Isı Bilimi ve Tekniği Dergisi, 32, 2, 31-41, 2012
J. of Thermal Science and Technology

ISSN 1300-3615

NUMERICAL ANALYSIS OF NATURAL CONVECTION HEAT TRANSFER FROM ANNULAR FINS ON A HORIZONTAL CYLINDER

Ayla DOGAN*, Sinan AKKUS and Şenol BASKAYA****
* Akdeniz University Department of Mechanical Engineering
07058 Campus, Antalya, Turkey, ayladogan@akdeniz.edu.tr
** Gazi University Department of Mechanical Engineering
06570 Maltepe, Ankara, Turkey, baskaya@gazi.edu.tr

(Geliş Tarihi: 18.10.2010, Kabul Tarihi: 18.02.2011)

Abstract: In this study, natural convection heat transfer from an annular fin on a horizontal cylinder was numerically investigated. The aim of the study was to determine the effects of geometric parameters like: fin diameter, fin spacing and base-to-ambient temperature difference on the heat transfer performance of fin arrays, and to find optimum parameters that maximize the heat transfer rate. Keeping the fin thickness at 1 mm, fin diameter was varied from 35 mm to 160 mm and fin spacing 3.5 mm to 146 mm. Surface of the cylinder as well as the surface of each fin were assumed to be at a uniform temperature. Air was selected as the working fluid. The problem was a three-dimensional natural convection phenomenon with open boundaries and was solved with a cylindrical coordinate system. The equations of mass, momentum and energy were solved using appropriate boundary conditions by means of PHOENICS. The obtained results have shown that the convection heat transfer from the fins depends on fin diameter, fin spacing and base to ambient temperature difference. Finally, a correlation was obtained for the optimum fin spacing depending on Rayleigh number and fin diameter.
Keywords: Natural convection, Annular fins, Fin diameter, Fin spacing, CFD.

YATAY BİR SİLİNDİR ÜZERİNDE BULUNAN DAİRESEL KANATÇIKLARDAN DOĞAL KONVEKSİYONLA ISI TRANSFERİNİN SAYISAL OLARAK İNCELENMESİ

Özet: Bu çalışmada, yatay bir silindir üzerine yerleştirilmiş dairesel kanatçıklardan doğal konveksiyon ile ısı transferi sayısal olarak incelenmiştir. Çalışmanın amacı, kanatçık çapı, kanatçık aralığı gibi geometrik parametrelerin ve taban-ortam sıcaklık farkının kanatçık dizilerinin ısı transfer performansı üzerine etkilerini belirlemek ve maksimum ısı transferini sağlayacak optimum parametreleri bulmaktır. Kanatçık kalınlığı 1 mm tutularak, kanatçık çapı 35 mm den 160 mm ye ve kanatçık aralığı 3.5 mm den 146 mm ye değiştirilmiştir. Silindir yüzeyinin yanı sıra her bir kanatçık yüzeyinin sabit sıcaklıkta olduğu kabul edilmiştir. Çalışma akışkanı olarak hava kullanılmıştır. Problem açık sınırlı, üç boyutlu silindirik koordinat sisteminde doğal konveksiyon problemidir. Kütle, momentum ve enerji denklemleri uygun sınır şartları kullanılarak PHOENICS programı ile çözülmüştür. Elde edilen sonuçlar, kanatçıklara transfer edilen ısının, kanatçık çapına, kanatçık aralığında ve taban-ortam sıcaklık farkına bağlı olduğunu göstermiştir. Çalışmanın sonunda elde edilen verilerden, Rayleigh sayısına ve kanatçık aralığına bağlı olarak, optimum kanatçık aralığı için bir bağıntı elde edilmiştir.
Anahtar kelimeler: Doğal konveksiyon, Dairesel kanatçıklar, Kanatçık çapı, Kanatçık aralığı, HAD.

NOMENCLATURE

A	Total heat transfer area [m^2]
C_p	Specific heat [J/kg°C]
d	Outer diameter of the cylinder [m]
d_i	Inner diameter of the cylinder [m]
D	Fin diameter [m]
g	Gravitational acceleration [m/s^2]
h	Convection heat transfer coefficient [W/m2°C]
k	Thermal conductivity [W/m°C]
L	Length of cylinder [m]
$\dot{m}$	Mass flow rate [kg/s]
Nu_D	Nusselt number based on fin diameter
Nu_S	Nusselt number based on fin spacing
P	Pressure [N/m^2]
Q_c	Total heat transfer rate [W]
$Q_c^{[1]}$	Convection heat transfer rate from fins in small-s limit [W]
$Q_c^{[2]}$	Convection heat transfer rate from fins in large-s limit [W]
$Q_{c\,[max]}$	Maximum heat transfer rate [W]
Q_0	Convection heat transfer rate from the cylinder [W]
Ra_S	Rayleigh number based on fin spacing
Ra_D	Rayleigh number based on fin diameter
s	Fin spacing [m]
t	Fin thickness [m]

T_a	Ambient temperature [°C]
T_w	Fin temperature [°C]
ΔT	Temperature difference [°C]
u	velocity component in θ-direction [m/s]
v	velocity component in r-direction [m/s]
w	velocity component in z-direction [m/s]

Greek letters

α	Thermal diffusivity [m^2/s]
β	Volumetric thermal expansion coefficient [K^{-1}]
ρ	Density [kg/m^3]
ν	Kinematic viscosity [m^2/s]
μ	Dynamic viscosity [Ns/m^2]

INTRODUCTION

Fins are used to enhance convective heat transfer in a wide range of engineering applications such as electronic equipment, heat exchangers, air conditioning, refrigeration, gas turbines, compressors etc. The geometry of the fins is the merely controllable variable to enhance the convective heat transfer rate. The more fins are used, the more heat is rejected. But there may be a problem which causes to resist the air flow. Therefore designers must focus on finding a technique not only to increase heat transfer, but also achieves high fin efficiency. They must also optimize carefully the size and spacing of the fin arrays to obtain best results.

There exist numerous studies on heat transfer enhancement in the literature. Aziz [1] considered the optimization problem for three shapes of convecting fins: rectangular, triangular and concave parabolic. His article was devoted to the review of the literature on optimum dimensions of extended surfaces losing heat by pure convection to the surroundings. Afterwards, the same author investigated the optimization of longitudinal fins of given profile shapes, dissipating heat to the environment purely by radiation [2]. The numerical solution of the laminar free convection of air around a horizontal cylinder with external longitudinal fins was reported by Haldar [3]. The numerical procedure adopted in this study was used for Gr up to 10^6, a fin length up to 0.6 times the radius of the cylinder, and the fin numbers up to eighteen, positioned at equal angular spacing. Another study was conducted numerically by Haldar et al. [4], who investigated laminar free convection about a horizontal cylinder with longitudinal fins of finite thickness. Although fins themselves contributed very little to the total heat transfer, their presence greatly had influenced the heat transfer from the uncovered area of the cylinder. The effect of fins on heat transfer from the cylinder was found negative except for very thin fins. Ullmann and Kalman [5] investigated increasing the heat dissipation of annular fins at a defined magnitude of mass. Four different cross-sections were examined; constant thickness, constant area for heat flow, triangular and parabolic fin shapes. Kundu and Das [6] examined the performance analysis and optimization of eccentric annular disk fins. They showed that eccentric annular disc fins could dissipate more heat compared to concentric annular fins of identical volume. Under determined circumstances they concluded that eccentric annular disc fins could be better alternatives. The fin efficiency together with the optimized dimensions were presented, which enables the design of the best fin for any practical use. Another study was conducted experimentally by Yıldız and Yüncü [7], who investigated natural convection heat transfer in annular fin arrays mounted on a horizontal cylinder. They found that the rate of convection heat transfer from fin arrays depended on the geometry of the fin array, fin diameter and fin spacing and base-to-ambient temperature difference. Kundu and Das [8] developed an analytical model to carry out the performance and optimum design analysis of convective fin arrays attached to flat and curved primary surfaces. The performance parameters, fin efficiency, fin effectiveness and augmentation factor were evaluated for a wide range of design variables. From their results, they highlighted optimum fin dimensions in fin arrays differ from that of the individual fins.

As a result of these studies, one can see that there are a large number of reports associated with fins available in the literature. But rather limited data are available for an array of annular fins. In this numerical study, a wide range of annular fin configurations mounted on a horizontal cylinder were investigated under steady state incompressible laminar natural convection conditions. The effects of geometric parameters fin diameter, fin spacing and base-to-ambient temperature difference on the heat transfer performance of fin arrays was investigated. The main goal of this study was to find optimum parameters that maximize the heat transfer rate.

NUMERICAL MODEL

Fin configuration and the geometry of the fin array and coordinate system of the problem under consideration are depicted in Fig. 1. Length of the cylinder is 325 mm. The outer diameter is 24.9 mm and the inner diameter is 21mm. The computational domain can be reduced to a quarter of the region between two fins with an extension towards one open end, considering the geometry and symmetry conditions of the problem. The size of the extended boundaries of the solution domain was determined by a detailed analysis according to the effects of this extension to the heat transfer results. Final extended boundaries did not alter the solutions. The additional distances used for various configurations were up to 5 times the fin height. The base temperature has been taken uniform and constant and the same temperature has been assumed to prevail in the fins, implying a highly conducting material.

The numerical code employed for the present study was rigorously verified. First, a trial study was carried out to ensure that the solution is grid independent. Trial solutions were obtained with a wide range of cell number combinations for grid independency checks.

Final simulations were performed with cell numbers up to 35x37x12 depending on the relevant sizes.

In order to check the adequacy of the present numerical approach, the obtained results are compared with experimental measurements by Yildiz and Yüncü [7].

As can be seen in Fig. 2, there is a good agreement between numerical and experimental results. The maximum difference between experimental and CFD values does not exceed 7%. This can be considered as a sufficient verification of the computational domain and numerical procedures applied in the present study.

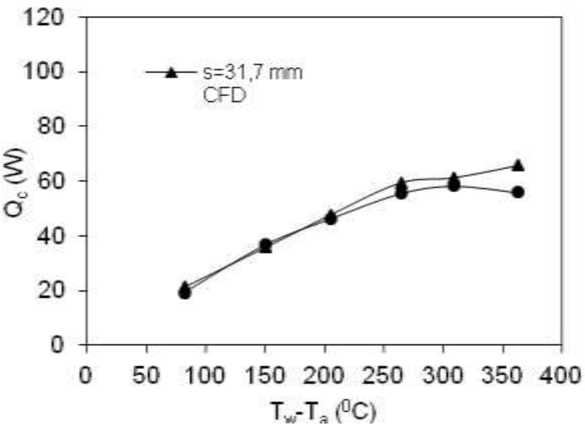


Figure 2. Comparison of present CFD results with experimental measurements by Yildiz and Yüncü [7] for D=35 mm, s=31.7 mm .

GOVERNING EQUATIONS

The continuity, momentum and energy equations for a three dimensional incompressible laminar flow have been solved using appropriate boundary conditions by means of the PHOENICS CFD code. Following assumptions have been made: there is no viscous dissipation, the gravity acts in the vertical direction, the fluid properties are constant and variations of fluid density are neglected, except in the buoyancy term, and radiation heat exchange was assumed negligible. And also heat conduction through the fin and cylinder materials was not simulated. Therefore, there is no need for thickness or material thermal conductivity values.

According to Yıldız and Yüncü [7] and to the other relevant literature [9, 10, 11] the Ra number can be defined as given below.

$$Ra_D = \frac{g\beta(T_w - T_a)D^3}{\alpha\nu} \tag{1}$$

Here T_w-T_a is the temperature difference between the fin and ambient temperatures and D is the fin diameter. According to the relevant literature the flow can be assumed to be laminar up to $Ra_D=10^9$. In the present study the temperature difference changes from 50 to 200 ^oC and the fin diameter between 35 and 160 mm. Hence, for the range of parameters studied, the Ra number various in the present study between $Ra_D=10^5$-10^7. As a result of this, the solutions were obtained with a laminar flow assumption.

At steady-state conditions using above assumptions, the governing equations for three-dimensional laminar flow in the cylindrical coordinate system can be expressed as given below:

Continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(vr) + \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

θ -component of the momentum equation;

$$\begin{aligned}&(v\frac{\partial u}{\partial r} + \frac{u}{r}\frac{\partial u}{\partial \theta} + \frac{uv}{r} + w\frac{\partial u}{\partial z})\\ &= -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \beta g_\theta (T - T_\partial)\\ &+ \nu\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) - \frac{u}{r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v}{\partial \theta} + \frac{\partial^2 u}{\partial z^2}\right]\end{aligned} \tag{3}$$

r -component of the momentum equation;

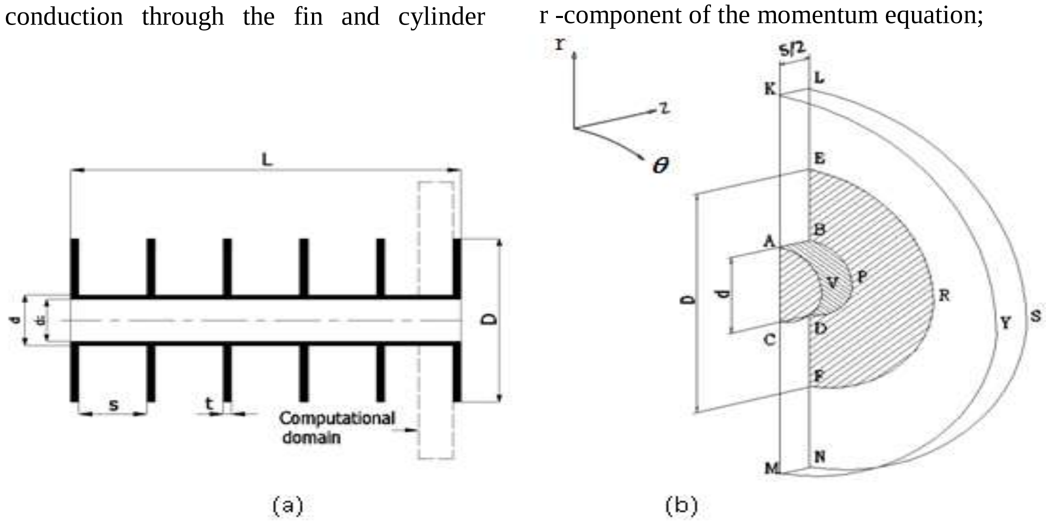


Figure 1. Schematic drawing of (a) fin configuration and (b) computational domain.

$$(v\frac{\partial v}{\partial r}+\frac{u}{r}\frac{\partial v}{\partial\theta}-\frac{u^2}{r}+w\frac{\partial v}{\partial z})$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\beta g_r(T-T_\partial) \tag{4}$$
$$+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v}{\partial r})-\frac{v}{r^2}+\frac{1}{r^2}\frac{\partial^2 v}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial u}{\partial\theta}+\frac{\partial^2 v}{\partial z^2}\right]$$

z -component of the momentum equation;

$$(v\frac{\partial w}{\partial r}+\frac{u}{r}\frac{\partial w}{\partial\theta}+w\frac{\partial w}{\partial z})$$
$$=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w}{\partial r})+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial^2 w}{\partial z^2}\right] \tag{5}$$

where ν, is the kinematic viscosity and β is the volumetric thermal expansion coefficient.

Energy equation;

$$v\frac{\partial T}{\partial r}+\frac{u}{r}\frac{\partial T}{\partial\theta}+w\frac{\partial T}{\partial z}$$
$$=\alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right] \tag{6}$$

where α is the thermal diffusivity.

Boundary Conditions

Velocity, pressure and temperature in the continuity, momentum and energy equations have been solved using three types of boundary conditions, wall (ABPDCVA, BERFDB, AVCA), symmetry (KLNMK, KYMCVAK, LSNFREL) and open boundaries (KLSYK, YSNM) as shown in the Fig. 1(b). Table 1 lists collectively all the boundary conditions used in the solution.
The PHOENICS code iteratively solves linear algebraic equations resulting from the finite volume integration of the partial differential equations. Due to the iterative process of the code, convergence was used as the monitor of achievement of the final solution. The criterion of convergence of the numerical solution is based on the absolute normalized residuals of the equations that were summed for all cells in the computational domain. Convergence was considered as being achieved when these residuals become less than 10^{-6}, which was the case for most of the dependent variables. Iterative convergence was also checked by terminating the solution only when the progressive single cell values of pressure, velocity and temperature showed little change per iteration as the calculation progressed. Furthermore, checks for the achievement of a final solution were made based on the conservation of mass, momentum and energy. Spot values were also controlled.

Dimensionless numbers affecting the heat transfer are given below. The values of Nusselt number based on fin diameter and fin spacing have been calculated by using the following definitions:

$$Nu_D=\frac{hD}{k} \tag{7}$$

$$Nu_s=\frac{hs}{k} \tag{8}$$

where h is the convection heat transfer coefficient obtained from the present numerical study and is defined by the following equation:

$$h=\frac{Q_c}{A\Delta T} \tag{9}$$

Where Q_c is the total convection heat transfer rate and A is the total heat transfer area (cylinder and fin).
The Rayleigh numbers based on fin diameter and fin spacing are defined as:

$$Ra_D=\frac{g\beta(T_w-T_\partial)D^3}{\alpha\nu} \tag{10}$$

Table 1. The boundary conditions for the computational domain

ABPDCVA (wall)	u=0	v=0	w=0	$T=T_w$
BERFDB (wall)	u=0	v=0	w=0	$T=T_w$
AVCA (wall)	u=0	v=0	w=0	$T=T_w$
KLNMK (symmetry)	u=0	$\frac{\partial v}{\partial\theta}=0$	$\frac{\partial w}{\partial\theta}=0$	$\frac{\partial T}{\partial\theta}=0$
KYMCVAK (symmetry)	$\frac{\partial u}{\partial z}=0$	$\frac{\partial v}{\partial z}=0$	w=0	$\frac{\partial T}{\partial z}=0$
LSNFREL (symmetry)	$\frac{\partial u}{\partial z}=0$	$\frac{\partial v}{\partial z}=0$	w=0	$\frac{\partial T}{\partial z}=0$
KLSYK (open)	$\frac{\partial u}{\partial r}=0$	$\frac{\partial v}{\partial r}=0$	$\frac{\partial w}{\partial r}=0$	$\frac{\partial T}{\partial r}=0$
YSNM (inlet)	$\frac{\partial u}{\partial r}=0$	$\frac{\partial v}{\partial r}=0$	$\frac{\partial w}{\partial r}=0$	$T=T_a$

$$Ra_s = \frac{g\beta(T_w - T_\partial)s^3}{\alpha v} \tag{11}$$

where, β is the volumetric coefficient of thermal expansion.

Solution Algorithm

The numerical model is based on a control volume-finite difference formulation. The above equations are integrated over each control volume to obtain a set of discretized linear algebraic equations of the form:

$$a_P\phi_P = \sum a_{nb}\phi_{nb} + s \tag{12}$$

Equations in the format given above are called finite volume equations. Finite volume equations describe processes affecting the value of ϕ, in cell P, in relation to its neighbor cells, together with the source terms. These equations were solved by the widely used commercial CFD package PHOENICS (Rosten and Spalding [12]) employing the SIMPLEST algorithm (Spalding [13]) for the pressure correction process along with the solution procedure for the hydrodynamic equations. The package uses a staggered grid arrangement. Implicit temporal differencing is employed, and for the discretization of convective-diffusive transport, the hybrid scheme is the default scheme within the code (see Patankar [14] for details). This scheme combines the stability of the upwind-scheme with the approximation accuracy of the central-difference-scheme. In the hybrid-scheme, diffusion is cut off when the cell Peclet number (Pe=RePr, i.e. the ratio of heat convection to heat conduction) equals 2.0. In other words, the convective transport is assumed to dominate diffusive transport, and the hybrid-scheme reduces to the upwind formulation, with diffusion terms being neglected. The central-difference-scheme leads to second–order truncation error in the approximations, whereas the upwind-scheme gives only first order accuracy. The discretized equations are solved by the TDMA (Tri-Diagonal-Matrix-Algorithm).

RESULTS AND DISCUSSION

In this study, natural convection heat transfer rates from annular finned horizontal cylinders have been investigated numerically. The effect of fin diameter (D), fin spacing (s) and base-to-ambient temperature difference (ΔT) on convection heat transfer has been examined. The results have shown that the convection heat transfer has been largely affected by fin spacing, fin diameter and base-to-ambient temperature difference.

Effect of Fin Spacing

Fig. 3 represents variations of convection heat transfer with fin spacing as a function of base-to-ambient temperature difference, for fin diameters of 160, 125, 90, 60 and 35mm, respectively. Convection heat transfer rate increases with increase of temperature difference for all fin spacing arrangements. Normally, for small values of fin spacing, total number of fin along the horizontal cylinder is increasing. Therefore it is expected that the heat transfer rate increases from the fin arrays. But as can be seen from the figures, when the fin diameter is very high (D=160 mm) and the fin spacing is very low (3.5 mm), convection heat transfer rate decreases. The reason for this may be the resistance to air flow, which causes a deceleration of the air flow and thus heat transfer rate decreases. When the smallest and largest values of fin spacing (s=3.5 mm and s=146 mm) are compared, there is clear difference in the heat transfer rate. This means that there is an optimum value for the fin spacing at a given fin diameter and base-to-ambient temperature difference, for which the convective heat transfer is maximized. With the increase in temperature difference, the convection heat transfer rates diverge but at low temperature differences convection heat transfer values approach each other for all fin spacing arrangements.

Effect of Fin Diameter

The effect of fin diameter and base-to-ambient temperature difference on convection heat transfer rate at various fin spacing (s=31.5, 14.5, 5 and 3.6 mm) is shown in Fig. 4. These figures depict that while both fin diameter and temperature differences increase, the convection heat transfer rate increases from the fin arrays. At higher temperature differences, it is observed that convective heat transfer rate increases, and at low temperature differences heat transfer rates are very close to each other. When the figure for s=3.6 mm (Fig. 4(d)) is examined carefully, one can see that the convection heat transfer rates from fins which have the diameter of 125 mm are less than that of the fins with the diameter of 90 mm and 60 mm. For cylinders with smaller fin spacing and larger fin diameter, the air circulation in the channel formed by fin spacing arrangements is not completed and air pockets occur in the channel. This gives rise to local temperature increments (hot spots) on the heat transfer surface.

Effect of Base-to-Ambient Temperature Differences

Fig. 5 was drawn in order to show the effect of base-to-ambient temperature difference on convection heat transfer rate form fin arrays at different fin diameters of D=160, 125, 90, 60 and 35 mm, respectively. As can be seen at first, convection heat transfer rate from a fin array increases rapidly up to a maximum value and then it starts to decrease. From all the figures, with increase of the temperature differences, the effect of buoyancy driven flow increases the convection heat transfer rate in

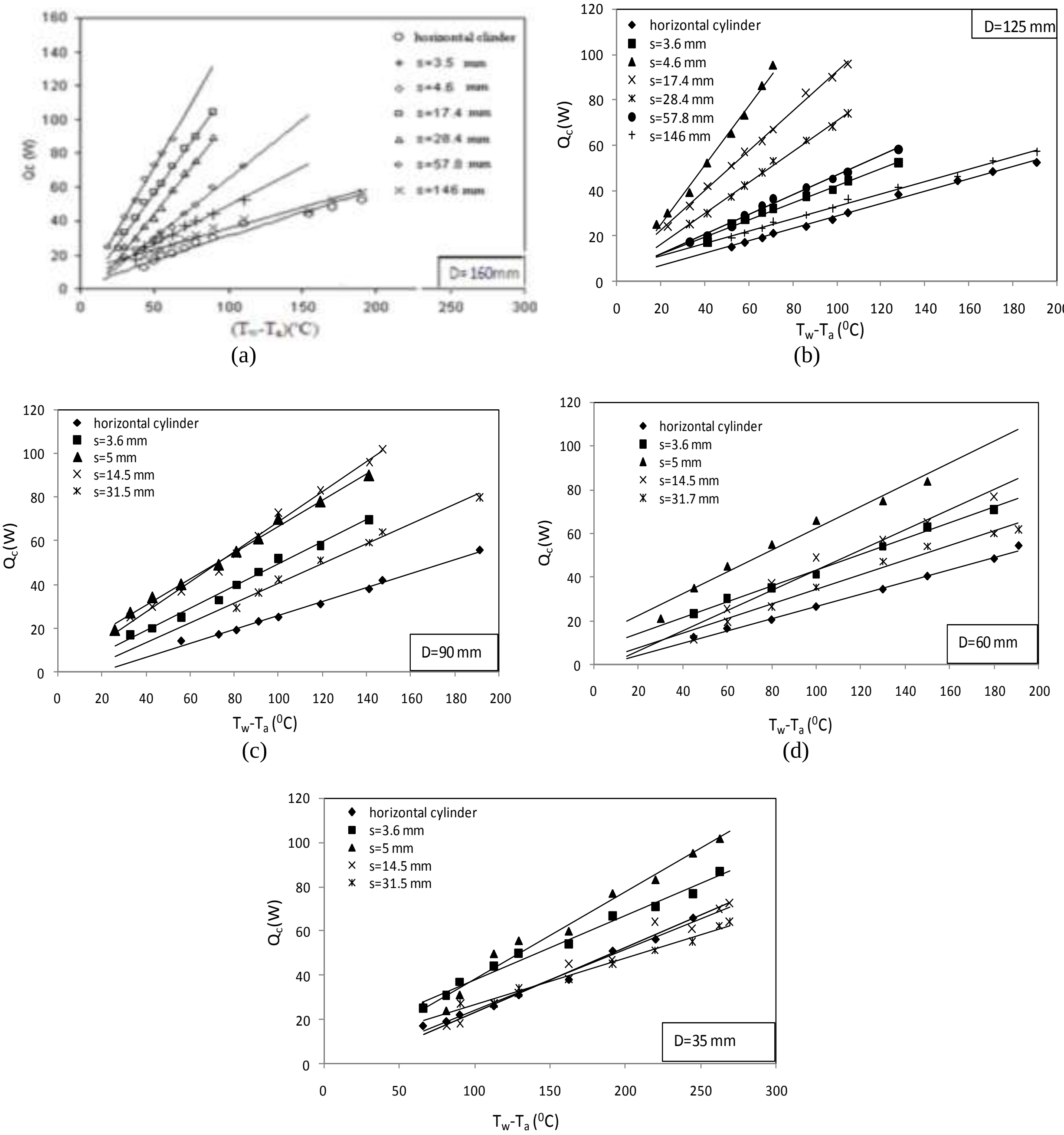


Figure 3. Effect of fin spacing on convection heat transfer rate at fin diameters of D=160, 125, 90, 60 and 35 mm.

the fluid layer surrounding the system. This is the result of buoyancy forces which are a function of difference between finned cylinder temperature and ambient air temperature. In this way the air circulation increases, and this gives rise to increase in the amount of fresh air and so convection heat transfer rate is increased. When all figures are examined collectively, there is one maximum value of convection heat transfer rate from the fin arrays for each fin diameter. This means that there exists an optimum value of fin spacing depending on fin diameter and base-to-ambient temperature differences.

For all investigated cases, velocity vector and temperature distributions were drawn. It is impossible to show all these results. Therefore, as an example, the temperature contours and velocity vectors for D=125 mm, s=3.6 and 17.4 mm and $\Delta T=100^0$C are given in Figs. 6-7.

As can be seen from Fig. 6, cold air enters the solution domain at the bottom and moves along and around the fin and cylinder. The heated air leaves the geometry at the top section of the computational domain. According to the simulation results, as the fin spacing is decreased the velocities also decrease. For a small fin spacing (s=3.6 mm), the velocity distribution and temperature contours do not change very much. For this case it can be said that the flow is similar to fully developed channel flow. In Fig. 7 the distributions for a large fin spacing (s=17.4 mm) are shown. Because the fin spacing is large enough, cold air moves easily inside the

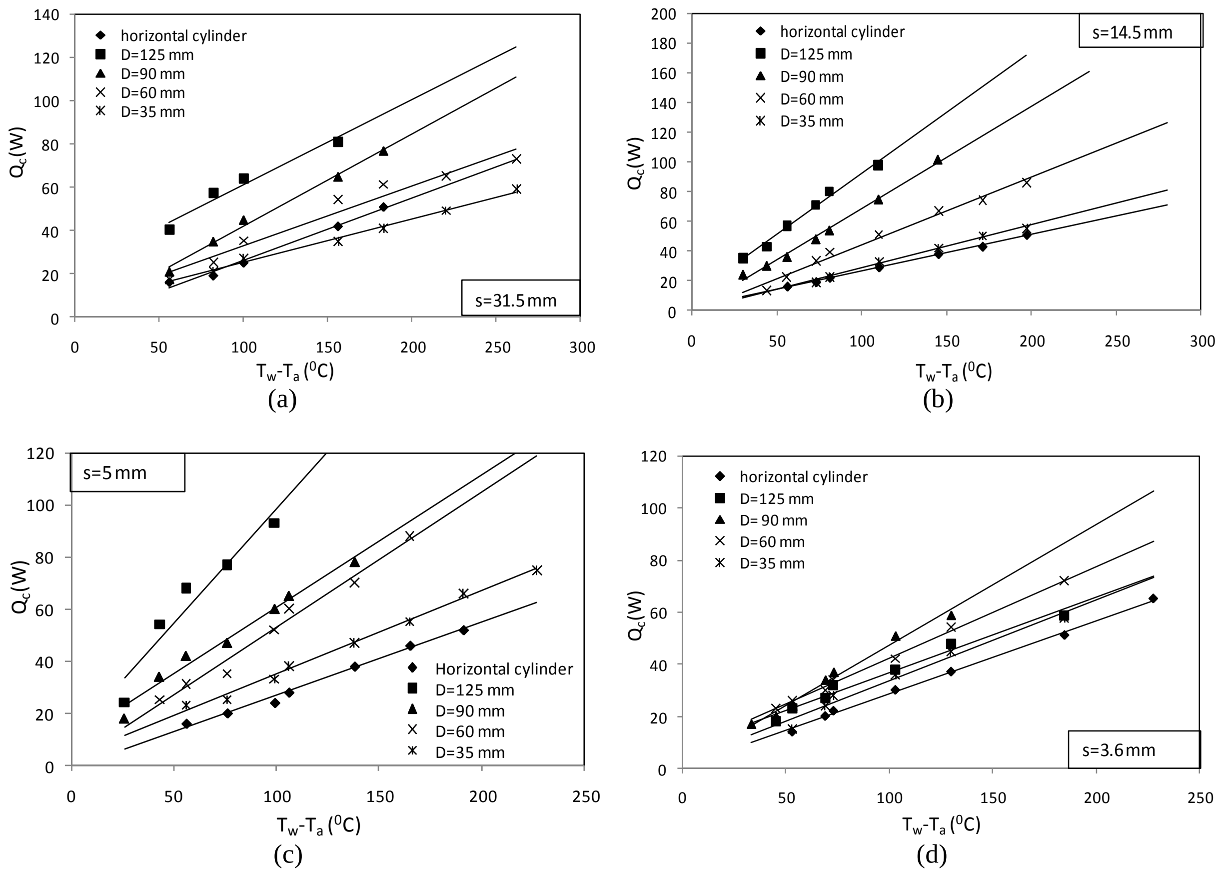


Figure 4. Effect of fin diameter on convection heat transfer rate at a fin spacing of s=31.5, 14.5, 5, 3.6 mm.

fin spacing and then moves upward along the fin diameter. With increase in fin spacing, the cold air rate entering the channel also increases.

OPTIMUM FIN SPACING (s_{opt})

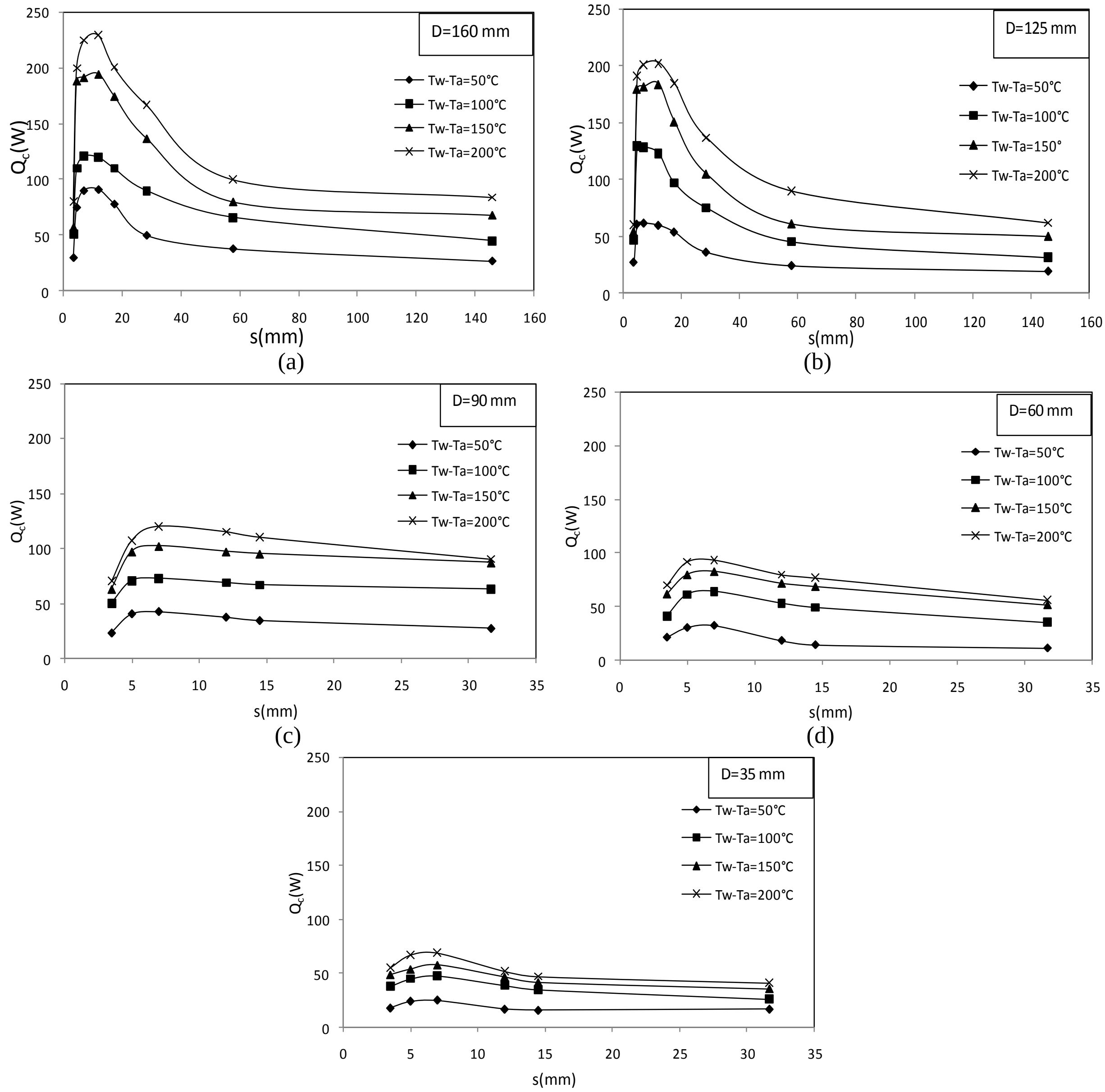
According to the obtained results, for a given fin diameter and temperature difference, with increase in the fin spacing, the convection heat transfer first increases up to a maximum value and after that it continuously decreases. The values of optimum fin spacing for a given fin diameter and base-to-ambient temperature difference are listed in Table 2. The values of optimum fin spacing which maximize the convection heat transfer vary between 7.9 mm and 9.5 mm. From the obtained results, it can be concluded that the fin spacing which maximizes the convection heat transfer is around 8.7 mm, for fin diameters from 35 mm to 160 mm, and for base-to-ambient temperature differences from 50^0C to 200^0C.

Optimum fin spacing ($s_{opt.}$) depends on fin diameter and base-to-ambient temperature difference, but this dependence is not very strong. Yıldız and Yüncü [7] made a scale analysis in their experimental study for a horizontal cylinder of length L, temperature T_w, and for a constant ambient air temperature of T_a. Fins with diameter D and spacing s are used to maximize the total convection heat transfer rate from the array to the ambient air. The thickness of the fins is neglected. The flow is assumed to be laminar. To find an optimum fin spacing, they estimated the boundary in which the fin spacing is minimized, flow was accepted to be fully developed channel flow and the condition in which the boundary layer thickness was very large, the flow was accepted to be boundary layer flow.

The mentioned scale analysis is shown in Fig. 8. The results from the scale analysis, according to their experimental study, in the case of small-s limit, total heat transfer rate is directly proportional to s^2, and in the case of large-s limit, it is inversely proportional with s.

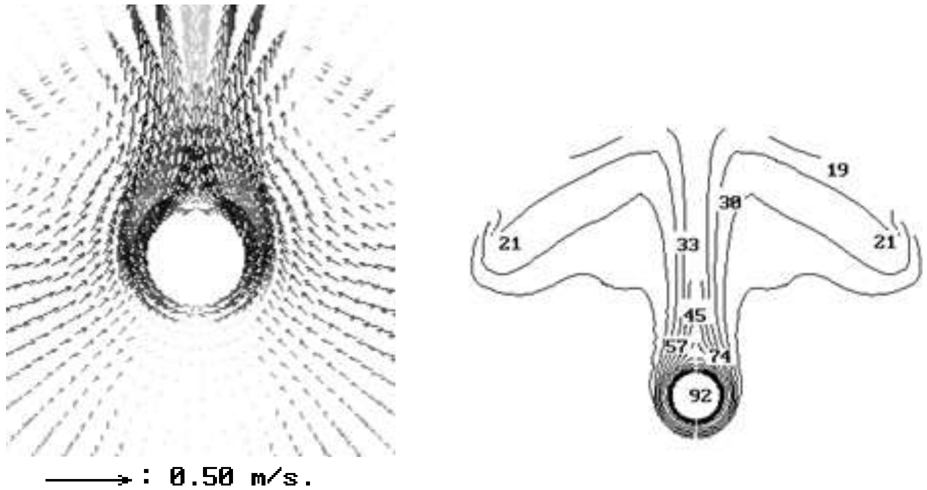
Table 2. The optimum fin spacing (mm)

ΔT(˚C)	D=35mm	D=60 mm	D=90 mm	D=125 mm	D=160 mm
50	8,6	8,7	9	9,3	9,5
100	8,3	8,5	8,8	9,1	9,3
150	8,2	8,2	8,7	9	9,2
200	7,9	8	8,5	8,7	8,9



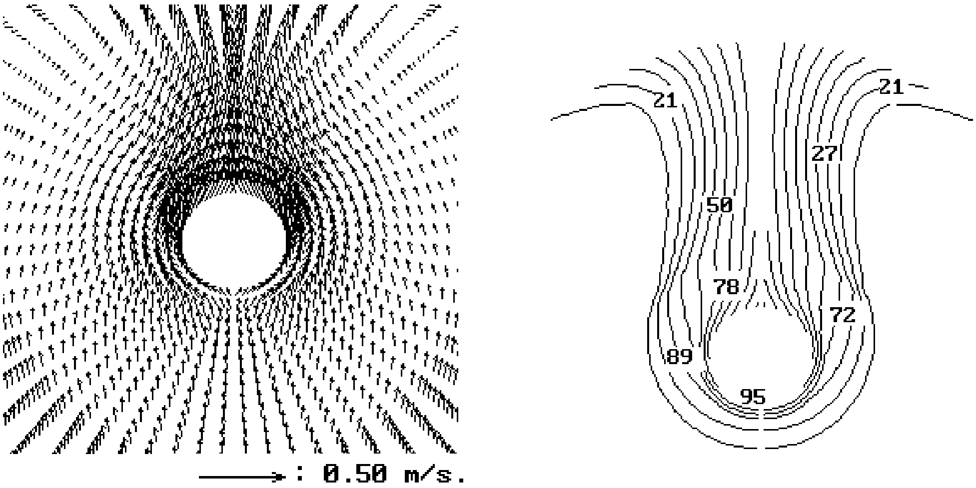
(e)

Figure 5. Effect of base-to-ambient temperature difference on convection heat transfer rate at fin diameters of D=160, 125, 90, 60 and 35mm.



(a) (b)

Figure 6. Velocity vectors and temperature contours for D=125 mm, s=3.6 mm and ΔT =100 $^{\circ}C$ a) velocity vectors, b) temperature contours



(a) (b)

Figure 7. Velocity vectors and temperature contours for D=125 mm, s=17.4mm and ΔT =100 $^{\circ}C$ a) velocity vectors, b) temperature counters

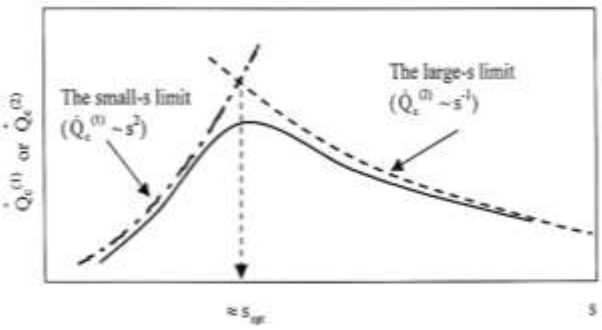


Figure 8. Dimensional analysis of an experimental study by Yıldız and Yüncü [7]

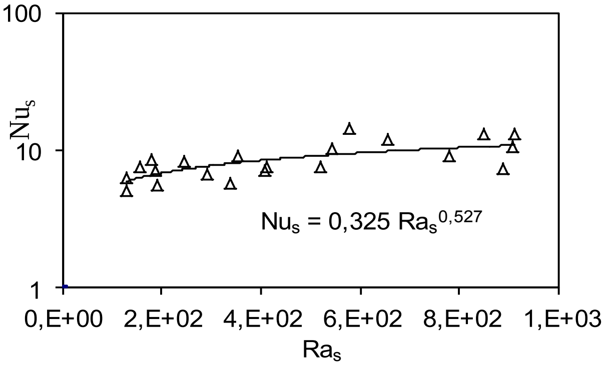


Figure 9. Variation of Nu_s number with Ra_s number for $s<s_{opt}$.

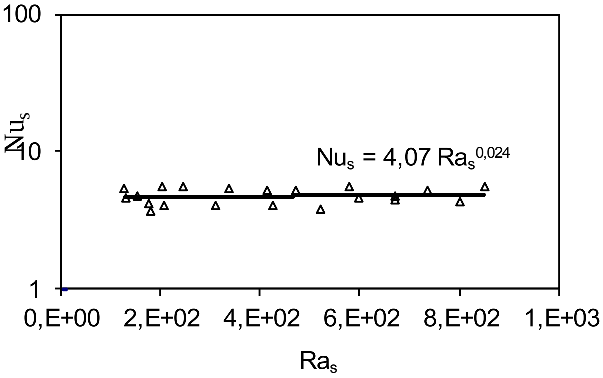


Figure 10. Variation of Nu_s number with Ra_s number for $s>s_{opt.}$

In the figure, $\dot{Q}_c^{(1)}$ is the total convection heat transfer rate from the fin arrays and the small-s limit was expressed as given below (Yıldız and Yüncü [7]):

$$Q_c^{(1)} \approx \frac{\rho.g.\beta.s^3.\Delta T}{v}.D.C_p.\Delta T.\frac{L}{s} \tag{13}$$

The large-s limit was expressed as given below (Yıldız and Yüncü [7]):

$$Q_c^{(2)} \approx \frac{g.\beta.D^3.\Delta T}{v.\alpha}.D.k.\Delta T.\frac{L}{s} \tag{14}$$

where ρ is the density, g is the gravitational acceleration, C_p is the specific heat at constant pressure, ν is the kinematic viscosity, α is the thermal diffusivity, k is the thermal conductivity, ΔT is the base-to-ambient temperature, L is the length of the horizontal cylinder and s is the fin spacing.

According to all calculated numerical results for ΔT=15-300^{0}C and D=35-160 mm, correlations were obtained as shown in Figs. 9-12. If the fin spacing is smaller than its optimum value, Nu_s increases with increasing Ra_s (Fig. 9). If the fin spacing is larger than its optimum value, variation of Nu_s with Ra_s is very small (Fig. 10). For the same temperature differences and fin diameter intervals, correlations with Nu_D and Ra_D are obtained for fin spacings which are both smaller (Fig. 11) and larger than the relevant optimum value (Fig. 12).

Fig. 13 presents correlation and variation of Ra_D number with $s_{opt.}/D$. As shown in the Figure $s_{opt.}/D$ decreases continuously with the increase of the Rayleigh number.

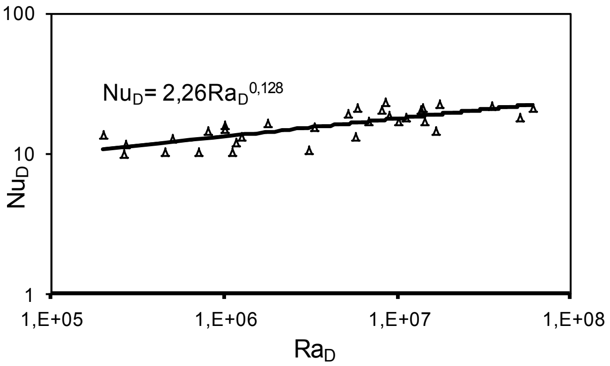


Figure 11. Variation of Nu_D number with Ra_D number for $s>s_{opt.}$

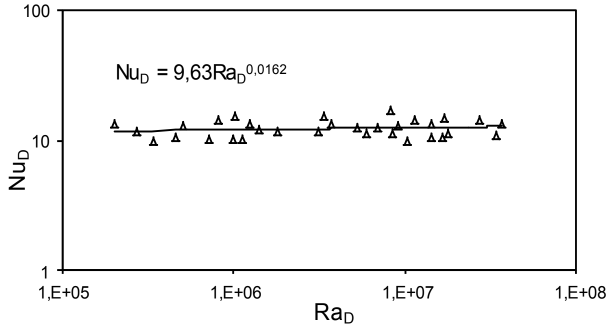


Figure 12. Variation of Nu_D number with Ra_D number for $s<s_{opt}$

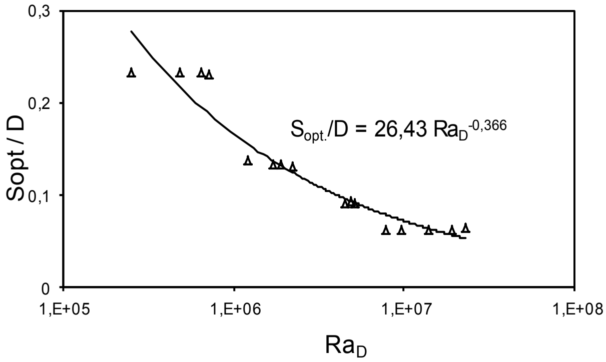


Figure 13. Variation of Ra_D number with $s_{opt.}/D$

The obtained results for all fin diameters and optimum fin spacing, the variation of maximum heat transfer with Rayleigh number and the related correlation are given in Fig. 14.

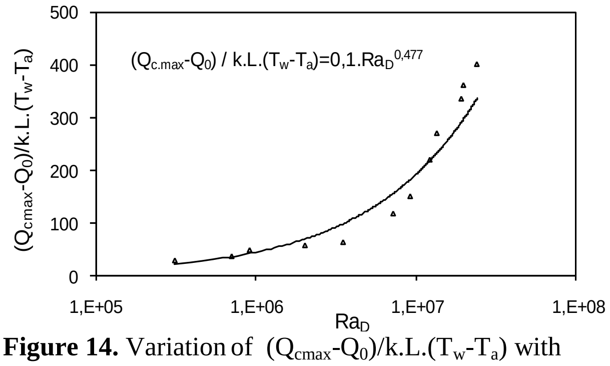


Figure 14. Variation of $(Q_{cmax}-Q_0)/k.L.(T_w-T_a)$ with Ra_D number

Fig. 13 gives a relation between the Rayleigh Number based on fin diameter and ratio of optimum spacing for the case of maximum convection heat transfer rate from the fins. Maximum convection heat transfer rates are further correlated and presented in Fig. 14. In both figures developed correlations are plotted in the same graphs with numerically calculated values. Both results and resulting equations can be applied in the design and analysis of annular fins attached to a horizontal cylinder.

CONCLUSIONS

Three-dimensional steady-state laminar natural convection heat transfer from a finned cylinder has been numerically investigated. The results show that the rate of convection heat transfer from the fin arrays depends on fin diameter, fin spacing and base-to-ambient temperature difference.

For all fin arrangements, convection heat transfer rate from the fin arrays increases with the increase in fin diameter. At low temperature differences, this increase is lower. However, at higher temperature differences, the values obtained for convection heat transfer are significantly different from each other. Besides, for the larger fin diameters, as the fin spacing is increased, convection heat transfer rates decrease and approach that of a horizontal cylinder. Additionally, at a very high fin diameter (D=160 mm) and at a very low fin spacing (3.5 mm), convection heat transfer rate decreases. Due to the fact that air circulation is not fully completed, air pockets cause local temperature increments on the surfaces between the fin arrays. At a given fin diameter, the convection heat transfer rate from the fin arrays increase with decrease in fin spacing, and it reaches a maximum value and then it begins to decrease with decrease in fin spacing. The value of fin spacing maximizing the heat transfer is called the optimum fin spacing. For the optimum fin spacing, a correlation was obtained as given below:

$$\frac{s_{opt}}{D} = 26.43 Ra_D^{-0.366} \qquad (15)$$

The above equation gives a relation between Ra number based on fin diameter and optimum spacing ratio for maximum heat transfer rate from the fin arrays. As a result of the present numerical study, optimum fin spacing maximizing the convection heat transfer is found between 7.9 mm and 9.5 mm. The fin diameter was varied from 35 mm to 160 mm and the base-to-ambient temperature difference was varied from 50^0C to 200^0C. Additionally, as the temperature differences increase, buoyancy induced flow is more effective and as a result of this, the heat removed by cold fluid from the ambient increases. In addition, because of increase in the thermal boundary layer thickness along the cylinder, surface temperature increases.

REFERENCES

Aziz, A., Optimum Dimensions of Extended Surfaces Operating in a Convective Environment, *Appl. Mech. Rev.* 45, 155-173, 1992.

Aziz, A., Kraus, A. D., Optimum Design of Radiating and Convecting-Radiating Fins, *Heat Transfer Engineering* 17, 44-78, 1996.

Chen H. T, Hsu W. L Estimation of Heat Transfer Coefficient on The Fin of Annular-Finned Tube Heat Exchangers in Natural Convection for Various Fin Spacings, *Int. J. Heat Mass Transfer* 50, 1750-1761, 2007.

Haldar, S. C., Laminar Free Convection Around a Horizontal Cylinder with External Longitudinal Fins, *Heat Transfer Engineering* 25, 45-53, 2004.

Haldar, S. C., Kochhar, G.S., Monohar, K., Sahoo, R. K., Numerical Study of Laminar Free Convection About a Horizontal Cylinder with Longitudinal Fins of Finite Thickness, *Int. J. Thermal Sci*. 46, 692-698, 2007.

Karabacak, R., The Effects of Fin Parameters on The Radiation and Free Convection Heat Transfer from a Finned Horizontal Cylindrical Heater, *Energy Convers. Mgmt.* 33, 997-1005, 1992.

Kundu, B., Das, P. K., Performance Analysis and Optimization of Eccentric Annular Disk Fins, *J. Heat Transfer*, 121, 128-135, 1999.

Kundu, B., Das, P. K., Performance Analysis and Optimum Design Analysis of Convective Fin Arrays Attached to Flat and Curved Primary Surfaces, *Int. J. Refrigeration* 32, 430-443, 2009.

Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere, New York, 1980.

Rosten, H., Spalding, D. B., *The PHOENICS Beginners Guide*, CHAM 1, Ltd. London, 1987.

Spalding, D. B., *The PHOENICS Encyclopedia*, CHAM Ltd. London, 1994.

Ullmann, A., Kalman, H., Efficiency and Optimized Dimensions of Annular Fins of Different Cross-Section Shapes, *Int. J. Heat Mass Transfer* 32, 1105-1110, 1989.

Yildiz, Ş., Yüncü, H., An Experimental Investigation on Performance of Annular Fins on a Horizontal Cylinder in Free Convection Heat Transfer, *Heat Mass Transfer*, 40, 239-251,2004.

Wang, C.-S., Yovanovich, M. M., Culham, J. R., General Model for Natural Convection: Application to Annular-Fin Heat Sink, *National Heat Transfer Conference* 5, 119-128, 1997.