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EFFECTS OF MHD AND THERMAL RADIATION ON FORCED CONVECTION FLOW ABOUT A PERMEABLE HORIZONTAL PLATE EMBEDDED IN A POROUS MEDIUM

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Abstract
In this study, forced convection heat transfer about a permeable horizontal plate in the presence of magnetic field and thermal radiation effects in the porous medium are investigated. The fluid is assumed to be incompressible and dense. The nonlinear parabolic partial differential equations governing the flow are transformed into the non-similar boundary layer equations. The Keller box method is used to solve these equations. The governing dimensionless parameters arising from dimensionless governing equations are as follows: the porosity ε, the inertia parameter γ, the magnetic parameter Mn, the radiation–conduction parameter R_d, the surface temperature parameter θ_w, and the suction/injection parameter f_w. The effects of these parameters on the local skin friction and local heat transfer parameters as well as the velocity and temperature profiles are analyzed. The validity of the solution methodology and the results are questioned by comparing the findings obtained with those available in the literature and, a fairly good agreement is reached.
Keywords: Thermal radiation, MHD flow, Forced convection, Suction/injection effect.

GÖZENEKLİ BİR ORTAMDA TUTULAN GEÇİRGEN BİR YATAY PLAKA ÜZERİNDEN ZORLANMIŞ TAŞINIMLA OLAN AKIŞA MHD VE ISIL IŞINIMIN ETKİSİ

Özet
Bu çalışmada, gözenekli bir ortam içerisinde tutulan geçirgen yatay bir plaka üzerinden zorlanmış taşınımla olan akışa manyetik alan ve ısıl ışınımın etkileri araştırılmıştır. Akışkanın sıkıştırılamaz ve yoğun olduğu kabul edilmiştir. Lineer olmaya parabolik kısmi diferansiyel denklemleri benze olmayan sınır tabaka denklemlerine dönüştürülmüştür. Dönüştürülen bu denklemeler Keller-box yöntemi kullanılarak çözülmüştür. Denklemlerin boyutsuzlaştırılmasıyla ortaya çıkan boyutsuz parametreler şunlardır: Gözeneklilik (veya porozite) ε, boyutsuz atalet γ, Manyetik alan Mn, ısıl ışınım R_d, yüzey sıcaklık θ_w ve emme/üfleme f_w parametreleri. Bu parametrelerin yerel sürtünme ve yerel ısı transfer parametrelerinin yanında hız ve sıcaklık profilleri üzerine olan etkileri analiz edilmiştir. Çözüm yönteminin doğruluğu literatürde mevcut verilerle karşılaştırılarak sınanmış ve çok uyumlu sonuçlara ulaşılmıştır.
Anahtar Kelimeler: Isıl ışınım, MHD akışı, Zorlanmış taşınım, Emme/üfleme etkisi.

NOMENCLATURE

c_p	specific heat of the convective fluid
f	dimensionless stream function
f_w	suction/injection parameter $\left[=-2K^{1/2}V_w/\nu\right]$
F	inertial coefficient
K	permeability of the porous medium [m^2]
Mn	magnetic parameter $\left[=\sigma B_0^2 K/\mu\right]$
Pr	Prandtl number $\left[=\mu c_p/k\right]$
q_r	the component of radiative flux in *y* direction [w/m^2]
R_d	Radiation parameter $\left[=k\alpha_R/4\sigma T_\infty^3\right]$
T	temperature
u, υ	velocities in x and y directions, respectively
x,y	coordinates in horizontal and vertical directions, respectively
Greek symbols	
α_R	Rosseland mean absorption coefficient
η	pseudo similarity variable $\left[=y\,\mathrm{Re}_x^{1/2}/x\right]$
ε	porosity
ξ	non-similarity variable $\left[=\nu x/K u_\infty\right]$
γ	dimensionless inertia effect $\left[=F\,K^{1/2}u_\infty/\nu\right]$
σ	Stefan–Boltzmann constant [W/m^2K^4]
ρ	fluid density
μ	dynamic viscosity
ν	kinematic viscosity
θ	dimensionless temperature profile in Eq. (6)
θ_W	temperature ratio $\left[=T_w/T_\infty\right]$
Subscripts	
W	wall
∞	free stream

INTRODUCTION

Studying transport phenomena in porous media has received a great deal of research interest due to its wide and important applications in environmental, geophysical and energy related engineering problems. Prominent applications are the utilization of geothermal energy, the migration of moisture in fibrous insulation, drying of porous solid, food processing, casting and welding in manufacturing processes, the dispersion of chemical contaminants in different industrial processes, the design of nuclear reactors, chemical catalytic reactors, compact heat exchangers, solar power, etc. Forced convection over a horizontal plate in a fluid-saturated porous medium is a good representative geometry for many of the areas mentioned above. For an extensive survey of the literature, the reader is referred to the excellent reviews by Kaviany (1995), Pop and Ingham (2001), Ingham and Pop (1998, 2002) and Nield and Bejan (1999).

The study of magnetohydrodynamic flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth (Damseh et al. 2006; Chen 2004). The thermal radiation effect becomes very important when high temperatures are encountered in these application areas.

There are a few studies investigating combined effects of magnetic field and thermal radiation in external convection studies. For some specific external convection geometries under some other additional effects, Damseh et al. (2006), Chamkha et al. (2003), Duwairi (2005), and Aydın and Kaya (2008a) investigated these two effects together for a clear fluid while Abbas and Tasawar (2008), Alam et al. (2008), Yih (2001), Al-Odat et al. (2005) and Murthy et al. (2004) investigated for a porous medium.

The aim of the present study is focused at studying combined effects of gas radiation and magnetic field on the forced convection flow from a permeable horizontal plate embedded in a porous medium.

ANALYSIS

Consider steady, incompressible, laminar, two-dimensional, electrically conducting and radiative heat transfer from a semi-infinite horizontal plate embedded in a Newtonian fluid-saturated porous medium at a uniform temperature T_w. The porous medium is considered to be homogeneous and isotropic (i.e. uniform with a constant porosity and permeability) and is saturated with a fluid which is in local thermodynamic equilibrium with the solid matrix. Far above the plate, the velocity and the temperature of the uniform main stream are u_∞ and T_∞. A uniform magnetic field is assumed to apply in the y-direction causing a flow resistive force in the x-direction. It is assumed that the induced magnetic field, the external or imposed electric field and the electric field due to the polarization of charges (i.e. Hall effect) are negligible. The properties of the fluid and the porous media, such as viscosity, thermal conductivity, specific heat and permeability, are assumed to be constant. In order to study transport through non-Darcian media, the original Darcy model is improved by including the convective and inertia effects. Also, the fluid is assumed to be a grey, emitting and absorbing, but non-scattering medium. The following assumptions are made in the analysis: (i) viscous dissipation effects are negligible, and (ii) the radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction, where the physical coordinates (x, y) are chosen such that x is measured from the leading edge in the streamwise direction and y is measured normal to the surface of the plate. The coordinate system and the flow configuration are shown in Figure 1.

Under the usual Boussinesq approximation, the conservation equations for the steady, laminar, two-dimensional boundary-layer flow problem under consideration can be written as (Chamkha 1997; Takhar and Beg 1997; Mahmud and Fraser 2003):

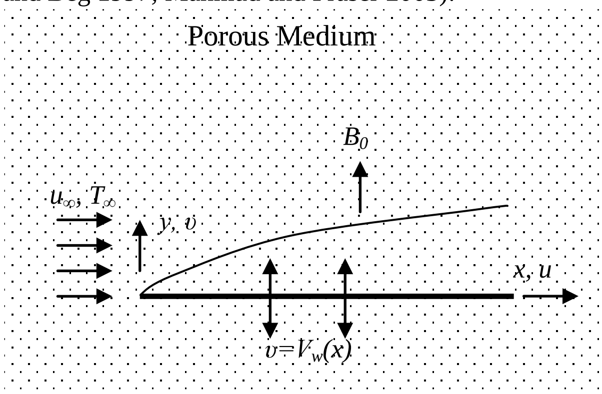


Figure 1. The schematic of the problem.

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$\frac{1}{\varepsilon^2}\left[u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}\right]=\frac{\nu}{\varepsilon}\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{K}\left(u-u_\infty\right)-\frac{F}{K^{1/2}}\left(u^2-u_\infty^2\right)-\frac{\sigma\beta_0}{\rho}\left(u-u_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\left(\frac{\nu}{\mathrm{Pr}}\right)\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(q_R\right) \tag{3}$$

The above equations are called Brinkman–Forchheimer-extended-Darcy equations (Lauriat and Ghafir 2000). Here u and υ are the velocity components in the x and y direction, respectively, T is the temperature of the fluid, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity of the fluid, B_0 is the magnetic flux density, ε is the porosity, K is the permeability and F is the inertial coefficient which depend on permeability and microstructure of the porous matrix.

The quantity q_r on the right-hand side of equation (3) represents the radiative heat flux in the y-direction. For simplicity and comparison, the radiative heat flux term in the energy equation is analyzed by utilizing the Rosseland diffusion approximation (Sparrow and Cess 1961) for an optically thick boundary layer as follows:

$$q_r = -\frac{4a}{3\alpha_R}\frac{\partial T^4}{\partial y} \quad \text{and} \quad \frac{\partial q_r}{\partial y} = -\frac{16\sigma}{3\alpha_R}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right) \tag{4}$$

where σ is the Stefan–Boltzmann constant, α_R is the Rosseland mean absorption coefficient. This approximation is valid at points optically far from the bounding surface, and is good only for intensive absorption, that is, for an optically thick boundary layer (Hossain et al. 2001).

The appropriate boundary conditions for the velocity and temperature of this problem are:

$$\begin{array}{lllll} x=0 & y>0 & T=T_\infty & u=u_\infty & \\ x>0 & y=0 & T=T_w & u=0 & \upsilon=\pm V_w(x) \\ y\to\infty & & T\to T_\infty & u\to u_\infty & \end{array} \tag{5}$$

The minus sign for the conductive gray fluid vertical velocity means the suction from the porous wall, where the plus sign means the gray fluid injection. To seek a solution, the following dimensionless variables are introduced:

$$\xi(x) = \frac{vx}{Ku_\infty}, \qquad \psi(x,y) = (vu_\infty x)^{1/2} f(\xi,\eta),$$
$$\eta = y\left(\frac{u_\infty}{vx}\right)^{1/2}, \ \theta = \frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

where $\psi(x,y)$ is the free stream function that satisfies Eq. (1) with $u=\partial\psi/\partial y$ and $\upsilon=-\partial\psi/\partial x$.

In terms of these new variables, the velocity components can be expressed as

$$u = u_\infty f', \tag{7}$$
$$\upsilon = -\frac{(vu_\infty x)^{1/2}}{x}\left\{\frac{1}{2}f + \xi\frac{\partial f}{\partial \xi} - \frac{\eta}{2}f'\right\} \tag{8}$$

The transformed momentum and energy equations together with the boundary conditions, Eqs. (2), (3) and (5), can be written as

$$\frac{1}{\varepsilon}f''' + \frac{1}{2\varepsilon^2}ff'' - \xi(f'-1) - \xi\gamma(f'^2-1)$$
$$-Mn\xi(f'-1) = \frac{\xi}{\varepsilon^2}\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \tag{9}$$

$$\frac{1}{\Pr}\theta'' + \frac{1}{2}f\theta' + \frac{4}{3\Pr R_d}\left\{\left[\theta(\theta_w-1)+1\right]^3\theta'\right\}'$$
$$= \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \tag{10}$$

with the boundary conditions;

$$f(\xi,0) + 2\xi\frac{\partial f}{\partial \xi} = f_w\xi^{1/2}, \ f'(\xi,0)=0, \ \theta(\xi,0)=1,$$
$$f'(\xi,\infty)=1, \qquad \theta(\xi,\infty)=0 \tag{11}$$

where $f_w = -\frac{2K^{1/2}V_w}{v}$, the case $f_w>0$ designates suction while $f_w<0$ indicates injection or blowing, $f' = \frac{\partial f}{\partial \eta}$ is the derivation of f and $\theta' = \frac{\partial \theta}{\partial \eta}$ is the derivation of θ.

The corresponding dimensionless numbers that appeared in the governing equations defined as:

$$Mn = \frac{\sigma B_0^2 K}{\mu}, \qquad \gamma = \frac{F K^{1/2} u_\infty}{v}, \qquad \Pr = \frac{\mu c_p}{k} = \frac{v}{\alpha},$$
$$R_d = \frac{k\alpha_R}{4\sigma T_\infty^3}, \qquad \text{and} \qquad \theta_W = \frac{T_w}{T_\infty} \tag{12}$$

where Mn is the magnetic parameter, γ is the dimensionless inertia effect, Pr is the Prandtl number, R_d is the Planck number (radiation-conduction parameter), θ_W is the surface temperature ratio to the ambient fluid.

In the above system of equations, the radiation conduction parameter is absent from the MHD-forced convection heat transfer problem when $R_d\to\infty$, ε=1.0 and γ=0.0. It should be mentioned that the optically thick approximation should be valid for relatively low values of the radiation-conduction parameter, R_d. According to Ali et al. (1984), some values of R_d for different gases are: (1) R_d=10-30: carbon dioxide (100-650^{o}F) with corresponding Prandtl number range 0.76-0.6; (2) R_d=30-200: ammonia vapor (120-400^{o}F) with corresponding Prandtl number range 0.88-0.84; (3) R_d=30-200: water vapor (220-900^{o}F) with corresponding Prandtl number 1.

NUMERICAL SOLUTION

The system of transformed equations under the boundary conditions, Eqs. (9) and (10), has been solved numerically using the Keller box scheme along with the Newton's linearization technique, which is proved to be an efficient and accurate finite-difference scheme (Cebeci and Bradshaw 1977). In this method, any quantity g at point $\left(\xi_n, \eta_j\right)$ is written as g_j^n. Quantities and derivatives at the midpoints of grid segments are approximated to second order as

$$\left.\begin{aligned} g_j^{n-1/2} &= \frac{1}{2}\left(g_j^n + g_j^{n-1}\right) \\ g_{j-1/2}^n &= \frac{1}{2}\left(g_j^n + g_{j-1}^n\right) \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} \left(\frac{\partial g}{\partial \xi}\right)_j^{n-1/2} &= \frac{1}{\Delta\xi}\left(g_j^n - g_j^{n-1}\right) \\ \left(g'\right)_{j-1/2}^n &= \frac{1}{\Delta\eta}\left(g_j^n - g_{j-1}^n\right) \end{aligned}\right\} \tag{14}$$

where g is any dependent variable and n and j are the node locations along the ξ and η directions, respectively. First the third-order partial differential equation is converted in the first order by substitutions $f' = s$ and $s' = w$, the difference equations that are to approximate the previous equations are obtained by averaging about the midpoint $\left(\xi_n, \eta_{j-1/2}\right)$, and those to approximate the resulting equations by averaging about $\left(\xi_{n-1/2}, \eta_{j-1/2}\right)$. At each line of constant ξ, a system of algebraic equations is obtained. With the nonlinear terms evaluated at the previous station, the algebraic equations are solved iteratively (Duwairi 2005). The same process is repeated for the next value of ξ and the problem is solved line by line until the desired ξ (ξ is taken 0.01 for this study) value is reached. The effect of the grid size $\Delta\eta$ and $\Delta\xi$ and the edge of the boundary layer η_∞ (η_∞ is taken 16 for this study) on the solution had been examined. The results presented here are independent of the grid size.

In the calculations, a uniform grid of the step size 0.01 in the η-direction and a non-uniform grid in the ξ-direction with a starting step size 0.1 and an increase of 0.01 times the previous step size were found to be satisfactory in obtaining sufficient accuracy. For a given value of ξ, the iterative procedure is stopped when the difference in computing the velocity and the temperature in the next iteration is less than 10^{-6}, i.e. when $\left|\delta f_i\right| \le 10^{-6}$, where the superscript denotes the iteration number. The details of the computational procedure have been discussed further in the book by Cebeci and Bradshow (1977) and Takhar et al. (1997).

In order to verify the accuracy of the numerical results, the validity of the numerical code developed has been checked for a limiting case. We compare our $-\theta'\left(\xi,0\right)$ results with those given by Lloyd and Sparrow (1970) and Chang (2006) for ε=1, Mn=0.0, Pr=10, γ=0, $R_d\to\infty$ and θ_W=0.0 (Table 1).

Table 1. Comparison of the values $-\theta'\left(\xi,0\right)$ for various values ξ with ε=1, Pr=10, γ=0, f_w=0, $R_d\to\infty$, and θ_W=0.0

ξ	Lloyd and Sparrow (1970)	Chang (2006)	Present Study
0.00000	0.7281	0.7280	0.7278
0.00125	0.7313	0.7291	0.7291
0.00500	0.7404	0.7373	0.7328
0.01250	0.7574	0.7566	0.7556
0.05000	0.8259	0.8351	0.8351
0.12500	0.9212	0.9412	0.9432
0.25000	1.0290	1.0603	1.0603

Also, we compare our $-\theta'\left(\xi,0\right)$ results with those given by Aydın and Kaya (2008) for various values ξ and ε with Pr=1, γ =0.5, f_w=0, $R_d\to\infty$, and θ_W=0.0 (Table 2). As it is seen from Tables 1 and 2, our results correspond very well with theirs.

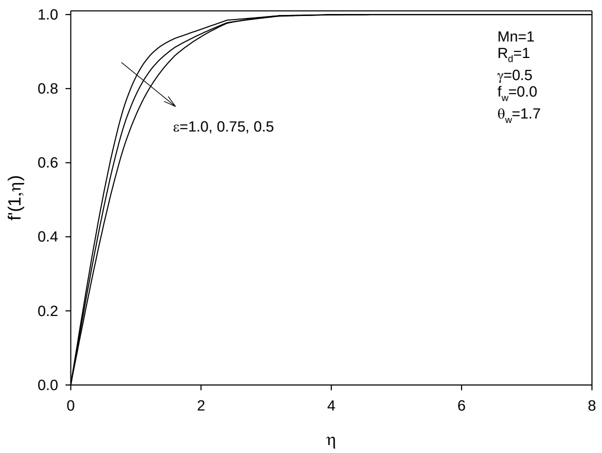
Table 2. Comparison of the values $-\theta'\left(\xi,0\right)$ for various values ξ and ε with Pr=1, γ =0.5, f_w=0, $R_d\to\infty$, and θ_W=0.0

	ε=0.5		ε=0.75		ε=1.0	
ξ	Aydın and Kaya (2008b)	Present Study	Aydın and Kaya (2008b)	Present Study	Aydın and Kaya (2008b)	Present Study
0.0	0.3665	0.3665	0.3463	0.3463	0.3320	0.3320
0.01	0.3680	0.3680	0.3495	0.3495	0.3377	0.3377
0.02	0.3694	0.3694	0.3526	0.3526	0.3429	0.3429
0.03	0.3708	0.3708	0.3556	0.3556	0.3477	0.3477
0.04	0.3722	0.3722	0.3584	0.3584	0.3522	0.3522
0.05	0.3735	0.3735	0.3612	0.3612	0.3564	0.3564
0.06	0.3749	0.3749	0.3638	0.3638	0.3604	0.3604
0.07	0.3761	0.3761	0.3663	0.3663	0.3642	0.3642
0.08	0.3774	0.3774	0.3687	0.3687	0.3677	0.3677

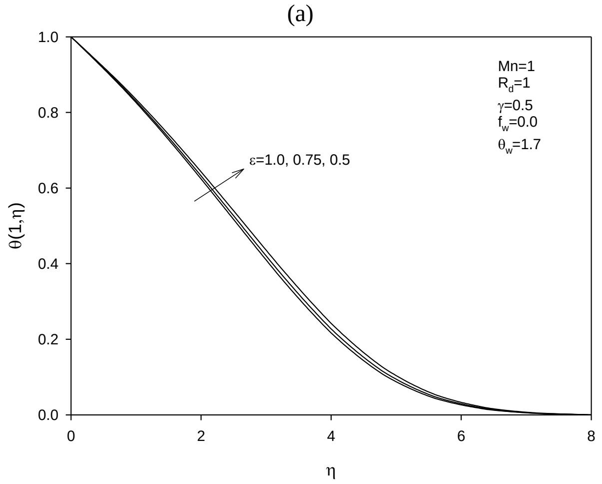
RESULTS AND DISCUSSION

In this article, the forced convection effects of ionized gas adjacent to radiate porous wall embedded in a porous media are investigated including the magnetic field. The temperature of the plate is assumed to be constant (Figure 1). The following ranges of the main parameters are considered: *Pr*=1.0; porosity ε=0.5, 0.75, 1.0; magnetic parameter *Mn*=0.0, 0.25, 0.5, 0.75, 1.0; inertia parameter γ=0.0, 0.5, 1.0, 1.5, 2.0; radiation parameter R_d=1, 3, 5, and 10; surface temperature parameter θ_W=1.7, 2.0, and 2.3 and suction/injection parameter f_w=-0.1, 0.0, 0.1. The porosity ε, the magnetic parameter *Mn*, the inertia parameter γ, the radiation parameter R_d, the surface temperature parameter θ_W and the suction/injection effects f_w on the momentum and heat transfer are analyzed and discussed.

Figure 2 shows the dimensionless velocity and temperature profiles inside the boundary layers for different values of the porosity ε. The increasing of the porosity decreases momentum and thermal boundary layer thickness and increases velocity and temperature profiles. Increasing the velocity and temperature profiles increases local skin friction (Figure 3a) and local heat transfer (Figure 3b) parameters.

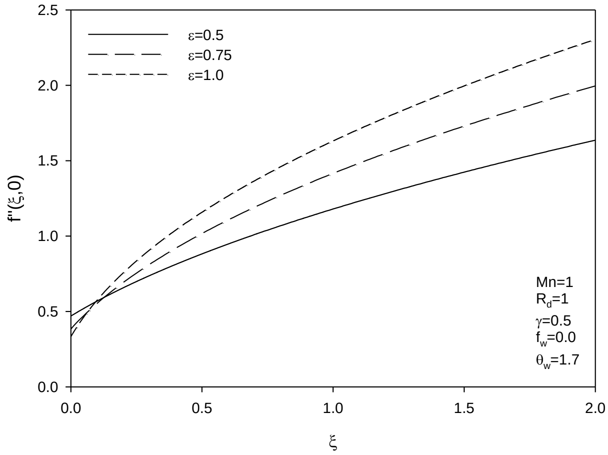


(a)

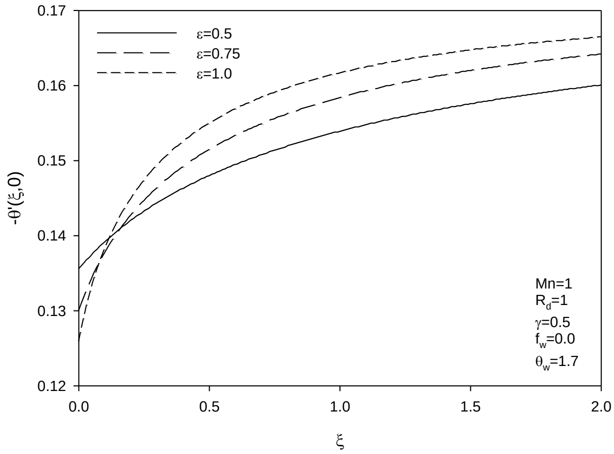


(b)

Figure 2. Dimensionless velocity (a) and temperature (b) profiles for different ε while *Mn*=1, γ=0.5, ξ=1, R_d=1, *Pr*=1.0, θ_W =1.7, and f_w=0.0.



(a)

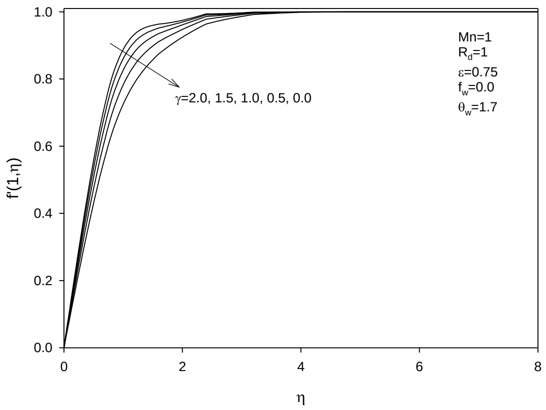


(b)

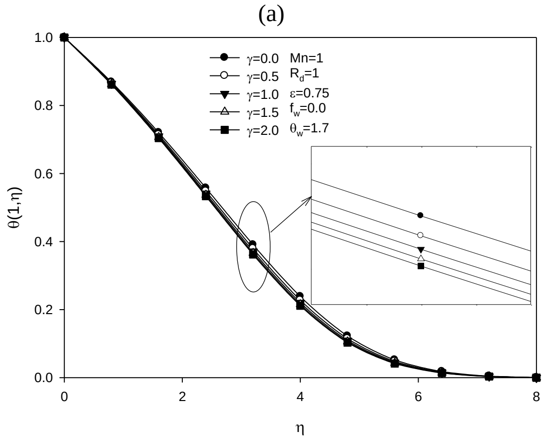
Figure 3. Numerical values of local skin friction (a) and local heat transfer (b) against non-similar parameter for different ε while *Mn*=1, γ=0.5, R_d=1, *Pr*=1.0, θ_W =1.7, and f_w=0.0.

Figure 4 shows the effect of the Forcheimmer parameter γ on the dimensionless velocity and temperature profiles. With an increase in γ, the velocity and temperature gradients decrease (Figures 4a and b), while both the local skin friction and the local heat transfer parameters increase (Figures 5a and b).

The dimensionless velocity and temperature profiles inside the boundary layer for different values of the magnetic parameter *Mn* are shown in Figures 6 and 7. The increasing of the magnetic parameter *Mn* decreases the momentum and the temperature boundary layer thicknesses while it increases both the local skin friction and the local heat transfer parameters (Figure 8). Since the magnetic parameter *Mn* is multiplied by ξ [see Eq. (9)], the effect of *Mn* intensifies with the streamwise distance ξ. In order to understand the effect of the magnetic parameter *Mn*, we should examine Eq. (2). The sign of the last term in the right hand side of Eq. (2), $\left(\sigma B_0^2/\rho\right)\left(u-u_\infty\right)$ is directly related with the sign of $\left(u-u_\infty\right)$. For the forced convection regime this term will always be negative since $\left(u_\infty > u\right)$. Therefore, it will generate a force in the main flow direction, which will aid the main flow. In fact, this term has two

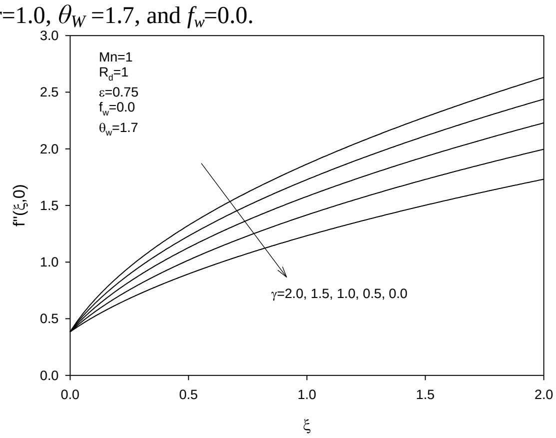


(a)

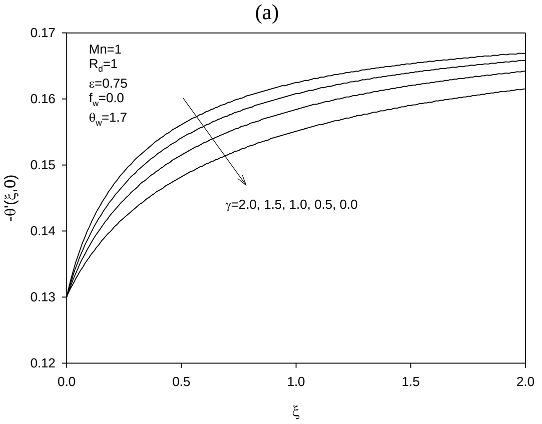


(b)

Figure 4. Dimensionless velocity (a) and temperature (b) profiles for different γ while Mn=1, ε =0.75, ξ=1, R_d=1, Pr=1.0, θ_W =1.7, and f_w=0.0.



(a)



(b)

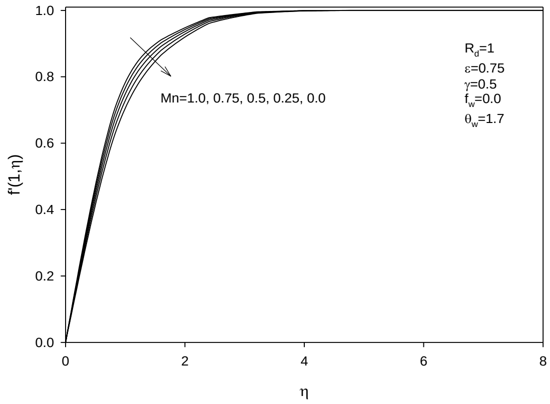
Figure 5. Numerical values of local skin friction (a) and local heat transfer (b) against non-similar parameter for different γ while Mn=1, ε =0.75, R_d=1, Pr=1.0, θ_W =1.7, and f_w=0.0.

components: the first one, $\left(\sigma B_0^2/\rho\right)(u)$, represents the imposed pressure force in the inviscid region of the conducting fluid, while the second one, $\left(\sigma B_0^2/\rho\right)(u_\infty)$, represents the Lorentz force imposed by a transverse magnetic field to an electrically conducting, which slows down the fluid motion in the boundary layer region. When the imposed pressure force overcomes the Lorentz force, i.e. $(u_\infty > u)$, the effect of the magnetic interaction parameter is to increase velocity. Similarly, when the Lorentz force dominates over the imposed pressure force, i.e. $(u_\infty < u)$, the effect of the magnetic interaction parameter will decrease velocity (Aydın and Kaya, 2009).

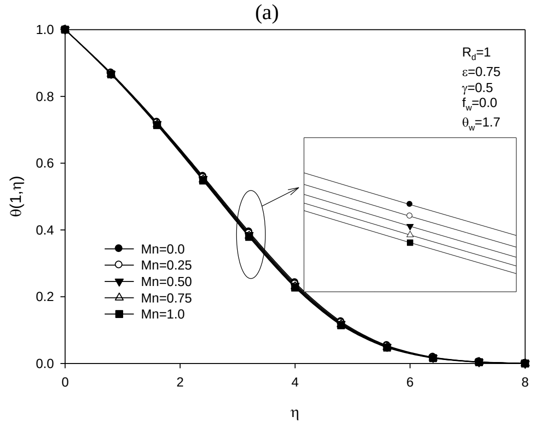
For the forced convection case, as seen in Eqs. (2) and (3), the radiation parameter R_d and the surface temperature parameter θ_w does not have any influence on velocity profile since the momentum and energy equations are not coupled, however, it does on the temperature profile. Figure 8a shows the dimensionless temperature profiles inside the boundary layer for different values of the radiation parameter R_d. The increasing of the radiation parameter R_d increases temperature gradients near the porous wall, which increases heat transfer rates (Figure 8b), this is due to the fact that radiation effects increase temperatures of ionized gases and the absence of radiation defines small temperatures (Duwairi, 2005).

The effect of surface temperature parameter θ_w on temperature profiles is shown in Figure 9a. The increasing of this parameter heats the conductive gray fluid and broadens the temperature inside the boundary layer, and consequently the increasing of this parameter reduces the temperature gradient at the wall (Figure 9b).

Figure 10 shows the velocity (a) and temperature (b) profiles for different values of the suction/injection parameter, f_w. Remember that f_w > 0 corresponds to suction, f_w < 0 corresponds to injection and f_w = 0 represents the flow over an impermeable surface. Injecting fluid into the boundary layer broadens the velocity distribution and increases the hydrodynamic boundary layer thicknesses as shown in Figure 10, while the suction reverses this trend. Figure 10 also shows that injection broadens the temperature distribution, decrease the wall temperature gradient, and hence reduce the heat transfer rate. On the other hand, the thermal boundary layer becomes thinner and the wall temperature gradient becomes larger when suction is applied. Figure 11 shows the local skin friction parameter (a) and local heat transfer parameter (b) for different values of f_w at Mn=1, γ = 0.5, ε = 0.75, R_d = 10.0, Pr = 1.0, and θ_W =1.7. It is seen that increasing f_w increases local skin friction and local heat transfer parameter.

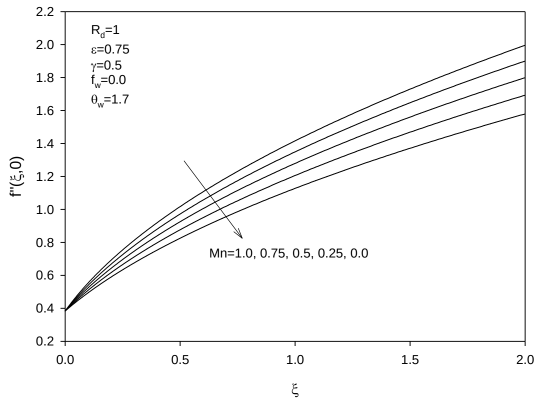


(a)

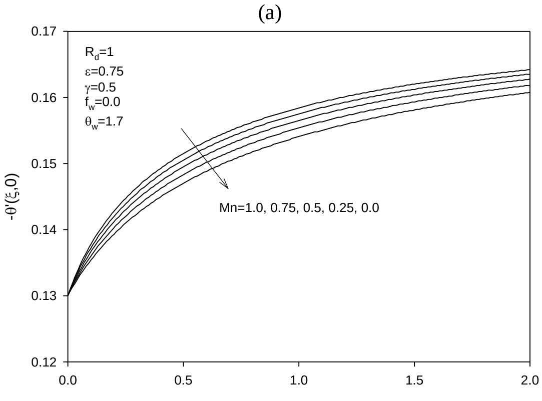


(b)

Figure 6. Dimensionless velocity (a) and temperature (b) profiles for different *Mn* while ε =0.75, γ=0.5, ξ=1, R_d=1, *Pr*=1.0, θ_W =1.7, and f_w=0.0.

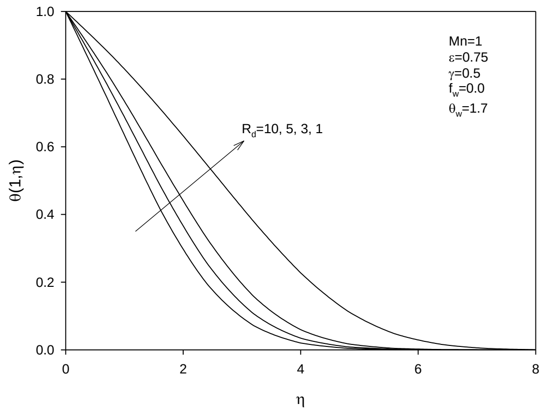


(a)

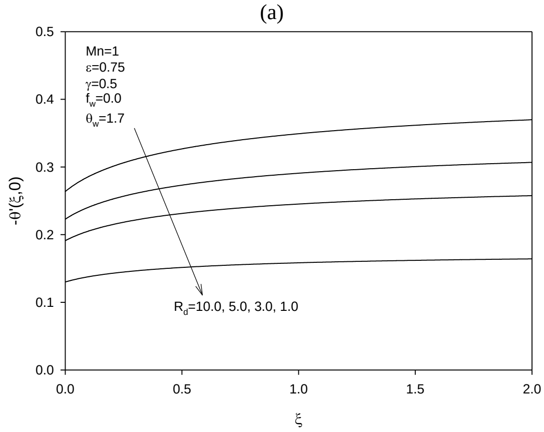


(b)

Figure 7. Numerical values of local skin friction (a) and local heat transfer (b) against non-similar parameter for different *Mn* while ε =0.75, γ=0.5, R_d=1, *Pr*=1.0, θ_W =1.7, and f_w=0.0.

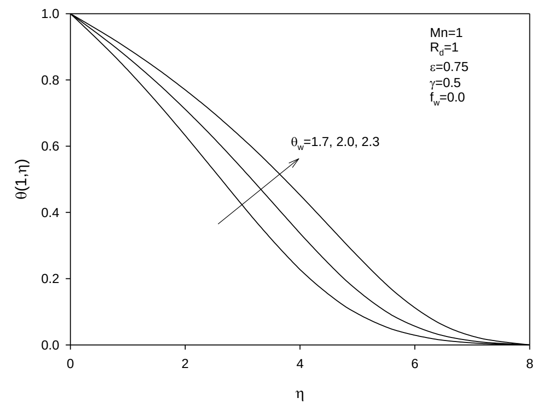


(a)

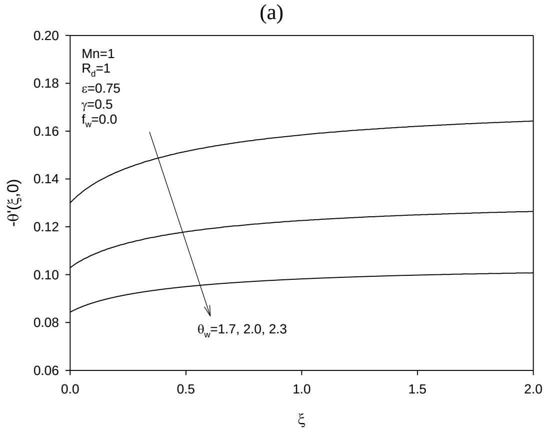


(b)

Figure 8. Dimensionless temperature profile (a) and local heat transfer for different R_d while ε =0.75, *Mn*=1, γ=0.5, *Pr*=1.0, θ_W =1.7, and f_w=0.0.



(a)



(b)

Figure 9. Dimensionless temperature profile (a) and numerical values of local heat transfer (b) for different θ_w while ε =0.75, *Mn*=1, γ=0.5, ξ=1, R_d=1, *Pr*=1.0, and f_w=0.0.

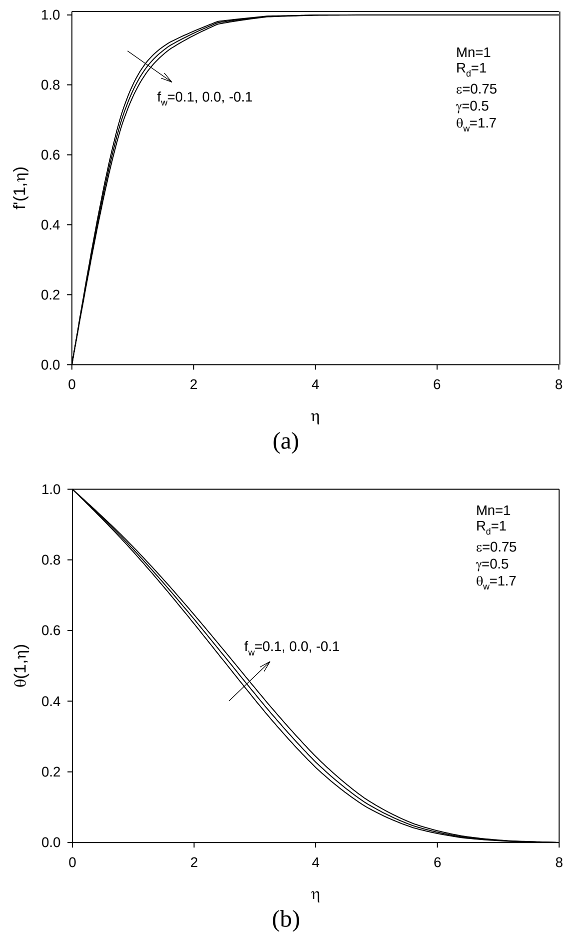


Figure 10. Dimensionless velocity (a) and temperature (b) profiles for different f_w while Mn=1, ε =0.75, γ=0.5, ξ=1, R_d=1, Pr=1.0, and θ_W =1.7.

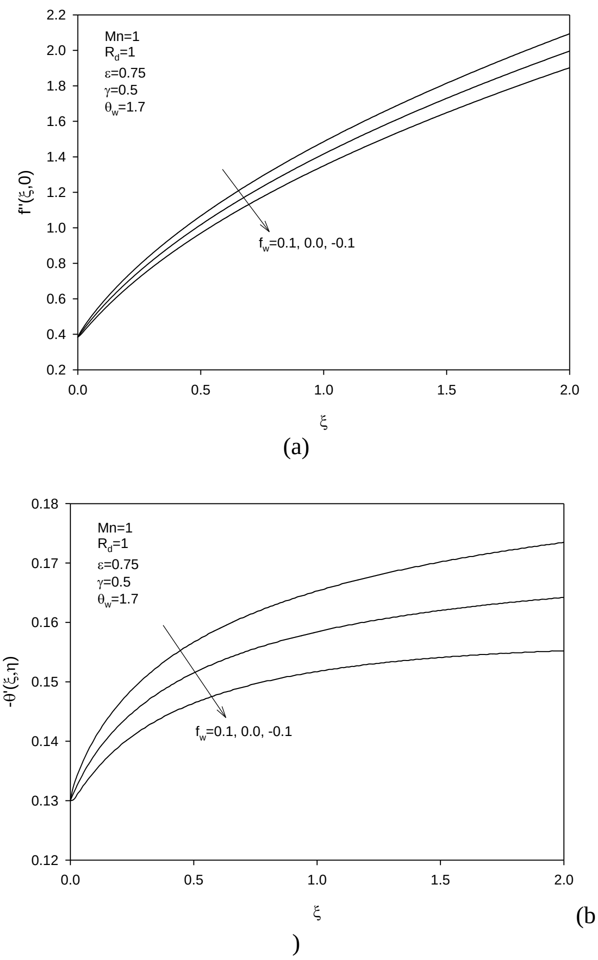


Figure 11. Numerical values of local skin friction (a) and local heat transfer (b) against non-similar parameter for different f_w while Mn=1, ε =0.75, γ=0.5, R_d=1, Pr=1.0, and θ_W =1.7.

CONCLUSIONS

In this article, we have studied numerically the effects of suction/injection and thermal radiation on a steady MHD forced convective flows about a permeable horizontal plate embedded in a porous medium. A transformed set of non-similar equations have been solved using the Keller box scheme. From the present numerical investigation, the following conclusions can be drawn:

1. An increase in the porosity, the Forcheimmer and the magnetic parameters increases the local skin friction and the local heat transfer parameters.

2. An increase in the radiation parameter increases the local heat transfer parameter.

3. An increase in the surface temperature parameter decreases the local heat transfer parameter.

Suction makes the thermal boundary layer thinner and therefore enhances the heat transfer, while the injection results in an opposite effect.

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