



MHD SLIP FLOW BETWEEN PARALLEL PLATES HEATED WITH A CONSTANT HEAT FLUX

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Abstract: The steady fully developed laminar flow and heat transfer of an electrically conducting viscous fluid between two parallel plates heated with a constant heat flux is investigated analytically in the presence of a transverse magnetic field. The momentum and energy equations are solved under the first order boundary conditions of slip velocity and temperature jump. The effects of the Knudsen number, Brinkman number and Hartmann number on the velocity and temperature distribution and heat transfer characteristics are discussed.

Keywords: Parallel plate, MHD, slip-flow, viscous dissipation

SABİT ISI AKISINA MARUZ PARALEL PLAKALAR ARASINDAN MHD KAYMA AKIŞI

Özet: Sabit ısı akısına maruz iki paralel plaka arasından, elektrik iletkenliği olan bir viskoz akışkanın, daimi, tam gelişmiş laminer akışı ve ısı transferi akışa dik bir manyetik alanın varlığında analitik olarak incelenmiştir. Momentum ve enerji denklemleri birinci mertebe kayma hızı ve sıcaklık sıçraması sınır şartları altında çözülmüştür. Knudsen sayısı, Brinkman sayısı ve Hartmann sayısının hız, sıcaklık dağılımı ve ısı transferi karakteristikleri üzerindeki etkisi tartışılmıştır.

Anahtar Kelimeler: Paralel plaka, MHD, kayma akışı, viskoz yayılım

NOMENCLATURE

B_0	magnetic field strength [Tesla]
Br	Brinkman number $[= \mu u_m^2 / k(T_s - T_c)]$
Br_q	modified Brinkman number $[= \mu u_m^2 / Hq_w]$
c_p	specific heat at constant pressure [J /kg K]
F_v	tangential momentum accommodation coefficient
F_t	thermal accommodation coefficient
h	convective heat transfer coefficient [W/ m ² K]
H	half distance between plates [m]
k	thermal conductivity [W/ m K]
Kn	Knudsen number $[= \lambda / 2H]$
M	Hartmann number $[= (\sigma B_0^2 H^2 / \mu)^{1/2}]$
Nu	Nusselt number $[= h(2H)/k]$
p	pressure [Pa]
P	dimensionless pressure
Pr	Prandtl number $[= \mu c_p / k]$
q	heat flux [W/m ²]
T	temperature [K]
u	velocity component in the x-direction [m/s]
U	dimensionless velocity component in the X-direction
x	dimensional axial coordinate [m]
X	dimensionless axial coordinate
y	dimensional normal coordinate [m]
Y	dimensionless normal coordinate

Greek symbols

α	thermal diffusivity of the fluid [m ² /s]
γ	specific heat ratio
λ	molecular mean free path [m]
μ	dynamic viscosity [Pa s]
ρ	density [kg/m ³]
σ	electrical conductivity of the fluid [1/(Ω m)]
ν	kinematic viscosity [m ² /s]
θ	dimensionless temperature $[= (T_s - T)/(T_s - T_c)]$
θ_q	modified dimensionless temperature $[= (T - T_s) / (\frac{q_w H}{k})]$
$\tilde{\theta}$	dimensionless temperature $[= (T_w - T)/(T_s - T_c)]$
$\tilde{\theta}_q$	modified dimensionless temperature $[= (T - T_w) / (\frac{q_w H}{k})]$

Subscripts

c	centerline
m	mean
s	fluid properties at the wall
w	wall

INTRODUCTION

The researchers have recently focused their attention on flow and heat transfer in microchannels because of several application areas such as electronic industry, microfabrication technology and biomedical engineering. Flow and heat transfer characteristics in microscale exhibit major differences compared to macroscale. For example, Navier-Stokes and energy equations with no slip and no temperature jump are no longer valid if the characteristic length scale is in micron range. In micron range, gas no longer reaches the velocity or the temperature of the surface and therefore a slip condition for the velocity and a jump for temperature have to be adopted. Another parameter that should be taken into account in flows in micron range is viscous dissipation. As a result of shear stress, viscous dissipation behaves as an energy source and therefore affects the temperature distribution.

There are four different flow regimes depending on the value of Knudsen number, which is defined as the ratio of the gas mean free path to the characteristic dimension in the flow field.

Continuum flow	$\text{Kn} < 10^{-3}$
Slip Flow	$10^{-3} < \text{Kn} < 10^{-1}$
Transition flow	$10^{-1} < \text{Kn} < 10$
Free molecular flow	$10 < \text{Kn}$

The Knudsen number is a measure of rarefaction and as Kn increases, rarefaction effects become more important, and eventually continuum approach breaks down.

A great deal of research have been undertaken to study the fundamentals of the flow and heat transfer in microchannels. (Beskok et al., 1996) proposed models and a computational strategy to simulate gas micro flow in the slip flow region for Knudsen number less than 0.3. (Arkilic et al., 1997) made an analytic and experimental investigation into gaseous flow with slight rarefaction through long microchannels. Their results showed that the analytic solution for the streamwise mass flow corresponds well with the experimental results. Convective heat transfer for steady laminar hydrodynamically developed flow in circular microtubes and rectangular microchannels were investigated by (Tunc and Bayazitoglu, 2001, 2002). They found that the temperature jump effect diminishes as the Prandtl number increases. (Hadjiconstantinou and Simek, 2002) studied the constant wall temperature convective heat transfer characteristics of a model gaseous flow in two-dimensional micro and nanochannel under hydrodynamically and thermally fully developed conditions and concluded that the Nusselt number decreases monotonically with increasing Knudsen number both in slip flow and transition regime. The results also showed that axial heat conduction increases the Nusselt number in slip flow regime. (Hadjiconstantinou, 2003) studied the effect of viscous dissipation in small scale gaseous flow

and presented predictions for the dissipation in terms of mean flow velocity in pressure-driven and gravity-driven Poiseuille flows. Viscous dissipation effects in microtubes and microchannels were studied by (Koo and Kleinstreuer, 2003, 2004) using dimensional analysis and experimentally validated computer simulations. Their results showed that ignoring viscous dissipation could affect accurate flow simulations and measurements in microconduits. Laminar slip-flow forced convection in rectangular microchannels was studied analytically by (Yu and Ameen, 2001) by applying a modified generalized integral transform technique and found that increasing temperature jump reduces heat transfer and thermal entrance length and Nusselt number shows a decrease with increasing aspect ratio. The wall effect on heat transfer characteristics was investigated by (Li et al., 2000) for laminar flow through microchannels. Their investigation showed that the change in thermal conductivity of gas in wall adjacent layer has a significant influence on the heat transfer when passage size is small. (Aydn and Avci, 2007) investigated forced convective heat transfer in a micro channel heated with a constant heat flux or a constant wall temperature. They determined the interactive effects of the Brinkman number and the Knudsen number on the Nusselt numbers analytically. Heat and fluid flow in a rectangular microchannel filled with a porous medium was analyzed by (Hooman, 2008). The results showed that as the permeability is decreased, velocity distribution tends to be more uniform and therefore Nusselt number increases. (Shams et al., 2009) studied slip flow in rhombus microchannels and concluded that aspect ratio and Knudsen number have important effect on Poiseuille number and Nusselt number. The results also showed that Reynolds number has also considerable influence on Nu number at low Re numbers. (Darbandi et al., 2008) presented a theoretical approach to predict the temperature field in micro-Poiseuille channel flow with constant temperature. Their formulations predict the temperature field in the channel readily compared to the past analytical solutions. Heat transfer characteristics of gaseous flows in microchannel with negative heat flux were investigated by (Hong and Asako, 2007) and a correlation was proposed for the prediction of the wall temperature. Flow and heat transfer characteristics in micro-Couette flow was investigated by (Xue and Shu, 2003). Their results showed that the solution of Burnett equations is superior to that of the Navier-Stokes equations at relatively high Kn numbers in the slip flow regime. Viscous dissipation effect on heat transfer characteristics of rectangular microchannels was studied by (Aynur et al., 2006). It was found that the viscous dissipation is negligible for gas flows in microchannels since the contribution of this effect on Nu number is about 1%. (Yu and Ameen, 2001) found a universal Nusselt number for laminar slip flow heat transfer at the entrance of a conduit. They also obtained that the Nu number expression is valid for both isothermal and isoflux thermal boundary conditions and for any conduit geometries. (Wang et al., 2008a, 2008b) used inverse

temperature sampling method to deal with diatomic results showed that this model can accurately simulate the flow under uniform heat flux boundary condition. They also concluded that gaseous rarefaction and compressibility increase with the increase of the wall heat flux. (Shams et al., 2009) performed a numerical simulation for incompressible and compressible fluid flows through microchannels in slip flow regime and found that the effect of compressibility is more noticeable when relative roughness increases. (Zhang et al., 2010) studied the effect of viscous heating on heat transfer performance in microchannel slip flow region and found that the viscous heating causes severely distortions on the temperature profile. (Aydin and Avci, 2006) performed an analysis on forced convection heat transfer in a micropipe in slip flow and analytically determined the Nusselt number as a function of Brinkman number and Knudsen number. The Knudsen number was shown to decrease the Nusselt number for low values of the Brinkman number. Laminar forced convection slip-flow in a micro-annulus between two concentric cylinders was studied by (Avci and Aydin, 2008). The results showed that an increase at Kn number decreases the Nu due to the temperature jump at the wall. Furthermore, Nusselt number decreases with increasing Brinkman number for the hot wall and increases with increasing Brinkman number for the cold wall. They also found that Br is more effective on Nu for lower values of Kn than for higher values of Kn. As is seen from the literature given above, there are many studies on slip flow in microchannels. In these studies, various parameters such as geometry and boundary conditions have been into consideration to reveal the effect of these parameters on flow and heat transfer. But there is only a little number of studies on the effect of magnetic field on the flow. In one of these studies, (Soundalgekar, 1966) studied MHD Couette flow of a rarefied gas and found that velocity profile is affected considerably by magnetic field. (Soundalgekar, 1967) also investigated MHD slip flow and heat transfer in a channel with walls having different temperatures and found that Nusselt number decreases with an increase in the slip parameter and temperature jump coefficient. (Makinde and Chinyoka, 2010) analyzed MHD transient flow and heat transfer of a dusty fluid numerically in a channel with Navier slip condition and found that an increase in the Hartmann number causes a decrease in the velocity and therefore in the dissipation. A new approximate solution for the velocity profile of steady incompressible MHD flow in a rectangular microchannel was proposed by (Kabbani et al., 2008). Their results showed that the proposed solutions agree better with existing experimental and computational reports than previous approximations. (Smolentsev, 2009) studied MHD duct flows under hydrodynamic slip condition and concluded that the Hartmann layers still exhibit in the duct flows with the slip. (Srinivas and Muthuraj, 2010) examined the problem of MHD flow in a vertical wavy porous space in the presence of a temperature-dependent heat source with slip flow boundary condition. They concluded that cross velocity profiles exhibit oscillatory character.

gaseous flow and heat transfer in a microchannel. Their In this study, MHD slip flow between two parallel plates heated with a constant heat flux was studied and the effect of rarefaction, viscous dissipation and magnetic field on the flow and heat transfer was revealed.

ANALYSIS

In this study, slip flow and heat transfer between parallel plates heated with a constant heat flux is investigated analytically in the presence of a transverse magnetic field. The geometry and the coordinate system are shown in Figure 1. The flow is assumed to be steady, laminar, incompressible and fully developed and fluid properties to be constant. The magnetic Reynolds number is assumed very small and the induced magnetic field due to motion of the electrically conducting fluid is neglected. Hall effect and Joule heating are also neglected. The coordinate system is located at the center of the microchannel. The x-axis is taken along the centerline of the microchannel and y-axis is taken normal to it. u is the dimensional velocity component in the x-direction. Under these conditions, the governing equations including the viscous dissipation effect are as follows:

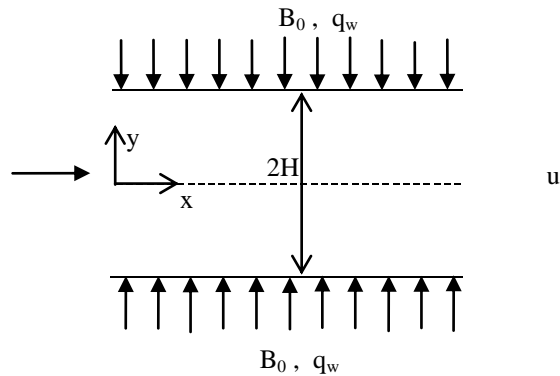


Figure 1. Geometry and the coordinate system.

$$\frac{d^2u}{dy^2} = \frac{1}{\mu} \frac{dp}{dx} + \frac{\sigma B_0^2}{\mu} u \quad (1)$$

$$u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{du}{dy} \right)^2 \quad (2)$$

Here μ , p , B_0 , α , v , c_p , T , σ , are the dynamic viscosity, pressure, magnetic field strength, thermal diffusivity, kinematic viscosity, specific heat, temperature and electrical conductivity of the fluid, respectively. In a slip flow regime, due to the rarefaction effect, flow will slip and the temperature will jump at the wall. First order velocity-slip and temperature-jump boundary conditions are defined as (Gad-el-Hak, 2006):

$$u_s = - \frac{2 - F_v}{F_v} \lambda \frac{\partial u}{\partial y} \Big|_{y=H} \quad (3)$$

$$T_s - T_w = - \frac{2 - F_t}{F_t} \frac{2\gamma}{\gamma + 1} \frac{\lambda}{Pr} \frac{\partial T}{\partial y} \Big|_{y=H} \quad (4)$$

where u_s is the slip velocity, λ the molecular mean free path, F_v the tangential momentum accommodation coefficient, T_s the temperature of the gas at the wall, T_w the wall temperature, F_t thermal accommodation coefficient, γ the specific heat ratio and Pr the Prandtl number. F_v and F_t are dependent on the gas and material of the surface and their values near unity are typical for engineering applications (particularly for air) (Zhang, 2007). Therefore, they are taken as unity.

Velocity Profile

Dimensionless variables are defined as:

$$X = \frac{x}{H}, Y = \frac{y}{H}, U = \frac{u}{u_m}, U_s = \frac{u_s}{u_m}, P = \frac{pH}{\mu u_m} \quad (5)$$

where u_m is the mean velocity and is defined as

$$u_m = \frac{1}{2H} \int_{-H}^H u dy \quad (6)$$

The dimensionless momentum equation in the x -direction and associated boundary conditions for hydrodynamically fully-developed MHD Poiseuille flow take the following form:

$$\frac{d^2U}{dY^2} = \frac{dP}{dX} + M^2U \quad (7)$$

$$\frac{dU}{dY} \Big|_{Y=0} = 0 \quad (8)$$

$$U = U_s = -2Kn \frac{dU}{dY} \Big|_{Y=1} \quad (9)$$

where M is the Hartmann number defined as:

$$M = \left(\frac{\sigma B_0^2 H^2}{\mu} \right)^{\frac{1}{2}} \quad (10)$$

The analytical solution of Eq.(7) subjected to the boundary conditions given in Eqs.(8-9) is obtained as follows:

$$U = \frac{\Delta_1}{\Delta_2} \quad (11)$$

$$\Delta_1 = M[\cosh(MY) - \cosh(M) - 2KnM \sinh(M)] \quad (12)$$

$$\Delta_2 = \sinh(M) [1 - 2KnM^2] - M \cosh(M) \quad (13)$$

Temperature Distribution

The constant heat flux is assumed at the wall.

$$q_w = k \frac{\partial T}{\partial y} \Big|_{y=H} \quad (14)$$

The temperature gradient for the fluid is equal to the temperature gradient of the wall for the constant heat flux case:

$$\frac{\partial T}{\partial x} = \frac{dT_w}{dx} = \frac{dT_s}{dx} \quad (15)$$

Dimensionless temperature can be defined as:

$$\theta = \frac{T_s - T}{T_s - T_c} \quad (16)$$

Dimensionless energy equation then takes the following form:

$$\frac{\partial^2 \theta}{\partial Y^2} = aU + Br \left(\frac{dU}{dY} \right)^2 \quad (17)$$

where $a = -\frac{H^2 u_m}{\alpha(T_s - T_c)} \frac{dT_s}{dx}$ and Br is the Brinkman number, which shows the effect of viscous dissipation and it is defined as:

$$Br = \frac{\mu u_m^2}{k(T_s - T_c)} \quad (18)$$

Dimensionless thermal boundary conditions therefore take the following form:

$$\theta = 1, \quad \frac{d\theta}{dY} \Big|_{Y=0} = 0 \quad (19)$$

$$\theta = 0 \text{ at } Y = 1 \quad (20)$$

The solution of Eq. (17) under thermal boundary conditions given in Eqs. (19-20) are:

$$\theta(Y) = \frac{T_s - T}{T_s - T_c} = \left[-Br \frac{\Delta_4}{\Delta_2} + 1 \right] \frac{\Delta_5}{\Delta_3} + Br \frac{\Delta_6}{\Delta_2} - Y + 1 \quad (21)$$

$$\Delta_3 = \cosh(M) \left[\frac{M}{2} - \frac{1}{M} \right] + KnM^2 \sinh(M) + \frac{1}{M} \quad (22)$$

$$\Delta_4 = \frac{M^2}{8} [-\cosh(2M) + 2M^2 + 1] \quad (23)$$

$$\Delta_5 = \frac{1}{M} \left[\cosh(MY) - \frac{1}{2} \cosh(M) Y [M^2(Y-1) + 2] \right] + \left[-KnM^2 Y \sinh(M) + \frac{1}{M} \right] [Y-1] \quad (24)$$

$$\Delta_6 = \frac{M^2}{8} [\cosh(2MY) + Y [-\cosh(2M) - 2M^2(Y-1) + 1] - 1] \quad (25)$$

We can also use the modified Brinkman number defined as:

$$Br_q = \frac{\mu u_m^2}{H q_w} \quad (26)$$

In this case, temperature distribution takes the following form:

$$\theta_q(Y) = \frac{T - T_s}{\frac{q_w H}{k}} = \left[Br_q \frac{\Delta_7}{\Delta_2} + 1 \right] \frac{\Delta_8}{\Delta_2} + Br_q \frac{\Delta_9}{\Delta_2} \quad (27)$$

$$\Delta_7 = \frac{M^3}{4} [\sinh(2M) - 2M] \quad (28)$$

$$\Delta_8 = \frac{1}{M} \left[\cosh(MY) - \frac{1}{2} \cosh(M) [M^2(Y^2 - 1) + 2] \right] - KnM^2 \sinh(M) [Y^2 - 1] \quad (29)$$

$$\Delta_9 = \frac{M^2}{8} [-\cosh(2MY) + \cosh(2M) + 2M^2(Y^2 - 1)] \quad (30)$$

Eqs. (21) and (27) are in terms of T_s . These equations can be transformed into the equations in terms of T_w using the following conversion formulas:

$$\frac{T_s - T_w}{T_s - T_c} = \frac{4\gamma}{\gamma + 1} \frac{Kn}{Pr} \frac{\partial \theta}{\partial Y} \Big|_{Y=1} \quad (31)$$

$$\frac{T_s - T_w}{\frac{q_w H}{k}} = -\frac{4\gamma}{\gamma + 1} \frac{Kn}{Pr} \frac{\partial \theta_q}{\partial Y} \Big|_{Y=1} \quad (32)$$

Therefore, Eqs. (21) and (27) become as follows:

$$\tilde{\theta}(Y) = \frac{T_w - T}{T_s - T_c} = \left[-Br \frac{\Delta_4}{\Delta_2^2} + 1 \right] \frac{\Delta_5}{\Delta_3} + Br \frac{\Delta_6}{\Delta_2^2} - Y + 1 - \frac{4\gamma}{\gamma + 1} \frac{Kn}{Pr} \left\{ \left[-Br \frac{\Delta_4}{\Delta_2^2} + 1 \right] \frac{\Delta_{10}}{\Delta_3} + Br \frac{\Delta_{11}}{\Delta_2^2} - 1 \right\} \quad (33)$$

$$\Delta_{10} = -\frac{1}{2M} \cosh(M) [M^2 + 2] + \sinh(M) [1 - KnM^2] + \frac{1}{M} \quad (34)$$

$$\Delta_{11} = \frac{M^2}{8} [2M \sinh(2M) - \cosh(2M) - 2M^2 + 1] \quad (35)$$

and

$$\tilde{\theta}_q = \frac{T - T_w}{\frac{q_w H}{k}} = \left[Br_q \frac{\Delta_7}{\Delta_2^4} + 1 \right] \frac{\Delta_8}{\Delta_2} + \frac{4\gamma}{\gamma + 1} \frac{Kn}{Pr} \quad (36)$$

For fully developed flows, mean fluid temperature is usually used instead of the centerline temperature in defining the Nusselt number. The mean temperature is defined as:

$$T_m = \frac{\int \rho u T dA}{\int \rho u dA} \quad (37)$$

The dimensionless mean temperature in terms of Brinkman and modified Brinkman number can be written as :

$$\tilde{\theta}_m = \frac{T_m - T_w}{T_s - T_c} = \left[-Br \frac{\Delta_4}{\Delta_2^2} + 1 \right] \frac{\Delta_{12}}{\Delta_2 \Delta_3} + Br \frac{\Delta_{13}}{\Delta_3^2} + \frac{\Delta_{14}}{\Delta_2} - \frac{4\gamma}{\gamma + 1} \frac{Kn}{Pr} \left\{ \left[-Br \frac{\Delta_4}{\Delta_2^2} + 1 \right] \frac{\Delta_{10}}{\Delta_3} + Br \frac{\Delta_{11}}{\Delta_2^2} - 1 \right\} \quad (38)$$

$$\Delta_{12} = \frac{1}{24M^2} [\cosh(2M) [-4Kn^2M^6 - 48KnM^2 - M^4$$

$$+ 12M^2 + 12] + 4Kn^2M^6 + 48KnM^2 - M^4 + 24M^2 + 36] + \frac{1}{M^2} \cosh(M) [M^2 - 2] + \frac{1}{12M} \sinh(2M) [-2KnM^4 + 12KnM^2 - 15] + 2KnM \sinh(M) \quad (39)$$

$$\Delta_{13} = \frac{M^2}{96} [\sinh(3M) [6KnM^2 - 7] + \sinh(M) [-8KnM^4 + 6KnM^2 - 39]] + \frac{M}{32} [\cosh(3M) [-2KnM^2 + M^2 + 2] + \frac{1}{3} \cosh(M) [6KnM^2 - 4M^4 + 33M^2 - 6]] + \frac{M}{8} [-\cosh(2M) + 2M^2 + 1] \quad (40)$$

$$\Delta_{14} = \frac{1}{2M} \cosh(M) [-M^2 + 2] - KnM^2 \sinh(M) - \frac{1}{M} \quad (41)$$

$$\tilde{\theta}_{q,m} = \frac{T_m - T_w}{\frac{q_w H}{k}} = \frac{Br_q}{\Delta_2^3} \left[\frac{\Delta_{15}}{4\Delta_2} + \Delta_{16} \right] - \frac{4\gamma}{\gamma + 1} \frac{Kn}{Pr} \quad (42)$$

$$\Delta_{15} = \frac{M^3}{6} [\sinh(2M) - 2M^4] \left[[\cosh(2M) [-4Kn^2M^4 - 12Kn - M^2 + 6] + \frac{1}{2M} \sinh(2M) [-8KnM^4 + 24KnM^2 - 15] + 4Kn^2M^4 + 12Kn - M^2 + 9] \right] \quad (43)$$

$$\Delta_{16} = \frac{M^2}{16} [M \cosh(3M) [Kn - 1] + \frac{M}{3} \cosh(M) [-3Kn + 8M^2 - 27] + \frac{1}{6} [\sinh(3M) [-12KnM^2 + 7] + \sinh(M) [32KnM^4 + 12KnM^2 + 39]]] \quad (44)$$

The forced convective heat transfer coefficient is defined as:

$$h = \frac{q_w}{T_w - T_m} \quad (45)$$

Therefore Nusselt number can be given as:

$$Nu = -\frac{2 \frac{d\tilde{\theta}}{dY} \Big|_{Y=1}}{\tilde{\theta}_m} \quad (46)$$

$$Nu = -\frac{2}{\tilde{\theta}_{q,m}} \quad (47)$$

After the necessary substitution, the Nusselt number is obtained as

$$\text{Nu} = \left\{ \frac{2}{\Delta_3} \left[\text{Br} \frac{\Delta_4}{\Delta_2^2} - 1 \right] \Delta_{17} - \text{Br} \frac{\Delta_{18}}{4\Delta_2^2} + 2 \right\} \left\{ -\text{Br} \frac{\Delta_4}{\Delta_2^2} + 1 \right\} \frac{\Delta_{12}}{\Delta_2 \Delta_3} + \text{Br} \frac{\Delta_{13}}{\Delta_2^3} + \frac{\Delta_{14}}{\Delta_2} - \frac{4\gamma}{\gamma + 1} \frac{\text{Kn}}{\text{Pr}} \left\{ \left[-\text{Br} \frac{\Delta_4}{\Delta_2^2} + 1 \right] \frac{\Delta_{10}}{\Delta_3} + \text{Br} \frac{\Delta_{11}}{\Delta_2^2} - 1 \right\}^{-1} \quad (48)$$

$$\Delta_{17} = \sinh(M) - \cosh(M) \left[\frac{M}{2} + \frac{1}{M} \right] - \text{Kn} M^2 \sinh(M) + \frac{1}{M} \quad (49)$$

$$\Delta_{18} = M^2 [2M \sinh(2M) - \cosh(2M) - 2M^2 + 1] \quad (50)$$

Using the modified Brinkman number, the Nusselt number can also be written as follows:

$$\text{Nu} = -2 \left\{ \frac{\text{Br}_q}{\Delta_2^3} \left[\frac{\Delta_{15}}{4\Delta_2} + \Delta_{16} \right] - \frac{4\gamma}{\gamma + 1} \frac{\text{Kn}}{\text{Pr}} \right\}^{-1} \quad (51)$$

Asymptotic Behavior of the Velocity and Temperature Distribution

The asymptotic behavior of the flow can be obtained by using the Taylor series expansions.

$$U = \frac{3}{2} \frac{\left[1 - Y^2 + 4\text{Kn} \left(1 + \frac{M^2}{6} \right) \right]}{\left[1 + 6\text{Kn} \left(1 + \frac{M^2}{6} \right) \right]} \quad (52)$$

For $M \rightarrow 0^+$

$$\lim_{M \rightarrow 0^+} U = \frac{3}{2} \frac{[1 - Y^2 + 4\text{Kn}]}{[1 + 6\text{Kn}]} \quad (53)$$

As is seen, the velocity distribution for MHD slip flow approaches asymptotically to that for the ordinary slip flow. If we follow the same procedure, equation for temperature distribution can be written as:

$$\begin{aligned} \theta(Y) = & 3\text{Br} \left[\frac{M^2}{8} + 6\text{Kn} \left(1 + \frac{M^2}{6} + \frac{M^4}{120} \right) + \frac{3}{2} \right] [Y^4 \\ & - Y^2] \left[\frac{M^2}{10} + 6\text{Kn} \left(1 + \frac{M^2}{6} + \frac{M^4}{120} \right) + 1 \right]^{-2} \left[\frac{M^2}{2} \right. \\ & + 24\text{Kn} \left(1 + \frac{M^2}{6} + \frac{M^4}{120} \right) + 5 \left. \right]^{-1} + \left[Y^2 [Y^2 - 6] - \left[\frac{M^2}{2} \right. \right. \\ & + 24\text{Kn} \left(1 + \frac{M^2}{6} + \frac{M^4}{120} \right) \left. \left. [Y^2 - 1] + 5 \right] \right] \left[\frac{M^2}{2} \right. \\ & + 24\text{Kn} \left(1 + \frac{M^2}{6} + \frac{M^4}{120} \right) + 5 \left. \right]^{-1} \quad (54) \end{aligned}$$

For $M \rightarrow 0^+$

$$\lim_{M \rightarrow 0^+} \theta = \frac{1}{5 + 24\text{Kn}} \left[\frac{9}{2} \text{Br} \frac{[1 + 4\text{Kn}]}{[1 + 6\text{Kn}]^2} [Y^4 - Y^2] + Y^2 [Y^2 - 6] - 24\text{Kn} [Y^2 - 1] + 5 \right] \quad (55)$$

As is shown, temperature distribution for MHD slip flow approaches asymptotically to that for the ordinary slip flow (Aydm and Avci, 2007).

RESULTS AND DISCUSSION

In this study, hydrodynamically and thermally fully developed MHD slip flow between parallel plates heated with a constant heat flux is studied analytically. The results are obtained for the Knudsen numbers ranging from 0 to 0.1, for the Brinkman number ranging from -0.1 to 0.1 and for the Hartmann number ranging from 0 to 2. Air is taken as the working fluid with $\text{Pr}=0.71$. The dimensionless axial velocity profile for various values of Knudsen number and Hartmann number are shown in Figure 2. As it can be seen from the figure, slip velocity of fluid on the microchannel wall increases as the Knudsen number increases. Furthermore, higher slip is associated with the lower core velocities, as expected. As shown in Figure 2, presence of a transverse magnetic field leads to a decrease at velocity with an increase of the velocity gradients near the walls.

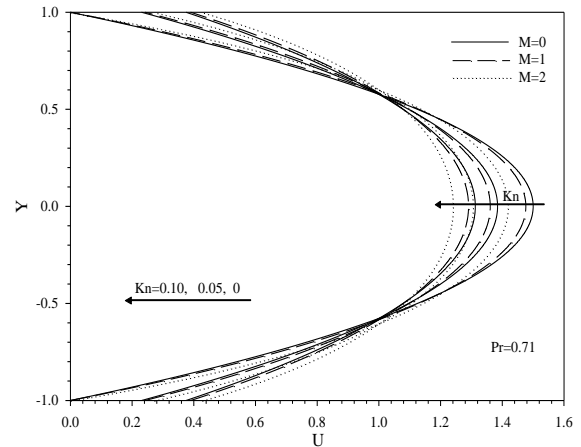


Figure 2. Fully developed velocity profiles for various values of Hartmann number.

The dimensionless temperature distributions are shown in Figure 3 for different values of the governing parameters. As was stated before, velocity takes lower values with increasing of Kn and M numbers. Therefore, temperature takes higher values.

Variation of the Nusselt number with the Knudsen number is shown in Figures 4 and 5 for various values of the Hartmann number, Brinkman number and modified Brinkman number. As the Knudsen number increases, temperature jump at the microchannel walls increases due to the rarefaction effect. Temperature jump at the wall reduces the Nusselt number. The percentage decrease in the Nusselt number in the slip

flow range taken into consideration was given in Table 1. As it can be observed, decrease in the Nusselt number is more pronounced for negative values of the Brinkman number, particularly for higher values of Hartmann number.

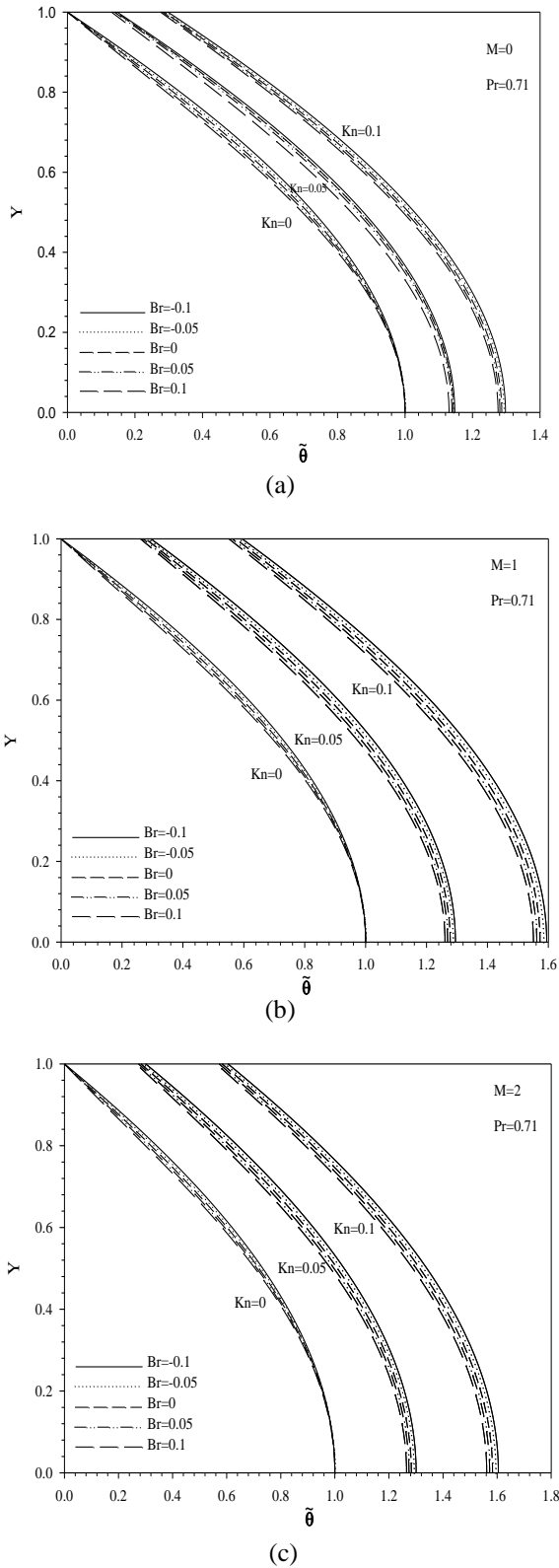


Figure 3. Variation of the temperature profiles for a) $M=0$, b) $M=1$ and c) $M=2$.

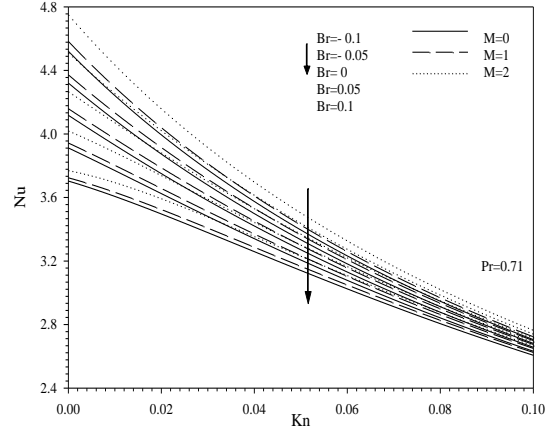


Figure 4. Variation of the Nu number with Kn number for various values of Br and M.

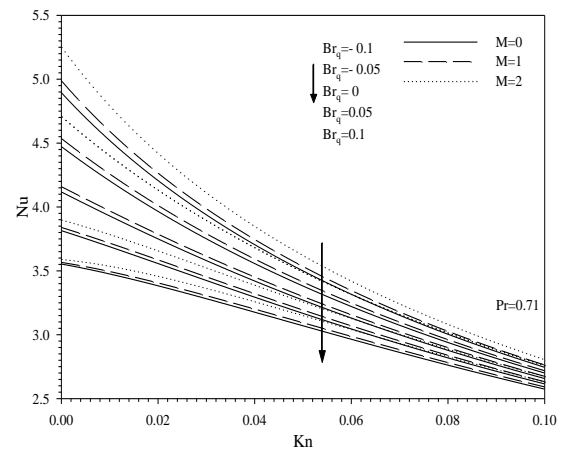


Figure 5. Variation of the Nu number with Kn number for various values of Br_q and M.

Table 1. Percentage decrease in Nusselt number between $Kn=0$ and $Kn=0.1$ (slip flow range).

Br	M=0	M=1	M=2
	%	%	%
-0.1	40.16	40.58	41.77
-0.05	37.94	38.27	39.20
0	35.48	35.68	36.89
0.05	32.72	32.82	33.03
0.1	29.62	29.54	29.25

Positive values of the Brinkman number mean that the fluid is being heated by a hot wall, while the opposite is true for negative values of it. For the cold wall case, viscous dissipation increases the temperature difference between the wall and the fluid. Therefore, the Nusselt number increases with an increase of the Brinkman number in the negative direction. On the other hand, viscous dissipation decreases the temperature difference between the wall and the fluid for the hot wall case. Therefore, the Nusselt number takes lower values with increasing Brinkman number. Because of the increase in the velocity gradient on the microchannel walls, Nusselt number increases with an increase of the Hartmann number. A comparison between Figures 4 and 5 shows that the behavior of the Nusselt number versus the Knudsen number for different Br_q is similar to for different Br . The variation of the Nusselt number with

Brinkman number and modified Brinkman number is shown in Figures 6 and 7 for various values of Knudsen number and Hartmann number.

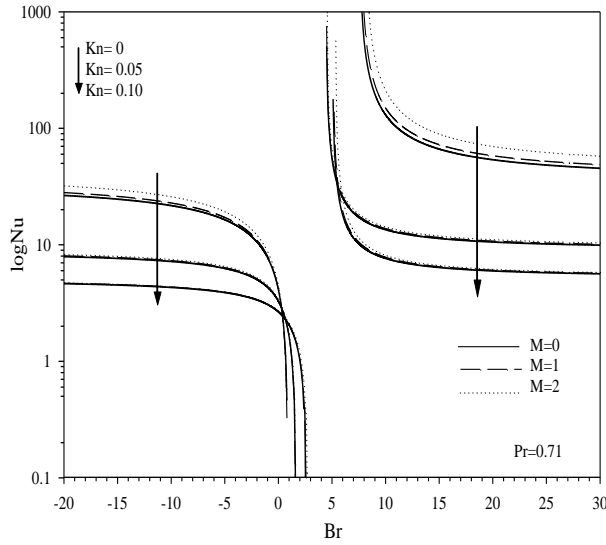


Figure 6. Variation of the Nu number with Br for various values of Kn and M.

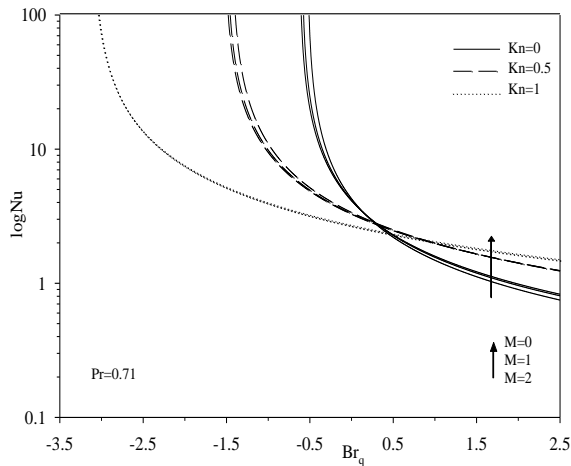


Figure 7. Variation of the Nu number with Br_q for various values of Kn and M.

As it can be observed from the figures, a singularity is observed at a specific Br for each Knudsen number as other investigators (Aydın and Avcı, 2007, Zhang et al., 2010) have reported and for each magnetic parameter. The reason for this behavior is that heat transfer between the wall and the fluid is equal to the internal heat generation due to the viscous dissipation for these values of the Brinkman number. The Brinkman numbers that singularities appear in Nusselt number were reported on Table 2. As can also be observed from figures, the Nusselt number reaches the same asymptotic value when the Brinkman number goes to the infinity for either the hot wall or the cold wall case.

Table 2. Brinkman numbers that singularities appear in Nusselt number.

Kn	M=0	M=1	M=2
0	7.555	7.7122	8.0787
0.05	4.4493	4.4653	4.4766
0.1	5.0278	5.1040	5.2758

In order to validate the analysis, the results of this study for $M=0$ were compared with the results available in the literature (see Table 3). As it can be observed from Table 3, there is a good agreement between the results.

Table 3. The fully developed Nusselt numbers for $M=0$ ($Pr=0.71$).

Kn	Hadjiconstantinou, Aydın and Avcı, (2007)					
	Present		Hadjiconstantinou, (2003)		Aydın and Avcı, (2007)	
	Br					
	0	0.01	0	0.01	0	0.01
0	4.118	4.077	4.118	4.086	4.118	4.077
0.05	3.271	3.258	3.271	3.258	3.271	3.258
0.1	2.657	2.652	2.657	2.651	2.657	2.652

CONCLUSION

In this study, the effect of transverse magnetic field in slip flow in a microchannel between two parallel plates heated with a constant heat flux is studied analytically. It was found that the velocity and temperature field were strongly affected by the presence of magnetic field and the Kn number. As the Knudsen number increases, Nusselt number decreases because of the increasing temperature jump at the channel walls. The Nusselt number increases with an increase of the Brinkman number in the negative direction for the cold wall case and decreases with an increase of the Brinkman number for the hot wall case. The Nusselt number takes higher values with an increase of the Hartmann number.

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