In these tables, the difference between the analytically (eq. 31) and numerically obtained I values (eq. (19)) is named as error Er:

$$Er = \frac{I_{analytical} - I_{numerical}}{I_{numerical}} 100\,[\%] \quad (46)$$

Depending on the determination of a according to eqs. (35), (42) and (44), Er is designated as Er1/2, ErG and ErL, respectively.
It can be seen from these tables that eq. (35) (Er1/2) always yields better results. However for temperatures between 20-70 °C, logarithmic and geometric mean values used in eqs. (42) and (44) yield as good values as obtained using eq. (35). Geometric mean values yield slightly better results than logarithmic mean values. However using geometric mean values one always has an error of less than -4.54 % and +1.73 % even for very high $\varepsilon / \varepsilon_{\max}$ values till 0.9 and cooling ranges between 0-16 °C.

Table 2. Errors between numerically (exact) and analytically obtained values for high ε values.

$T_o - T_i$ (°C)	$h_{a,o}$ (kJ/kg)	$\dot{Q}$ (kW)	$\dot{M}_a$ (kg/s)	$\varepsilon_{\max}$	ε	ErG (%)	ErL (%)	$Er1/2$ (%)
10-14	11.15	500	20.0	1.000	0.886	0.71	0.71	-0.69
10-18	11.15	500	15.0	1.000	0.836	4.3	4.3	-0.08
10-26	11.15	500	9.0	0.991	0.798	14.6	14.8	-0.23
20-24	36.43	500	16.0	1.000	0.869	0.40	0.40	-0.29
20-28	36.43	500	11.0	1.000	0.849	2.20	2.21	-0.11
20-36	36.43	500	7.0	0.986	0.716	5.80	5.90	0.13
30-34	75.86	500	12.0	1.000	0.884	0.12	0.12	-0.23
30-38	75.86	500	8.0	1.000	0.835	0.94	0.95	-0.11
30-46	75.86	750	7.0	0.969	0.718	2.60	2.70	0.49
40-44	99.38	1,000	11.0	1.000	0.871	-0.41	-0.41	-0.43
40-48	99.38	2,000	15.0	1.000	0.891	-0.20	-0.20	-0.19
40-56	99.38	2,000	9.0	0.997	0.809	-1.7	-1.6	-0.51
50-54	99.38	2,000	10.0	1.000	0.840	-0.40	-0.40	-0.30
50-58	99.38	3,000	11.0	1.000	0.865	-0.59	-0.58	-0.16
50-66	99.38	2,750	7.0	1.000	0.729	-2.70	-2.70	-0.35
60-64	115.8	2,000	5.0	1.000	0.878	-1.10	-1.10	-0.90
60-68	115.8	3,000	6.0	1.000	0.834	-1.10	-1.10	-0.18
60-76	115.8	7,500	10.0	1.000	0.713	-5.50	-5.50	-0.59
70-74	115.8	4,500	6.0	1.000	0.824	-0.60	-0.60	-0.30
70-78	115.8	5,000	6.0	1.000	0.683	-1.60	-1.60	-0.14
70-86	115.8	10,000	6.0	0.990	0.690	-13.60	-13.40	-1.30

Table 3. Errors between numerically (exact) and analytically obtained values for low ε values.

$T_o - T_i$ (°C)	$h_{a,o}$ (kJ/kg)	$\dot{Q}$ (kW)	$\dot{M}_a$ (kg/s)	$\varepsilon_{\max}$	ε	ErG (%)	ErL (%)	$Er1/2$ (%)
10-14	11.15	500	40.0	1.000	0.443	0.74	0.74	0.00
10-18	11.15	500	30.0	1.000	0.418	2.34	2.35	0.01
10-26	11.15	500	18.0	0.991	0.399	6.91	7.00	0.22
20-24	36.43	500	32.0	1.000	0.435	0.37	0.38	0.00
20-28	36.43	500	22.0	1.000	0.425	1.19	1.19	0.02
20-36	36.43	500	14.0	0.986	0.358	3.40	3.50	0.38
30-34	75.86	500	24.0	1.000	0.442	0.23	0.23	0.00
30-38	75.86	500	16.0	1.000	0.418	0.56	0.56	0.04
30-46	75.86	750	14.0	0.969	0.359	1.70	1.80	0.66
40-44	99.38	1,000	22.0	1.000	0.436	0.01	0.01	0.00
40-48	99.38	2,000	30.0	1.000	0.446	-0.04	-0.03	-0.03
40-56	99.38	2,000	18.0	0.997	0.405	-0.14	-0.10	0.38
50-54	99.38	2,000	20.0	1.000	0.420	-0.05	-0.05	0.00
50-58	99.38	3,000	22.0	1.000	0.433	-0.23	-0.23	0.00
50-66	99.38	2,750	14.0	1.000	0.365	-1.10	-1.10	0.25
60-64	115.8	2,000	10.0	1.000	0.439	-0.11	-0.11	0.00
60-68	115.8	3,000	12.0	1.000	0.417	-0.51	-0.50	-0.01
60-76	115.8	7,500	20.0	1.000	0.357	-2.60	-2.50	0.35
70-74	115.8	4,500	12.0	1.000	0.412	-0.20	-0.20	0.00
70-78	115.8	5,000	12.0	1.000	0.342	-0.99	-0.99	0.00
70-86	115.8	10,000	12.0	0.990	0.345	-6.10	-6.00	0.90

$h_{a,w}$ is not an exponential function for T_w<20 °C and T_w>70 °C. For these temperature ranges, the application of eq. (35) yields acceptably good results. The difference between numerical and analytical values is less than -1.3 % and 0.9 % in these cases, respectively. For the validity of the calculations, *a* must have positive values. Therefore, one gets from eqs. (42) and (43),

$$h_{io}^{*\,2} - 10\,h_{io}^{*} + 9 \le 0 \tag{47}$$

$$h_{io}^{*} \le 9 \quad (48$$

Using eqs. (44) and (45), the following equation can be written:

$$h_{io}^{*} = 1 + \frac{\ln h_{io}^{*}}{3}\left(h_{io}^{*} + 2\right) \tag{49}$$

$$h_{io}^{*} \le 8.578 \tag{50}$$

In the range of 20-70 °C, eq. (38) can be used and from this equation, one obtains

$$\Delta T = \ln h_{io}^{*} \,/\, c \tag{51}$$

where ΔT is the cooling tower range:

$$\Delta T = T_{wi} - T_{wo} \tag{52}$$

Using eqs. (48), (50) and (51) with c=0.0522 from table 1, the following results are obtained:

$$\Delta T \le 42.1\ ^{o}C \tag{53}$$

$$\Delta T \le 41.2\ ^{o}C \tag{54}$$

for the validity of the analytical calculations.
Eq. (55) can be obtained from eq. (35) for $a \ge 0$

$$h_{a,w1/2} \,/\, h_{a,wo} \ge \frac{h_{io}^{*} - 1}{4} \tag{55}$$

Using eqs. (38) and (55), one gets

$$\exp(c\,\Delta T) - 4\exp(c\,\Delta T/2) \le 1 \tag{56}$$

$$c\,\Delta T \le 2.887 \tag{57}$$

$$\Delta T \le 54.4\ ^{o}C \tag{58}$$

It can be seen that in the range of 20-70 °C, cooling tower range ΔT can be as high as 42.1 °C for geometric mean and as high as 41.2 °C for logarithmic mean values approach. For mean temperature approach, the cooling range can be as high as 54.4 °C. The errors demonstrated in tables 2 and 3 are in accordance with these ranges.

Comparison between analytical, LMED and Effectiveness-NTU Methods

Using eqs. (1) and (2), it follows from eq. (6)

$$Me = \int_{a,i}^{a,o} \frac{dh_a}{h_{a,w} - h_a} \tag{59}$$

where *Me* can be named as the Merkel number and is defined as follows:

$$Me = \frac{\alpha_a\ F_T}{c_{pa}\ \dot{M}_a} \tag{60}$$

In simple Logarithmic Mean Enthalpy Difference Method (LMED), Me number is calculated as:

$$Me = \frac{h_{a,o} - h_{a,i}}{\Delta h_m} \tag{61}$$

where Δh_m is the logarithmic mean enthalpy difference

$$\Delta h_m = \frac{\left(h_{a,wi} - h_{a,o}\right) - \left(h_{a,wo} - h_{a,i}\right)}{\ln \dfrac{h_{a,wi} - h_{a,o}}{h_{a,wo} - h_{a,i}}} \tag{62}$$

A correction factor δ is introduced for the correction of the nonlinearity of saturated enthalpy of air $h_{a,w}$ as follows:

$$\delta = \left(h_{a,wi} + h_{a,wo} - 2\ h_{a,wm}\right)/\,4 \tag{63}$$

where $h_{a,wm}$ is saturated enthalpy of air at mean water temperature:

$$T_{wm} = \frac{T_{wi} + T_{wo}}{2} \tag{64}$$

The inlet and outlet enthalpies are corrected then as follows:

$$h'_{a,wi} = h_{a,wi} - \delta \tag{65}$$

$$h'_{a,wo} = h_{a,wo} - \delta \tag{66}$$

In LMED-C Method, the corrected values $h'_{a,wi}$ and $h'_{a,wo}$ are used instead of $h_{a,wi}$ and $h_{a,wo}$ for the determination of Δh_m.

LMED and very similar Effectiveness-NTU (E-NTU) Methods are clearly described by Jaber and Webb [2]. Between the Me number and N number defined in eq. (9), there is the following relationship:

$$Me = N\ \frac{\dot{M}_w\ c_{pw}\left(T_{wi} - T_{wo}\right)}{\dot{M}_a\ \left(h_{a,wi} - h_{a,wo}\right)} \tag{67}$$

LMED and E-NTU Methods give nearly the same results [2]. Therefore, only results obtained by LMED method is compared with the analytical and numerical results obtained in this work. Simple LMED method uses only inlet and outlet water temperatures and air inlet enthalpy. This is very similar to eqs. (43) and (45) which also use only the inlet and outlet values. LMED method with correction (LMED-C) uses $h_{a,w1/2}$ ($h_{a,wm}$) values at the arithmetic mean water temperature in addition to $h_{a,wi}$ and $h_{a,wo}$ values. Therefore LMED-C values are similar to the analytically obtained values which uses $h_{a,w1/2}$ in eq. (35).

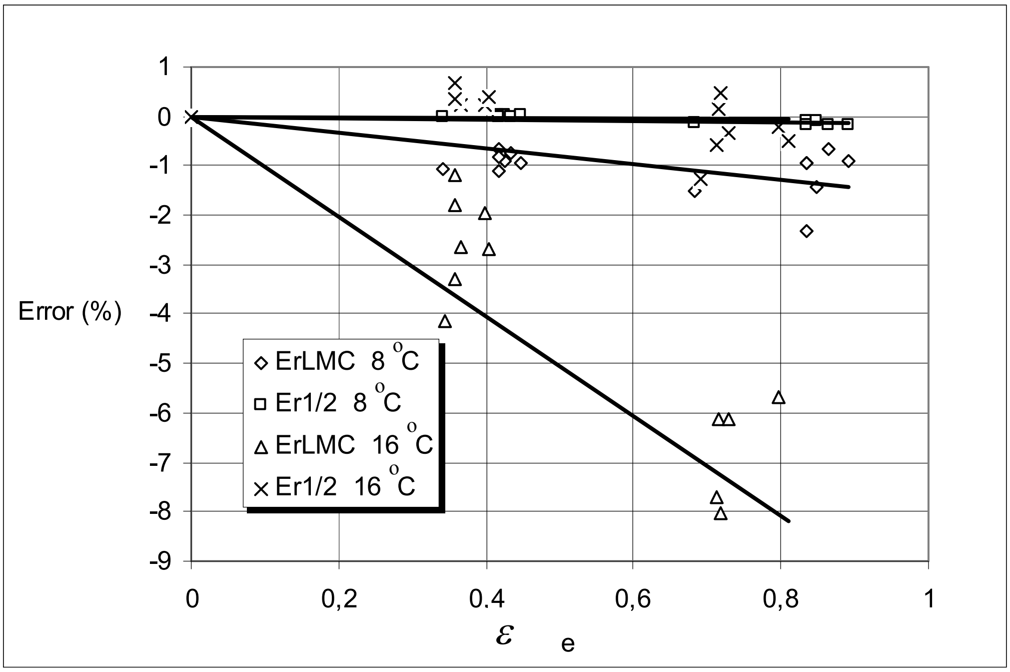

Figure 4. Variation of ErLM and ErG with ε for 8 °C and 16 °C cooling ranges between 20-70 °C water temperatures (20 °C<T_w<70 °C).

The errors for LMED and LMED-C are named as ErLM and ErLMC. In figures 4 and 5, the errors are shown as a function of the efficiency ε for different cooling ranges. ErG and ErLM are compared in figure 4, because both need only inlet and outlet conditions. From the results given in figure 4, one can see that analytical results are always much more accurate than the results obtained using LMED method. Root mean square error (RMSE) of Me numbers determined according to LMED and eq. (43) are 5.64% and 0.96% for 8 °C cooling range and 18.27% and 2.88% for 16 °C cooling range, respectively, between 20 °C – 70 °C water temperatures. LMED-C method and analytical results using eq. (35) both need saturated air enthalpies at the water inlet-outlet temperatures and at arithmetic mean water temperature. Er1/2 and ErLMC are compared in figure 5. The analytical results are much more accurate than the results obtained using LMED-C method as can be seen in figure 5.

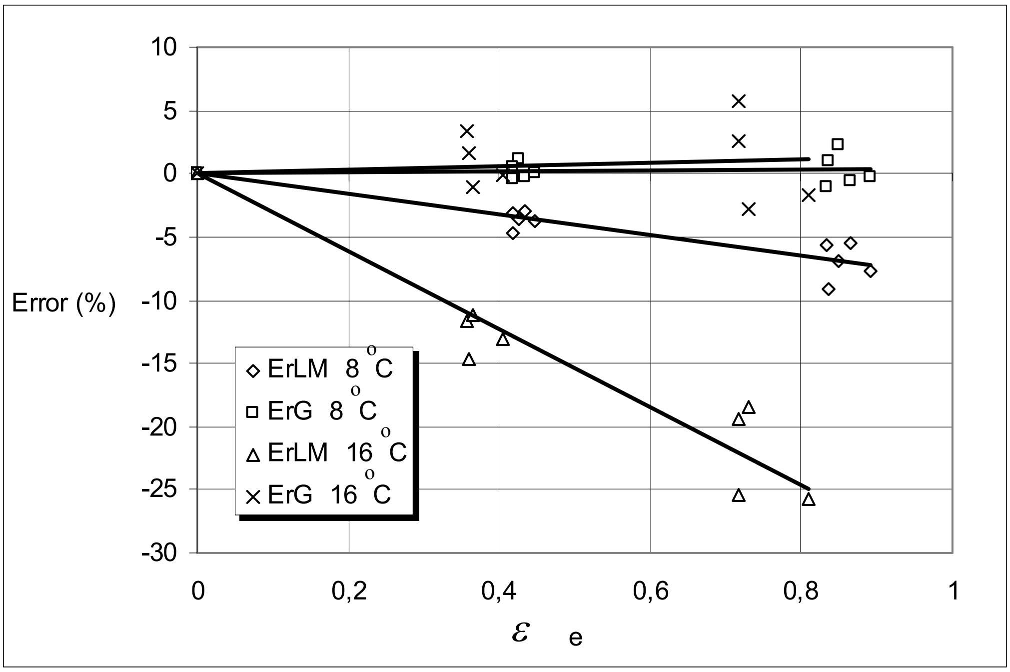

Figure 5. Variation of ErLMC and Er1/2 with ε for 8 °C and 16 °C cooling ranges between 10-86 °C water temperatures (10 °C<T_w<86 °C).

Especially for outlet temperatures over 70 °C and inlet temperatures below 20 °C, LMED-C method gives much bigger differences compared to the numerically obtained values especially at large cooling ranges and at high efficiencies, whereas the analytical method using eq. (35) yields very resonable values. Root mean square error (RMSE) of Me numbers determined according to LMED-C and eq. (35) are 1.15% and 0.10% for 8 °C cooling range and 6.38% and 0.68% for 16 °C cooling range, respectively, between 10 °C – 86 °C water temperatures.

Comparison between analytical and experimental results

Milosavljevic and Heikkilae 2001 carried out measurements at an industrial cooling tower which has 180 m^2 cross-sectional area and 2.4 m height of the packing. They used different packing materials at the top (0.6 m) and bottom (1.8 m) of the cooling tower. Only the experimental results given for the top part are compared with the analytical solution because of the lack of information about the filling material at the bottom part.

In table 4a, the measured (Milosavljevic and Heikkilae 2001) data are given. Some parameters calculated using these values are presented in table 4b. In this table, ($\alpha_a\ F_T$) value is calculated from the measured data at the pilot-scale cooling tower.

Table 4a. Measured data at industrial cooling tower [9].

A	m^2	180
H	m	0.6
T_{wi}	^oC	40
T_{wo}	^oC	29.7
$T_{a,i}$	^oC	25.5
$T_{a,o}$	^oC	30.7
$x_{a,i}$	$kg\ H_2O/kg\ air$	0.0190
$x_{a,o}$	$kg\ H_2O/kg\ air$	0.0290
$\dot{M}_w$	kg/s	450
$\dot{M}_a$	kg/s	650

Table 4b. Calculated data using the measured data.

$\dot{Q}$	kW	20150
$h_{a,wi}$	kJ/kg	166.7
$h_{a,wo}$	kJ/kg	98.5
$h_{a,i}$	kJ/kg	74.0
$h_{a,o}$	kJ/kg	105.0
$\alpha_a\ F_T$	$kW/^oC$	511.2

In table 5, the analytically determined values are given for the conditions given in table 4a. It can be seen that, there is almost no difference between the a values calculated using eqs. (42) and (44) which are determined via geometric and logarithmic mean enthalpies, respectively.

Number of transfer units that is calculated analytically is found to be N=1.830. This means that ($\alpha_a\ F_T$) value is 553.3. The corresponding measured value is 511.2 [9]. The difference between analytical and experimental results is only 7.9%. This is a very good agreement because ($\alpha_a\ F_T$) values are obtained at a pilot-scale cooling tower whereas the capacity measurements are carried out at an industrial cooling tower.

In figures 6 and 7, the water temperature and the air enthalpy profiles along the cooling tower are demonstrated, respectively. The analytically-obtained temperatures are calculated using eqs. (8), (17), (18) and (19). The analytical results in figure 7 are determined according to eq. (23). Experimental values are taken from the above explained industrial cooling tower. The agreement between the analytically calculated results and the experimentally measured data is very good.

Table 5. Analytically calculated data for industrial cooling tower.

$\dot{Q}_{\max}$	kW	Eq. (25)	60260
ε	-	Eq. (24)	0.334
$h_{a,wg}$	kJ/kg	Eq. (45)	128.1
$h_{a,wl}$	kJ/kg	Eq. (47)	129.6
a	-	Eq. (44)	0.736
a	-	Eq. (46)	0.738
$h^*_{a,wo}$	-	Eq. (14)	0.359
$I = N$	-	Eq. (31)	1.830
$\Delta T_{aw,e}$	^oC	Eq. (10)	66.4
$\alpha_a\ F_T$	$kW/^oC$	Eq. (9)	553.3

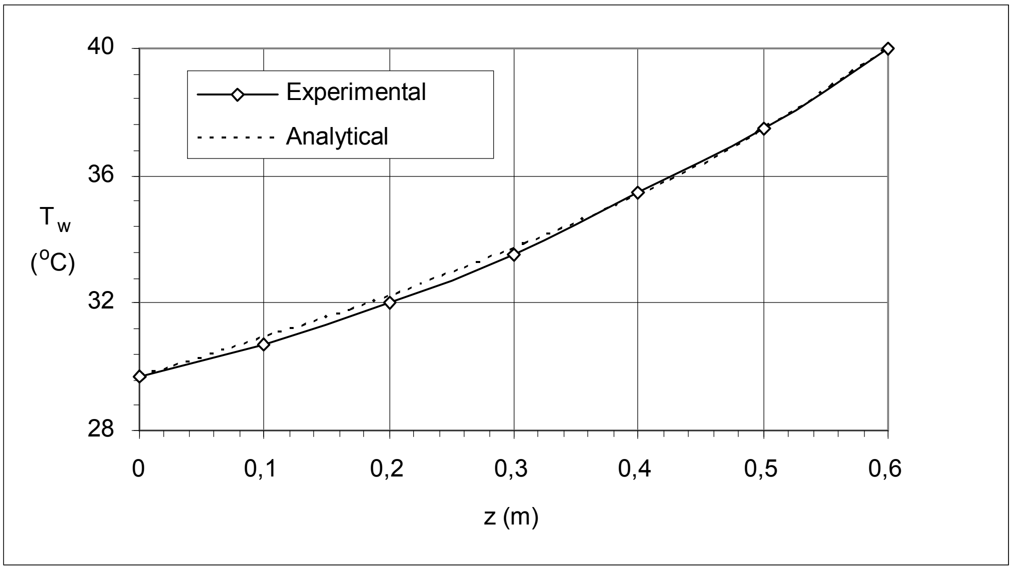

Figure 6. Analytically and experimentally obtained temperature profiles along the cooling tower.

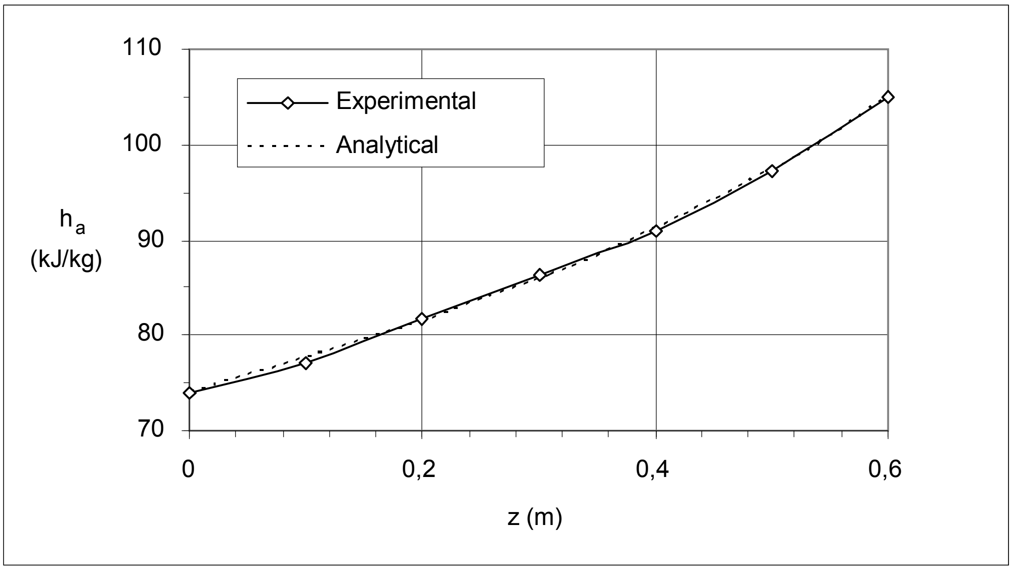

Figure 7. Analytically and experimentally obtained enthalpy profiles along the cooling tower.

CONCLUSIONS

Simple analytical equations can be used to calculate cooling tower performance without any numerical integrations. Between 20-70 ^oC, one needs only saturation enthalpies of air at water inlet and outlet temperatures. For water inlet-outlet temperatures less than 20 ^oC or greater than 70 ^oC, the saturation enthalpy of water at the arithmetic mean water temperature is needed, besides air saturation enthalpies at water inlet and outlet temperatures for analytical calculations. The analytical equations derived in the present study render results which compare well with the experimental and numerical ones. The presented analytical method yields much more accurate results compared to the results obtained using the well-known LMED and LMED-C methods.

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